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On the dynamics of k-essence models

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Abstract. We investigate cosmological dynamics of models with higher-order corrections to the canonical (second-order) kinetic lagrangian for a scalar field, which have been termed "k-essence". We study the qualitative dynamics of simple purely kinetic k-essence models and find that the simplest attempts to construct non-singular cosmological models by including higher-order terms in the kinetic lagrangian fail because of a different type of singularity where the scalar field theory becomes ill-defined.

1. Introduction

We investigate cosmological dynamics of models with higher-order corrections to the canonical (second-order) kinetic Lagrangian for a scalar field, which have been termed "k-inflation" at first [1] and, more recently, called "k-essence" (we adopt the latter designation in this work).

These models are of interest to cosmologists in at least two contexts. One is the search for a "natural" model of dark energy which appears to dominate the energy density of the universe at present [1, 2, 3, 4, 5, 6, 7, 8]. "k-essence" extends the range of models available beyond the standard quintessence based on canonical scalar fields. The second context in which extensions beyond canonical scalar fields may be required is to construct non-singular bounce cosmologies, where the present expanding universe originates not from a singular big bang, but from a non-singular bounce from an earlier pre-big bang phase [9, 10, 11, 12, 13].

In both cases a characteristic of the k-inflation models is the existence of regimes where the null energy condition $\rho + P \geq 0$ is violated. Indeed this is a pre-requisite for a non-singular bounce in spatially-flat FRW cosmologies in general relativity.

In this paper we investigate the qualitative dynamics of simple purely kinetic k-inflation models and find that the simplest attempts to construct non-singular cosmological models by including higher-order terms in the kinetic Lagrangian for a scalar field fail because of the existence of a different type of singularity where the scalar field theory becomes ill-defined. We prove a general no-go theorem for Lagrangians that are a function solely of the kinetic energy with the canonical low-energy limit.

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2. K-essence models

K-essence models are scalar field cosmological models based on the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{R}{6} + \mathcal{L}\left(\varphi, \nabla\varphi\right)\right] , \tag{1}$$

(we take $8\pi G = c = 1$) where the scalar field Lagrangian, $\mathcal{L}$,

$$\mathcal{L} = P(\varphi, X) , \tag{2}$$

has the distinctive feature that we allow for a dependence on higher-order powers of the kinetic term

$$X \equiv -\frac{1}{2}(\nabla\varphi)^2 = \frac{1}{2}(\varphi_{;\eta}\varphi^{;\eta}) . \tag{3}$$

The idea that non-standard (i.e. non-quadratic), kinetic terms in the scalar field Lagrangian might be of interest to inflation was pointed out by C. Armendáriz-Picón, T. Damour and V. Mukhanov in [1] and is string inspired. Subsequently, it was realized that this class of models could also play an important role in the dark energy question, providing an alternative to quintessence with the power to avoid the cosmic coincidence problem [2, 3]. Indeed the now called *k-essence* component only behaves as a negative pressure component after the matter-radiation equality, and exhibits tracking attractor solutions during the radiation epoch making the cosmic evolution rather insensitive to initial conditions. Models with $p(\varphi, X) = K(\varphi)\tilde{p}(X)$, with $\tilde{p}(X) \sim \mathrm{const} + \mathrm{X} + \mathrm{O}(\mathrm{X}^2)$ for small X were considered with this purpose (more generally, functions $\tilde{p}(X)$ that increase monotonically with X, and requiring $\rho'(X) > 0$ to guarantee the stability of the k-essence background translated by a positive sound velocity were studied).

General features of the k-essence models were analysed by Malquarti, Copeland, Liddle and Trodden [6], and by Malquarti, Copeland and Liddle [7]. They have shown that in certain dynamical regimes it is possible to establish an equivalence between k-essence and quintessence models (see also [14]). This sheds some light on the tracking behaviour of k-essence. The study of the dynamics of k-essence models has been developed in various works [15, 16, 17, 18, 19, 20], among these Chimento and Feinstein [16] have analysed the class of models that yield power-law behaviour and found that the models previously discussed in the literature where the scalar field evolves linearly with time and the k-field potential is an inverse square of the field are only particular cases of a large class of potentials giving a constant barotropic index.

An important issue which undermines the k-essence models is the possibility of a divergence of the sound velocity in spite of this not being a well-defined concept in classical scalar fields. This question was discussed in Garriga and Mukhanov [4] and in Malquarti, Copeland and Liddle [6], and was further analysed in [7] where it is argued that the theory is valid up to a certain cutoff which excludes the singularity (divergence of sound speed). In [17] the problem of the divergence of the velocity of sound was addressed by studying a particular class of theories for which the sound velocity is always divergent.

Recently the caveats arising from a divergent speed of sound have also been analysed by Abramo and Pinto-Neto and by Bonvin, Caprini and Durrer [21, 22] raising doubts about the physical scope of these theories. These works [21, 22] bear a connection to our present analysis in that we obtain a no-go result regarding non-singular cosmologies that arises from the singular behaviour associated with the divergence of the speed of sound.

We can formally write the energy-momentum tensor as a perfect-fluid

$$T_{\mu\nu} = (\rho + P)\, v_\mu v_\nu - P g_{\mu\nu} , \tag{4}$$

where the pressure P is defined by

$$P = P(\varphi, X) \equiv \mathcal{L}(\varphi, X) , \tag{5}$$

the energy density ρ results from the Legendre transformation

$$\rho = \rho(\varphi, X) \equiv 2X\frac{\partial P}{\partial X} - P \tag{6}$$

and where the 4-velocity $v_\mu$is given by

$$v_\mu \equiv \sigma \frac{\varphi_{;\mu}}{\sqrt{2X}}\,, \tag{7}$$

σ being the signal of $\varphi_{;0} = \dot\varphi$ (we assume $\varphi_{;\mu}$ to be time-like (i. e. $X > 0$)). $\rho(X)$ is given in terms of the function $P(X)$ and its derivatives in Eq. (6), so $\rho(X)$ will be continuous for any analytic function $P(X)$.

Although k-essence models arise from any choice of a lagrangian $\mathcal{L} = P(\varphi, X)$ with a non-linear dependence on X, the theory which has attracted most interest considers

$$P(\varphi, X) = V(\varphi)\, F(X)\ , \tag{8}$$

and has been called *purely kinetic k-essence* [8]. For this theory, the equation of state only depends on $F(X)$ since

$$\omega = \frac{F(X)}{2X\, F'(X) - F(x)} \tag{9}$$

where the prime stands for the derivative with respect to X. The same happens with the effective velocity of sound [4] as

$$c^2_{sk} = \frac{\frac{\partial P(\varphi,X)}{\partial X}}{\frac{\partial \rho}{\partial X}} = \frac{F'(X)}{F'(X) + 2X\, F''(X)}\,. \tag{10}$$

3. Friedmann models with purely kinetic Lagrangians

We consider a flat FRW universe characterised by the line element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left\{\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right\}\,.$$

The Einsteins' equations, then read

$$3H^2 = \rho\,, \tag{11}$$

$$\dot H = -\frac{1}{2}\,(p + \rho)\ . \tag{12}$$

where the dot stands for time derivation and $H = \dot a/a$. We also have the equation

$$\dot\rho = -3H(\rho + P)\,, \tag{13}$$

which translates the energy-momentum balance, $T^{\mu\nu}{}_{;\mu} = 0$, and which is not an independent equation since it can be derived from the former two.

From these equations it is immediate to see that the necessary conditions for a bounce are that ρ vanishes at some point and that, when this occurs, the null energy condition be violated, i.e., $\rho + p < 0$, which in this case only requires $p < 0$. Matter fields under these circumstances have been dubbed *phantom* [23] .

We first consider Lagrangians that are only a function of the kinetic terms: $\mathcal{L} = P(X)$, that is, we take $V(\varphi)$ to be a constant in Eq. (8). It follows that both the pressure and energy density are also just functions of the kinetic energy,

$$P = P(X) = \mathcal{L}(X) \tag{14}$$

$$\rho = \rho(X) = 2X\, P'(X) - P\,, \tag{15}$$

and, in this case, the effective speed of sound

$$c_{sk}^2 = \frac{P'(X)}{\rho'(X)} = \frac{P'(X)}{P'(X) + 2XP''(X)} \,. \tag{16}$$

coincides with the *adiabatic sound speed* defined as

$$c_s^2 \equiv \frac{\mathrm{d}P}{\mathrm{d}\rho} = \frac{\dot{P}}{\dot{\rho}} \,. \tag{17}$$

From Eqs. (14) and (15) we see that these models correspond to perfect fluid models where the pressure and density are defined in parametric form.

We can write the system, Eq. (11) and Eq. (13), as

$$3H^2 = \rho(X) = 2X\,P'(X) - P(X) \tag{18}$$

$$\dot{X} = -6HX\frac{P'(X)}{P'(X) + 2XP''(X)} = -6HX\frac{P'(X)}{\rho'(X)} = -6HXc_{sk}^2 \,. \tag{19}$$

So we find that the scalar field cosmology is singular whenever ρ has a local maximum or minimum, $\rho'(X_c) = 0$, and $P'(X_c) \neq 0$. That is, respectively, when

$$P_c' + 2X_cP_c'' = 0 \qquad \text{and} \qquad \frac{\rho_c + P_c}{2X_c} \neq 0 \,.$$

These are not curvature singularities since the energy density, pressure and the expansion rate all remain finite, but the adiabatic sound speed diverges and the evolution terminates abruptly. The solution is singular in the sense that the evolution is not determined at this point. We will refer to this as a *terminating singularity.*

3.1. Quadratic purely kinetic Lagrangian with $V(\varphi)$ constant.

If we think of the non-canonical kinetic terms in the Lagrangian as arising from high-energy corrections, it is natural to consider first the lowest-order terms in an expansion about $X = 0$. With this in mind, we consider a quadratic Lagrangian of the form

$$\mathcal{L} = -V_0 + X + \frac{C_0}{2}X^2 \,, \tag{20}$$

where C_0 and V_0 are constants. At small $X \ll \sqrt{2}/|C_0|^{1/2}$ this reproduces the standard evolution for a canonical scalar field with constant potential energy V_0 . Note that any positive coefficient different from unity in front of the linear term can simply be absorbed in a re-scaling of the scalar field φ. We deliberately exclude models with a negative coefficient in front of the linear term in order to ensure that the vacuum state, $X = 0$, is a local minimum of the energy density.

With this Lagrangian the pressure and energy-density are given by

$$P = -V_0 + X + \frac{C_0}{2}X^2$$

$$\rho = V_0 + X + \frac{3C_0}{2}X^2 \,.$$

and the system (11,13) becomes

$$3H^2 = V_0 + X + \frac{3C_0}{2}X^2 \tag{21}$$

$$\dot{X} = -6HX\frac{1 + C_0X}{1 + 3C_0X} \tag{22}$$

From Eq. (21) we see that H vanishes at

$$X = -\frac{1}{3C_0}\left(1 \pm \sqrt{1-6C_0\,V_0}\right)\,, \tag{23}$$

which are the values of X where a bounce (or a recollapse) can occur. They can only be positive, as required by the definition of X, for $C_0 < 0$. We also see that we must have $C_0V_0 \leq 1/6$ for the roots to be real. In Table 1 we display the domains where the energy density is non-negative for all the possible choices of C_0 and V_0.

$C_0 > 0$	$V_0 \geq 0$	$X \in [0, +\infty[$
	$V_0 < 0$	$X \in \left[\frac{-1+\sqrt{1-6C_0V_0}}{3C_0}, +\infty\right[$
	$V_0 \geq 0$	$X \in \left[0, \frac{-1-\sqrt{1-6C_0V_0}}{3C_0}\right]$
$C_0 < 0$	$\frac{1}{6C_0} \leq V_0 < 0$	$X \in \left[\frac{-1+\sqrt{1-6C_0V_0}}{3C_0}, \frac{-1-\sqrt{1-6C_0V_0}}{3C_0}\right]$
	$V_0 < \frac{1}{6C_0}$	—
$C_0 = 0$	$V_0 \geq 0$	$X \in [0, +\infty[$
	$V_0 < 0$	$X \in [-V_0, +\infty[$

Table 1. Domains where the condition $\rho \geq 0$ is satisfied.

Hereafter we shall restrict to the $V_0 > 0$, $C_0 = -|C_0| < 0$ case in order to discuss the possibility of having a successful bounce associated with the transition from a situation where matter behaves normally to another situation where matter displays phantom behaviour, or conversely.

We only need to consider the Eq. (22) and notice that it defines a one dimensional dynamical system whose right-hand side vanishes at three places $X_0 = 0$, at $X_1 = 1/|C_0|$ and at the value of $X = X_+$ where $H(X)$ vanishes. The latter is given by the rightmost root of (23) (notice that the leftmost root falls in the unphysical $X < 0$ domain, and is thus of no importance). The two values X_1 and X_+, although defining fixed points of this one-dimensional system, do not correspond to static solutions, since $\dot{X} = 0$ implies $\varphi \propto t$. They are points where the kinetic energy is constant. At X_1 the pressure $P(X)$ exhibits a maximum. This point is associated with $\rho + p = 0$ and hence to , $\omega = -1$. As ρ given by Eq.(21) is nonvanishing, but rather $\rho = V_0 - 1/(2|C_0|)$, the scale factor has a de Sitter exponential growth. At X_+ the model has a bouncing point, since, for $X_+ > X_1$, $\rho(X) + P(X) = 2X\,P'(X) < 0$, and thus the two conditions required by a bounce are fullfilled. Finally, the value $X_0 = 0$ corresponds to the usual inflationary de Sitter behaviour where $\dot{\varphi} = 0$ and the scale factor expands exponentially $a(t) \propto \exp\left(\sqrt{V_0/3}\,t\right)$. We also realize that the system is not defined at $X_c = 1/(3|C_0|)$, the location where $\rho'(X) = 0$ and hence where the energy density has an extremum. On the left of X_c, the model is characterised by $c^2_{sk} > 0$ as well as by $\rho + P > 0$, and is thus *well-behaved*. On the right of X_c, in the domain (X_c, X_1) the model exhibits a negative effective velocity of sound, $c^2_{sk} < 0$. In the domain (X_1, X_+) the model has phantom behaviour, $\rho + p < 0$, although having a positive velocity of sound $c^2_{sk} > 0$.

Considering the possibility of having a successful bounce requires that H be negative at first and so the right-hand side of Eq. (22) has its negative sign transformed into a positive one and the origin, X_0, is an unstable fixed point. A simple analysis of the sign of the right-hand side of Eq. (22) reveals that X_1 is also unstable, and that X_+ is an attracting point. We thus realize that X_c attracts all trajectories on both sides, since it is located between X_0 and X_1 which are repellors. This means that all the trajectories on both sides of X_c end up with a diverging sound velocity. Most importantly, we see that it is impossible to have the system going from a *well-behaved* physical initial state to the bounce that might occur at X_+ with phantom behaviour. The singularity at $X = X_c$ is thus a *terminating singularity*. The only possibility that survives is rather unnappealing and corresponds to having a bounce from an initial phantom state into another phantom state, that is, from an initial condition in the region (X_1, X_+) approaching the attractor point X_+ and then leaving it, since the sign of H changes at the bounce and the latter point then becomes a repellor.

3.2. More general quadratic, purely kinetic k-essence models

We now consider the models with $V = V(\varphi) > 0$, that is, those models characterised by (8) where $F(X)$ is given by (20). From Eqs. (9) and (10), we see that a non-constant $V(\varphi)$ does not modify neither c_{sk}^2 nor the equation of state. However it modifies the evolution equation for the scalar field which now becomes

$$(F'(X) + 2X\,F''(X))\,\ddot{\varphi} + 3H\,F'(X)\,\dot{\varphi} + (2X\,F'(X) - F(X))\,\frac{\partial \ln V}{\partial \varphi} = 0\;. \tag{24}$$

We see that the major modification with respect to the constant-V case lies in the last term on the left-hand side. Since $\dot{\varphi} = \sqrt{2X}$ we recast the latter equation as

$$\dot{X} = -6HX\,\frac{F'}{F' + 2XF''} - \sqrt{2X}\,\frac{2XF' - F}{F' + 2XF''}\left(\frac{\partial \ln V}{\partial \varphi}\right)\,, \tag{25}$$

which better reveals the differences with regard to the former case. The relevant point is that the divergence of the speed of sound still occurs at $F' + 2XF'' = 0$ and this condition only depends on X. This means that, for our choice of a quadratic $F(X)$ given by Eq. (20), X_c is still at $X_c = 1/(3|C_0)|)$ and thus corresponds to a vertical line on the plane $\{x, \varphi\}$ separating the left region, where $\rho + p > 0$ and $c_{sk}^2 > 0$ and hence matter is well-behaved, from the right region where matter has either $c_{sk}^2 < 0$ or $\rho + p < 0$. If V does not have any extrema, the "fixed" points are also still given by the $X = 0$, and by X_+, since the latter makes H to vanish and hence also $2XF' - F = \rho/V = 0$. The vertical line $X = X_1$, where $\rho + p = 0$ vanishes, is no longer a line where X remains fixed.

We see that, in spite of the model becoming more complicated, a model contracting from a well behaved initial condition still has the unstable, invariant line $X = 0$ and has the boundary line $X = X_c$ which cannot be crossed, thus forbidding the evolution of the model from a well-behaved initial state towards the bounce. Therefore, the no-go result remains valid, and the same holds for more general choices of $F(X)$ satisfying the necessary conditions for a bounce.

Before presenting a general reasoning to support this latter statement, we wish to make two remarks. The first is that the class of models characterized by

$$V(\varphi) \propto (\varphi - \varphi_*)^{-2}\,, \tag{26}$$

where φ_* is an arbitrary constant, are such that the dependence on φ in Eq. (25) factors out, and so the analysis previously made for the constant V case also applies to the present case. The second remark is that when $V(\phi)$ exhibits a local extremum, at say $\varphi = \varphi_0$, there will be a fixed point on the $X = 0$ line at the φ_0 location, which will be a sink if the extremum is a minimum and which will be a unstable fixed point otherwise.

3.3. No-go theorem for non-singular cosmologies

We now display a general proof of the no-go result for non-singular cosmologies in purely kinetic k-essence models.

Suppose we wish to construct a non-singular cosmological model starting from a "well-behaved" initial state with $\rho + P > 0$ and $c_{sk}^2 > 0$. This requires that we have both $dP/dX > 0$ and $d\rho/dX > 0$ initially.

Without loss of generality we consider that the universe is collapsing, $\dot{a} < 0$, at this initial time. (An expanding universe with $\dot{a} > 0$ just corresponds to the time-reversed solution.) In a collapsing universe with $\rho + P > 0$ the continuity equation (13) requires $d\rho/dt > 0$ and hence we have $dX/dt > 0$ initially.

In order for a bounce to occur the Friedmann constraint, Eq. (11), requires $\rho = 0$. This cannot occur until after some time $t = t_0$ which we define to be the first time $\dot{\rho} = 0$, which for $\dot{a} \neq 0$ requires $\rho + P = 0$ and hence $dP/dX = 0$. At this time t_0 we have $d\rho/dX = 2X(d^2P/dX^2)$. As we approach $dP/dX = 0$ from above we must have $d^2P/dX^2 < 0$ and hence $d\rho/dX < 0$. But as we started from an initial state with $d\rho/dX > 0$ and have reached a state with $d\rho/dX < 0$ any continuous function $\rho(X)$ must have passed through the terminating singularity where $d\rho/dX = 0$ and $\rho + P \neq 0$.

4. Conclusions

In this work we have investigated the possibility of a non-singular cosmological model in the framework of k-essence with a purely kinetic lagrangian.

We have proved the following no-go theorem for any k-inflation model described by an analytic function $\mathcal{L} = V(\varphi)\,P(X)$ in a spatially-flat FRW cosmology: *any "well-behaved" state evolves to a terminating singularity in a collapsing universe, and any expanding universe evolves from a terminating singularity.*

This result tells us that, although the presence of non-standard kinetic terms in the scalar field lagrangian opens the possibility of having a primordial bounce in a flat Friedmann universe, this is not fullfilled, as the model cannot evolve from a well behaved initial state to the phantom stage required by the bounce. Our result being a direct consequence of the existence of a diverging effective velocity of sound in the k-essence models draws attention to this underlying feature of the models, an issue which has recently been addressed from different viewpoints in[21, 22] as well as in [24].

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References

[1] Armendariz-Picon C, Damour T and Mukhanov V F 1999 *Phys. Lett.* B **458** 209 (*Preprint* hep-th/9904075)
[2] Armendariz-Picon C, Mukhanov V F and Steinhardt P J 2001 *Phys. Rev.* D **63** 103510 (*Preprint* astro-ph/0006373)
[3] Armendariz-Picon C, Mukhanov V and Steinhardt P J 2000 *Phys. Rev. Lett.* **85** 4438 (*Preprint* astro-ph/0004134)
[4] Garriga J and Mukhanov V F 1999 *Phys. Lett.* B **458** 219 (*Preprint* hep-th/9904176)
[5] Damour T and Mukhanov V F 1998 *Phys. Rev. Lett.* **80** 3440 (*Preprint* gr-qc/9712061)
[6] M. Malquarti, E. J. Copeland, A. R. Liddle and M. Trodden 2003 *Phys. Rev.* D **67** 123503 (*Preprint* astro-ph/0302279)
[7] M. Malquarti, E. J. Copeland and A. R. Liddle 2003 *Phys. Rev.* D **68** 023512 (*Preprint* astro-ph/0304277)
[8] Scherrer R J 2004 *Phys. Rev. Lett.* **93** 011301 (*Preprint* astro-ph/0402316)
[9] Allen L E and Wands D 2004 *Phys. Rev.* D **70** 063515 (*Preprint* astro-ph/0404441)
[10] Gasperini G M and Veneziano G 2003 *Phys. Rept.* **373** 1 (*Preprint* hep-th/0207130)
[11] Veneziano G 2004 *JCAP* **0403**, 004 (*Preprint* hep-th/0312182)
[12] Khoury J, Ovrut B A, Seiberg N, Steinhardt P J and Turok N 2002 *Phys. Rev.* D **65** 086007 (*Preprint* hep-th/0108187)

[13] Senovilla J M M 1990 *Phys. Rev. Lett.* **64** 2219
[14] Aguirregabiria J M, Chimento L P and Lazkoz R 2005 *Phys. Lett.* B **631** 93 (*Preprint* astro-ph/0411258)
[15] Chiba T 2002 *Phys. Rev.* D **66** 063514 (*Preprint* astro-ph/0206298)
[16] Chimento L P and Feinstein A 2004 *Mod. Phys. Lett.* A **19** 761 (*Preprint* astro-ph/0305007)
[17] Chimento L P and Lazkoz R 2005 *Phys. Rev.* D **71** 023505 (*Preprint* astro-ph/0404494)
[18] Aguirregabiria J M, Chimento L P and Lazkoz R 2004 *Phys. Rev.* D **70** 023509 (*Preprint* astro-ph/0403157)
[19] Chimento L P, Forte M and Lazkoz R 2005 *Mod. Phys. Lett.* A **20** 2075 (*Preprint* astro-ph/0407288)
[20] Rendall A D 2006 *Class. Quant. Grav.* **23** 1557 (*Preprint* gr-qc/0511158)
[21] Abramo L R and Pinto-Neto N 2006 *Phys. Rev.* D **73** 063522 (*Preprint* astro-ph/0511562)
[22] Bonvin C, Caprini C and Durrer R 2006 *Phys. Rev. Lett.* **97** 081303 (*Preprint* astro-ph/0606584)
[23] Caldwell R R, Kamionkowski M and Weinberg N N 2003 *Phys. Rev. Lett.* **91**, 071301 (*Preprint* astro-ph/0302506)
[24] Chimento L P and Lazkoz R 2006 *Phys. Lett.* B **639** 591 (*Preprint* astro-ph/0604090)