Non-Gaussianities from ekpyrotic collapse with multiple fields

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Abstract

We compute the non-Gaussianity of the curvature perturbation generated by ekpyrotic collapse with multiple fields. The transition from the multi-field scaling solution to a single-field dominated regime converts initial isocurvature field perturbations to an almost scale-invariant comoving curvature perturbation. In the specific model of two fields, ϕ_1 and ϕ_2, with exponential potentials, $-V_i \exp(-c_i\phi_i)$, we calculate the bispectrum of the resulting curvature perturbation. We find that the non-Gaussianity is dominated by non-linear evolution on super-Hubble scales and hence is of the local form. The non-linear parameter of the curvature perturbation is given by $f_{NL} = 5c_j^2/12$, where c_j is the exponent of the potential for the field which becomes sub-dominant at late times.

1 Introduction

Recently, there has been progress in generating a scale-invariant spectrum for curvature perturbations in the ekpyrotic scenario with more than one field, which we will refer to as the new ekpyrotic scenario [1, 2, 3]. If these fields have steep negative exponential potentials, there exists a scaling solution where the energy densities of the fields grow at the same rate during the collapse. In this multi-field scaling solution background, the isocurvature field perturbations have an almost scale-invariant spectrum, owing to a tachyonic instability in the isocurvature field.

The multi-field scaling solution in the new ekpyrotic scenario can be shown to be an unstable saddle point in the phase space and the late-time attractor is the old ekpyrotic collapse dominated by a single field [4]. But the transition from the multi-field scaling solution to the single-field-dominated solution also provides a mechanism to automatically convert the initial isocurvature field perturbations about the multi-field scaling solution into comoving curvature perturbations about the late-time attractor [5].

On the other hand, the non-Gaussianity of the distribution of primordial curvature perturbations in the inflationary scenario has been extensively studied by many authors (see e.g. [6] for a review). Thus, as a natural extension of the study performed in [4, 5], in this paper [7] we compute the non-Gaussianity of the primordial curvature perturbations generated from the contracting phase of the multi-field new ekpyrotic cosmology.

2 Model and Homogeneous dynamics

We first review the model and the background dynamics of the new ekpyrotic cosmology with multiple scalar fields. During the ekpyrotic collapse the contraction of the universe is assumed to be described by a 4D Friedmann equation in the Einstein frame with n scalar fields with negative exponential potentials

$$3H^2 = V + \sum_j^n \frac{1}{2}\dot{\phi}_j^2 , \quad \text{where} \quad V = -\sum_j^n V_j e^{-c_j\phi_j} , \tag{1}$$

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and we take $V_i > 0$ and set $8\pi G$ equal to unity. From now on, for simplicity, we concentrate our attention on the case of two fields. In this case, it will be easier to work in terms of new variables,

$$\varphi = \frac{c_2\phi_1 + c_1\phi_2}{\sqrt{c_1^2 + c_2^2}}\,, \quad \chi = \frac{c_1\phi_1 - c_2\phi_2}{\sqrt{c_1^2 + c_2^2}}\,. \tag{2}$$

The potential can then be simply re-written as

$$V = -U(\chi)\, e^{-c\varphi}\,, \quad \text{with} \quad U(\chi) = V_1\, e^{-(c_1/c_2)c\chi} + V_2\, e^{(c_2/c_1)c\chi}\,, \quad \frac{1}{c^2} \equiv \sum_j \frac{1}{c_j^2}\,. \tag{3}$$

It can be shown that $U(\chi)$ has a minimum at $\chi = \chi_0$ and the multi-field scaling solution corresponds to the classical solution along this minimum $\chi = \chi_0$, while φ is rolling down the exponential potential. It is worth noting that the potential for χ has a negative mass-squared $m_\chi^2 \equiv \partial^2 V/\partial\chi^2 = c^2 V < 0$ around $\chi = \chi_0$ which makes the multi-field scaling solution unstable. Furthermore, the χ field evolution is nonlinear, with the cubic interaction being given by

$$V^{(3)} \equiv \frac{\partial^3 V}{\partial \chi^3} = \tilde{c} m_\chi^2\,, \quad \text{where} \quad \tilde{c} \equiv \frac{c_2^2 - c_1^2}{\sqrt{c_1^2 + c_2^2}}\,, \tag{4}$$

which becomes important when we consider the non-Gaussianity later in this paper. Another important solution is the single-field dominated scaling solution which is also appeared in the old ekpyrotic scenario. In this paper, we consider the case in which the background evolves from the multi-field scaling solution to the ϕ_2-dominated scaling solution without loss of generality.

3 Statistical correlators and δN-formalism

In the two-field new ekpyrotic cosmology, the isocurvature fluctuations acquired by the field χ during the multi-field scaling regime, play a crucial role to generate a scale-invariant spectrum of perturbations. On the other hand, the fluctuations of the field φ are negligible on large scales, because of its very blue spectral tilt. Thus, in the following we neglect $\delta\varphi$ fluctuations. To relate the non-Gaussianity of the scalar field fluctuations to observations, we need to calculate the three-point functions of the comoving curvature perturbation ζ. In order to do that, we can use the δN-formalism [8, 9]. In the δN-formalism, the comoving curvature perturbation ζ evaluated at some time $t = t_f$ coincides with the perturbed expansion integrated from an initial *flat* hypersurface at $t = t_i$, to a final *uniform density* hypersurface at $t = t_f$, with respect to the background expansion, i.e.,

$$\zeta(t_f, \mathbf{x}) \simeq \delta N(t_f, t_i, \mathbf{x}) \equiv N(t_f, t_i, \mathbf{x}) - N(t_f, t_i)\,, \tag{5}$$

with

$$N(t_f, t_i, \mathbf{x}) \equiv \int_{t_i}^{t_f} \mathcal{H}(\mathbf{x}, t) dt\,, \quad N(t_f, t_i) \equiv \int_{t_i}^{t_f} H(t) dt\,, \tag{6}$$

where $\mathcal{H}(\mathbf{x}, t)$ is the inhomogeneous Hubble expansion. We will choose the initial time t_i to be *during* the multi-field scaling regime. Furthermore, since φ is unperturbed, δN can be expanded in series of the initial field fluctuations $\delta\chi_i$. Retaining only terms up to second order, we obtain

$$\delta N = N_{,\chi_i}\delta\chi_i + \frac{1}{2} N_{,\chi_i\chi_i}(\delta\chi_i)^2\,, \tag{7}$$

where $N_{,\chi}$ denotes the derivative of N with respect to χ.

The bispectrum of the curvature perturbation ζ, which includes the first signal of non-Gaussianity, is defined as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(\sum_j \mathbf{k}_j) B_\zeta(k_1, k_2, k_3)\,, \tag{8}$$

where the left hand side of Eq. (8) can be evaluated by the δN-formalism using Wick's theorem,

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = N_{,\chi_i}^3 \langle \delta\chi_{i\mathbf{k}_1} \delta\chi_{i\mathbf{k}_2} \delta\chi_{i\mathbf{k}_3} \rangle + \frac{1}{2} N_{,\chi_i}^2 N_{,\chi_i\chi_i} \langle \delta\chi_{i\mathbf{k}_1} \delta\chi_{i\mathbf{k}_2} (\delta\chi_i \star \delta\chi_i)_{\mathbf{k}_3} \rangle + \text{perms}\,. \tag{9}$$

In the above equation, a star $\star$ denotes the convolution and we have neglected correlators higher than the four-point.

Observational limits on the non-Gaussianity of the primordial curvature perturbations are usually given on the nonlinear parameter f_{NL} defined by

$$\frac{6}{5}f_{NL} \equiv \frac{\prod_j k_j^3}{\sum_j k_j^3}\frac{B_\zeta}{4\pi^4\mathcal{P}_\zeta^2}\,, \tag{10}$$

where $\mathcal{P}_\zeta$ is the power spectrum of the curvature perturbation ζ. If the non-Gaussianity is local, one can write ζ as

$$\delta N = \zeta_L + \frac{3}{5}f_{NL}\zeta_L^2\,, \tag{11}$$

where ζ_L is a Gaussian variable.

4 Non-Gaussianities

We consider the situation in which χ_i is perturbed on the $t = t_i$ hypersurface, while H_i assumes on this hypersurface a constant value. This is justified by the fact that the $t = t_i$ hypersurface is flat and since χ is an isocurvature field its fluctuations do not affect the local Hubble expansion. Furthermore, we assume that the transition into the single-field-dominated scaling solution at the time $t = t_T$, happens *instantaneously* on the hypersurface $\chi = \chi_T =$ const., where H_T is perturbed.

Under these assumptions, the expansion N defined by Eq. (6) can be split into

$$N = \int_{t_i}^{t_T} H dt + \int_{t_T}^{t_f} H dt\,, \tag{12}$$

where t_f is set sufficiently later than the transition time t_T. In Eq. (12), the first integral is over the multi-field scaling evolution and the last integral is over the ϕ_2-dominated phase.

The first term on the right hand side of Eq. (12) can be expressed as $-(1/\epsilon)\ln(H_i/H_T)$, where $\epsilon = c^2/2$, while the second term becomes $-(1/\epsilon_2)\ln(H_T/H_f)$, where $\epsilon_2 = c_2^2/2$. Then, for a fixed t_i and t_f, the expansion N can be expressed as

$$N = \frac{2}{c_1^2}\ln|H_T| + \text{const.}, \tag{13}$$

which depends only on the parameter c_1, besides the transition time t_T.

During the multi-field scaling regime, the linear evolution equation of χ on large scales is given by

$$\ddot\chi + 3H\dot\chi + m_\chi^2\chi = 0\,. \tag{14}$$

Including the cubic self-interaction $V^{(3)}$ given in Eq. (4), the large scale evolution equation for χ in the multi-field scaling regime becomes

$$\ddot\chi + 3H\dot\chi + m_\chi^2\chi = -\frac{1}{2}\tilde c m_\chi^2\chi^2. \tag{15}$$

The above evolution equation can be solved perturbatively. Given the solution to the linear equation (14), i.e., $\chi_L \propto H$, the growing-mode solution for χ is

$$\chi = \chi_L + \frac{1}{4}\tilde c\chi_L^2 = \alpha H + \frac{1}{4}\tilde c\alpha^2 H^2\,, \tag{16}$$

where α is a constant parameter whose value distinguishes the different trajectories and shown to be close to Gaussian. Then, the simplest way to compute f_{NL} is to calculate the δN corresponding to the fluctuation $\delta\alpha$, i.e.,

$$\delta N = N_{,\alpha}\delta\alpha + \frac{1}{2}N_{,\alpha\alpha}(\delta\alpha)^2\,. \tag{17}$$

In order to compute $N_{,\alpha}$ and $N_{,\alpha\alpha}$ we want to use Eq. (13), and for this we need to know how H_T varies as a function of α at the transition from multi-field scaling to single-field ϕ_2-dominated scaling solution. Inverting Eq. (16) (to leading order in $\tilde{c}\chi$) gives

$$\alpha = \frac{\chi}{H}\left(1 - \frac{1}{4}\tilde{c}\chi\right) . \tag{18}$$

Assuming as in the linear case that the transition corresponds to a critical value of the tachyon field $\chi = \chi_T$, on the transition surface (constant χ_T) we have from (18) that $\alpha \propto H_T^{-1}$ and hence we find

$$\delta N = -\frac{2}{c_1^2}\frac{\delta\alpha}{\alpha} + \frac{1}{c_1^2}\left(\frac{\delta\alpha}{\alpha}\right)^2 , \tag{19}$$

which means

$$N_{,\alpha} = -\frac{2}{c_1^2}\frac{1}{\alpha}, \qquad N_{,\alpha\alpha} = \frac{2}{c_1^2}\frac{1}{\alpha^2} . \tag{20}$$

Taking $\delta\alpha$ to be a Gaussian random variable and comparing with Eq. (11) with $\zeta_L = -2\delta\alpha/(c_1^2\alpha)$ we obtain the nonlinear parameter for the curvature perturbation after the transition:

$$f_{NL} = \frac{5}{6}\frac{N_{,\alpha\alpha}}{N_{,\alpha}^2} = \frac{5}{12}c_1^2 . \tag{21}$$

5 Conclusion

In this paper we have studied the nonlinear evolution of perturbations in the multi-field new ekpyrotic cosmology. We have studied the simplest model based on two fields with exponential potentials and considered the specific scenario in which the nearly scale-invariant comoving curvature perturbation is generated by the transition from the multi-field scaling solution to the single-field dominated attractor solution. We have applied the δN-formalism, which is widely adopted to study the non-linearity of the primordial curvature perturbation. We find that after the transition to the single-field attractor solution the non-Gaussian parameter $f_{NL} = 5c_1^2/12$, where $-V_1 \exp(-c_1\phi_1)$ is the potential of the field ϕ_1 which becomes subdominant at late time. Since the non-Gaussianity is mainly generated by the nonlinear super-Hubble evolution, it is of the local form, and the nonlinear parameter is k independent. Since c_1^2 must be large, in order to generate an almost scale invariant spectrum, the non-Gaussianity is inevitably large. Thus, the model is strongly constrained by observational bounds on the spectral index and non-Gaussianity.

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