

Constructing an Effective Neutrinoless Double-Beta Decay Operator in the Shell Model

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Abstract.

We are developing a formalism to generate effective neutrinoless double-beta decay ($0\nu\beta\beta$) operators within the shell model. We use lighter nuclei ($A = 8, 10$) as a laboratory. We start with the $A=6$ system where the wavefunctions are generated using the No-Core Shell Model (NCSM) with the CD-Bonn potential in a relatively large space. Thereafter, both the $0\nu\beta\beta$ operator and the two-nucleon interaction are renormalized to the p -shell using the Lee-Suzuki transformation scheme. These renormalized p -shell operator and interaction are then used in the $A = 8$ system and compared with the 'full' result (in the large space). Here we present preliminary results for the ${}^8\text{He}-{}^8\text{Be}$ transition. Also worth mentioning is the fact that we are simultaneously evaluating other renormalization schemes.

1. Introduction

Neutrinoless double-beta decay is a very slow lepton-number-violating nuclear process that occurs if neutrinos are Majorana particles. In this process, an initial nucleus (Z, A) decays to $(Z + 2, A)$ thereby emitting two electrons [1]. This process is of interest to physicists because apart from confirming whether neutrinos are Majorana or Dirac particles, it would also shed light on the absolute mass scale and hierarchy of neutrinos, and θ_{13} , to name a few.

The rate for such a process is given by

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q, Z) |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2, \quad (1)$$

where Q is the energy difference between the initial and final nuclei, Z is the charge of the initial nucleus, $G_{0\nu}(Q, Z)$ is a tabulated phase-space measure, $M_{0\nu}$ is the nuclear matrix element to which we turn shortly, and $m_{\beta\beta}$ is a linear combination of neutrino masses:

$$m_{\beta\beta} \equiv \left| \sum_{k=1}^3 m_k U_{ek}^2 \right|. \quad (2)$$

Here, m_k is the neutrino mass (these mass eigenstates are linear combinations of the electron-, mu- and tau-neutrinos) and U_{ek} is the element of the unitary mixing matrix that connects the k^{th}

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neutrino to the electron neutrino. The quantity of interest $m_{\beta\beta}$ is extracted from experiments. However, such an extraction relies on the nuclear matrix element $M_{0\nu}$, which must be calculated in some nuclear model and is a source of uncertainty.

The problem in calculating $M_{0\nu}$ comes from the complexity of dealing with nuclear structure itself. Most calculations of $M_{0\nu}$ are done either in the neutron-proton Quasiparticle Random Phase Approximation (QRPA) or in the shell model with results that do not quite agree. Both kinds of calculations use bare operators in evaluating the $0\nu\beta\beta$ matrix elements. Our aim is to renormalize the $0\nu\beta\beta$ -decay operator itself for implementation within the shell model.

With some approximations, one can write the nuclear matrix element $M_{0\nu}$ [2, 3] as

$$M_{0\nu} \approx M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^F \quad (3)$$

with g_V and g_A the vector and axial-vector coupling constants, and

$$M_{0\nu}^F = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \tau_a^+ \tau_b^+ | i \rangle, \quad (4)$$

$$M_{0\nu}^{GT} = \langle f | \sum_{a,b} H(r_{ab}, \bar{E}) \vec{\sigma}_a \cdot \vec{\sigma}_b \tau_a^+ \tau_b^+ | i \rangle. \quad (5)$$

Here $|i\rangle$ and $|f\rangle$ are the initial and final nuclear ground states, σ and τ have their usual meanings and a, b label nucleons. \bar{E} is an average excitation energy and r_{ab} is the inter-nucleon separation. (In the rest of the text we may refer to $M_{0\nu}^F$ and $M_{0\nu}^{GT}$ as the ‘Fermi’ or ‘Gamow-Teller’ matrix elements respectively.) H is given by

$$H(r, \bar{E}) = \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \bar{E} - (E_i + E_f)/2}. \quad (6)$$

The quantity R is the nuclear radius, inserted to make the matrix element dimensionless.

2. Present Formalism

Effective operators in shell model has a long history. The formalism [4] is based on the division of the many-body Hilbert space into a shell model space, P and the rest of the Hilbert space, Q . Then,

$$P = \sum_{i \in P} |i\rangle\langle i| \quad (7)$$

would define an operator that projects on to the P -space and similarly the operator,

$$Q = \sum_{\text{other } i} |i\rangle\langle i| \quad (8)$$

would project on to the Q -space. These operators have the properties:

$$P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0. \quad (9)$$

We employ the Lee-Suzuki (LS) transformation [5] to project an operator on to the P -space with the condition that the model space (P) and the excluded space (Q) are not coupled by the Hamiltonian. The transformed Hamiltonian is given by

$$\mathcal{H} = e^{-S} H e^S \quad (10)$$

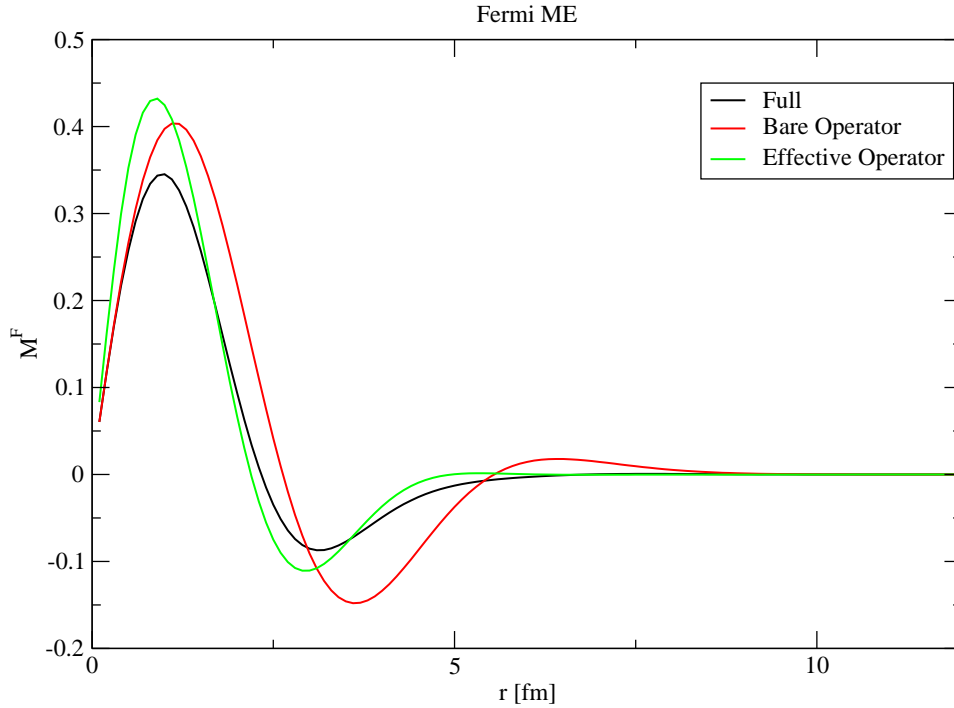


Figure 1. The Fermi matrix-element as a function of r . The black curve is the ‘full’ calculation in $N_{max} = 8$ space, the red curve is a p -shell shell model calculation with an effective interaction and the bare $0\nu\beta\beta$ -decay operator and the green curve is again a p -shell shell model calculation with an effective interaction but with an effective $0\nu\beta\beta$ -decay operator obtained from the $A = 6$ system.

where S is determined such that P and Q are decoupled ($P\mathcal{H}Q = 0$) and this ensure that the Hamiltonian is energy independent [6]. If S is determined such that even $Q\mathcal{H}P = 0$, then it can be shown that any effective operator

$$\mathcal{O} = e^{-S} H e^S \quad (11)$$

will also be energy independent. It is usual to write S in terms of ω where $S = \text{arctanh}(\omega - \omega^\dagger)$ and $Q\omega P = \omega$. Then the effective Hamiltonian and operator (any) can be written as–

$$H_{eff} = P\mathcal{H}P = \frac{P + Q\omega^\dagger P}{\sqrt{P + \omega^\dagger\omega}} H \frac{P + Q\omega P}{\sqrt{P + \omega^\dagger\omega}}. \quad (12)$$

$$O_{eff} = P\mathcal{O}P = \frac{P + Q\omega^\dagger P}{\sqrt{P + \omega^\dagger\omega}} O \frac{P + Q\omega P}{\sqrt{P + \omega^\dagger\omega}}. \quad (13)$$

ω is evaluated from

$$\langle \alpha_Q | \omega | \alpha_P \rangle = \sum_{k \in \mathcal{K}} \langle \alpha_Q | k \rangle \langle \tilde{k} | \alpha_P \rangle. \quad (14)$$

Here $|\alpha_P\rangle$ and $|\alpha_Q\rangle$ are the basis states in P and Q spaces respectively, $|k\rangle$ denotes states from a selected set \mathcal{K} of eigenvectors of the Hamiltonian in the full space and $\langle \alpha_P | \tilde{k} \rangle$ is the inverse of $\langle k | \alpha_P \rangle$.

Thus, once ω is determined it is fairly straightforward to generate any effective operator in the desired model space.

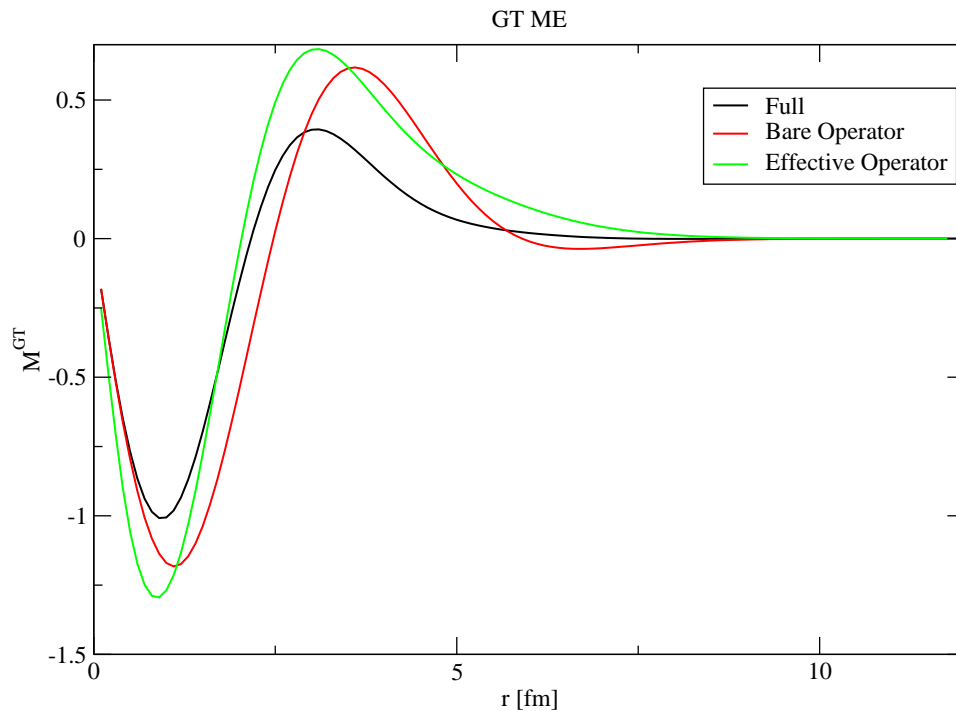


Figure 2. The Gamow-Teller matrix-element as a function of r . The legend is same as in the previous figure.

3. Preliminary Results

In this exploratory project we focus on p -shell nuclei as our laboratory to test effective $0\nu\beta\beta$ operator. The steps involved in our calculation are–

- Perform relatively large-space ($8\hbar\omega$) NCSM calculations for ${}^5\text{Li}$ and ${}^5\text{He}$ (to obtain single particle energies) and ${}^6\text{Li}$ to obtain the effective Hamiltonian. From this we generate an effective two-body interaction.
- Do the same for ${}^6\text{He}$ to obtain an effective two-body $0\nu\beta\beta$ operator.
- Use these effective interaction and operator to calculate the processes ${}^{8,10}\text{He} \rightarrow {}^{8,10}\text{Be}$.

If this prescription works for these p -shell nuclei then it could be tested for heavier nuclei. We present preliminary results for the ${}^{8,10}\text{He} \rightarrow {}^{8,10}\text{Be}$ in Fig. 1 and 2. Fig. 1 shows the Fermi matrix element whereas Fig. 2 shows the Gamow-Teller matrix element. In both figures the black curve represents a full $8\hbar\omega$ calculation for ${}^{8,10}\text{He} \rightarrow {}^{8,10}\text{Be}$, the red curve represents a calculation with the ‘bare’ (p -shell) operator and the effective interaction and the green curve represents similar calculation but with the effective $0\nu\beta\beta$ operator obtained by the procedure described above. The effective operator qualitatively ‘fixes’ the position of the peaks and also seems to do better at large r . The excess at the peaks seems to be caused by the effective interaction used. For example, if projections of the actual ground state of ${}^8\text{He}$ and ${}^8\text{Be}$ are used instead of an eigenstate of an effective interaction then the curves for the effective operator lie much closer to the full result. However, to draw any definite conclusion further investigation is necessary and is ongoing.

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References

- [1] F. T. Avignone III, S. R. Elliott, and J. Engel, *Rev. Mod. Phys.* **80**, 481 (2008).
- [2] J. Menendez, A. Poves, E. Caurier, and F. Nowacki, *Nucl. Phys.* **A818**, 139 (2009).
- [3] F. Simkovic, A. Faessler, V. Rodin, P. Vogel, and J. Engel, *Phys. Rev.* **C77**, (2008).
- [4] P. Navratil, M. Thoresen, and B. R. Barrett, *Phys. Rev.* **C55**, R573 (1997); A. F. Lisetskiy, M. K. G. Kruse, B. R. Barrett, P. Navratil, I. Stetcu, and J. P. Vary, *Phys. Rev.* **C80**, 024315 (2009).
- [5] K. Suzuki, S. Y. Lee, *Prog. Theor. Phys.* **64**, 2091 (1980); K. Suzuki, *Prog. Theor. Phys.* **68**, 246 (1982).
- [6] S. Okubo, *Prog. Theor. Phys.* **12**, 603 (1954); P. Navratil, H. Geyer, T. T. S. Kuo, *Phys. Lett.* **B315**, 1a (1993).