Glueballs

The current status of various glueball properties such as level ordering, mass, production, and decay is reviewed. Glueball candidates in the $J^{PC} = 0^{-+}$ and 2^{++} sectors are examined in detail for evidence that they are, in fact, glueballs. The best candidates at present are the $\phi\phi$ 2^{++} resonances, $g_T(2120)$, $g_T(2220)$, and $g_T(2360)$. Their flavor singlet decay characteristics must be verified via the detection of other decay modes and their subsequent comparisons, and it remains for the g_T's to be detected in the radiative decay of the J/ψ and/or Υ. The $\xi(2220)$ is still a possible glueball choice, although it seems to fit the $q\bar{q}$, $1 = 3$ assignment quite well. The $\theta(1720)$ is examined in detail, but with no firm conclusion about its classification, other than that it does not seem to be a glueball. The pseudoscalar sector is more complicated than ever; there is no clear evidence that $\iota(1460)$ is a glueball, nor is there evidence that the spectroscopy of the pseudoscalar system is at all understood.

I. INTRODUCTION

Glueballs[1-6]—colorless, flavorless composites of two or more gluons—should exist, according to the basic tenents of quantum chromodynamics. In this brief review, we summarize the current (~May 1984) status of theory and experiment for various aspects of glueball spectroscopy. We examine, in order of mass, the current list of glueball candidates, offering whatever gems of insight that we and others may have as to their suitability for the title "glueball."

Comments Nucl. Part. Phys.
1984, Vol. 13, No. 6, pp. 325–351
0010-2709/84/1306-0325/$25.00/0

II. SPECTROSCOPY

A. Level Order

The simplest models which give the order J^{PC} of glueball states arise from potentials for confinement. Unfortunately, we cannot compute masses in this approach. Although this approach has been presented in detail before, we mention the model and results briefly.

The model for constructing the two gluon glueball states has two gluons moving with a relative orbital angular momentum L.[6e,7] We require that the two gluon wavefunctions be totally symmetric under the interchange of the space, spin, and color coordinates of the gluons. Taking $J = L + S$, where S is the spin of the two gluons, the allowed $2g$ states are shown in Figure 1. Note that an oddball, 1^{-+}, occurs for the first time at the $L = 1$ level. Oddballs[8,9] are glueballs with J^{PC} quantum numbers that do not occur in $q\bar{q};L$ mesons, i.e., 0^{--}; 1^{-+}, 3^{-+}, . . . ; 0^{+-}, 2^{+-}, ...

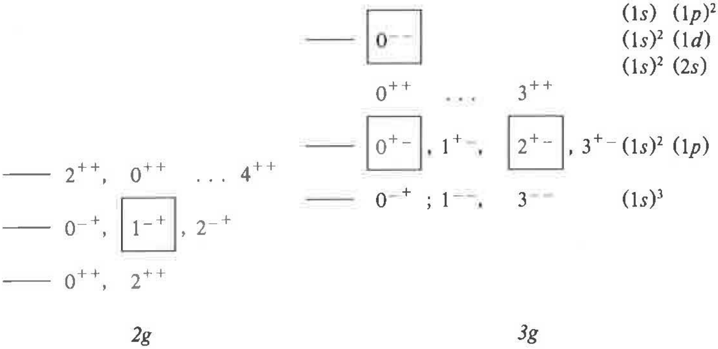

Fig. 1. $2g$ and $3g$ levels.

Also shown is the $3g$ spectrum obtained by putting three gluons in a potential well.[7b] No estimates for the masses have been made, so the exact normalization of the $3g$ sector with respect to the $2g$ sector is arbitrary. We note that the lowest $3g$ states are 0^{-+}; 1^{--}, 3^{--}. Also note that there are a large number of low-lying 2^{++} states, as well as 0^{-+} and 0^{++} states.

B. Masses

Three methods of estimating glueball masses continue to dominate the thinking of those interested in the field: lattice gauge theories, bag models, and massive constituent gluon theories. They all produce spectra with some similarities, i.e., a 0^{++} ground state, and low-lying 0^{-+} and 2^{++} states. They are also all dependent on the choice of some parameter to set the scale. We shall see that this is not an academic point when we confront the ultimate arbiter, experimental results.

1. Lattice gauge theories

Lattice gauge theories (LGT) offer us the possibility of carrying out a well-defined fully controllable nonperturbative QCD calculation of the glueball spectrum. The Monte Carlo technique has been the vehicle for carrying out this program. The two main groups of protagonists are Berg[10] and Billoire[11] and Ishikawa, Schierholz, and Teper[12,13] and their respective co-workers. These groups carried out the first extensive Monte Carlo Variational (MCV) calculations for the so-called mass gap, i.e., the value for $m(0^{++})$ and for the excited glueball states. The general views of the two groups are clearly and exhaustively represented in two excellent recent reviews by Berg[10] and by Teper.[12,13] These groups and others cited in the reviews[10,12] try to extract glueball masses by evaluating numerically two point correlation functions of operators with the glueball quantum numbers. The mass of the lowest state is extracted from the large distance exponential falloff of these operators over several (1, 2, and even 3) lattice spacings in time. The hope in these calculations is to replace continuum spacetime by a lattice of points with a spacing a. One takes $a \to 0$ in order to recover the continuum theory.

One might hope that there would be some consensus about the viability of MCV for calculating if not the entire glueball spectrum, at least the masses of the lowest lying glueballs, the ones with $J^{PC} = 0^{++}, 2^{++}, 0^{-+}$, and the important oddball 1^{-+}. Unfortunately, this is not the case except for possibly the 0^{++} case. An important problem is deciding when we are in the continuum limit, i.e., finding a lattice spacing sufficiently small so that the glueball mass calculated is that of the continuum limit. A criterion used is that if the calculation is redone with a smaller lattice spacing that the mass should be the same in physical units.[12] It is assumed that the lattice spacing is small enough for the perturbative formula for $a(\beta)$ to hold:

$$a(\beta) = \frac{83.5}{\Lambda_{\text{mom}}} e^{-(4\pi^2/33)\beta} \left(\frac{8\pi^2}{33}\beta\right)^{51/121} \tag{1}$$

For a given β, LGT gives the dimensionless product $m(\beta)a(\beta)$.

There seems to be general agreement that $m(0^{++})$ can be determined, i.e., that a mass gap exists. Teper finds that finite size effects are small, but a recent study by Patel and Gupta[14] claims that such effects are large. The physical value of $m(0^{++})$ is determined by what value of Λ_{mom} is chosen. Teper uses a value of 200 ± 30 MeV giving $m(0^{++}) = 770 \pm 30$ MeV. Berg and Billoire[15] use 750 MeV, in agreement with the foregoing. Not everyone agrees. Hamber and Heller[16] find that by two different methods, they get: $m(0^{++}) = (2.6 \pm 1.0)\sqrt{T} = 1080 \pm 400$ MeV, where $\sqrt{T} = 420$ MeV (T is the string tension); alternatively, via another method they obtain $m(0^{++}) = (1.1 \pm 0.2)\, m_\rho = 850 \pm 200$ MeV. Michael and Teasdale[17] obtain 1000 ± 300 MeV.

Which value for $m(0^{++})$ is adopted has important physics ramifications, because the choice sets the scale for glueball excitations. If a value at 770 MeV is chosen, then the low-lying glueball excitations lie in the 1–2 GeV range. The 1000 MeV choice puts these excitations in the 2–3 GeV range, which is very different phenomenologically. Consequently, unless otherwise specified, we shall express all glueball masses as ratios to $m(0^{++})$.

There is not an impressive amount of agreement on the excited state spectrum. Listed in Table I are the results of Teper, Schierholz, and co-workers[12,13] as well as the ratios of their excited states to the ground state. Note that their pseudoscalar glueball lies below the 2^{++} state, and that they have a very light 1^{-+} oddball. Berg and Billoire[10,15] find that their 2^{++} calculations do not scale but quote a value $m(2^{++}) \simeq 2.3\, m(0^{++})$ in agreement with the result above; however, their lowest excited state is 2^{++}, not 0^{-+}. Their best evaluation for $m(0^{-+})$ has $m(0^{-+}) \simeq 2.5\, m(0^{++})$ with the proviso that they feel that scaling behavior has not been reached. With some caveat they find, in another calculation, the results of which are also shown below in Table I, that their oddball 1^{-+} state with

TABLE I
Low-Lying Glueball States

	J^{PC}*	Mass (MeV)*	Continuum Scaling*	Finite Size Effects*	$\frac{m(J^{PC})}{m(0^{++})}$ T*	$\frac{m(J^{PC})}{m(0^{++})}$ BB†
	0^{++}	770 ± 30	yes	small	1	1
	0^{-+}	1450^{+240}_{-150}	not checked		1.9	2.9
	2^{++}	1665^{+110}_{-87}	yes	small	2.2	2.3
Oddball:	1^{-+}	1800 ± 230	not checked		2.3	3.2

*The levels, masses, and properties of states calculated by Teper *et al.*[12]
†$m(J^{PC})/m(0^{++})$ calculated by Berg and Billoire.[15]

$m(1^{-+}) = 3.2\ m(0^{++})$ is somewhat higher than the multiple of 2.3 given above. From the foregoing, it is clear that lattice gauge theories offer only a tentative guide to glueball spectroscopy at present. However, once the approximations and calculational techniques are in hand, they should become useful ways for learning about glueball properties.

2. Bag models

Several bag model calculations[4,18–20] for glueballs have been carried out recently, that update and refine earlier ideas.[21,22] In the bag model, the lowest lying glueball states are constructed from two or three valence gluons in the lowest transverse electric (TE) and transverse magnetic (TM) cavity modes. Chanowitz and Sharpe[4,20] (CS) predict glueball masses for $C_{TE}/C_{TM} = ½$, 1, and 2 where the C's are gluon self-energies. Carlson, Hansson, and Peterson[18] (CHP) consider only the ratio of ½ because they claim that the other two values correspond to the situation of having the TE gluon self-energy > TE kinetic energy. Remembering that CS have *assumed* the $\iota(1440)$—they used 1440 as the ι—to be a glueball, their low 0^{++} and 2^{++} states fall at 670 and 1750 MeV, close to the masses of Teper *et al.*[12,13] CHP[18] make this comparison as well as calculating the same levels in their own approach.[19] Their model assumes that the QCD vacuum is locally well described as a color and spin singlet $(TE)^2$ gluon state. Pursuing their version of the bag model,[18,19] they find good agreement with CS and with Teper *et al.*[12,13] Correspondingly, they must disagree with Berg and Billoire,[10,11] whose 0^{-+} state lies above the 2^{++} state.

The CHP 0^{++} ground state falls at 780 MeV with an error of $\pm$ 150 MeV. CHP feel that the agreement between the lattice gauge theory predictions and the bag model calculations may suggest that a two or three valence gluon structure for the glueballs is correct.

3. Massive constituent gluons

Cornwall and Soni[23] have considered glueballs to be bound states of massive gluons. They describe the gluon dynamics as massive spin 1 fields interacting through a breakable string. Their string potential is $V_S(r) = 2m(1 - e^{-r/r_0})$, where m is an effective gluon mass, determined by Bernard[24] to be $\simeq$ 500 MeV; r_0 is taken to be 0.6. With these parameters, they fit the mass of $\iota(1460)$ and $\theta(1720)$ fairly well, although they have a very low-lying 1^{-+} oddball just above the ι. In addition, their 0^{++} ground state is about 1200 MeV. A different effective gluon mass, say of 750 MeV as obtained earlier by Parisi and Petronzio[25] raises all of their glueball masses by 500 MeV. Cornwall and Soni have continued this program. In recent calculations, Hou and Soni[26] have used this model to examine the three gluon bound states. They show the the masses of the lowest lying three-gluon glueballs with $J^{PC} = 0^{-+}$, 1^{--}, and 3^{--} are degenerate, with a value of $\simeq 4.8m = 2400$ MeV. They discuss glueball decays and suggest that total decay widths of glueballs are those characteristic of ordinary hadron decays. Cornwall and Soni[27] have also derived a set of QCD sum rules for the operator $\int dx G^{\nu}_{\mu} G^{\mu}_{\nu}$ which they take as a generalization of the trace anomaly. They try to estimate the couplings of the lightest scalar and pseudoscalar glueballs to quarks and hadrons using these sum rules.

4. Conclusions

Unfortunately, we do not have the analog of the potential models which have been so successful in treating heavy quarkonia, with detailed predictions of masses and couplings. The three classes of theories discussed above are valiant efforts in that direction, but their predictive powers are minimal.

C. Hermaphrodites—Meiktons

The bag model has been the only vehicle for studying $q\bar{q}g$ composites,[4] until a recent paper by Cornwall and Tuan[28] invoked the idea of constituent gluon mass. Proposed by Horn and Mandula,[29] meiktons[4]—also called hermaphrodites or hybrids—have a somewhat different spectroscopy than glueballs because some of them may have isospin and also strangeness. In a bag model, the lowest meiktons are obtained by coupling a $q\bar{q}$ pair (0^{-+} or 1^{--}) to a TE gluon (1^{+-}) giving four nonets of $J^{PC} = 0^{-+}, 1^{-+}, 1^{--}$, and 2^{-+}. The parameters chosen for the bag yield states lying in the 1200–2000 MeV range for these low-lying states. Particularly interesting is the occurrence of a 1400–1800 MeV oddball (1^{-+}) nonet. Inasmuch as Chanowitz[4] has reviewed this fascinating composite system recently in an excellent and comprehensive study, we shall not pursue the subject further.

D. Widths

The "standard" view of glueball widths follows from the assumption that glueballs decay to quarkonia through their coupling of gluonic constituents to pure quark states. Robson[7a] introduced this "$\sqrt{\text{OZI}}$ rule," which is based on the idea that in perturbative QCD, OZI violating amplitudes are mediated by intermediate gluons. With such reasoning a "typical width" is of the O(10–30) MeV.[8] Not everyone agrees with such estimates. For example, Cornwall and Soni[27] are at the opposite end of the width spectrum and would estimate $\Gamma(0^{++}) \sim 250$–1000 MeV (comparable to the mass), and $\Gamma(0^{-+}) > 50$ MeV (50–200 MeV). Of course, mixing in $q\bar{q}$ states, also broadens glueball states.[7b] At present, we do not know with any degree of certainty exactly how broad glueballs are.

III. GLUEBALL PRODUCTION

The two main ways to produce glueballs are via the radiative decays of vector mesons and as final state products in Zweig-forbidden processes. In addition, double pomeron exchange dominated processes yield final states with the quantum numbers of glueballs. In the discussion that follows we shall emphasize the latest experiments and the attendant theoretical interpretations, rather than rehashing what is by now quite familiar and is well covered in the reviews cited earlier.

The radiative decays of vector mesons (production in hard gluon channels) is the way in which the first glueball candidates, the $\iota(1460)$[30,31] and the $\theta(1720)$[32,33] were found. [They were originally $\iota(1440)$ and $\theta(1640)$.] This "classic" method looks attractive because the estimate $\Gamma(\psi \to \gamma gg)/\Gamma(\psi \to ggg) = 16\alpha/5\alpha_S$ implies that $B(\psi \to \gamma G) \simeq 6\%$–$10\%$, and is in fact, the source of all of the glueball candidates except for the three 2^{++} g_T states found in Brookhaven National Laboratory[34,35] to be discussed below. Many new results have been produced in the past year as a result of the beautiful work of the MARK III group. So many resonances have now been seen in the J/ψ radiative decay that the earlier penchant for labelling all such states as glueballs has abated.

After several years of theoretical analyses, but no new experimental results forthcoming for radiative decays, the logjam was broken within the past year by reports of work from the MARK III detector at Spear. These results are contained in preliminary reports by Hitlin,[3] Wermes,[36] Burnett,[37] and Wisniewski,[38] from which we quote liberally. The Mark III detector has as its goal the complete reconstruction of exclusive final states. The final states are $K^+K^-\pi^0$, $K_s^0K^\pm\pi^\pm$, $K_s^0K_s^0\pi^0$; $\gamma\rho$; $\rho^0\rho^0$, $\rho^\pm\rho^\pm$; $\omega\omega$; $K\bar{K}$; and $\eta\pi\pi$.

Before summarizing the pseudoscalar system and in particular the ι, we consider below an important new state which is relevant to our discussion.

A. $P(1600$–$1900)$[36–38]

Motivated by MARK II's observation of a large $\gamma\rho^0\rho^0$ signal with structure in the $\rho^0\rho^0$ spectrum near 1600 MeV,[39] the MARK III group is investigating the channels $J/\psi \to \gamma 4\pi$ for a $\gamma\rho\rho$ signal, and also $J/\psi \to \gamma 6\pi$ for a $\gamma\omega\omega$ signal. They find strong evidence for both decays. They find[36–38]:

(1) In the $\rho\rho$ case there is structure in the 1500–1900 MeV region; a spin-parity analysis suggests that the $\rho\rho$ enhancement is primarily 0^-(50%), but there may also be some 2^{++}. This also implies that the θ is not the dominant $\rho\rho$ decay mode:

$$B(J/\psi \to \gamma X(0^- \text{ projection}))B(X \to \rho\rho) = (7.7 \pm 3.0) \times 10^{-4}.$$

(2) In the $\omega\omega$ case there is structure in the 1600–1900 MeV region; a $J =$ even, $P = -$ assignment is suggested by χ angle distributions; a spin-parity analysis is under way:

$$B(J/\psi \to \gamma X)B(X \to \omega\omega) = (6.7 \pm 1.7 \pm 2.4) \times 10^{-4}.$$

It is particularly tempting to assume that both enhancements correspond to different decay modes of a 0^{-+} resonance that we call $P(1600\text{–}1900)$. Whether or not the $N(1700)$,[40] a possible $\eta\pi\pi$ enhancement at 1700 MeV, is another decay mode of the same 0^{-+} particle as the P is a crucial question and awaits a spin-parity analysis of the $\eta\pi\pi$ enhancement.

B. 0^{-+} Isoscalars

The existing 0^{-+} resonances are $\eta(548)$, $\eta'(960)$, $\zeta(1275)$, $\iota(1460)$, the new P($\rho\rho$ and $\omega\omega$ peaks in the 1600–1900 region), and N. The η, η', and ζ are generally regarded as primarily quark states while the glueball candidates are ι, $P(1600\text{–}1900)$, and possibly $N(1700)$.

One of the ways to identify a glueball would be to know the ordinary $q\bar{q}$ spectroscopy so well that the interloping glueball (or hybrid) would stand out. Let us, therefore, look at some representative calculations of meson (and in particular pseudoscalar) spectra. The results of recent calculations by Godfrey and Isgur[41] (GI) and by Frank and O'Donnell[42] (FO) are shown in Table II. Godfrey and Isgur in a very comprehensive study of meson spectroscopy have two distinct sets of levels for the pseudoscalar system, differing mainly in the annihilation graph behavior. In their first solution they do not get a state to correspond to the $\zeta(1275)$, supposedly a radial excitation of $\eta(548)$, but they do get a state at 1430 MeV. Incidentally, this state has a large decay width to $\gamma\rho$, as we shall see does the ι. Solution 1 is clearly incorrect if the $\zeta(1275)$ exists. Solution 1 has an $s\bar{s}$ resonance at the position to correspond to the new $\omega\omega$ and $\rho\rho$ resonances, but their nonstrange quark structure renders this a bad choice. Solution 2 seems designed to accommodate the $\zeta(1275)$ as η_r. It has an $s\bar{s}$ state at 1540 MeV that could be associated with the iota with some stretching. GI express some concern as to the validity of the $\zeta(1275)$ as a resonance. They seem to be even more skeptical about the assignment of ι as a glueball. They say that our ignorance of the annihilation force means that it is possible for ι to be accommodated as a radial excitation in their model.

Frank and O'Donnell[42] present two calculations based on a nonrelativistic harmonic oscillator quark model that considers bound states made only of u, d, and s quarks. They have examined the isoscalar meson spectroscopy both with and without glueball mixing (see Table II), using the η,η' and their first two radial excitations,

Table II
$J^{PC} = 0^{-+}$, I = 0 Meson States (all masses in MeV)

State	GI-*P*1*		GI-*P*2	FO† without Glueballs	FO† with Glueballs	PPB‡	Experiment Only η and η' Assigned
1η	530			530	570	549	η (549)
$1\eta'$	960			1070	1050	960	η'(960)
$2\eta_r$	1430	$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$	1270	1390	1390	1275	ζ(1275)
$2\eta'_r$	1640	$s\bar{s}$	1540	2500	2100	1700	ι(1460)
$3\eta_{rr}$				2100	2800		*P*(1710)
$3\eta'_{rr}$				3200	3600		
G					1500	1460	
	$\iota \rightarrow \gamma\rho$ large	$\iota \rightarrow \gamma\rho$ small					

*Godfrey and Isgur.[41] GI-*P*1 is incorrect if ζ(1275) exists.
†Frank and O'Donnell.[42] If ζ(1275) doesn't exist η_r(1390) can be ι(1460).
‡Palmer, Pinsky, and Bender.[43,44]

six (seven) states in all. The nonglueball solution has a radial excitation at 1390 MeV that could be identified with $\iota(1460)$, but FO hesitantly identify this state with $\zeta(1275)$. This solution is rather similar to GI-$P1$ except that GI-$P1$ has $\eta_r = 1640$ MeV and is pure $s\bar{s}$, and FO have $\eta_r = 2500$ MeV and is a mixed quark state. FO also evaluate radiative decay amplitudes, and use these to rule out solution 1. (We shall discuss radiative decays of the ι in detail shortly.) The FO solution with glueball included (Table II) still has a state at 1390 MeV which they again identify with $\zeta(1275)$, but also has a state at 1500 MeV, 72% glueball, which they identify with $\iota(1460)$. This has more acceptable radiative decay amplitudes and is preferred by FO. There is however, no state in this solution that corresponds to the $P(1600–1900)$ state, a somewhat serious objection; there is also no room for the $N(1700)$ state, a not so serious objection because its J^{PC} is currently unknown.

The fact that $P(1600–1900)$ decays into $\rho\rho$ and $\omega\omega$ means that it is either made solely of $u\bar{u}$ and $d\bar{d}$ quarks or of a mixture of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ quarks but not $s\bar{s}$ quarks alone. It can't decay into $\phi\phi$ because the $\phi\phi$ mass is 2040 MeV. Such a state can decay into $\eta\pi\pi$, so that the Palmer, Pinsky, Bender[43,44] assumption of the N as a 0^{-+} state means that P and N could be different decay modes of the same particle. Even if it turns out that P and N are distinct particles, P could have an $\eta\pi\pi$ decay mode. N's other decay modes depend on its J^{PC}. Clearly, if it had $C = -$ it could not decay into $\rho\rho$ or $\omega\omega$.

It is clear that a more complete calculation is called for, in particular, one in which both the lowest 2-gluon glueball and the lowest 3-gluon glueball states are included. In addition, the inclusion of the lowest ω and ϕ hybrids would be necessary for a reasonably complete calculation. Using the work of Chanowitz and Sharpe[20] as a guide, and realizing that they have assumed $\iota(1440)$ to be a glueball, then depending on whether the value of $C_{TE}/C_{TM} = ½$, 1, or 2 the ω and ϕ hybrids can be 1200 and 1610; 1410 and 1820; or 1610 and 2030 MeV, respectively. The hybrid inclusion implies that a 10 state system would be the starting point and that a 10×10 mass matrix would have to be evaluated and diagonalized. This dwarfs the original 3×3 mixing calculation of Fishbane, Karl, and Meshkov,[2,5,45] and the 5×5 calculation of Palmer, Pinsky, and Bender[43,44] (both to be discussed below) as well as the 7×7 calculation of Frank and O'Donnell[42] already mentioned.

The analogous treatment of spectroscopy holds for the $J^{PC} = 2^{++}$ system. We defer the 2^{++} discussion until the detailed discussion of 0^{-+} experimental candidates below.

We give below a brief summary of what has been learned about the $\iota(1460)$, and how it is related to the other pseudoscalar states.

1. $\iota(1460)$ $J^{PC} = 0^{-+}$

This was the first glueball candidate and has remained a highly controversial resonance. It was first observed by the Mark II detector in the decay mode $J/\psi \rightarrow \gamma K_S K^{\pm}\pi^{\pm}$.[30] It was subsequently observed by the crystal ball in the decay $J/\psi \rightarrow \gamma K^+K^- \pi^0$.[31] MARK III has seen both of these decay modes and has also observed $J/\psi \rightarrow \gamma K_S^0 K_S^0 \pi^0$.[3] They give a product of branching ratios; $B(J/\psi \rightarrow \gamma\iota)B(\iota \rightarrow K\bar{K}\pi) = (5.6 \pm 0.4 \pm 1.3) \times 10^{-3}$. MARK III measures a mass of 1461 ± 5 MeV/c^2 with a width of 105 ± 11 MeV/c^2. The ι that they measure is slightly heavier and twice as wide as the original measurements because the Mark III analysis does not use a δ-cut. As of the time of writing this review they have not completed a spin-parity analysis, so we shall take ι to have $J^{PC} = 0^{-+}$.

The $K\bar{K}\pi$ decay mode is apparently dominated by the $\delta(980)\pi$ mode; since $\delta \rightarrow \eta\pi$ we might expect $J/\psi \rightarrow \gamma\eta\pi\pi$ to give a large signal at the ι mass. It doesn't in either the crystal ball or the Mark III experiments. The absence of $\iota \rightarrow \eta\pi\pi$ has been a continuing thorn for the ι glueball advocates. An explanation of this in terms of an interfering pole model has been advanced by Milton, Palmer, and Pinsky.[46] Whereas nothing is seen in the ι region in $\eta\pi\pi$, the crystal ball sees an impressive broad structure centering about 1700 MeV [which we have called $N(1700)$ earlier]. MARK III observes a prominant signal in this region but it is broader and not so sharply peaked as the $N(1700)$.[3] Pinsky, Palmer, and Bender[43,44] have used the N as their fifth pseudoscalar in the mixing analysis mentioned previously and described below. Other peaks appear in the preliminary MARK III $\eta\pi\pi$ work at the mass of the $D(1280)$, $\Gamma = 26$ MeV and at $M = 1378 \pm 4$ MeV with $\Gamma = 27 \pm 18$ MeV.[3] A J^{PC} analysis of the new structure has not been made, nor has any interpretation of the 1378 bump been made.

(a) $\iota \rightarrow \gamma\rho$ mode and other radiative processes:

One of the most intriguing problems in studying the formation and decay of glueballs is the description of their couplings to photons. In particular we are interested in the radiative decay $\iota \rightarrow \gamma\rho$ and in the two photon decay $\iota \to \gamma\gamma$. Since gluons do not couple directly to photons, the simplest view of glueball electromagnetic decays is that they should vanish. Similarly, we would not expect glueballs to be created by two photon production. Clearly, this is a lowest order result which has to be modified if (a) the state that we are discussing has a $q\bar{q}$ admixture or if (b) one of the glueball's constituent gluons pops a $q\bar{q}$ pair to one of which quarks the photon couples. In a recent paper, Liu[47] has claimed that two photon production is a great way to make glueballs at small x in the collision center of mass frame. C. E. Carlson[48] and we have concluded that this seems to be the case only at very high energies but not at SPEAR energies. On the other hand, one can expect processes that are hooking gluons to photons by popping quark pairs to be down by α_s^2.[49] Cornwall and Soni[50] estimate that two photon decays have widths comparable to the decay widths of quarkful mesons. This conclusion implies that we could not distinguish glueballs from quark states by their electromagnetic couplings, an unsavory prospect since it removes one of the small number of leading glueball indicators from our kit. In a similar vein, in a recent preprint, Donoghue[51] has considered the magnetic dipole radiative decay of a $J^{PC} = 0^{-+}$ glueball within the MIT bag model (for which $\alpha_s = 1 \to 2.2$) and concludes that such decays are expected to be larger than those of radially excited $q\bar{q}$ mesons. He urges caution in using electromagnetic transitions to interpret the status of glueball candidates. Clearly these recent developments call for further study on the question of coupling gluons to photons.

With the important caveats mentioned above, we examine the new MARK III $\gamma\rho$ results since various models for the $\iota(1460)$ give widely varying predictions, ranging from decay widths of zero for a pure glueball (without the complications described above) to 3.5 MeV (Table XII of Hitlin)[3] as predicted by Milton, Palmer, and Pinsky.[46] The process considered is $J/\psi \to \gamma X \to \gamma\gamma\rho \to \gamma\gamma\pi\pi$. The crystal ball and MARK III have both seen $\gamma\rho$ signals in the ι region. The crystal ball obtains:

$$M_X = 1398 \pm 22 \pm 15 \text{ MeV}$$
$$\Gamma_X = 155 \pm 75 \pm 20 \text{ MeV}$$
$$B(J/\psi \to \gamma X)B(X \to \gamma\rho) = (1.6 \pm 0.6 \pm 0.4) \times 10^{-4}.$$

Updated preliminary MARK III values[3,38] for the same quantities are:

$$M_X = 1434 \pm 14 \text{ MeV}$$
$$\Gamma_X = 133 \pm 32 \text{ MeV}$$
$$B(J/\psi \to \gamma X)B(X \to \gamma \rho) = (1.1 \pm 0.24 \pm 0.25) \times 10^{-4}.$$

These values result from fitting the data to two Breit–Wigner curves, the lower one setting a limit on $f(1270) \to \gamma\rho$, and corresponding to a small shoulder on the X (presumably ι—but not definitely identified because a spin–parity analysis is in progress) with a branching ratio product $B(J/\psi \to \gamma f)B(f \to \gamma\rho) < 4.7 \times 10^{-5}$ 90% CL. If the resonance seen in $\gamma\rho$ is identified with that seen in the $K\bar{K}\pi$ channel, then $\Gamma(\iota \to \gamma\rho) \approx 1.7$ MeV, consistent with the earlier value of 1.4 ± 0.6 MeV presented by Hitlin at Cornell.[3] This value is inconsistent with the pure glueball prediction of zero, but is consistent with the pole model prediction of Milton, Palmer, and Pinsky[46] of 3.5 MeV, with the η, η', ι mixing analysis of Rosenzweig[52] of 0.4–1.5 MeV and the recent Donoghue[51] estimates which range from 0.4 to 1.6 MeV. We shall consider the role of the $\iota \to \gamma\rho$ decay in the quarkonium–glueball mixing models immediately below.

2. Mixing Models

The earliest studies of the ι properties centered on the question of whether it was a glueball or a radial excitation, presumably of the $\eta'(960)$. An attractive way to settle the problem of identifying the degree to which states are to be considered glueballs is to allow bare quark and bare glueball states to mix via some appropriate QCD interactions. This is true for any J^{PC}, not just for the pseudoscalar sector. To be dubbed a glueball a state should be primarily made of a bare glueball, not just have small amounts of glueball mixed in. A simple model to examine the consequences of glueball–quarkonium mixing was used by Fishbane, Karl, and Meshkov[2,5,45] (as well as by Rosner,[53] Rosner and Tuan,[54] and Schnitzer[55] in a related calculation) in an attempt to see whether the ι, in the 0^{-+} system and the $\theta(1700)$ in the 2^{++} system could be considered as glueballs. These calculations, described in detail elsewhere, which allowed mixings both between different quark flavors and between quarks and glueballs were simple because they considered mixing only between the η, η', and the ι for the 0^{-+} system, and between $f(1270)$, $f'(1520)$, and $\theta(1700)$—then called $\theta(1670)$—for the 2^{++} system. It was concluded that neither ι nor θ were to be regarded as glueballs.

Palmer, Pinsky, and Bender[43,44] have improved on the 0^{-+} calculation by mixing a pseudoscalar octet η_8, a pseudoscalar singlet η_0, one ideally mixed radially excited light quark state, one radially excited $s\bar{s}$ state and one bare glueball. The quark states are only mixed with the glueball. They do not allow for interactions among the various quark states. Their mass matrix is shown in Table III. The parameters f and h describe the $SU(3)$ symmetric mixing of the glueball with the ground states and the radial excitations respectively. The parameter a mixes the bare η in a small $SU(3)$ asymmetric way. The mass matrix is required to have eigenvalues of the physical masses, $\mu = \eta(548)$, $\eta'(958)$, $\xi(1275)$, $\iota(1460)$ and a fifth pseudoscalar state, called NS in the 1600–1900 MeV range. They require f^2, g^2, and a^2 to be positive; the output of their model is the bare masses of the η', namely η_0, and m_G and the wavefunctions of the diagonalized matrix, which wavefunctions determine the quark or glue content of the physical states. They find two possible solutions which they test with various physical processes such as the decay $\iota \to \gamma\rho$, $\eta' \to \gamma\rho$, ψ radiative decays. These are presented in Table IV for two different values of μ_{NS}. Solution I has the ι being primarily $s\bar{s}$, with the NS state mostly glueball, whereas the converse is true for Solution II. Using the MARK III results described earlier they strongly prefer Solution II, which has the $\iota \simeq 68\%$ glueball, and for which NS is mostly $(ss)_r$.

There are a few caveats to this impressive analysis. If Solution II is correct and NS is mostly $s\bar{s}$ then it is difficult to have the P and N peaks correspond to different decay modes of the NS resonance

TABLE III
Pseudoscalar Mass Matrix

8	$m_{\eta_8^2}$		a		
0		$m_{\eta_0^2}$	f		
G	a	f	m_G^2	$\frac{h}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}h$
$(l\bar{l})_r$			$\frac{h}{\sqrt{3}}$	$m_{\eta_r'}$	
$(s\bar{s})_r$			$\sqrt{\frac{2}{3}}h$		m_{η_r}

TABLE IV
Two Solutions of the Palmer, Pinsky, and Bender Pseudoscalar Mass Matrix

μ_{NS}	$m(s\bar{s})_r$	$\left\lvert\frac{\langle G\vert\iota\rangle}{\langle G\vert\eta'\rangle}\right\rvert$	$\left\lvert\frac{\langle\eta_0\vert\iota\rangle}{\langle\eta_0\vert\eta'\rangle}\right\rvert^2$	$m_{\eta'}$	m_G
		Solution I			
1.58	1.46	0.64	0.036	1.06	1.47
1.70	1.45	0.5	0.01	1.04	1.61
		Solution II			
1.58	1.560	0.88	0.64	1.20	1.23
1.58	1.565	1.2	0.48	1.17	1.26
1.58	1.570	2.2	0.27	1.09	1.31
1.70	1.690	1.0	0.63	1.22	1.23
1.70	1.695	1.3	0.50	1.16	1.25
1.70	1.670	3.6	0.21	1.06	1.34

(whose J^{PC} has not been measured), inasmuch as a pure $s\bar{s}$ state cannot decay to $\rho\rho$ and $\omega\omega$. In addition, Solution II has a bare η_0 mass at 1200 MeV, higher that the heavy vector meson $\phi(1020)$. It seems unlikely that a pseudoscalar with quark structure $(u\bar{u} + d\bar{d} + s\bar{s})\sqrt{3}$ should be more massive than a pure vector $s\bar{s}$ state. The anomaly contribution would have to be quite large to be the source of this effect.

At this point we should recall the work of FO[42] who mix seven states, namely η, η', their first two radial excitations and a glueball. Their difficulty is that even with this complexity they cannot fit the masses of the 0^{-+} system let alone various decay and production amplitudes.

3. Conclusions

(a) It is clear that the pseudoscalar system is a mess, that individual resonances like the ι must be examined in conjunction with the other 0^{-+} states, i.e., the η, η', ζ, P and N as well as any other states that might lie nearby;

(b) It is crucial to determine whether $\zeta(1275)$ really does exist and is not an artifact of one experiment; if it does not exist, spectroscopic calculations and their subsequent interpretations will be changed.

(c) All of the spectroscopic calculations are seriously flawed either in missing one or more of the states listed in (l) and/or in not properly accounting for their decay modes.

(d) The spectroscopy of the region covered by our appellation P and even higher masses should be a matter of serious concern experimentally and theoretically, for it may well contain the clues that we need to solve the pseudoscalar puzzle.

(e) In view of (a)–(c), to speak of the ι as a glueball is still a bit premature.

(f) We must improve our understanding of mechanisms for connecting gluons to photons via higher order processes.

C. 2^{++} Isoscalars

The existing 2^{++} resonances are the $f(1270)$, $f'(1525)$, $\theta(1720)$, $f(1810)$, $g_T(2120)$, $g_T(2220)$, $g_T(2360)$,[35] and possibly $\xi(2220)$.[3] The J^{PC} of $\xi(2220)$ has not been determined, but because of its decays into K^+K^- and into $K_s^0K_s^0$, it can be 0^{++}, 2^{++}; or 4^{++}. The $f(1270)$ and $f'(1525)$ have been generally regarded as quark states, although it has been suggested[53,54] that some amount of glueball may be admixed to the $f(1270)$. This will be discussed below. The resonance which we call $f(1810)$ has been seen in only two experiments[56,57] and has been regarded as a possible quark state. The θ and the three g_T states are the glueball candidates in this group. $\xi(2220)$ has had a variety of possible assignments, but is most likely a quark state.

As for the pseudoscalars, one way to identify a glueball is to see whether the ordinary 2^{++} quark spectroscopy is well enough understood that any extra states stand out. Before doing that, let us examine the current experimental status of the various states.

1. θ(1720)

The $\theta(1720)$, the other original glueball candidate, was first observed by the crystal ball in the decay $J/\psi \rightarrow \gamma\eta\eta$.[32] In the original analysis the $f' \rightarrow \eta\eta$ term was not separated out. It was found that $M(\theta) = 1640 \pm 50$ MeV; $\Gamma(\theta) = 220^{+100}_{-70}$ MeV; and that $\theta(1640) -/\rightarrow \pi\pi$. The lack of $\pi\pi$ in θ decay cast doubt on the glueball interpretation of the θ, since glueballs are flavor singlets[58,59] and as such should go to $\pi\pi$ strongly, i.e., $\tilde{\Gamma}(\theta \rightarrow \pi\pi) = 3(\tilde{\Gamma}(\theta \rightarrow \eta\eta)$. $\tilde{\Gamma}$ is the reduced width, i.e., without phase space factors. Pire and Ralston[60] claim that $\pi\pi$ modes may be suppressed due to a pinch singularity, and that care must be taken to use properly adjusted relations when $\pi\pi$ decay modes are involved. θ was also found by MARK II in $J/\psi \rightarrow \gamma K^+K^-$;[33] its J^{PC} was measured by both groups to be 2^{++}. The Mark II mass was 1700 ± 30 MeV. MARK III has found clear evidence for the f' and θ in the K^+K^- system and has seen an analogous structure in $K_s^0K_s^0$.[38] They obtain:

$$m(f') = 1525 \pm 10 \text{ MeV};$$
$$\Gamma(f') = 85 \pm 25 \text{ MeV};$$
$$B(J/\psi \to \gamma f')B(f' \to K^+K^-) = (3.0 \pm 0.7 \pm 0.8) \times 10^{-4};$$

and

$$m(\theta) = 1720 \pm 10 \text{ MeV};$$
$$\Gamma(\theta) = 130 \pm 25 \text{ MeV}$$
$$B(J/\psi \to \gamma\theta)B(\theta \to K^+K^-) = (4.5 \pm 0.6 \pm 0.9) \times 10^{-4}.$$

The θ mass has climbed considerably from its early value of 1640 MeV. The $\pi\pi$ decay is still small, with a limit for $B(J/\psi \to \gamma\theta)$ $B(\theta \to \pi\pi)$ of $< 3 \times 10^{-4}$ at 90% CL. The θ is not a glueball (for an alternative view see Ref. 55).

An interesting and important product of this experiment has been a new value for $B(J/\psi \to \gamma f')$.[38] The new value fits in nicely with the prediction that $\Gamma(J/\psi \to \gamma f') \,/\, \Gamma(J/\psi \to \gamma f) = 0.5$ for an ideally mixed f and f'. The MARK III result is $\geqslant 0.35 \pm 0.14$. The new value for $B(J/\psi \to \gamma f')$ is more than a factor of three larger than the old MARK II value of $(0.9 \pm 0.28 \pm 0.46) \times 10^{-4}$ which led to an apparent violation of the ideal mixing value.[54] It no longer seems to be a problem.

A simple mixing analysis was carried out for the 2^{++} system,[2,5,45] mixing f, f', and θ in exact analogy with the pseudoscalar case (see also Refs. 54,55). Assuming a constant mass matrix it was found that the mass of the θ could be fitted but that the model could not account for the observed branching ratios. Using as input the experimental values for $\Gamma(f' \to \pi\pi)$ and $\Gamma(f' \to K\bar{K})$ it was predicted that $R = \Gamma(\theta \to \pi\pi)/\Gamma(\theta \to \eta\eta) \geqslant 49$; experimentally, $R < 1.1$. Once again, it was concluded that $\theta(1670)$ was not a glueball. Note that 1670 MeV was the θ mass when that calculation was carried out. Rosner and Tuan[54] also noted that in such a model a glueball assignment could not be given to $\theta(1670)$ and proposed an ad hoc model in which $f' = s\bar{s}$, unmixed with f and θ. However, they can accommodate the decay characteristics of f and θ if: $f =$ nonstrange quarks + 10% glueball and θ = glueball + 10% nonstrange quarks.

2. f(1810)

No firm results relevant for a discussion of the properties of this state have appeared since resonances in the $\pi\pi$ and KK systems were observed by Cason *et al.*,[56] and by Costa *et al.*[57] The MARK III spectrum[3] in the K^+K^- and $K_s^0K_s^0$ channels does not seem to exclude such a state, but no analysis has been carried out; the MARK III $\pi\pi$ spectrum shows no evidence for a state at 1800 MeV.[3]

3. $g_T(2120)$, $g_T(2220)$, and $g_T(2360)$ $\phi\phi$ resonances[35]

These three unusually broad states (Table V) are the glueball candidates produced in the Zweig-forbidden reaction $\pi^- p \rightarrow \phi\phi n \rightarrow K^+K^-K^+K^-n$ (Figure 2). Although apparently Zweig-forbidden, the cross section is comparable to Zweig allowed processes such as $K^-p \rightarrow \phi\phi(\Sigma^0, \Lambda)$, and $\pi^-p \rightarrow \phi K^+K^-n$. The strength of the Zweig-forbidden process is attributed to the formation and subsequent decay into two ϕ's of a glueball.

In a typical physics controversy, this view has been disputed. Lipkin[61] has claimed that two-step processes are important and may be the source of the enhancement. He further claims that no theory has succeeded in giving reliable quantitative predictions for cross sections or partial decay widths of new OZI-forbidden processes,[62] and that it is therefore risky to use the experimental observation of such processes as conclusive evidence for the existence of new particles, as, for example the g_T states. Lindenbaum[63] has countered with argument that Zweig diagrams, phenomenologically, are to be taken quite literally as one step processes, and that one may not connect disconnected diagrams via hard multigluon exchanges. He argues that, experimentally, the J/ψ and Υ systems demonstrate clearly that a double hairpin type disconnection in a Zweig diagram is strongly OZI suppressed. We shall continue to assume that the g_T states are viable glueball candidates whose glueball properties are to be verified. Some tests to be imposed are:

(a) Radiative modes of production and decay

A glueball should be readily produced in the decays of vector mesons, J/ψ or Υ. MARK III has a decay channel, $J/\psi \rightarrow \gamma\phi\phi$; unfortunately, the reconstruction efficiency plummets in the region 2100–2400 MeV (Figure 7 of Ref. 3).

Table V
Resonant States in $\pi^- p \rightarrow \phi\phi n$ ($\pm$) Refers to Sign of the Coupling

State	I^G	J^{PC}	Mass (MeV)	Γ (MeV)	S wave $S = 2$	D wave $S = 2$	D wave $S = 0$
$g_T(2120)$	0^+	2^{++}	2120^{+20}_{-120}	300^{+150}_{-50}	~50% +	~50% −	
$g_{T'}(2220)$	0^+	2^{++}	2220 ± 20	200 ± 50	~50% +	~50% −	
$g_{T''}(2360)$	0^+	2^{++}	2360 ± 20	200 ± 50			~100% +

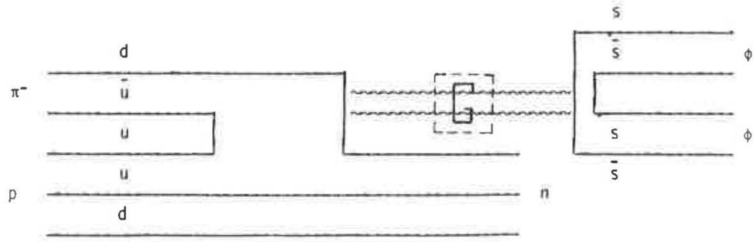

FIGURE 2. Zweig-forbidden process.

(b) $SU(3)$ decay ratios

Since glueballs are flavor singlets they should decay into two pseudoscalar final states with the ratios:

$$\frac{\tilde{\Gamma}(g_T \to \pi\pi)}{\tilde{\Gamma}(g_T \to \eta\eta)} = \frac{3}{1} \quad \text{and} \quad \frac{\tilde{\Gamma}(g_T \to K\overline{K})}{\tilde{\Gamma}(g_T \to \eta\eta)} = \frac{4}{1}.$$

The latter ratio is to be preferred because the masses of the final state particles are comparable, unlike case (1), where the possibility of corrections due to the lightness of the pions may be a problem.[60] Even so, once the pion amplitudes are correctly evaluated, the singlet fraction can be extracted and compared with either the $\eta\eta$ or $K\overline{K}$ amplitudes.

We defer further discussion of the glueball nature of the g_T states until the discussion of 2^{++} spectroscopy.

4. ξ(2220)[3,38]

Also seen in the MARK III K^+K^- and $K_S^0K_S^0$ spectrum was an enhancement ξ at $M = 2218 \pm 3 \pm 10$ MeV; $\Gamma < 40$ MeV; and $B(J/\psi \to \gamma\xi)B(\xi \to K^+K^-) = (6 \pm 2 \pm 2) \times 10^{-5}$.

Due to its narrow width, it has had a myriad of explanations. ξ has been considered as a glueball, a Higgs particle, or a quark state which is either a $^3F_2$ or a $^3F_4$ $s\bar{s}$ composite. Inasmuch as only limits have been placed on the other ξ decay modes, a definitive statement cannot be made at this time, but the data seems quite consistent with an interpretation as $L = 3$ $s\bar{s}$ state.[64] It should be noted that the decay data limits are also consistent with a $\sqrt{\text{OZI}}$ width glueball interpretation. Since our overall 2^{++} analysis follows, replete with discussions of possible quark and glueball assignments for the various states, we postpone any further discussion of the ξ.

5. Assignment of particles[2,5,41]

There are many possibilities for low-lying quark states and glueball states. These have been listed before, and correlations drawn. Listed below are the low-lying 2^{++} states which are theoretically possible, results from the recent GI calculation, and the assignments which seem plausible. In Table VI we list the lowest lying $J^{PC} = 2^{++}$ $q\bar{q}$, two-gluon glueballs, and three-gluon glueball states that we might expect. Also listed are the energy levels calculated by GI[41] together with their assignments, as well as the list of possible experimental candidates and our assignments of them.

The f and f' are clearly accommodated as $q\bar{q}$, $1 = 1$ nonstrange (NS) and strange (S) states, respectively. $f(1810)$ corresponds to the first radial excitation of the $f(1270)$, calculated by GI to be at 1790 MeV. The first radial excitation of the f' is calculated by GI to be at 2030 MeV with no apparent experimental candidate; the $\theta(1720)$ would satisfy the requirements for such an $s\bar{s}$ state, except for its mass being 300 MeV too low. The $\theta(1720)$ has all of the characteristics of an $s\bar{s}$ resonance, since it decays into $\eta\eta$ and $K\bar{K}$, but not $\pi\pi$. Its proper characterization is still a mystery.

Godfrey, Kokoski, and Isgur claim,[64] in a recent preprint, that the $\xi(2220)$ should be identified with the lowest $^3F_2$ state (which they calculate to be at 2270 MeV), rather than the $^3F_4$ state at 2210 MeV because its decay characteristics are more consistent with $^3F_2$ than the closer $^3F_4$. The $^3F_4$ NS and $^3F_4$ S states calculated by GI are shown in Table VI for reference sake.

The $\xi(2220)$ decay data limits on the $\eta\eta$ and $\pi\pi$ modes are also still consistent with its being a glueball. In addition, its narrow width, i.e., < 40 MeV, is characteristic of a $\sqrt{\mathrm{OZI}}$ width glueball. Its assignment as a Higgs particle is probably ruled out by recent Υ decay experiments which see a smaller than expected $\Upsilon \to \gamma\xi$ signal.[65]

The three g_T states may be identified with the three lowest-lying two gluon glueballs as shown in Table VI. A believable prediction of glueball masses would be most helpful here.

6. Conclusions

The best 2^{++} glueball candidates are the $g_T(2120)$, $g_T(2220)$, and $g_T(2360)$ $\phi\phi$ resonances. It is necessary, however, to verify their

TABLE VI
$J^{PC} = 2^{++}$, $I = 0$ States (all masses in MeV)

Experiment			Theory	GI
$f(1273)$	↔	1^3P_2	NS	1280
$f'(1516)$	↔	1^3P_2	S	1510
$f(1810)$	↔	$2^3P_2{}'$	NS	1790
$\theta(1720)$	?	$2^3P_2{}'$	S	2030
		1^3F_2	NS	2070
$\xi(2220)$	? ↔	1^3F_2	S	2270
		1^3F_4	NS	2060
		1^3F_4	S	2210
$g_T(2120)$		$G(2g)$	$L = 0, S = 2$	
$g_T(2220)$	? ↔		$L = 2, S = 0$	
$g_T(2360)$			$L = 2, S = 2$	
		$G(3g)$	$L = 1, S = 1$	
			$L = 1, S = 2$	

singlet decay characteristics, and it would be most reassuring to detect them in the radiative decay of the J/ψ and/or the Υ. The $\xi(2220)$ is still a possible glueball choice, although it seems to fit the $q\bar{q}$, $1 = 3$ assignment quite well. Better decay data, especially to the $\eta\eta$ mode will clarify its classification.

The $\theta(1720)$ is a puzzling state. Its decay characteristics seem to be those of an $s\bar{s}$ state, yet it lies about 80 MeV below its nonstrange counterpart $f(1810)$ in the GI QCD calculation. Perhaps it originally does lie higher than the $f(1810)$ but is preferentially depressed due to its interaction with numerous glueball states above it. This preferential interaction with an $s\bar{s}$ state compared to the $f(1810)$'s $(u\bar{u} + d\bar{d})$ structure could come about if glueballs like to couple more strongly to strange quarks than to nonstrange quarks.

A good theoretical attempt to simultaneously calculate the QCD spectrum of the quark states, the glueball states and the meikton states would be most useful. Similarly, some careful experimental spectroscopy to fill the gaps in Table VI is clearly needed.

D. 0^{++} States

The 0^{++} states are vexing. The difficulty in treating them theoretically has been discussed earlier. Experimentally, they are difficult to untangle; the best understood 0^{++} states are in the charmonium and upsilonium sectors. The best way (if not the only practical way) of finding 0^{++} glueballs is to do experiments in kinematic regions where we expect the dominance of double pomeron exchange reactions. This method yields final states with $J^{PC} = 0^{++}, 2^{++}, 4^{++}$, etc. Final states such as $\pi^+\pi^-$, K^+K^- are measured. Akesson *et al.*[66] studied $pp \rightarrow pp\pi^+\pi^-$ in the central region at $\sqrt{s} = 63$ GeV. Their $\pi\pi$ mass distribution shows evidence for S*(980) production, which might indicate some admixture of glueball, inasmuch as the double pomeron exchange process[67] has zero quark content in the initial state.

IV. SUMMARY

Having studied the glueball problem in detail, what can we say about its state of affairs? Almost everything that we have looked at has a caveat.

Our best candidates for the appellation "glueball" are the 2^{++} $\phi\phi$ resonances $g_T(2120)$, $g_T(2220)$, and $g_T(2360)$. Their interpretation as glueballs is under some theoretical challenge. At a more mundane level, it is necessary to verify that they have other decay modes and that these modes can be correlated to verify that they come from a flavor singlet. In addition, the g_T's still remain to be detected in the radiative decay of vector mesons, i.e., from J/ψ and or Υ. The $\iota(1460)$ is part of a pseudoscalar system that is growing in size and complexity, with the welcome discovery of new pseudoscalar resonances. The system is so interrelated and our understanding of it so confused that there are no analyses that explain its main features. It is certainly premature to speak of the $\iota(1460)$ as a glueball. Ordinarily we would say that its decay into $\gamma\rho$ would rule out such an assignment. Yet there are mixing models that assign the ι to a state that is $2/3$ glueball; in addition, we are being asked to rethink our interpretations of radiative transitions. We don't know with any degree of certainty that we have seen the first radially excited η, let alone a glueball.

The $\theta(1720)$ is a puzzling state. It was originally put forth as a glueball candidate, yet has very little to recommend it as such. It does not fit into any simple classification scheme for the 2^{++} system.

A new supply of states is emerging that will, doubtless, confuse the issue at first (as it is doing now). Together with new theoretical studies, they will lead us to a better understanding of glueball and meson spectroscopy, and ultimately of QCD.

Acknowledgments

The authors are grateful to S. Pinsky for innumerable enthusiastic and helpful discussions, to D. Hitlin for frequent guidance, and to T. Burnett, F. Porter, and W. Wisniewski for most useful discussions about their data. We thank M. Barnett, D. Caldwell, C. Carlson, J. M. Cornwall, S. J. Lindenbaum, P. J. O'Donnell, and A. Soni for their comments and interest.

P.M.F. is supported in part by the National Science Foundation under Grant No. PHY-8100257.

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