LIQUID NITROGEN COOLING OF THE LINEAR ELECTRON ACCELERATOR AT S AND C BAND*

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The considerantions underlying the choice of the L, S and C bands as operating frequencies for linear electron accelerators are sufficiently known (see (1), (2) and (3)).

Nevertheless it may be worth considering the construction and running costs for somewhat higher frequencies at lower temperatures than usual. The costs depend on the duty factor and accelerating field-strength, the beam loading being neglected in the analysis for reasons of simplicity.

1. SHUNT IMPEDANCE AND POWER CONSUMPTION

The shunt impedance per unit length of an accelerating structure is inversely proportional to the skin depth for given geometrical proportions. The skin depth is given by

$$\delta = 0.29 \sqrt{\lambda_o \rho} \text{ mm} \qquad [1]$$

in which λ_o is the free-space wavelength in cm and ρ the specific resistance of the cavity material in ohm cm. The shunt impedance for an OFHC copper waveguide at 40° and 10.5 cm wavelength in the $2\pi/3$ mode is about 57 MΩ/m (1).

If we write the shunt impedance as $r = K_1 (\lambda_o \rho)^{-1/2}$ we find for $K_1 = 2.53 \times 10^3 \, \Omega^{3/2}$.

The r.f. power for an optimally designed travelling wave accelerator with constant field and with negligible beam loading is given by

$$P_{QF} = 1.23 \frac{UE}{r} \qquad [2]$$

where P = r.f. input power in watts
U = energy in volts
E = accelerating field in volts per meter
r = shunt impedance in ohms per meter.

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The specific resistance of pure copper for a temperature T between 60°K and 350°K can empirically be written as:

$$\rho_T = 7 \times 10^{-9} (T - 47.2) \, \Omega \text{ cm} \qquad [3]$$

Substituting [3] and [1] in [2] gives for $K^1 = 2.53 \times 10^3$

$$P_{r.f.} = 4.62 \times 10^{-8} \text{ UE} \{\lambda_o (T - 47.2)\}^{1/2} \qquad [4]$$

With a dtuy factor d_c the mean r.f. power is:

$$\overline{P}_{r.f.} = d_c P_{r.f}$$

For temperatures below 290°K a refrigerator is necessary. The overall efficiency of a refrigerator is a fraction η_r of the Carnot efficiency η_T and η_T depends on the temperature T as:

$$\eta_T = \frac{T}{290 - T} \qquad [5]$$

In general η_r depends on the temperature too.

For the Philips refrigerator type C-215 the efficiency η_r is given in Table I.

To remove the mean r.f. power $\bar{p}_{rf}$ that is dissipated in the accelerating guide the refrigerator needs an input power:

$$P_r = (\eta_r \eta_T)^{-1} \overline{P}_{r.f.} \qquad [6]$$

Substituting [4] and [5] in [6]:

$$P_r = 4.62 \cdot 10^{-8} \text{ UE} \frac{1}{\eta_r} \times \frac{290 - T}{T} \{\lambda_o (T - 47.2)\}^{1/2} \qquad [7]$$

The input power for the r.f. tubes depends on the efficiencies of the rectifier and modulator η_m and the efficiency η_t of the tube itself:

$$P_i = (\eta_m \eta_t) \overline{P}_{r.f.}$$

or:

$$P_i = 4.62 \times 10^{-8} \text{ UE} \lambda_o^{1/2} (T - 47.2)^{1/2} \cdot \left\{ \frac{1}{\eta_m \eta_t} + \frac{1}{\eta_r} \frac{290}{T} - \frac{1}{\eta_r} \right\} \qquad [8]$$

We calculate now a temperature factor F_T:

$$F_T = (T - 47.2)^{1/2} \left\{ \frac{1}{\eta_m \eta_t} + \frac{1}{\eta_r}\frac{290}{T} - \frac{1}{\eta_r} \right\} \quad [9]$$

for $\eta_m \eta_t = 0.30$ and η_r using the data in Table I one finds the results as given in Table II.

The factor F is only slightly dependent on the temperature in the considered region, so that the total input power is almost constant with the working temperature of the accelerator.

2. CONSTRUCTION COSTS

The total costs of construction amounts to:

$$K_c = K_i + LK_l + N_b K_{bv} + N_K K_K$$

where:

K_i = initial costs for developing a prototype section, the costs of preparing the project and the fixed costs of the injection system

K_l = costs proportional to the length

K_{bv} = costs per tube station

K_K = costs per refrigerator inclusive of installation

N_b = number of tubes

N_K = number of refrigerators.

It can be shown that these costs, with a fixed K_i, are minimum for a field-strength:

$$E = \left\{ \frac{r K_l}{d_c \left(\frac{K_{bv}}{W_{rf}} + \frac{K_k}{W_k}\right)} \right\}^{1/2} \quad [10]$$

and the minimum construction costs are:

$$K_{c\,min} = K_i + 2.48\, U \left\{ \frac{d_c K_l}{r} \left(\frac{K_{bv}}{W_{rf}} + \frac{K_K}{W_K}\right) \right\}^{1/2} \quad [11]$$

We assume now that:

a) the price of a tube with supply system for a given frequency is determined by the average power.

b) the price of cooling-jackets for liquid nitrogen is the same as the price water-jackets inclusive the heat exchangers, and the costs of the building and shielding are independent of the method of cooling.

To estimate the cost we write [11] as:

$$K_{c\,min} - K_i = 2.48\, U (d_c K_l)^{1/2} K_T^{1/2} \quad [12]$$

in which:

$$K_T = \frac{1}{r} \left(\frac{K_{bv}}{W_{r.f.}} + \frac{K_K}{W_K}\right) \quad [13]$$

The price of tube and modulator is dependent *on the frequency and one can expect that the* price per delivered watt r.f. power will rise with the frequency. For a given frequency one has to compare the costs of the refrigerator per "removed watt" with the gain introduced by the shunt impedance. To illustrate this we calculate K_T for two frequencies and two temperatures, assuming the dependency of r on frequency and temperature as given in Table III and:

K_{bv}/W_{rt} = Hfl 6 per watt r.f. in the case of 3 Gcs
= Rfl 7 per watt r.f. in the case of 6 Gcs

K_K/W_K = Hfl 18 per removed watt

the calculated K_T is given in Table IV.

TABLE I

Efficiency of Philips C 215 refrigerator in respect to Carnot efficiency

T	r
77°K	0.40
100	0.43
150	0.42
200	0.35

TABLE II

Temperature factor for the power consumption

T	F_T
77°K	54.8
100	56.6
150	56.1
200	57.1
290*	52

* At 290° without refrigerator.

TABLE III

Shunt impedance 2 π/3 mode

r in M Ω/m

	3 Gcs	6 Gcs
+ 40°C	57	81
− 195°C	165	235

As the construction costs vary with $K_T^{1/2}$ these are not very dependent on temperature and frequency.

3. ANNUAL COSTS

We shall consider the main part of annual costs of the accelerator:

K_{jp} = staffing costs of operating and maintenance personnel, inclusive laboratory, tools, measuring equipment etc.

K_{jb} = replacement of the tubes.

K_e = electric power consumption.

K_w = cooling water consumption.

K_{jK} = depreciation and maintenance of the refrigerators.

K_{jl} = depreciation and maintenance of accelerator tube, buildings and shielding.

We shall now assume that:

1) the staffing cost are not dependent upon the accelerating field, duty cycle or frequency.

2) the depreciation and costs of maintaining accelerator guide, buildings and shielding can be disregarded a scompared with the other costs.

3) the overall electric power consumption of the r.f. tubes results in heating of the cooling water, so that the water consumption stands in a fixed ratio to the electric power consumption.

Adding the costs one finds:

$$K_j - K_{jp} = \frac{1.23\, UE\, d_c\, t_b}{r} \left\{ \frac{K_b}{\overline{W}_{HF}\, t_l} + \frac{K_K}{W_K \times t_K} + \frac{(1+\alpha)}{\eta_g \eta_b \eta_v} f_e + \frac{f_e}{\eta_K} + \frac{A_K f_K}{W_K} \right\} \quad [14]$$

in which:

η_g = efficiency of transporting the r.f. power from the tube to the accelerator guide (inclusive the mode transformer efficiency)

η_b = tube efficiency

η_v = efficiency of the power supply system (rectifier, modulator and pulse transformer)

and where α represents the wather consumption of the r.f. tube.

To compare the cost as a function of frequency and cooling we consider the factor:

$$K_\alpha = \frac{1}{r} \left\{ \frac{K_b}{\overline{W}_{rf}\, t_l} + \frac{K_K}{W_K\, t_K} + \frac{(1+\alpha)}{\eta_g \eta_b \eta_v} f_e + \frac{f_e}{\eta_K} + \frac{A_K f_w}{W_K} \right\}$$

r being taken as in Table III and assuming:

$$\frac{K_b}{\overline{W}_{rf}\, t_l} = 1.5 \times 10^{-3}\ \text{Hfl per watt hour for 3 Gcs}$$

$$= 2 \times 10^{-3}\ \text{Hfl per watt hour for 6 Gcs}$$

$$\frac{K_K}{W_K\, t_K} = 0.9 \times 10^{-3}\ \text{Hfl per watt hour;} \qquad \eta_g \eta_b \eta_v = 0.3$$

$\eta_K = 0.16$; $\alpha = 0.1$; $f_e = 7 \times 10^{-5}$ Hfl per watt hour

$A_K = 20\ m^3/hr$; $f_w = 0.2$ Hfl per m^3; $W_K = 20 \times 10^3$ watt;

With these figures we find for K_α the numbers as given in Table V.

The decrease in running costs is mainly due to the fact that the expected life-time of a refrigerator exceeds the mean life-time of a klystron by a factor of at least ten. As is pointed out in section 1, the total power consumption is practically independent of the working temperature. The gain in shunt impedance lowers the number of klystron stations by a factor of four going from 3 Gcs and 40°C to 6 Gcs and − 195°C.

It is evident from formula [14] that for reasons of economy a lower field strength than usual is necessary for accelerators working with duty factors ten to a hundred times greater than achieved up to now.

Note: It may be useful to take into account the anomalous skin effect, especially in the case of higher frequencies and lower temperatures than considered in this paper. Using the Reuter and Sondheimer theory (4) one can find correction factors for the "classical" skin depth. In Table VI these factors are given for copper at temperatures of 77°K and 50°K.

TABLE IV

Factor for investments

	K_T	
	3 Gcs	6 Gcs
+ 40°C	0.11	0.09
− 195°C	0.14	0.11

TABLE V

Factor for annual klystron, refrigerator and power consumption costs

$K_\alpha \cdot 10^5$

T	V	
	3 Gcs	6 Gcs
40°C	3.1	2.8
− 195°C	2.0	1.6

TABLE VI

Correction factor for anomalous skin effect

Gcs	$\frac{\delta}{\delta_{cl}}$	
	77°K	50°K
3	1.025	1.30
6	1.050	1.40
9	1.065	1.46

REFERENCES

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(2) Chodorow et al.: Rev. Sci. Instr. 26, *134* (1955).
(3) Eldredge, Loew and Neal: Slac. Report nr. 7 (1962).
(4) Reuter and Sondheimer: Proc. Royal Soc. A *195* 336 (1948).

DISCUSSION

KOMAR: What is material of the cavities?

GUGELOT: Copper. It seems to be the best material from the stand point of electrical and thermal conductivity at 77°K.

KRIENEN: Would not the beam loading affect your shunt impedance r?

GUGELOT: A rough estimate gives $\Delta V/\Delta i = 6\text{-}8$ MeV/mA.

SELF-CONSISTENT PARTICLE DISTRIBUTION AND THE LIMIT CURRENT IN LINEAR ACCELERATORS

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1. FORMULATION OF THE PROBLEM

The problem of the limit current attainable in a proton linear accelerator has acquired great importance in recent years and a number of papers has been devoted to it (1-4). The limit current is restricted by Coulomb space charge repulsion of accelerated particles. The simplest way of taking this effect into account would be an approximate representation of bunches by uniformly charged ellipsoids. This assumption results in an expression for the limit current which is in good agreement with experimental data (1, 4). It is shown to be (1, 2, 4)

$$I_M = \frac{3\,E_m\,R^2\varphi_s \sin\varphi_s}{\gamma\,\lambda\,M_z\,(S_M)} \cdot \frac{\tau S_M\,(1 - S_M)}{0{,}18} \qquad [1]$$

Here I_M is limit current value averaged over the pulse, amperes; E_m is accelerating field amplitude, kV/cm; λ, R – wavelength and the greatest radial dimension of the bunch respectively, cm; γ is the greater-to smaller transversal semi-axes ratio for the bunch; φ_s – equilibrium phase; M_z — factor depending on the bunch form; τ — the efficiency ratio of the separatrix; S – the repulsive-to-phasing longitudinal force ratio for small phase oscillations; S_M is S – value corresponding to the maximum of $S(1-S)/M_z(S)$.

However, in (3) some doubts are expressed as to what extent the assumption of uniformly charged ellipsoids can approximate self-consistent charge distribution in the bunch and whether the correlations given by this assumption correspond to the real behaviour of the bunch. Also an attempt was made in that paper to find a self-consistent charge distribution which proved to be not quite successful for some reasons (see below, sect. 3).

The present paper deals with the problem of self-consistent distribution of particles in linear accelerator. As usual we shall neglect the fields of the neighbouring bunches, the effect of tank walls and the electron and ion space charge of residual gas. Longitudinal axes on bunches are supposed to coincide with the accelerator axis.

2. REPRESENTATION OF BUNCHES BY UNIFORMLY CHARGED ELLIPSOIDS

Let us show that uniformly charge distribution within the ellipsoid is approximately self-consistent. Let X, Y, z be an accompanying coordinate system with the axes coinciding with those of