Systematic study of two proton decay of ^{42}Cr, ^{47}Co, ^{48}Ni and ^{56}Ga nuclei within the two-potential approach

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Introduction

In two-proton radioactivity, the nucleus emits two protons simultaneously in nuclei near the proton drip line. The studies on two-proton radioactivity provide information about the nucleus's structure, stability, and nucleon interactions. The experimental detection of two-proton decay is challenging due to its rarity. Several experimental studies have confirmed a few cases of two-proton radioactivity. Two-proton radioactivity has been successfully explained using different theoretical models, such as the screened electrostatic barrier, Gamow-like model (GLM), diproton model, three-body model, simultaneous and sequential decay model, continuous shell model, generalised liquid drop model (GLDM) and effective liquid drop model (ELDM). Additionally, several empirical formulas have been proposed to accurately estimate the half-life of two-proton decay. Buck et al [1] introduced a cluster model approach using a parameterized cosh-type nuclear potential for studying decay phenomena. In the present work, we studied two-proton radioactivity of ^{42}Cr, ^{47}Co, ^{48}Ni and ^{56}Ga isotopes via the two-potential approach.

The model

In the two-potential approach, the total interaction potential is the sum of the Coulomb potential, centrifugal potential, and nuclear potential. Which can be expressed as

$$V(r) = V_C(r) + V_l(r) + V_N(r) \qquad (1)$$

The Coulomb potential is calculated by considering the nucleus as a uniformly charged sphere, and it can be expressed as.

$$V_C(r) = \frac{Z_{2p} Z_d e^2}{2R}\left[3 - \frac{r^2}{R^2}\right] \qquad \text{for } r \le R \qquad (2)$$

$$V_C(r) = \frac{Z_{2p} Z_d e^2}{2r} \qquad \text{for } r > R \qquad (3)$$

The atomic numbers Z_{2P} and Z_D correspond to the 2p fragment nucleus and daughter nucleus, respectively, and A is the mass number of the parent nucleus. The sharp radius R is calculated using the formula $R = r_0 A^{1/3}$. In the current work, r_0 is taken as 1.2fm. The Langer-modified centrifugal barrier is utilized for calculating the centrifugal potential, and it can be expressed as follows.

$$V_{(l)} = \frac{\hbar^2 (l+\frac{1}{2})^2}{2\,\mu\, r^2} \qquad (4)$$

where μ and l denote the reduced mass of the decaying nuclear system and the orbital angular momentum, respectively.

In the current study, we investigated two-proton radioactivity using a cosh-parameterized nuclear potential, as proposed by Buck et al[1].

$$V_N(r) = -\lambda V_0 \frac{1+\cosh(r/a)}{\cosh(r/a)+\cosh(R/a)} \qquad (5)$$

Where λ, V_0, a, and R are the renormalization factor, depth of nuclear potential, diffuseness parameters, and sharp radius, respectively. For each case of decay, the Bohr-Sommerfeld quantization condition is used to find the renormalization factor.

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}\left[Q_{2P} - V(r)\right]}\,dr = (G - l + 1)\frac{\pi}{2} \qquad (6)$$

In the current calculation for two-proton decay, where G = 2n+l represents the principal quantum number, values of G = 4 or 5 are chosen for l = 0, 2, and 4. Γ_{2p} is the width of two proton decay it can be obtain by WKB method

$$\Gamma_{2p} = S_{2p}\frac{\hbar^2 FP}{4\mu} \qquad (7)$$

Where S_{2p} is the formation probability of two proton radioactivity. The normalized factor (F) can be expressed as

$$F\int_{r_1}^{r_2} \frac{1}{2k(r)}\,dr = 1 \qquad (8)$$

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The penetration probability (P) is calculated using the WKB approximation.

$$P = \exp\{-2\int_{r_2}^{r_3} k(r)dr \quad (9)$$

The classical turning points. r_1, r_2 and r_3 are calculated using the conditions V (r) = Q_{2p}. The wave number of the emitted two protons can be calculated as

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}[Q - V(r)]} \quad (10)$$

The half-life of two-proton radioactivity is calculated by the following equation:

$$T_{1/2} = \frac{\hbar ln2}{\Gamma_{2p}} \quad (11)$$

Result and Discussions

In this study, we investigated the two-proton radioactivity of nuclei ^{42}Cr, ^{47}Co, ^{48}Ni, and ^{56}Ga using the Two-Potential Approach (TPA) along with a cosh-type nuclear potential. In TPA, the total interaction potential is the sum of the Coulomb potential, centrifugal potential, and nuclear potential.

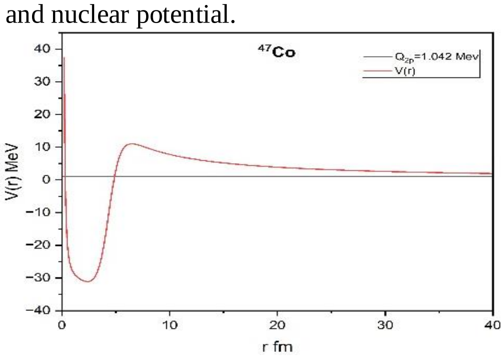

Fig. 1 The variation of the total potential energy of a ^{47}Co nucleus with separation radius (r)

We also investigated the potential energy characteristics of nucleus ^{47}Co by potting graph between potential energy with separation radius, as shown in Figure 1. From the figure, it is clear that the total potential coincided with the experimentally observed Q value of 1.042 MeV, which corresponds to the classical turning points and satisfies the condition $V(r)=Q_{2p}$. We computed the cosh-type nuclear potential using a depth parameter V_0 of 58.405MeV and a diffuseness value (a) of 0.537fm, which exhibited the lowest standard deviation with experimental half-lives. The formation probability plays an essential role in estimating the two-proton decay half-life. One method to measure the experimental formation probability (S_{2p}) is to compare the estimated two-proton radioactivity half-life to the observed experimental half-life. The experimental formation probability of two-proton decay is calculated as

$$S_{2P}^{exp} = S_0 T_{1/2}^{cal} / T_{1/2}^{exp} \quad (12)$$

We use the approximation S_0 =0.5[2] to compute the half-life of two-proton decay and use eqn.11 to find the corresponding experimental formation probability. In the absence of experimental half-lives for the selected nuclei, the half-lives are taken from Ref.[3] to compute the formation probabilities. The computed half-lives and formation probability values are given in Table 1. The half-life values obtained from the computations were compared with the previous predictions and is given in figure 2. From the figure, it is clear that predicted half-lives show good agreement with previous theoretical predictions.

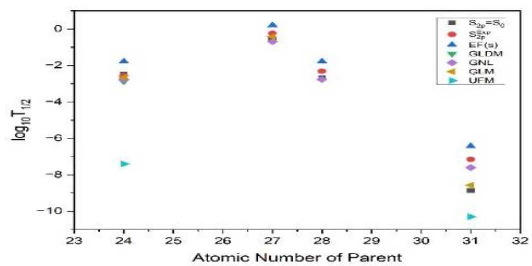

Fig. 2 Computed half-life is compared with the previous theoretical model predictions and also with two formula predictions.

Nuclei	Q_{2p}	l value	S_{2P}^{exp}	$\log_{10}T_{1/2}$ $S_{2p} = S_0$	$\log_{10}T_{1/2}$ S_{2P}^{exp}
^{42}Cr	1.002	0	0.614	-2.473	-2.562
^{47}Co	1.042	0	0.24	-0.555	-0.237
^{48}Ni	1.309	0	0.211	-2.695	-2.32
^{56}Ga	2.443	0	0.01	-8.861	-7.16

Table 1: Computed half-lives and preformation probabilities of ^{42}Cr,^{47}Co, ^{48}Ni, and ^{56}Ga nuclei.

References

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