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DCCT NOISE AND BEAM LIFETIME MEASUREMENT

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Abstract

Beam lifetime measurements are an important tool to characterize the key storage ring and machine performance parameters. They are usually derived from dc current transformer (DCCT) data, and their accuracy depends on DCCT noise and data duration period. However, accurate dc current and fast lifetime determination are in contradiction and have to be balanced carefully. In this contribution, a model is presented which relates the relative accuracy in lifetime determination and the DCCT noise with the acquisition time. For the PETRA IV project at DESY which aims to upgrade the present PETRA III synchrotron into an ultra low-emittance source, according to this model a lifetime determination to the level of 1 % should be possible within 5 to 6 s acquisition time.

INTRODUCTION

The PETRA IV project at DESY (Hamburg, Germany) aims at the construction of a diffraction limited ultra-low emittance light source operating at 6 GeV [1,2]. The storage ring will be built in the existing PETRA III tunnel, thus inheriting the original 8-fold symmetry of the former PETRA collider. The accelerator lattice is based on a modified hybrid six-bend achromat (H6BA) cell and, taking advantage of the 2.3 km circumference, it provides electron beams with 20 pm rad emittance. The machine will be operated in two different modes, a *timing mode* with 80 bunches and 1 mA/bunch and a *brightness mode* with a homogeneous fill pattern of 1920 bunches and about 0.1 mA/bunch (baseline parameters). The expected lifetimes are 5 h (dominated by Toschek scattering) resp. 10 h in both modes.

Beam lifetime is an important measure to characterize the key storage ring and machine performance parameters, therefore their precise knowledge is of utmost importance. It is derived from the measurement of a change of the beam intensity which can be done with any intensity monitor. Usually a DCCT is used for this purpose.

In order to provide the required user stability a precise dc current measurement is mandatory, i.e. the DCCT noise should be kept to a minimum. For the PETRA IV brightness mode with 200 mA and 1920 bunches and assuming 1 % top-up level, the minimum detectable change in the beam current while filling a single bunch should amount to 1 µA. To be on the safe side, this minimum current change should be a factor of two above the noise level which results in rms(I) ≤ 0.5 µA. Reducing the noise level below this limit is usually achieved by averaging which entails a bandwidth reduction. At the other hand, from machine physical aspects a fast lifetime determination is preferable which is in contradiction to the process of long-term averaging. Therefore, accurate dc current and fast lifetime determination are in contradiction and have to be balanced carefully.

In this contribution a simple formula is derived which takes into account the dependency of lifetime accuracy and monitor noise, thus enabling to balance both quantities. The procedure is applied to the PETRA IV parameter case, indicating that it will be possible to determine the lifetime to a level of 1 % within 5 to 6 s acquisition time. Finally, the possibility is discussed to use more than one intensity monitor in order to reduce the acquisition time for lifetime determination.

MODEL DESCRIPTION

The beam current reduction caused by multipe effects (elastic and inelastic gas scattering, quantum lifetime, Touschek effect ...) as function of time in a storage ring is usually described by an exponential $I(t) = I_0 \exp(-t/\tau)$ with τ the beam lifetime. The output signal of a beam intensity resp. current monitor is recorded at a fix sampling rate, i.e. $t = n\,T_s$ with T_s the sampling time and $I_n = I(n\,T_s)$. Under normal circumstances $n\,T_s \ll \tau$ such that the exponential can be expanded, resulting in

$$\Delta I_n = I_0 - I_n = I_0 \frac{T_s}{\tau}\, n \,. \qquad (1)$$

Eq. (1) represents a linear relation between a sample point n and the corresponding intensity loss ΔI_n with the lifetime information encoded in its slope. In order to derive a best guess for lifetime and lifetime uncertainty from a data set of N samples ($n = 0, \ldots, N-1$), this equation has to be analyzed which is conventionally done by means of a χ^2 minimization.

Parameter Estimation

Having N data samples (x_n, y_n) with indexing as before, for the case of a linear dependency $f(x_n) = m \cdot x_n + b$ the parameter estimation with corresponding error deviation can be performed analytically, see e.g. Ref. [3]:

$$\begin{aligned} \chi^2 &= \sum_{n=0}^{N-1} \frac{[y_n - f(x_n)]^2}{\sigma_n^2} \\ &= F - 2mE - 2bC + m^2 D + 2mbA + b^2 B \,, \end{aligned} \qquad (2)$$

with the abbrevations

$$\begin{aligned} A &:= \sum_{n=0}^{N-1} \frac{x_n}{\sigma_n^2} \qquad B := \sum_{n=0}^{N-1} \frac{1}{\sigma_n^2} \qquad C := \sum_{n=0}^{N-1} \frac{y_n}{\sigma_n^2} \\ D &:= \sum_{n=0}^{N-1} \frac{x_n^2}{\sigma_n^2} \qquad E := \sum_{n=0}^{N-1} \frac{x_n\, y_n}{\sigma_n^2} \qquad F := \sum_{n=0}^{N-1} \frac{y_n^2}{\sigma_n^2} \,. \end{aligned} \qquad (3)$$

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For the parameter estimation (m_0, b_0), Eq. (2) has to be minimized, defining a set of equations

$$\frac{\partial \chi^2}{\partial m} = -2E + 2m_0 D + 2b_0 A \stackrel{\text{def}}{=} 0$$
$$\frac{\partial \chi^2}{\partial b} = -2C + 2m_0 A + 2b_0 B \stackrel{\text{def}}{=} 0$$

which is solved for m_0 (the slope is solely considered because of the τ dependency). The result is

$$m_0 = \frac{EB - AC}{BD - A^2} \,. \quad (4)$$

Parameter Error Estimation

The parameter errors are extracted from the diagonal elements of the covariance matrix $\mathscr{C}$ which is the inverse of a matrix $\mathscr{A}$ containing the second derivatives of Eq. (2) with respect to the parameters c

$$\mathscr{C} = \begin{pmatrix} \sigma_1^2 & \dots & \mathrm{Cov}(c_1, c_r) \\ \vdots & \ddots & \vdots \\ \mathrm{Cov}(c_r, c_1) & \dots & \sigma_r^2 \end{pmatrix} = \mathscr{A}^{-1}$$

with σ_m^2 the variance of parameter c_m and

$$\mathscr{A}_{l,m} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial c_l \partial c_m}\bigg|_{c_{l0}, c_{m0}} ,$$

see Ref. [3]. Applying the formalism to the case of the linear dependency results in a covariance matrix of the form

$$\mathscr{A} = \frac{1}{BD - A^2} \begin{pmatrix} B & -A \\ -A & D \end{pmatrix} = \begin{pmatrix} \sigma_{m_0}^2 & \mathrm{Cov}_{b_0, m_0} \\ \mathrm{Cov}_{m_0, b_0} & \sigma_{b_0}^2 \end{pmatrix}$$

from which the expression of the error in m_0 can be extracted:

$$\sigma_{m_0} = \sqrt{\frac{B}{BD - A^2}} \,. \quad (5)$$

Data Sampling with Constant Rate

In the case under discussion with intensity data sampled by a single device at constant sampling frequency, Eqn. (4,5) can be simplified because the device error is constant ($\sigma_n = \sigma$) and the data sample is the sample index itself, i.e. $x_n = n$. Then it is possible to express the abbrevations in Eq. (3) which depend only on x_n, σ_n in a closed form:

$$A = \frac{1}{\sigma^2} \frac{N(N-1)}{2}, \qquad B = \frac{N}{\sigma^2} \qquad \text{and}$$
$$D = \frac{1}{\sigma^2} \frac{(N-1)N(2N-1)}{6} \,.$$

Inserting the abbrevations in Eqn. (4,5), slope estimate and its corresponding error are expressed as

$$m_0 = \frac{6}{(N-1)N(2N-1)} \sum_{n=0}^{N-1} y_n (2n - N + 1), \quad (6)$$
$$\sigma_{m_0} = 2\sigma \sqrt{\frac{3}{(N+1)N(N-1)}} \,. \quad (7)$$

Comparing the linear functional dependency with Eq. (1), i.e. $x_n = n$, $y_n = \Delta I_n$, and $m_0 = I_0 \frac{T_s}{\tau}$, the lifetime is given by

$$\tau = \frac{T_s}{6} \frac{(N+1)N(N-1)}{\sum_{n=0}^{N-1} \frac{I_0 - I_n}{I_0} (2n - N + 1)} \,. \quad (8)$$

Having in mind that the relation between lieftime and slope is given by $\tau = I_0 \frac{T_s}{m_0}$, the lifetime uncertainty can be derived from error propagation as

$$\sigma_\tau = I_0 \frac{T_s}{m_0^2} \sigma_{m_0} = \frac{\tau^2}{I_0 T_s} \sigma_{m_0} \,.$$

Inserting the slope error Eq. (7) in the equation above, the relative lifetime uncertainty is

$$\sigma_\tau / \tau = 2\sqrt{3} \frac{\tau}{T_s} \frac{\sigma}{I_0} \frac{1}{\sqrt{(N+1)N(N-1)}} \,,$$
$$\sigma_\tau / \tau \approx 2\sqrt{3} \frac{\tau}{T_s} \frac{\sigma}{I_0} \frac{1}{\sqrt{N^3}} \,. \quad (9)$$

Expressing the total intensity loss based on Eq. (1) in the whole acquisition period as $\Delta I_N = I_0 N \frac{T_s}{\tau}$, Eq. (9) can be rewritten as

$$\sigma_\tau / \tau = 2\sqrt{3} \frac{\sigma}{\Delta I_N} \frac{1}{\sqrt{N}},$$

which agrees with the expression for the lifetime uncertainty in Ref. [4]. However, the following dicussion of the acquisition time will be based on Eq. (9).

The acquisition time is determined by the number of samples and the sampling time, i.e. $t_{acq} = N T_s$. Expressing N by Eq. (9) results in the following equation for t_{acq}:

$$t_{acq} = \sqrt[3]{12\, T_s\, \tau^2 \left(\frac{\sigma}{I_0} \frac{1}{\sigma_\tau / \tau} \right)^2} \,. \quad (10)$$

As can be seen, for a desired relative lifetime uncertainty σ_τ / τ the acquisition time depends on machine parameters (current I_0 and lifetime τ) and parameters which can be optimized (monitor noise σ and sampling time T_s). As longer the lifetime, as longer the required t_{acq} because it takes longer to measure a significant change in the beam current with a given monitor resolution.

As an example, t_{acq} is calclated for the machine parameters in Ref. [4]. With a required accuracy of $\sigma_\tau / \tau = 1\,\%$, $I_0 = 100\,\mathrm{mA}$, $\sigma = 1\,\mu\mathrm{A}$, $\tau = 50\,\mathrm{h}$, and $T_s = 1\,\mathrm{s}$, the acquisition time amounts to 73 s which is in agreement with the value stated in Ref. [4].

APPLICATION TO PETRA IV

As mentioned above, the acquisition time can be optimized by balancing the parameters σ and T_s of the intensity monitor. In the following this will be investigated for the brightness mode operation of PETRA IV because of its longer expected lifetime of 10 h. As discussed in the introduction, the DCCT should have a noise level of rms(I) = $\sigma \leq$

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0.5 µA to be capable to detect the minimum expected beam current change of 1 µA, assuming 1 % top-up level.

For this purpose, a new commercial New Parametric Current Transformer (NPCT) from the company *Bergoz* which is specified as Very High Resolution model was recently installed in the existing PETRA III ring at DESY and compared to the about 35 years old existing Parametric Current Transformer (PCT). Readout of the monitor data was performed with a 7.5 digit Digital Volt Meter (DVM, *Keithley DMM 7510*) whose noise contribution was tested to be significantly smaller than the DCCT noise such that σ is dominated by the current monitor and not by the DVM.

A series of measurements was performed without beam at PETRA III in order to compare the noise level performance of both monitors. Instead of quoting directly the sampling time T_s in seconds, the measurements were performed as function of the Number of Power Line Cycles (NPLC). This is convenient because the accuracy of a DC voltage measurement (as done with the DVM) is typically reduced by power line induced AC noise, thus using an NPLC of 1 or greater increases the AC noise integration time and by this the measurement resolution and accuracy. Having in mind that the nominal power line frequency (according to the European standard) is 50 Hz, 1 NPLC corresponds to $T_s = 20$ ms.

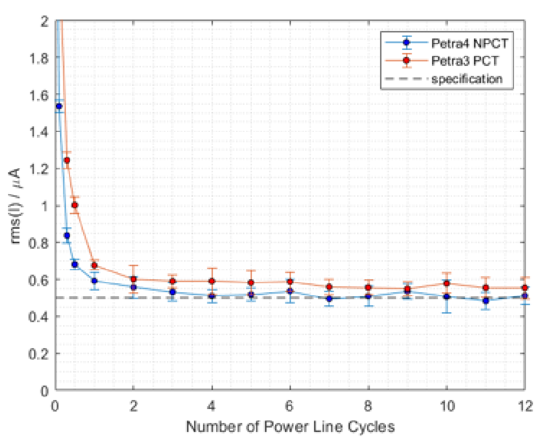

Figure 1: DCCT monitor noise (rms(I) or σ) as function of sampling time expressed as NPLC. Blue line: NPCT for PETRA IV, red line: standard PCT for PETRA III. In addition the requested specification level is indicated as dashed line.

Figure 1 shows the measured monitor resolution as function of the sampling time expressed as NPLC. As can be seen, small NPLCs result in a steep increase of the noise level as expected due to reduced averaging and suppression of 50 Hz and its harmonics. The fact that the noise reaches a constant level with increasing NPLC instead of following a $(\mathrm{NPLC})^{-1/2}$ scaling as expected for pure Gaussian noise raises suspicions that a low-frequency interfering signal contributed to the measurement. As can be seen, the noise performance of the NPCT for PETRA IV is slightly better than the one of the old PETRA III PCT, but the improvement level is marginal. Nevertheless the NPCT reaches the noise

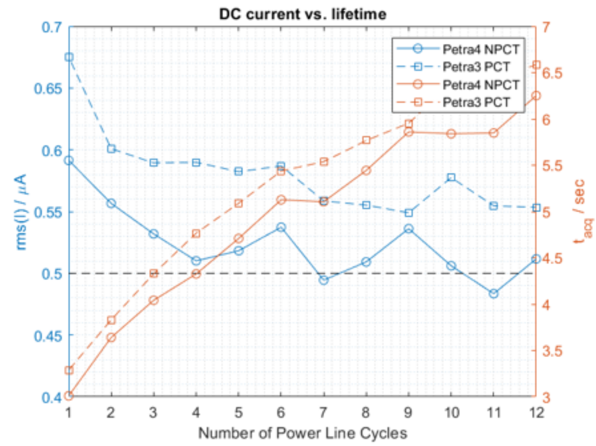

Figure 2: Acquisition time t_{acq} and monitor resolution as function of sampling time expressed as NPLC. Orange lines indicate the t_{acq} dependency (right ordinate), blue ones the rms(I) dependency (left ordinate). Solid lines correspond to the PETRA IV NPCT, dashed ones to the PETRA III PCT.

specification while the PCT noise level is always above the required limit.

In the next step the corresponding acquistion times are calculated based on Eq. (10), assuming the parameters of brightness mode operation, i.e. $I_0 = 200$ mA and $\tau = 10$ h, and that the requested relative livetime accuracy is $\sigma_\tau/\tau = 1$ % as before. The monitor noise level values σ are taken from the measurements depicted in Fig. 1.

The results for t_{acq} are plotted in Fig. 2 for both monitors together with the monitor noise levels of Fig. 1 in a limited range of interest. As can be seen, for a reasonable monitor resolution (5 - 10 NPLCs) the acquistion time is in the order of 5 to 6 s which seems to be sufficient fast values for machine studies.

MULTIPLE MONITORS

In Ref. [5] it was proposed to use multiple intensity monitors in order to further reduce the acquistion time resp. to increase the accuracy in lifetime determintion for fix t_{acq}. For this purpose an intensity signal can be generated by summing up the four pickup signals of each beam position monitor (BPM). The number of BPMs in a storage ring is typically large (> 50 and more), therefore the gain in accuracy resp. the reduction in t_{acq} could be substantial.

This case is considered here based on the formalism derived in the previous sections. Starting point is Eq. (9) which is slightly modified:

$$\sigma_\tau = 2\sqrt{3}\,\frac{\tau^2}{T_s}\,\frac{\sigma}{I_0}\,\frac{1}{\sqrt{N^3}}\;. \tag{11}$$

Considering the case of M identical monitors, each with the same error σ, the total error is given by $\bar{\sigma}_\tau = \frac{\sigma_\tau}{\sqrt{M}}$ resp.

$$\bar{\sigma}_\tau = 2\sqrt{3}\,\frac{\tau^2}{T_s}\,\frac{\sigma}{I_0}\,\frac{1}{\sqrt{M N^3}}\;. \tag{12}$$

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Re-arranging Eq. (12) as it was done for Eq. (10) results in the equation for the acquistion time for M identical monitors

$$t_{acq} = \sqrt[3]{\frac{12}{M}\,T_s\,\tau^2\left(\frac{\sigma}{I_0}\frac{1}{\bar{\sigma}_\tau/\tau}\right)^2}\;. \qquad (13)$$

As can be seen from the comparison of Eq. (10) and Eq. (13), the acquistion time reduction scales with the factor $M^{1/3}$.

For better illustration, the previous example of PETRA IV brightness mode operation is considered again under the assumption that the BPM sum signal (SUM, total number of BPMs about $M = 800$) will be used for lifetime determination. The sum signals are extracted from the slow acquisition data path with 0.1 s sampling time of the *Libera* BPM system described in Ref. [6]. The ratio σ/I_0 was estimated based on an existing *Libera Brilliance* module in the PETRA III ring using the maximum SUM signal and the SUM signal variation as $\sigma_{SUM}/SUM_{max} \approx 2.23 \times 10^{-5}$. For these parameters the acquisition time is calculated to be $t_{acq} = 2.1$ s which is about a factor of 3 shorter than for lifetime determination based on the single NPCT. Probably this factor might even be higher because the noise level in the about 15 years old *Libera Brilliances* operated at PETRA III is larger than in the newest generation of BPM modules described in Ref. [6], however a drastic improvement in the acquisition time reduction is not to be expected.

SUMMARY AND CONCLUSION

In this paper, a model is described to calculate beam lifetime and its corresponding uncertainty based on a χ^2 minimization method of beam intensity monitor data. Both parameters can be described by simple expressions, see Eqs. (8) and (9), which were derived under the assumption of a linear approximation of the exponential beam intensity decay. Strictly speaking this linearization is not necessary, an exact expression can be derived by taking the natural logarithm of the beam intensity decay resulting in

$$\underbrace{\ln\left(\frac{I_0}{I_n}\right)}_{y_n} = \underbrace{\frac{T_s}{\tau}}_{m_0}\,\underbrace{n}_{x_n}\;.$$

Applying the same formalism as described before results in a slightly modified expression for the beam lifetime

$$\tau = \frac{T_s}{6}\,\frac{(N+1)N(N-1)}{\sum_{n=0}^{N-1}\ln\left(\frac{I_0}{I_n}\right)(2n-N+1)}\;. \qquad (14)$$

Based on the equation for the lifetime uncertainty an expression for the acquisition time was derived and analyzed in view of the future PETRA IV parameters. It was demonstrated that it should be possible to measure the beam lifetime with 1 % accuracy and a single NPCT within 5 to 6 s.

The model was extended to the case of M identical intensity monitors for lifetime determination. It was shown that the acquistion time reduction will scale with a factor of $M^{1/3}$ for multiple monitors. Such a case was proposed and realized in Ref. [5] by using the BPM sum signal as intensity signal because the number of BPMs in a storage ring is typically quite large. However, this acquisition time reduction is moderate because of the $M^{1/3}$-scaling, a reduction by a factor of 10 would require 1000 identical monitors. Lifetime determination was implemented already in *Libera Brilliance* modules, see Ref. [7], but the formula for lifetime calculation stated in this reference slightly differs from the ones derived in this paper, see Eq. (8) resp. Eq. (14).

Another option to reduce the acquisition time might be to split the monitor tasks of providing beam current and lifetime data at the same time. The minimum reasonable sampling time is 1 NPLC in order to suppress the power line frequency and its harmonics. For this sampling time, t_{acq} would amount to 3 s according to Fig. (2) with the drawback that rms(I) is out of specifications. At the other hand, a beam current update is not necessarily required with this update rate such that averaging of beam current data could be performed over a longer time interval e.g. via the control system, such that the required accuracy for current measurements will be achieved.

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