《原子核物理评论》
www.npr.ac.cn Nuclear Physics Review
Started in 1984

重论超子化与芯冠转变对中子星潮汐形变的效应

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在线阅读 View online: https://doi.org/10.11804/NuclPhysRev.41.QCS2023.16

引用格式:
李牛, 叶婧, 韦斯纳, 杨荣瑶, 蒋维洲. 重论超子化与芯冠转变对中子星潮汐形变的效应[J]. 原子核物理评论, 2024, 41(3):858–862. doi: 10.11804/NuclPhysRev.41.QCS2023.16
LI Niu, YE Jing, WEI Sina, YANG Rongyao, JIANG Weizhou. Revisiting Effects of Hyperonization and Core–crust Transition on Tidal Deformability of Neutron Stars[J]. Nuclear Physics Review, 2024, 41(3):858–862. doi: 10.11804/NuclPhysRev.41.QCS2023.16

第 41 卷 第 3 期
2024 年 9 月

原 子 核 物 理 评 论
Nuclear Physics Review

Vol. 41, No. 3
Sep., 2024

Article ID: 1007-4627(2024)03–0858–05

Revisiting Effects of Hyperonization and Core-crust Transition on Tidal Deformability of Neutron Stars

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Abstract: Neutron star tidal deformability extracted from gravitational wave data provides a novel probe to the neutron star structures that encode the information of the nuclear equation of state (EOS). In this contribution to QCS2023 special issue, attention is paid to the role of the hyperon compositions and core-crust transition in the tidal deformability in a relativistic mean-field approach with the density-dependent parametrizations. Since both the core-crust transition density and the hyperon compositions involve large uncertainties in neutron stars, it is demonstrated that various core-crust transition density and hyperon compositions allowed by the parametrization can both have significant effects on the tidal deformability of neutron stars. These results are instructive for further experimental and theoretical works.

Key words: core-crust transition; hyperons; neutron star tidal deformability; relativistic mean-field models

CLC number: O572.21　　**Document code:** A

DOI: 10.11804/NuclPhysRev.41.QCS2023.16　　**CSTR:** 32260.14.NuclPhysRev.41.QCS2023.16

0 Introduction

Explorations on neutron star interior and the associated nuclear equation of state (EOS) are important issues related to the phase structures and transitions of strongly interacting matter. With the establishment of new observatories such as Ligo/Virgo[1−2], NICER[3], and eXTP[4], the quantitative investigations of these issues are on the new track to fuel new impetus. For instance, the extraction from the gravitational wave data of the binary merger gives the tidal deformability of $\Lambda_g = 190^{+390}_{-120}$ for the canonical neutron star[5], which is very informative to the neutron star structures. Together with the terrestrial experiments, the multi-messengers will shed new lights on the texture of dense matter and structures inweaved with the fundamental interactions in the medium that belong essentially to the many-body problems.

The tidal deformability is just moderately affected by the stiffness of the symmetry energy around saturation density, as there is a counteraction between the Love number and the neutron star radius[6]. The sensitivity to the tidal deformability arises more from phase transitions in the interior of neutron stars[7−8], when new degrees of freedom come forth to change the energetics of the system. As hyperons can be important constituents in the neutron star interior[9−14], we here report the influence of the hyperon emergence on the tidal deformability. With the inclusion of hyperons, our density-dependent relativistic mean-field (RMF) parametrizations with chiral limits can meet the $2M_\odot$ constraint without the upset of the hyperon puzzle[10, 15].

In addition to the probe of the tidal deformability to the core region of neutron stars, it is significant to check whether the exterior core-crust transition can have imprints on the tidal deformability, since the neutron star surface and core-crust interface are associated with direct observations and have been attached arresting attention[3−4, 16−20]. Similar to the uncertainty of the interior neutron stars, the various treatments of the transition from the homogeneous core to the inhomogeneous crust can produce rather large uncertainty for the transition density[21−26]. Even, the core-crust transition can be affected significantly by the fluctuation in the Dirac sea[27−28]. In practice, large flexibility exists in choosing the core-crust transition density[29].

In this report, we revisit the effect of the hyperons and core-crust transition on the tidal deformability of neutron stars. The remainder of the paper is organized as follows. In Sec. I, we present some main formulas for asymmetric nuclear matter and the methods to determine the core-crust transition density. Some numerical results and discussions are given in Sec. II. A brief summary is given in Sec.III.

Received date: 01 Jul. 2024;　**Revised date:** 15 Aug. 2024

Foundation item: National Natural Science Foundation of China (11775049, 12375112)

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1 Methodology

We adopt the density-dependent RMF models that feature the chiral limits based on the Brown-Rho scaling[30-31] and herein invoke directly the energy density and pressure from the previous RMF models SLC and SLCd[10]:

$$\mathcal{E}=\frac{1}{2}m_{\omega}^{*2}\omega_0^2+\frac{1}{2}m_{\rho}^{*2}b_0^2+\frac{1}{2}m_{\phi}^{2}\phi_0^2+\frac{1}{2}m_{\sigma}^{*2}\sigma^2+\frac{1}{2}m_{\sigma^*}^{2}\sigma^{*2}+2\sum_i\int_0^{k_{Fi}}\frac{\mathrm{d}^3k}{(2\pi)^3}E_i^*,\tag{1}$$

$$p=\frac{1}{2}m_{\omega}^{*2}\omega_0^2+\frac{1}{2}m_{\rho}^{*2}b_0^2+\frac{1}{2}m_{\phi}^{2}\phi_0^2-\frac{1}{2}m_{\sigma}^{*2}\sigma^2-\Sigma_0^R\rho-\frac{1}{2}m_{\sigma^*}^{2}\sigma^{*2}+\frac{2}{3}\sum_i\int_0^{k_{Fi}}\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{\boldsymbol{k}^2}{E_i^*},\tag{2}$$

where i runs over the species of baryons and leptons considered in neutron star matter, $E_i^*=\sqrt{\boldsymbol{k}^2+m_i^{*2}}$ with m_i^* being the Fermion effective mass of species i, and Σ_0^R is the rearrangement term, originating from the density-dependent parameters[32]. In Eqs. (1)~(2), there are also contributions from the strange mesons ϕ (1 020 MeV) and σ^* (*i.e.* f_0, 975 MeV), in addition to the normal mesons. The meson coupling constants and masses with asterisks denote the density dependence, given by the Brown-Rho (BR) scaling Refs. [10, 30−31]. For the nucleonic sector, we take the scaling functions for the coupling constants of scalar and vector mesons as $\Phi_{\sigma N}(\rho)=(1-y_i\rho/\rho_0)/(1+x_i\rho/\rho_0)$ with coefficients x_i and y_i given in Refs. [30−31]. For hadron masses, the scaling function is given as $\Phi(\rho)=1-y\rho/\rho_0$. The parameters for strange mesons σ^* and ϕ are assumed to be density independent. We study neutron stars with compositions including baryons (N,Λ,Ξ) and leptons (e,μ). Σ baryons are not taken into account for its repulsive potential which is energetically unstable.

The parametrization for hyperons follows Ref. [15] to include the density dependence of the hyperon-meson coupling constants:

$$\begin{aligned}\Phi_{\omega\Lambda}(\rho)&=\left(\frac{1}{3}-\alpha\right)\Phi_{\omega N}(\rho_0)+\left(\frac{2}{3}+\alpha\right)\Phi_{\omega N}(\rho),\\ \Phi_{\omega\Xi}(\rho)&=\left(\frac{2}{3}-\alpha\right)\Phi_{\omega N}(\rho_0)+\left(\frac{1}{3}+\alpha\right)\Phi_{\omega N}(\rho),\\ \Phi_{\sigma Y}(\rho)&=(1-f_{\sigma Y})\Phi_{\sigma N}(\rho_0)+f_{\sigma Y}\Phi_{\sigma N}(\rho),\end{aligned}\tag{3}$$

where ρ_0 is saturation density, and α and $f_{\sigma Y}$ are adjustable constants. The parameter α is invoked to tune the density dependence in Eq. (3), considering the in-medium effect is also existent in the hyperon sector. The scaling function $\Phi_{\rho\Xi}$ for the ρ meson are taken the same as that of the ω meson. Note that $\Phi_{\omega N}(\rho_0)$ and $\Phi_{\sigma N}(\rho_0)$ are just constants. In Eq. (3), the parameters α and $f_{\sigma Y}$ do not change the hyperon potentials at saturation density that are set at empirical values[10]

$$U_{\Lambda}^{(N)}=-30\ \mathrm{MeV},\ U_{\Xi}^{(N)}=-18\ \mathrm{MeV}.\tag{4}$$

The resulting ratio parameters $X_{\sigma Y}$, $X_{\omega Y}$, and $X_{\rho Y}$ ($X_{iY}=g_{iY}/g_{iN}$) are now density dependent. For the strange mesons, we adopt the density-independent coupling constants for simplicity, just the same as what was done in Ref. [10].

In the low density region, there are no hyperons and even no muons. The EOS of this density region comprises two pieces: the inner and outer crustal ones. In the inner crust, we adopt a phenomenological EOS $p(r)=a+b\mathcal{E}(r)^{4/3}$ [26] with constants a and b being determined by the continuous condition at the core-crust transition density ρ_t and the density $\rho_1=2.57\times10^{-4}\ \mathrm{fm}^{-3}$ with the energy density $\mathcal{E}=0.24\ \mathrm{MeV\cdot fm^{-3}}$ and $p=4.87\times10^{-4}\ \mathrm{MeV\cdot fm^{-3}}$ which connects to the outer crust[26, 33]. However, if the thermodynamic consistency is admitted in the inner crust, we may encounter a discontinuity in the energy density at ρ_t. The following formulas hold for the thermodynamic consistency:

$$p=\mu\rho-\mathcal{E},\ \rho=\partial p/\partial\mu,\tag{5}$$

with μ being the chemical potential. The number density can be further written as

$$\rho(\mathcal{E})=\rho_1\exp\left(\int_{\mathcal{E}_1}^{\mathcal{E}}\frac{\mathrm{d}\mathcal{E}}{\mathcal{E}+p}\right).\tag{6}$$

Usually, there is the inequality $\rho_t\neq\rho(\mathcal{E}_t)$, which gives rise to a density jump ($\Delta\rho$) at the core-crust transition. The energy density jump $\Delta\mathcal{E}$ can be obtained from $\Delta\rho$ according to the density profile of the energy density. This interfacial discontinuity can be imprinted in the spacetime perturbation or the tidal deformability through the following relation

$$y^+=y^--\frac{\Delta\mathcal{E}}{\rho(r_t)+3\rho_{Mt}},\tag{7}$$

where $y=H'/H$ with H being the spacetime perturbation[15], and ρ_{Mt} is the average density in the sphere with a radius r_t: $\rho_{Mt}=M(r_t)/V(r_t)$. The tidal deformability is given by

$$\Lambda_g=\frac{2}{3}k_2\left(\frac{R}{M}\right)^5,\tag{8}$$

with k_2, R and M being the Love number[15], neutron star radius and mass, respectively. In fact, the apparent dependence of Λ_g on R^5 can be cancelled largely by the Love number.

The core-crust transition density ρ_t is the lowest density of uniform phase. We use the thermodynamic method and relativistic random phase approximation (RRPA)[26-27] to determine ρ_t. In the thermodynamic method, the stability of matter requires the convex energy against the volume[34-35]. In RRPA, the transition density is the largest density determined by the relation of the dielectric function

$$\epsilon_L = \det\left[1 - D_L(q)\Pi_L(q)\right]_{q_0=0} \leqslant 0, \qquad (9)$$

for any q in the static field with $q_0 = 0$ [26-27]. Here, D_L and Π_L are the longitudinal meson propagator and polarization matrices[27], respectively.

2 Results and discussions

With the inclusion of hyperons, our parametrizations SLC and SLCd models can give rise to the maximum mass of neutron stars in accord with the $2M_\odot$ constraint[10, 15]. Here, we highlight that the hyperon onset density can be modulated sensitively by parameters α and $f_{\sigma Y}$ in Eq. (3), which adjust the vector and scalar hyperon potentials, respectively. Table 1 gives the hyperon onset densities for various parameters α and $f_{\sigma Y}$. Apart from the hyperon onset density around $2\rho_0$ in usual mean-field models, our models shift the onset density above $2.5\rho_0$.

Table 1 Onset densities of the Λ hyperon with various parameters α and $f_{\sigma\Lambda}$ in SLC and SLCd. The density is in unit of ρ_0 (=0.16 fm^{-3}).

Model	$f_{\sigma\Lambda}$	$\alpha = 0$	$\alpha = -0.05$	$\alpha = 0.05$
SLC	0.8	2.63	2.70	2.57
	0.9	2.78	2.88	2.69
SLCd	0.8	2.85	2.97	2.77
	0.9	3.06	3.24	2.93

The core-crust transition densities with thermodynamic method and RRPA are presented in Table 2. It is seen that ρ_t is quite different for two approaches. This difference stems from the different underlying physics in two approaches. In the thermodynamic method, ρ_t is determined by the variation of the mean field, while the RRPA involves the correlation effect which usually shows both the momentum and density dependence. In the practical calculation, obtaining the large RRPA ρ_t is relevant to the rather large momentum q at which the inequality (9) breaks down. In deed, arbitrariness to choose the core-crust transition density was considered in the literature[29], and our results seem to justify such choices.

Table 2 Core-crust transition density with two approaches in the parametrizations of SLC and SLCd. The density is in units of fm^{-3}. The value in the bracket is for $\Delta\rho$.

Model	RRPA	Thermodynamic
SLC	0.143 2 (0.006 7)	0.090 4 (0.001 4)
SLCd	0.144 3 (0.004)	0.091 5 (0.000 65)

Now, we present in Table 3 the tidal deformability of the canonical neutron star for various cases. It can be observed that various values of α, associated with different hyperon inclusions, give rise to the appreciable variation in the tidal deformability. The difference in α can account for the departures in hyperon onset densities, see Table 1, and hyperon fraction in neutron stars[15]. Here, all the predicted tidal deformability fall nicely in the experimentally constrained region 190^{+390}_{-120} [5]. A larger hyperon fraction (correspondingly, the larger α) supports a smaller tidal deformability, which is more close to the central experimental value.

Table 3 Tidal deformability with the continuous connection and density jump at the core-crust interface for various parametrizations. The result is for the canonical neutron star with $M = 1.4M_\odot$. Here, $f_{\sigma\Lambda} = 0.8$.

Model	RRPA (DJ/CC)	Therm. (DJ/CC)
SLCd, $\alpha = 0.5$	165.77 / 168.54	161.15 / 163.52
$\alpha = 0.0$	197.04 / 200.47	191.02 / 193.91
$\alpha = -0.5$	218.54 / 222.45	211.62 / 214.97
SLC, $\alpha = 0.5$	226.01 / 229.28	217.03 / 220.30
$\alpha = 0.0$	285.81 / 290.16	251.24 / 255.19
$\alpha = -0.5$	401.17 / 407.68	345.65 / 351.52

Varying the transition density ρ_t significantly may have prominent or moderate effects on the tidal deformability, depending on the models whether they predict a larger or smaller neutron star radius. The big difference of the transition density can affect Λ_g appreciably in the model that predicts a large radius of neutron stars, especially when the hyperon fraction is smaller. This is the case of $\alpha = 0$ and −0.5 with the SLC, as seen in Table 3. In this case, the radius of neutron stars with the RRPA is much larger than that with the thermodynamic method. For instance, the radius of the canonical star is 15.07 (12.99) km with ρ_t obtained with the RRPA (thermodynamic method). Though the Love number and star radius have a counteracting role in the tidal deformability, sufficiently large radii can win out to cause a significant increase in the tidal deformability. In contrast, whether it is a continuous connection (CC) or the inclusion of a density jump (DJ) at the core-crust interface just causes a small difference in the tidal deformability, which is overall at the level within 2%.

The radii of the canonical neutron star are presented in Table 4. Here, the cases of CC and DJ are not distinguished, because the radii are almost the same for them. For $\alpha = 0.0$ and −0.5, the large ρ_t obtained with the RRPA in SLC results in a unfavorably large radius. This is not the problem in obtaining ρ_t with the RRPA, but the disfavored stiff symmetry energy in the SLC, since the unique difference between the SLC and SLCd is the stiffness of the symmetry energy[30−31]. The large neutron star radius associated with the stiff symmetry energy is not supported by the large ρ_t obtained with the RRPA.

Table 4　Canonical neutron star ($M = 1.4M_{\odot}$) radii with ρ_t of the RRPA and thermodynamic method. Here, the radius is in units of km, and $f_{\sigma\Lambda} = 0.8$.

Model	RRPA	Therm.
SLCd, $\alpha = 0.5$	12.11	11.28
$\alpha = 0.0$	12.40	11.52
$\alpha = -0.5$	12.57	11.66
SLC, $\alpha = 0.5$	13.79	12.14
$\alpha = 0.0$	14.34	12.46
$\alpha = -0.5$	15.07	12.99

3　Summary

In this contribution to QCS2023 special issue, we investigated the effects of the hyperon and core-crust transition on the tidal deformability of neutron stars with density-dependent RMF models. The models can accommodate hyperons without breaking the $2M_{\odot}$ constraint, while the hyperon fraction can be adjusted by invoking free parameters without changing the hyperon potentials at saturation density. The modulation of the hyperon fraction in a moderate range can effectively affect the tidal deformability of neutron stars. This provides another example, instead of the first-order hadron-quark transition, that the tidal deformability can signal the interior structure (composition) of neutron stars, albeit with some degeneracy in quark or hyperon appearances. On the other hand, we checked the effect of the core-crust interface on the tidal deformability. The significant difference in the core-crust transition densities has generally moderate effects on the tidal deformability, while the effects can be amplified significantly in the models that have the stiff symmetry energy and large neutron star radius. In contrast, the strategy whether a continuous connection or a density jump is adopted at the core-crust interface does not affect the tidal deformability significantly.

Acknowledgements This work was supported in part by the National Natural Science Foundation of China (Grant No.11775049 and No.12375112). The Big Data Computing Center of Southeast University is acknowledged for providing the facility support for the partial numerical calculations of this work.

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重论超子化与芯冠转变对中子星潮汐形变的效应

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摘要: 从引力波数据提取的中子星潮汐形变为包含核态方程信息的中子星结构提供了新颖的探针。利用相对论平均场密度依赖的参数，在这篇QCS2023专刊文章中，重点关注中子星的超子组份与芯冠转变对潮汐形变的作用。鉴于两者都存在大的不确定度，阐明了变化参数化模型所许可的芯冠转变密度与超子组份两者都可以对中子星潮汐形变有显著的效应。这些结果对进一步的实验与理论工作具有一定的启示作用。

关键词: 芯冠转变；超子；中子星潮汐形变；相对论平均场模型

收稿日期: 2024-07-01；　**修改日期:** 2024-08-15

基金项目: 国家自然科学基金资助项目 (11775049, 12375112)

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