Maximal efficiency of collisional Penrose process with spinning particles

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1. Introduction

It is known that the center of mass energy can take arbitrarily large value if a collision occurs near the event horizon of an extreme rotating black hole[1]. However, it does not mean that an ejected particle will have very high energy at infinity because of red-shift. Hence, how large energy we will observe at infinity has been evaluated for various situations[2]. Most of those results were obtained for a point test particle. But a matter around a black hole normally has rotation. Since the equation of motion of a spinning particle is totally different from the geodesic equation, and the effect of spin is nontrivial, it is important to study the efficiency of the extracted energy ($\eta =$ (extracted energy)/(input energy)) in the collision of spinning particles. We focus on the collisional Penrose process of spinning particles near the horizon of an extreme Kerr black hole and evaluate the maximal value of the energy efficiency for various processes.

2. The orbit of a spinning particle

In the Kerr spacetime, there are two Killing vectors, and then we have two conserved quantities; the particle energy E and the particle total angular momentum J. For simplicity, we assume that a spinning particle is orbiting on the equatorial plane ($\theta = \pi/2$) and the direction of a particle spin is parallel to the rotation of a black hole. The spin parameter s is positive when a particle is parallelly spinning to the rotation of the black hole, while s is negative when it is counter-rotating.

By this setting, we obtain the specific momentum $u^{(a)} = p^{(a)}/\mu$, where $p^{(a)}$ is the momentum and μ is the particle mass, as

$$u^{(0)} = \frac{\left[(r^3 + a(a+s)r + aMs)E - (ar + Ms)J\right]}{\mu r^2 \sqrt{\Delta}\left(1 - \frac{Ms^2}{r^3}\right)}, \tag{1}$$

$$u^{(3)} = \frac{[J - (a+s)E]}{\mu r\left(1 - \frac{Ms^2}{r^3}\right)}, \tag{2}$$

where M and a are the black hole mass and its specific angular momentum, respectively. The suffix (a) denotes the tetrad components. From the definition of the particle mass, we obtain the radial component $u^{(1)}$ as

$$u^{(1)} = \sigma\sqrt{(u^{(0)})^2 - (u^{(3)})^2 - 1}, \tag{3}$$

where $\sigma = \pm 1$ denotes the direction of the radial motion of the particle, i.e., a particle is moving outward for $\sigma = +1$ while it does inward for $\sigma = -1$.

Before the discussion of the collisional Penrose process, we should note a few important points as follows:

(1) Since the four-velocity of a spinning particle $v^{(a)} = dx^{(a)}/d\tau$ is not always parallel to the four-momentum $u^{(a)}$, we have to impose the timelike condition $v^{(a)}v_{(a)} < 0$ in our analysis.

(2) In order for a particle to reach the horizon, the condition $J_- \le J \le J_{cr}$ must be satisfied. Here, $J_{cr}(> 0)$ and $J_-(< 0)$ are the maximum and minimum values of the particle angular momentum, respectively. This condition is obtained from $(u^{(1)})^2 > 0$ for any radius of $r \ge r_H = M$.

(3) To obtain arbitrarily large center-of-mass energy, the collision must take a place near the horizon and one of collisional particles must have the critical angular momentum $J_{cr} = 2EM$, and the other particle must have sub-critical angular momentum $(J_- <)J < J_{cr}$.

3. Collisional Penrose process

We assume that two particles (the particle 1 [critical, $J_1 = 2E_1M$] and the particle 2 [sub-critical, $J_2 < J_{cr}$]) plunge from infinity, and collide at the point $r_c = r_H/(1 - \epsilon)$ $(0 < \epsilon \ll 1)$, where $r_H = M$ is the horizon radius. After collision, the particle 3 (= the particle 1) is coming back to infinity, while the particle 4 (= the particle 2) will fall into a black hole.

At the collision, we have the following onservation laws:

$$E_1 + E_2 = E_3 + E_4, \quad J_1 + J_2 = J_3 + J_4, \tag{4}$$

$$s_1 + s_2 = s_3 + s_4, \quad p_1^{(1)} + p_2^{(1)} = p_3^{(1)} + p_4^{(1)}. \tag{5}$$

From these conservation laws, we find that particle 3 must have near-critical angular momentum $(J_3 = 2E_3M + O(\epsilon))$. We consider only the head-on collision with $\sigma_1 = \sigma_3 = +1$ $\sigma_2 = \sigma_4 = -1$, $s_2 = s_4$, and $s_1 = s_3$, because it gives the maximum efficiency.

We expand the above conservation equations in terms of ϵ and solve the energies E_2 and E_3 by use of the expansion coefficients. We then evaluate the energy efficiency $\eta = E_3/(E_1 + E_2)$. The detailed analysis is given in Ref. 4.

In Table 1, we show the maximal efficiencies and the input and output energies for the following three cases : [1] Collision of two massive particles, [2] "Compton" scattering (the particle 1: massless, the particle 2: massive), and [3] "Inverse Compton" scattering (the particle 1: massive, the particle 2: massless).

In the case of the collision of two massive particles, the maximal efficiency η is about 15.01, while it is about 26.85 for the case of the "Compton" scattering in the limit of $E_1 \to \infty$. The maximal efficiency becomes the largest for the "Compton" scattering, which in fact is the same as the case for without spin. Compared with the spinless case, these maximal efficiencies become twice larger than the spinless case. We then conclude that a spin plays an important role in the context of energy extraction. Note that the efficiency does not change significantly in the case of the "inverse Compton" scattering because the absorbed massless particle is spinless and the escaped massive particle with large energy cannot have a large spin.

Table 1. The maximal efficiencies and energies for three cases of collisions of particles plunging from infinity. We include the result for the nonspinning case (Ref. 3) for comparison. The maximal efficiencies and maximal energies are enhanced when the spin effect is taken into account.

collisional process	spin (s_1, s_2)	input energy (E_1, E_2)	output energy (E_3)	maximal efficiency
Collision of two massive particles	non-spinning	(μ, μ)	12.66μ	6.328
	$(0.01379\mu M, -0.2709\mu M)$		30.02μ	15.01
"Compton" scattering	non-spinning	$(+\infty, \mu)$	$+\infty$	13.93
	$(0, -0.2709\mu M)$		$+\infty$	26.85
"Inverse Compton" scattering	non-spinning	$(\mu, 0)$	12.66μ	12.66
	$(0.02679\mu M, 0)$		15.64μ	15.64

The present setting may be very unlikely in more realistic astrophysical situations. Since there are many particles in an accretion disc around a black hole, it may be more natural for a particle plunging from infinity will collide particles in the innermost stable circular orbit (ISCO). The result in this case will be published elsewhere[5]. The ansatz of an extreme black hole is also not natural. The study for a non-extreme black hole is in progress.

References

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