P-WAVE MESON EMITTING DECAYS OF HEAVY FLAVOR HADRONS

A

THESIS

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IN

PHYSICS

By

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CERTIFICATE

It is certified that Ms. Neelesh Sharma has worked on the thesis entitled "**P-WAVE MESON EMITTING DECAYS OF HEAVY FLAVOR HADRONS**", to be submitted to the Punjabi University, Patiala, for the award of the degree of Doctor of Philosophy in Physics in the Faculty of Physical Sciences, under my supervision. No part of this work has been used by the candidate or anyone else for the award of any degree of this or any other university so far. The work presented in the thesis is original work of the candidate and is worth consideration for the award of the above said degree.

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DECLARATION

I hereby affirm that the work presented in this thesis is exclusively my own and there are no collaborators. It does not contain any work for which a degree/diploma has been awarded by any other University/Institution. A part of this work has already been published.

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LIST OF PUBLICATIONS

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ABSTARCT

At present, there exist rich amount of experimental data for the weak decays of heavy flavor hadrons. Weak leptonic and semileptonic decays are reasonably well understood in the Standard Model of fundamental interactions, however, weak hadronic decays have posed serious problems for the model as these decays experience strong interaction interference due to the gluons exchange among the quarks involved. In this thesis, two-body weak hadronic decays of heavy flavor mesons have investigated in the framework of the Standard Model. It has been found experimentally that two-body decays dominate the decay spectrum. Theoretical focus has, so far, been on the *s*-wave meson (i.e. pseudoscalar (P) and vector mesons (V)) emitting weak decays. However, charm and bottom mesons, being heavy, can also emit *p*-wave mesons, i.e. axial-vector (*A*), tensor (*T*) and scalar (*S*) mesons. Naively, the *p*-wave mesons emitting decays of the hadrons are expected to be suppressed kinematically due to the large mass of these meson resonances. However, now reasonable amount of experimental data has become available for branching ratios of the *p*-wave emitting decays of heavy flavour mesons which are found to be quite large, and require theoretical understanding. In this thesis, such weak decays of bottom mesons (B^-, $\overline{B}^0$ and B_s), which are the bound state of bottom quark and a light anti- quark, and of a unique bottom-charm (B_c) meson made up of heavy quarks only, have been investigated using the improved ISGW –II (Isgur, Scora, Grinstein and Wise) quark model. It has been the first model to calculate the form factors for *s*-wave meson to *p*-wave meson transitions using the constituent quark picture. Firstly, form factors for *s*-wave meson to *p*-wave meson transitions have been determined in this thesis using this model. Finally, branching ratios of weak hadronic decays involving $b \to c$ and $b \to u$ transitions are predicted that are found to be in good agreement with the available experimental data for bottom mesons (B^-, $\overline{B}^0$ and B_s). Since B_c meson is recently observed, and measurements for its weak decays are expected in future experiments, it is hoped that the predictions made in this thesis would help the experimentalists to identify the *p*-wave meson emitting decays of the heaviest bottom meson.

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CHAPTER 1

INTRODUCTION AND MOTIVATION

In the early sixties, Gell-Mann and Zweig independently put forth the idea of quark structure of the hadrons. They suggested that mesons and baryons are composites of three flavors of the quarks called up, down and strange (*u, d, s*) and their antipartners called antiquarks [1]. On the leptonic side, at that time four leptons, electron (*e*), muon (μ) and their respective neutrino partners (ν_e, ν_μ), had been observed. Inspired by the quark-lepton analogy, Bjorken and Glashow proposed the existence of the fourth flavor of quark named charm (*c*) in 1964[1]. Later, in 1970, mass of the charm quark was estimated through Glashow, Illiopoulos and Maiani (GIM) mechanism, which explained the observed suppression of certain processes, like $K^0 \to \mu^+\mu^-$. Discovery of the J/ψ $(c\bar{c})$ having mass 3.1 GeV in 1974, at SLAC and Brookhaven laboratory finally confirmed the existence of the charm quark [2], i.e. the first heavy flavor quark. Subsequently, evidence for even heavier quark called bottom quark (*b*) was obtained in 1977 with the discovery of another narrow resonance Υ $(b\bar{b})$ meson carrying mass 9.5 GeV. Around the same time, a heavy lepton namely tau (τ) was added to the list of the leptons. Again quark-lepton analogy suggested existence of the sixth quark called top quark (*t*) which eluded its discovery for

[1] *In 1965, Greenberg introduced the new property of the quark that is color charge and suggested that the hadrons are color neutral.*

some time. Finally in 1994, existence of the top quark with mass around 175 GeV has been established at Fermilab Tevatron collider [2, 3].

At the present energy scale, the fundamental constituents of the matter are pointlike quarks and leptons carrying spin half. The six quarks are grouped in three generations as (*u*, *d*), (*c*, *s*), (*t*, *b*) similar to the six leptons (e, ν_e), (μ, ν_μ), (τ, ν_τ). On the basis of mass pattern, quarks are classified as the light (*u*, *d*, *s*) and heavy (*c*, *b*, *t*) flavors [4]. The heavy flavor hadrons contain at least one heavy flavor quark. It may be remarked here that quarks are not observed as free particles, experimentally baryons and mesons, the bound states of these quarks[2], are produced.

Study of the heavy flavor hadrons is a very rich source of information for the fundamental interactions. There are four types of the fundamental interactions; strong, electromagnetic, weak and gravitational, in terms of which we can understand, in principle, all the processes occurring in nature from the elementary particles to the extra-galactic level. At the present energy scale of high-energy accelerators, the gravitational interactions are not relevant in the study of hadrons. The electromagnetic interactions are mediated by photon (γ), the weak interactions are carried out by exchange of three intermediate bosons $(W^{\pm}, Z^0)$, and the strong interactions among the quarks are mediated by eight gluons (g). After the development of the well-tested quantum electrodynamics (QED), through the independent works of Feynman, Schwinger and Tomonaga by 1950, a major step in this direction was taken by Weinberg and Salam in 1967, who independently developed the unified electroweak quantum gauge field theory based on the $SU(2)_L \times U(1)$ symmetry originally suggested by Glashow in 1964.This theory predicted the existence of the three bosons mediating the weak interactions[3]. In 1973, $SU(3)$ based quantum field theory of the

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[2] *The top quark cannot form bound states because of its short life time.*
[3] *They also predicted an additional scalar boson called the Higgs Boson that has not yet been observed.*

strong interactions at the quark level was formulated by Politzer, Gross and Wilczek, which is similar in structure to the quantum QED. Since the strong interaction deals with color-charge, it is called quantum chromodynamics (QCD) [5], in which gluons act as massless quanta of the strong-interactions. In QCD, diminution of the the strong interaction charge occur at the short distances. So a perturbative theory could be successfully employed in the high energy domain, but at large distances $(\approx 1 fm)$ quarks are subjected to the confining forces, which have not yet been derived from the first principles. Finally, all these theoretical efforts culminated in the development of the 'Standard Model' (SM) of the strong and electroweak interactions among the quarks and leptons, which is based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ relativistic quantum gauge field theory [6].

Though the Standard Model [7] has achieved a remarkable success in understanding various phenomena involving the elementary particles, it does not yield the final picture. For instance, the model has many free parameters, like Cabibbo-Kobayashi-Maskawa (CKM) weak mixing angles, which are empirically determined from the weak hadronic decays. Study of properties and decays of the heavy flavor hadrons can provide useful information on these parameters and to investigate the strong interaction effects at low energies. An intense activity on theoretical and experimental studies of the decays of the heavy flavor hadrons have been going on for the last few decades. Soon after the discovery of $J/\psi(c\bar{c})$ meson, weakly decaying pseudoscalar charm mesons (D^0, D^+ and D_s^+) and their excited states were produced [4]. Data on their masses and decays have been collected at electron-positron collider and fixed target experiments. After the discovery of $\Upsilon(b\bar{b})$ state, naked bottom states (B^0, B^+ and B_s^+) came into observation, and their masses were observed in such experiments [4]. However, major progress for measurements of their decays could occur only in the last few decades. At present, there exist rich amount of experimental data for the weak decays of heavy flavor mesons particularly for low lying spin zero particles [4].

Weak quark and lepton currents in the Standard Model generate leptonic, semileptonic and hadronic weak decays. The lifetime of the hadrons, their exclusive leptonic and semileptonic decays are reasonably well understood in this model [8, 9]. However, theoretical description of the exclusive weak hadronic decays confronts serious problems as these decays experience strong interaction interference due to the gluons exchange among the quarks involved. Fortunately, the short time-scale of weak decays allows one to separate the possible corrections from the strong interactions into short and long distance parts [8]. The asymptotic freedom property of the QCD allows a perturbative calculation of the effects of hard-gluon exchange on the weak Hamiltonian. The short distance effects can be resummed in the QCD coefficients, and the effective weak Hamiltonian has been constructed [8]. However, evaluation of matrix elements of the weak Hamiltonian between initial and final hadron states is not straightforward, due to the nonperturbative nature of the confinement mechanism responsible for forming the hadrons out of the interacting quarks. [8, 9]. Due to the lack of exact dynamics of the long distance strong interactions, hadronization of the quarks is generally studied through phenomenological approaches [8-19] like quark models, QCD sum rules, heavy quark effective theory (HQET) and lattice QCD.

Experimental data for the weak hadronic decays of the charm and bottom mesons, show the dominance of two-body decay modes. Initially, one expected their weak decays to have less interference due to the strong interactions, their measurements have revealed the contrary. The present data on these decays have posed serious problems for theory, which have led to several theoretical efforts [8-19] incorporating new ideas. At present, all over the world, several groups [20-22] at Fermilab, Cornell, CERN, DESY, KEK and

Beijing Electron Collider etc. are working to ensure wide knowledge of the heavy flavor physics. Thus, in the near future a large quantity of new and more accurate data on decays of the heavy flavor hadrons, including B_c, J/ψ and Υ, can be expected which calls for their comprehensive theoretical analysis. One of the goals of heavy flavor hadron physics is to elucidate the relationship among the particles of different generations. The b quark is specially interesting in this respect as it has W-mediated transitions to both first generation (u) and second generation (c) quarks. Therefore, in this thesis, we have investigated the two-body weak hadronic decays of heavy flavor mesons in the framework of standard model.

In chapter 2, we lay down the physical and mathematical preliminaries which have been applied for the study of weak decays of mesons emitting the s-wave mesons, pseudoscalar (P) and vector (V) mesons. To start with, we present the hadron spectroscopy upto the bottom level and classification of the weak decays into leptonic, semileptonic and nonleptonic decays. In general, these weak decays proceed through exchange of virtual W-boson between the charged weak (V-A) currents. Since leptons do not participate in the strong interactions, leptonic decays remain unaffected by the strong interaction effects and thus are well understood in the standard model [23]. We discuss the semileptonic decays of the bottom (B) mesons as they provide information about binding of the quarks. Since these decays proceed via spectator quark diagrams, their decay amplitudes can easily expressed in terms of the matrix elements of the hadronic weak currents between the parent and daughter meson states, which are usually calculated from the phenomenological models [8, 10, 11]. This forms the basis of the *'factorization approach'*, later applied to the weak nonleptonic decays. Theoretically, the two-body nonleptonic decays occur through several quark level processes, like W-emission (spectator diagram), W-exchange, W-annihilation and penguin diagrams. Out of these, W-emission diagrams are found to be dominant, as the

W-exchange and *W*-annihilation processes are helicity and color suppressed at the tree level. Weak decay amplitudes arising through the spectator diagrams can be expressed in terms of products of appropriate meson decay constants and the same form factors that are required for the semileptonic decays. We use the $B \to P$ form factors obtained in the Bauer, Stech and Wirbel (BSW) quark model framework [8]. Majority of these decay modes are seen to result in a large variety of *s*-wave mesons [18, 19, 24]. However, *B* mesons being heavy can also emit p-wave mesons like axial-vector (*A*), tensor (*T*) and scalar mesons (*S*) along with a pseudoscalar meson [25-27] which have attracted the attention of the experimentalist in the last few decades and the branching ratios of some of such decays have been measured. Therefore, we investigate *p-wave meson emitting decays of heavy flavor hadrons* decays in the following chapters.

In chapter 3, we extend factorization approach to study two-body hadronic weak decays of bottom emitting pseudoscalar and axial-vector mesons, i.e. $B / \overline{B}^0 / B_s \to PA / PA'$. After describing the spectroscopy of the two kinds of axial-vector mesons, i.e. $A\,(J^{PC} = 1^{++})$ and $A'(J^{PC} = 1^{+-})$, we proceed to obtain the weak decay amplitudes in the Standard Model framework. Similar to the *s*-wave mesons emitting decays, here also two kinds of the spectator diagrams, color-favored and color-suppressed diagrams, can contribute to $B \to PA / PA'$ decays. Using the factorization scheme, decay amplitudes are expressed in terms of the meson to meson form factors and meson decay constants. Though the meson decay constants are now reasonably known, the form factors are not properly understood. Isgur, Scora, Grinstein and Wise (ISGW I) model has been the first to calculate the form factors for *s*-wave meson to *p*-wave meson transitions [10] needed for $B \to PA / PA'$ decays [25]. However, the form factors evaluated in this model are reliable only at the maximum momentum transfer, whereas the weak hadronic decays require them at relatively lower momentum transfer. This model has now been improved, called as ISGW II model [10], in which the form factors provide a more realistic behavior. Therefore, we adopt this model for our purpose and calculate the $B \to A / A'$ transition form factors in the ISGW II model [10]. Consequently, we predict branching ratios of $B \to PA$ decays involving $b \to c$ and $b \to u$ transitions in the CKM-favored and CKM-suppressed modes. Experimentally [4], at present, branching ratios of eleven decays have been measured and upper limits are also available for five other decays. We compare our theoretical predictions with the available experimental measurements and also with other theoretical works.

In chapter 4, we have studied hadronic weak decays of bottom mesons emitting pseudoscalar and tensor mesons [26]. We first calculate the decay amplitudes in terms of the form factors and appropriate meson decay constants. Decay constants of tensor mesons vanish due to the tracelessness of the polarization tensor of spin 2 meson and its auxiliary condition. Therefore, either color-favored diagram or color-suppressed diagram can contribute to these decays and thus analysis of these decays becomes free of the interference between these diagrams. Here also, we employ ISGW II model [10] to determine the $B \to T$ transition form factors appearing in the decay matrix elements of weak currents involving $b \to c$ and $b \to u$ transitions. Consequently, we predict the branching ratios of $B \to PT$ decays in the CKM-favored and CKM-suppressed modes. Experimentally [4], branching ratios of only six decay modes have been measured and upper limits are available for five other decays. We compare the predicted branching ratios with the experimental results and with other theoretical values.

In chapter 5, we have studied hadronic weak decays of bottom mesons emitting pseudoscalar and scalar involving $b \to c$ and $b \to u$ transitions. We extend our model by employing the ISGW II model to determine the form factors appearing in the decay matrix element of weak currents for $B \to S$ transition. Consequently, we calculate the decay amplitude and predict branching ratios in the CKM-favored and CKM-suppressed modes. Though, for these decays both kinds of the spectator diagrams, color-favored and color-suppressed diagrams, can contribute, usually one of these gets suppressed due to the small values of the scalar meson decay constants. Experimentally not much data exist for $B \to PS$ decays, only three measured branching ratios decays are available [4]. We compare our results with other theoretical calculations.

In chapter 6, we study hadronic weak decays of uniquely observed bottom-charm (B_c) meson. In 1998, B_c meson, a unique state, composed of the two heavy quarks, bottom and charm, has been observed by the CDF collaboration [28]. Later, it announced an accurate determination of the B_c meson mass, m_{B_c} = (6.2857 ± 0.0053 ± 0.0012) GeV and its life time $\tau_{Bc} = 0.45^{+0.12}_{-0.10} \pm 0.12\,\text{ps}$ [29] in conformity with theoretical predictions. A peculiarity of the B_c decays, with respect to the decays of B and B_s mesons, is that both the quarks (b and $\bar{c}$) may decay weakly, thereby generating bottom changing and bottom conserving decay modes, respectively. The investigation of the B_c meson is of special interest as unlike its diagonal heavy quarkonium $(\bar{b}b,\ \bar{c}c)$ partners it decays only through weak interactions. Study of B_c^+ meson is becoming one of the most interesting topics of research in high-energy physics (HEP) both on experimental and theoretical side. Already there exists an extensive literature for the semileptonic and nonleptonic decays of B_c emitting *s*-wave mesons, pseudoscalar and vector mesons. However, relatively less work has been done on the *p*-wave meson emitting weak decays of B_c meson. Therefore, we extend our analysis to B_c meson decays emitting a pseudoscalar meson and a *p*-wave meson ($B_c \to PA / PT / PS$) [30]. In case of B_c meson decays, one naively expects the bottom conserving (and charm changing) decay modes to be kinematically suppressed in comparison to the bottom changing mode [30]. On the contrary, we find that the bottom conserving decays have branching ratios larger than that of the bottom changing modes due to the significant difference in the corresponding CKM factors.

Summary and conclusions of the work done are given in the last chapter.

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CHAPTER 2

PRELIMINARIES AND GENERAL FORMALISM

In this chapter, we give the mathematical and physical preliminaries that will be relevant to our work. We first present the basic ingredient of the Standard model which has been established to describe the interaction of fundamental particles. We then develop the weak Hamiltonian responsible for flavor changing weak decays and introduce QCD modifications at different levels. After giving the main features of the spectroscopy for mesons carry charm and bottom quantum number, we introduce the basic techniques for computation of the weak decay rates involving spectator and non spectator processes. The possible strong interaction effects which can modify the naïve weak processes are dealt with in the following sections.

2.1 MATTER CONTENT

There are 12 elementary particles of spin ½ known as the fermions [1], comprising of six leptons and six quarks, represented by spinor fields. The leptons which occur in pairs l and ν_l are divided into three families or generations as,

$$\begin{pmatrix} e^- \\ \nu_{e^-} \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_{\mu^-} \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_{\tau^-} \end{pmatrix} \qquad \textbf{Equation Chapter 2 Section 2} \qquad (2.1)$$

and undergo only electroweak transitions.

The six quarks calssified analogously are

$$\begin{pmatrix} u \\ b \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \tag{2.2}$$

The defining property of quarks is that they carry color charge and hence can also interact via strong interactions. Various properties of leptons and quarks are given in the Appendix A.

2.2 FORCE QUANTA

Interactions in physics are the ways that particles influence each other via fields. The Standard Model explains such forces in terms of force mediating spin 1 particles, bosons, associated with the gauge invariance groups, which are responsible for the exsisting strong, electromagnetic and weak interactions. Typical examples of these are shown below in Figures 2.1 (a), 2.1 (b) and 2.1 (c).

i) For the electromagnetic interactions: $\gamma-$photon

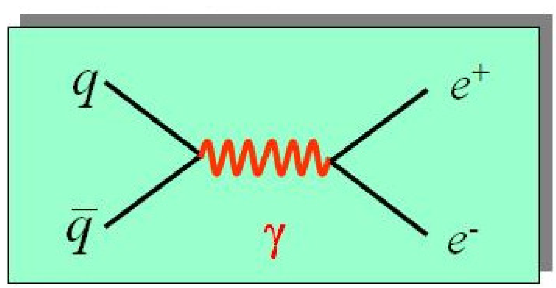


Figure 2.1(a) Electromagnetic Interactions

ii) For the weak interactions: $W^{\pm}$, Z^{0} – gauge bosons

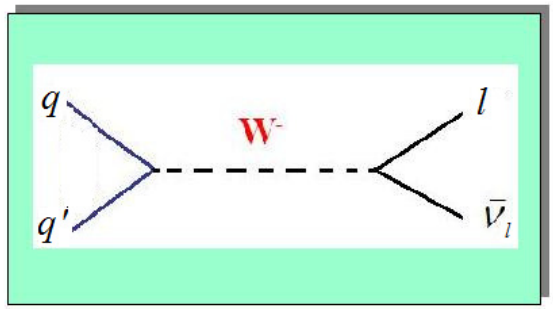


Figure 2.1(b) Weak Interactions

iii) For the strong interactions: 8-glouns

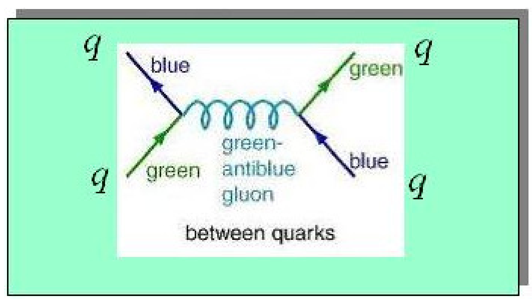


Figure 2.1(c) Strong Interactions

Last but not least, an essential ingredient of the Standard Model, a particle called the Higgs boson, has yet to be found in an experiment. The race is on to hunt for the Higgs- the key to the origin of particle mass. Finding it would be a big step for particle physics, although its discovery would not write the final ending to the story.

2.3 PARTICLE INTERACTIONS

2.3.1 ELECTROMAGNETIC INTERACTIONS

Quantum Electrodynamics (QED) is the relativistic quantum field theory of electromagnetic (*em*) interactions involving electrons and photons. It gives a spectacularly accurate description of the electron's properties in terms of only two parameters, the electron mass, m_e, and the fine structure constant, α. The electromagnetic interactions, arising out of the coupling of the photon γ with the charged particles, are represented by the simplest group, $U(1)_{em}$. The coupling of the electromagnetic quark and lepton current to the massless photon is given by

$$L_l = -ej_\mu^{em} A^\mu, \tag{2.3}$$

with

$$j_\mu^{em} = \sum_{i=u,d,s,c,b,t} Q_i \overline{q}_i \gamma_\mu q_i + \sum_{i=e^-,\mu^-,\tau^-} Q_i l \gamma_\mu l, \tag{2.4}$$

A^μ is the vector field and Q_i is the charge of the fermions in units of e.

First order electromagnetic interactions of fermions describing the emission or absorption of γ with a strength e. two such processes combine together to give general electromagnetic phenomena of second order, like the lepton-lepton and lepton-quark scattering, occurring with the strength given by the fine structure constant,

$$\alpha = \frac{e^2}{4\pi}. \tag{2.5}$$

2.3.2 WEAK INTERACTIONS

Fundamental weak interactions occur for all fundamental particles except gluons and photons. Weak interactions involve the exchange or production of *W* or *Z* bosons. The interaction responsible for all processes in which flavor changes and hence for the instability of heavy quarks and leptons and particles that contain them. Weak interactions that do not change flavor (or charge) have also been observed. In particle physics, the electroweak interaction is the unified description of two of the four known fundamental interactions of nature: electromagnetism and the weak interaction. The theory of electroweak interaction was developed around 1968 by Glashow, Salam and Weinberg (see *W* and *Z* bosons). The weak interaction lagrangian is

$$L_W = gJ_a^{\mu}W_{\mu}^{a} + g'J^{Y,\mu}B_{\mu}, \tag{2.6}$$

gives the coupling of weak currents to the vector bosons W_{μ}^{a} and B_{μ} with strengths g and g' corresponding to the $SU(2)_L \times U(1)_Y$ group. These W bosons physically occur as $W^{\pm}$, the linear combination of W_1 and W_2 and Z^0 which is a combination of W_3 and B_{μ} leaving the orthogonal combination for the photon field A_{μ},

$$W_{\mu}^{\pm} = \frac{W_{1\mu} \pm iW_{2\mu}}{\sqrt{2}} \ ,$$

$$Z_{\mu}^{0} = W_{3\mu}\cos\theta_W - B_{\mu}\sin\theta_W \text{ , and} \tag{2.7}$$

$$A_{\mu} = W_{3\mu}\sin\theta_W + B_{\mu}\cos\theta_W \ , \tag{2.8}$$

with θ_W is the Weinberg mixing angle satisfying

$$\cos\theta_W = \frac{M_W}{M_Z} = \frac{g}{(g^2 + g'^2)^{1/2}} \quad . \tag{2.9}$$

The electromagnetic coupling *e* is then simply $g'\cos\theta_W$.

Weak current J_{μ}^{a} consists of the weak charged currents $J_{\mu}^{\pm}$ coupling to the bosons and the weak neutral current J_{μ}^{nc} coupling to the Z^0 respectively. The weak neutral current is given by the left handed J_{μ}^{3} and the pure vector current

$$J_{\mu}^{nc} = J_{\mu}^{3} - \sin^2\theta_W J_{\mu}^{em} \tag{2.10}$$

is the charge of the fermions in units of *e* and

$$J_\mu^3 = \frac{1}{4}[(\bar{u}\,\bar{c}\,\bar{t})\gamma_\mu(1-\gamma_5)\begin{pmatrix} u \\ c \\ t \end{pmatrix} - (\bar{d}\,\bar{s}\,\bar{b})\gamma_\mu(1-\gamma_5)\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$(\bar{\nu}_e\,\bar{\nu}_\mu\,\bar{\nu}_\tau)\gamma_\mu(1-\gamma_5)\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} - (\bar{e}\,\bar{\mu}\,\bar{\tau})\gamma_\mu(1-\gamma_5)\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}]. \tag{2.11}$$

Infact, in the Standard model there exist no flavor changing neutral current. Experimental evidence to support this comes from the upper limit on generic decays like $B(D^+ \to \pi^+ e^+ e^-) < 7.4\times10^{-6}$.

Since the weak bosons are massive ≈ 80 GeV and M_q and $M_{lep} < M_W$ (except for the top or *t*-quark) there can be no possible first order weak processes observable at low energies. The virtual *W*-bosons lead to a variety of second order weak transitions through the numerous combinations. Thus, in general the weak Hamiltonian involves a transition matrix element of the type $\langle f | H_W | i \rangle$, where *f* and *i* denotes the final and initial states, respectively.

The weak charged *V-A* currents for the flavor changing interaction are

$$J_{\mu^-}^{w} = J_\mu + l_\mu ,$$

where

$$J_\mu = (\bar{u}, \bar{c}, \bar{t}\ )\gamma_\mu(1\text{-}\gamma_5)\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \tag{2.12}$$

$$l_\mu = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau\ )\gamma_\mu(1\text{-}\gamma_5)\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix}, \tag{2.13}$$

and $J_{\mu^+} = J_{\mu^-}^{\dagger}$.

In the current J_μ, the weak eigenstates d', s' and b' are related to the mass eigenstates d, s and b as follows:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (2.14)$$

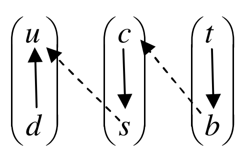
where

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}, \quad (2.15)$$

is a unitary matrix known as the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix[4] [2]. It can be parameterized in many representations due to the arbitrariness of phases in the quark fields. The most commonly used form in the literature [2] is,

$$V = \begin{bmatrix} c_1 & -s_1 c_3 & -s_1 c_3 \\ s_1 c_2 & c_1 c_2 c_3 - e^{i\delta} s_2 s_3 & c_1 c_2 s_3 + e^{i\delta} s_2 c_3 \\ s_1 s_2 & c_1 s_2 c_3 + e^{i\delta} c_2 s_3 & c_1 s_2 s_3 - e^{i\delta} c_2 c_3 \end{bmatrix}, \quad (2.16)$$

where $c_i = cos\theta_i, s_i = sin\theta_i, (i = 1,2,3)$ are the three Euler angles and δ is the phase factor generating the CP violation. The mixing is, in fact, responsible for the variety of weak decays of hadrons arising from transitions between generations. There appears to exist an hierarchy in the weak transitions between the different families. The heavier the neighboring families, the lesser they communicate, and families which are farthest, communicate the least. The dominant decay chain follows $t \to b \to c \to s \to u$.



The elements of the CKM matrix are the parameters of the Standard Model and are determined empirically [1] as,

$$V = \begin{bmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{bmatrix}. \quad (2.17)$$

[4] *The color structure is omitted here for the sake of simlicity.*

The errors given here arise due to the fact that these parameters denote the quark weak couplings, while the decay rates are measured at the hadron level.

Since the leptonic part is not strongly interacting, it can be calculated in straight-forward manner. However, the hadronic part cannot be treated so easily and has to be studied separately. This decoupling of the leptonic and quark currents into separate terms appears naturally within the semileptonic decays, and forms the basis for the so-called factorization approach. Replacing the leptonic current l_μ by the weak quark current J_μ, the weak Hamiltonian for nonleptonic decays is constructed, which would experience significant modification due to the strong interaction among the participation quarks. This will be discussed in detail in subsequent sections.

2.3.3 STRONG INTERACTIONS

Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons is $SU(3)_C$ component of the Standard Model [3]. It is developed in analogy with QED by generating the gauge invariant lagrangian for $U(1)$ to $SU(3)$. Corresponding to the electric charge in QED the quarks in QCD carry three possible color charges which are designated in the literature as red (R), blue (B) and green (G) and form an $SU(3)$ color triplet

$$\mathbf{3} = \begin{pmatrix} q_R \\ q_B \\ q_G \end{pmatrix}. \tag{2.18}$$

Leptons are color neutral and do not participate in the strong interactions. The color interactions are mediated by eight massless gluons, the gauge bosons corresponding to the eight generators of the unbroken $SU(3)_C$ and form a color octet.

The quark-gluon interaction lagrangian in QCD is

$$L_{\text{int}} = g_s \sum_i \sum_a \frac{1}{2} \overline{q}_i \lambda_a G_\mu^a \gamma^\mu q_i, \qquad (2.19)$$

where i is the flavor index, g_s is the dimensionless strong coupling constant, i.e. $\alpha_s = \frac{g_s}{4\pi}$, $\lambda_a (a = 1, 2, ..., 8)$ are the Gell-Mann traceless and hermitian color matrices and G_μ^a are the gluon fields.

Conversely, $\alpha_s(\mu^2)$ grows as grows μ^2 decreases, leading to a breakdown of the perturbation theory. The gauge coupling becomes strong at these large distances or small moments, with a probable power like potential $V(r) \approx r$ between quarks implying infrared slavery or confinement of quarks and gluon within the hadrons within the hadrons. Thus, $\mu^2 = \Lambda_{QCD}^2$ nonperturbative effects start dominating. At present, there is no completely satisfactory description of this long range force. Though the lattice gauge theories are expected to give some clues, these are still in their nascence and dependent on the computational limitations. Lack of knowledge of the strong interactions at low energies, limits the evaluation of the hadron dynamics from the first principles, and one often has to rely on phenomenological models to understand them. We hope that the study of the weak decays of hadrons will provide some information on these long distance QCD effects.

2.4 STANDARD MODEL

The standard model is a theory concerning the electromagnetic, weak and strong interactions, described in previous sections, which mediate the dynamics of known subatomic particles that includes $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups. The Standard Model developed from the Weinberg-Salam electroweak theory [4] and QCD of strong interaction [3]. It describes matter and its interactions in terms of a few building blocks;

quarks and leptons and the intermediate gauge bosons associated with the respective gauge groups [5].

Inspite of the remarkable successes of the Standard Model particularly for leptonic and semileptonic processes, it does not provide the final picture [6-9]. For instance, the model has many free parameters, like weak mixing angles. Study of weak decays, particularly in the heavy flavor sector, can provide useful information on these parameters, and to investigate the strong interaction effects at low energies. An intense activity on theoretical and experimental studies of the weak decays of the charm and bottom hadrons have been going on for the last three decades. Weak currents in the Standard Model generate leptonic, semileptonic and hadronic decays of the heavy flavor hadrons. Theoretical description of the exclusive weak hadronic decays based on Standard Model is not yet obtained as these experiences strong interaction interference. Since the quarks are confined inside the colorless hadrons, matching between theory and experiment requires an exact knowledge of the low energy strong interactions.

2.5 CONSTRUCTION OF THE EFFECTIVE FLAVOR CHANGING WEAK HAMILTONIAN

All the flavor changing weak decays involve the charged current $J_\mu^\pm$ and neglecting the q^2 dependence of the W propagator at low energies, as the momentum transfer is much smaller than the W mass, we can use the approximation

$$D_{\mu\upsilon}(M_W; x-y) \approx \frac{g_{\mu\upsilon}}{M_W^2}\delta(x-y),$$

shown in Figure 2.2. This reduces the weak Hamiltonian to the current $\otimes$ current form,

$$H_W = \frac{G_F}{\sqrt{2}}\int dx \frac{1}{2}\{J_\mu(x), J^{\dagger\mu}(x)\}, \tag{2.20}$$

where the weak current J_μ arises from group $SU(2)_L$ group and can have a vector or an axial-vector V, A character such that

$$J_\mu = V_\mu - A_\mu = \bar{q}_j \gamma_\mu (1-\gamma_5) q_i , \tag{2.21}$$

and the Fermi coupling constant is,

$$G_F = \frac{g^2}{4\sqrt{2} M_W^2} . \tag{2.22}$$

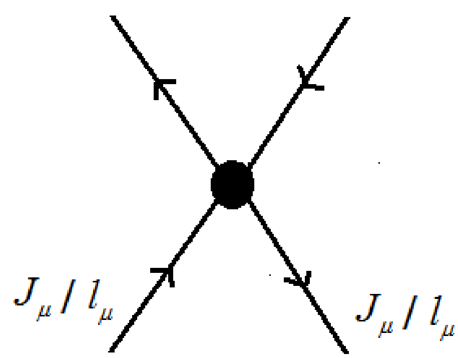


Figure 2.2 Current ⊗ Current form of weak interactions

2.6 CLASSIFICATION OF WEAK DECAYS

As quarks and leptons both can participate in the weak interactions (Figure 2.2), in general, weak decays are classified into three broad categories, pure leptonic, semileptonic and nonleptonic decays [10-14], which proceed through exchange of virtual[5] W-bosons between the weak currents.

a) Pure Leptonic Weak Decays

In leptonic weak processes, all the fermions involved in the weak interactions are leptons like, $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ (Figure 2.3). Since leptons do not participate in the strong interactions, such decays are uncomplicated by strong interaction effects and thus are well

[5] *The W-boson is generally virtual except in the case of decays of t quark, whose mass is greater than W.*

understood in the standard model [10-14]. In fact, μ-decay has been used to fix the weak coupling constant strength [8] $G_F = 1.166\times10^{-5}\,\text{GeV}^{-2}$ from the μ-lifetime, $\Gamma_\mu = \frac{h}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}$, obtained from the lowest order weak leptonic Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}}\{l^\mu, l_\mu^\dagger\}, \tag{2.23}$$

where $l_\mu = \sum_{l=e,\mu,\tau} \bar{l}\,\gamma_\mu(1-\gamma_5)\nu_l$ is the weak leptonic (V-A) current.

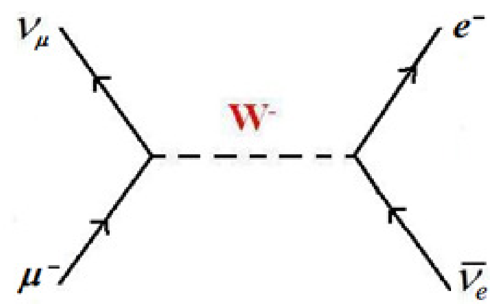


Figure 2.3 Lowest order of Feynman diagram for pure leptonic weak decay of muon

b) Semileptonic Weak Decays

Semileptonic weak decays can have either purely leptonic final states or both leptonic and hadronic parts. For instance,

i) In the leptonic decays of the bottom mesons, no hadrons appear in the final states and therefore, occur through the annihilation of the quark and antiquark in the initial state meson [15] as shown in Figure 2.4.

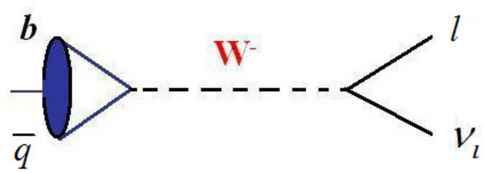


Figure 2.4 Weak annihilation diagram for leptonic decay of bottom mesons

Thus, due to the absence of flavor changing neutral currents in the Standard Model only charged mesons decay to lepton and its neutrino partner, for example, $B^- \to \tau^- \bar{\nu}_\tau$. Historically, the striking 10^{-4} suppression of the kinematically favored leptonic decay $\pi^- \to e^- \bar{\nu}_e$ relative to $\pi^- \to \mu^- \bar{\nu}_\mu$ was successfully explained by the weak interactions.

ii) In the semileptonic decays, one quark decays through emission of *W*-boson whereas the other constituent quark remains spectator [15]. For example, $B^- \to D^0 e^- \bar{\nu}_e$, proceed via spectator quark diagram as shown in Figure 2.5.

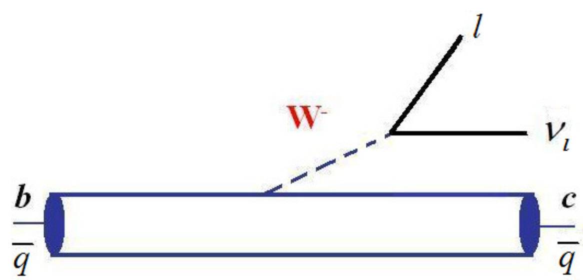


Figure 2.5 Spectator diagram for semileptonic decay of bottom mesons

c) Nonleptonic Weak Decays

Nonleptonic weak processes, the decay involves all the hadrons in the initial and final states for example, $B^- \to D^0 \pi^-$, as shown in Figure 2.6. The nonleptonic decays of bottom mesons are more complicated as compared to the leptonic and semileptonic decays due to the strong interaction interference in the hadronic final states [16].

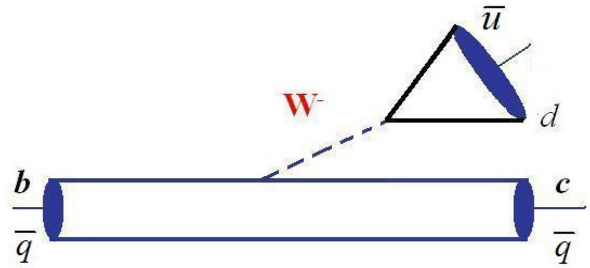


Figure 2.6 Spectator diagram for nonleptonic decay of bottom mesons

In this thesis, we investigate the nonleptonic decays of the heavy flavor hadrons, for which the required spectroscopy of the *s*-wave meson is presented in the following section.

2.7 *S*-WAVE MESON SPECTROSCOPY

According to the quark model, *s*-wave mesons are bound states of quarks[6] q and $\overline{q}$, where q = *u, d, s, c* or *b*. These quarks belong to the irreducible representation **5** of $SU(5)$ such that

$$\mathbf{5} = \begin{pmatrix} u \\ d \\ s \\ c \\ b \end{pmatrix} \tag{2.24}$$

and antiquarks belong to

$$\mathbf{5}^* = (\overline{u} \quad \overline{d} \quad \overline{s} \quad \overline{c} \quad \overline{b}). \tag{2.25}$$

The spins of the quarks q and $\overline{q}$ will give rise to *s*-wave pseudoscalar (0^-) and vector (1^-) states. Since

$$\mathbf{5} \otimes \mathbf{5}^* = \mathbf{24} \oplus \mathbf{1}, \tag{2.26}$$

[6]*The top quark is unique in that, unlike the other quarks, it is massive enough to decay to a real W boson. The decay time is so short, as compared to the typical timescale of hadronic interactions, thatno bound states would be seen.*

the mesons belong to SU(5) singlet or 24-plet.

Under the $SU(5) \supset SU(4) \otimes U(1)_b$ branching of the quark multiplet is given by

$$\mathbf{5} \supset \mathbf{4_0} \oplus \mathbf{1_{-1}},$$

i.e.
$$\begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix}_{\mathbf{0}} \oplus (b)_{-1},$$

where the subscripts denote the bottom quantum number belonging to the U(1)$_b$ group. Thus, the meson multiplet have the following decomposition under SU(4):

$$\mathbf{24} \supset \mathbf{4_1^*} \oplus (\mathbf{15} \oplus \mathbf{1})_{\mathbf{0}} \oplus \mathbf{4_{-1}},$$

$$\mathbf{1} \supset \mathbf{1_0}.$$

The naked bottom mesons containing a single *b* quark lie in SU(4)-sub multiplet **4** and the antimesons of these states lie in **4***.

2.7.1 PSEUDOSCALAR MESONS

The pseudoscalar mesons containing single *b* quark under $SU(4) \supset SU(3) \otimes U(1)_c$ branching are identified as

$$\mathbf{4} \supset \mathbf{3_0}\ (B^+, B^{\mathbf{0}}, B_s^{\mathbf{0}}) \oplus \mathbf{1_{+1}}\ (B_c^+), \tag{2.27}$$

where the subscript denotes the charm quantum number.

Similarly, $\mathbf{4}^*$ contains

$$\mathbf{4}^* \supset \mathbf{3_0^*}\ (B^-, \overline{B}^{\mathbf{0}}, \overline{B}_s^{\mathbf{0}}) \oplus \mathbf{1_{-1}^*}\ (B_c^-). \tag{2.28}$$

15-plet mesons are decomposed as given below

$$\mathbf{15} \supset \mathbf{8_0} \oplus \mathbf{1_0}\ (K^+, K^{\mathbf{0}}, \pi^+, \pi^{\mathbf{0}}, \pi^-, \eta_{\mathbf{8}}, \eta_{\mathbf{15}}, K^-, \overline{K}^{\mathbf{0}})$$
$$\oplus\ \mathbf{3_1^*}\ (D^+, D^{\mathbf{0}}, D_s^+) \oplus \mathbf{3_{-1}}\ (D^-, \overline{D}^{\mathbf{0}}, D_s^-), \tag{2.29}$$

and singlet state is given by

$$\mathbf{1} \supset \mathbf{1_0}\ (\eta_0). \tag{2.30}$$

In the real world, mixture of the states η_8, η_{15} and η_0 are identified with the physical states η, η' and η_c through

$$\eta = \eta_8 \cos\theta_P - \sin\theta_P\,(\eta_{15}\cos\phi_P + \eta_0 \sin\phi_P),$$

$$\eta' = \eta_8 \sin\theta_P + \cos\theta_P\,(\eta_{15}\cos\phi_P + \eta_0 \sin\phi_P),$$

$$\eta_c = (-\eta_{15}\sin\phi_P + \eta_0 \cos\phi_P). \tag{2.31}$$

The mixing angles θ_P and ϕ_P are determined from the empirical properties of these states. η_c is normally considered to be $c\bar{c}$ state, which corresponds to $\phi_P = 60^o$ (ideal mixing), and $\theta_P = -10^o$, -23^0 follows from the quadratic and linear mass formulas, respectively [1]. Similarly, η_b is taken to be $b\bar{b}$ state in the limit of ideal mixing. Experimentally, all these mesons have been detected and their masses are well measured.

2.7.2 VECTOR MESONS

The vector mesons are similarly described by the following replacement

a) Isovector : $\pi \to \rho$

b) Isodoublets : $K \to K^*, D \to D^*\ B \to B^*$

c) Isosinglets : $D_s \to D_s{}^*, \eta \to \phi, \eta' \to \omega, \eta_c \to \psi$ as

$$\mathbf{4} \supset \mathbf{3_0}\ (B^{*+}, B^{0*}, B_s^{*0}) \oplus \mathbf{1_1}\ (B_c^{*+}), \tag{2.32}$$

$$\mathbf{4}^* \supset \mathbf{3}_0^*\ (B^{*-}, \bar{B}^{0*}, \bar{B}_s^{*0}) \oplus \mathbf{1_{-1}}\ (B_c^{*-}), \tag{2.33}$$

$$\mathbf{15} \supset \mathbf{8_0} \oplus \mathbf{1_0}\ (K^{*+}, K^{*0}, \rho^+, \rho^0, \rho^-, V_8, V_{15}, K^{*-}, \bar{K}^{*0})$$
$$\oplus\ \mathbf{3}_1^*\ (D^{*+}, D^{*0}, D_s^{*+}) \oplus \mathbf{3_{-1}}\ (D^{*-}, \bar{D}^{*0}, D_s^{*-}), \tag{2.34}$$

and

$$\mathbf{1} \supset \mathbf{1_0}\ (V_0). \tag{2.35}$$

The physical states ω, ϕ and ψ are the mixture of diagonal states V_8, V_{15} and V_0. These are given by

$$\omega = \mathrm{V}_8 \sin\theta_\mathrm{V} + \cos\theta_\mathrm{V} (\mathrm{V}_{15} \cos\phi_\mathrm{V} + \mathrm{V}_0 \sin\phi_\mathrm{V}),$$

$$\phi = \mathrm{V}_8 \cos\theta_\mathrm{V} - \sin\theta_\mathrm{V} (\mathrm{V}_{15} \cos\phi_\mathrm{V} + \mathrm{V}_0 \sin\phi_\mathrm{V}),$$

$$\psi = (- \mathrm{V}_{15} \sin\phi_\mathrm{V} + \mathrm{V}_0 \cos\phi_\mathrm{V}). \quad (2.36)$$

Ideal mixing [6], fixes $\theta_V = 35.3^o$ and $\phi_V = 60^o$. Thus, J/ψ is purely $c\bar{c}$ state, $\phi = s\bar{s}$ and $\omega = \frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$. Similarly, ideal mixing fixes Υ to be $b\bar{b}$ state. We wish to point out that except for B_c^*, all the meson masses are available experimentally. Theoretical estimates for hyperfine splitting $m_{B_c^*} - m_{B_c}$ obtained in different quark models [17, 18] range from 65 to 90 MeV. For the present work, we take $m_{B_c^*} - m_{B_c} = 73 \pm 15$ MeV from [19], which has been quite successful in giving charmonium and bottomium mass spectra.

In the following, we discuss the main aspects of the leptonic, semileptonic and nonleptonic weak decays of bottom mesons.

2.8 LEPTONIC WEAK DECAYS OF *B* MESONS

A key feature of leptonic and semileptonic decays of bottom mesons is their relative simplicity a consequence of the fact that here the effects of the strong interactions can be isolated. In case of leptonic decays, the two initial state quarks must annihilate to generate lepton and its neutrino partner (Fig. 2.4). Thus, only charged bottom meson can decay to such leptonic pairs for instance $B^- \to e^-\bar{\nu}_e / \mu^-\bar{\nu}_\mu / \tau^-\bar{\nu}_\tau$, out of these e^- and μ^- emitting decays are helicity suppressed, but for $B^- \to \tau^-\bar{\nu}_\tau$ decay the large τ mass reduces the helicity suppression. The experimental branching ratio for $B^- \to \tau^-\bar{\nu}_\tau$ decay is (1.4 ± 0.4)×10^{-4} [1]. The matrix element of the hadronic current to the vacuum is given by

$$\langle 0|J_\mu|B\rangle = i\, f_B P_B\,, \tag{2.37}$$

where P_B is the four-momentum of pseudoscalar meson. The decay constant f_B measures the amplitude of the quarks to have zero separation. In the heavy quark limit the pseudoscalar decay constant is given by the formula [15],

$$f_B^2 \;=\; \frac{12\,|\psi(0)|^2}{M_B}\,, \tag{2.38}$$

which yield,

$$f_B = 0.260\,\text{GeV},$$

taking $|\psi(0)|^2$ from the Table 2.1, which is in nice agreement with the experimental observation:

$$f_B = (0.247 \pm 0.069)\ \text{GeV},$$

derived form the decay rate formula [11],

$$\Gamma(B \to l\,\bar{\nu}) = \frac{G_F^2}{8\pi}\left|V_{bq}\right|^2 m_B\, m_l^2 f_B^{\,2}\left(1-\frac{m_l^2}{m_B^2}\right)^2, \tag{2.39}$$

where V_{bq} is the CKM matrix element, m_l and m_B are masses of the lepton and charged meson, respectively. Note that $B^- \to \tau^-\bar{\nu}_\tau$ decay is sensitive to non-Standard Model contributions from the charged Higgs boson mediated amplitudes [1]; it can either increase or decrease the expected branching ratio. Theoretical predictions for f_B range from 0.120 to 0.290 GeV [15, 20]. Similarly, we find f_{B_s} = (0.345 ± 0.069) GeV, whose theoretical predictions varies from 204 to 300 [15, 20].

2.9 SEMILEPTONIC WEAK DECAYS OF *B* MESONS

We now discuss the semileptonic decays $B \to X\, l\,\bar{\nu}_l$ of heavy mesons in which the hadronic system *X* is a single meson, usually a pseudoscalar (*P*) or a vector (*V*) particle.

Semileptonic decays play a prominent role in heavy quark physics, as they provide information about the binding of quarks. These decays proceed via the spectator diagram, already shown for b quark decays in Fig 2.5. Models must account for strong interaction effects only among three primary quarks (b, $\overline{q}$ and c or u) rather than five in the more complicated case of the hadronic decays. Thus, hadronic physics enters through a single hadron-hadron matrix element, which is expressed in terms of a few form factors [15].

The semileptonic decays of bottom mesons can proceed through the selection rules [21]

i. Bottom changing and charm conserving,

$$\Delta b = 1, \Delta C = 0, \Delta S = 0 \quad \text{for } b \to u\, l \nu_l ; \tag{2.40}$$

ii. Bottom changing and charm changing,

$$\Delta b = 1, \Delta C = 1, \Delta S = 0 \quad \text{for } b \to c\, l \nu_l . \tag{2.41}$$

Out of these, $b \to c$ transition is dominant among the b quark decays due to the larger CKM factor.

In general, the semileptonic decay amplitude $A(B \to X\, l \overline{\nu}_l)$ can be expressed as

$$A_{SL}(B \to X) = \frac{G_F}{\sqrt{2}} \left| V_{Qq} \right|^2 L^{\mu} H_{\mu} , \tag{2.42}$$

where

$$L^{\mu} = \overline{u}(k_2) \gamma^{\mu} (1 - \gamma_5) \upsilon(k_1),$$
$$H_{\mu} = \langle X | J_{\mu} | B \rangle, \tag{2.43}$$

For $X = P$ or V and $k_1 = k_l$, $k_2 = k_\nu$ if the decaying quark is a b quark. $\left| V_{bq} \right|^2$ is the appropriate CKM matrix elements for $b \to q$ transition. For example, $B \to D l^- \nu_l$, is described by the product of two terms, a hadronic current and a leptonic current as $\langle D | J_{\mu} | B \rangle \times \langle \nu | \gamma_{\mu} (1 - \gamma_5) | l \rangle$. The leptonic piece is straight forward to calculate. Using the

Lorenz invariance the hadronic weak currents are expressed in terms of form factors, which have certain q^2 dependence, where $q^2 = (p_B^\mu - p_D^\mu)$, p_B^μ and p_D^μ being the *B* and *D* momenta four-vectors, respectively.

2.9.1 $B \to Pl\bar{\nu}_l$ DECAY: METHODOLOGY

From Lorentz invariance, one finds the most general form factor decompositions for pseudoscalar mesons can be given in terms of matrix elements of the currents defined [15, 21] as,

$$\langle P \left| J_\mu \right| B \rangle = (P_B + P_P - \frac{m_B^2 - m_P^2}{q^2} q)_\mu F_1(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0(q^2)\,, \qquad (2.44)$$

where $q_\mu = (P_B - P_P)_\mu$ and $F_1(0) = F_0(0)$ The form factors $F_0(q^2)$ and $F_1(q^2)$ can be associated with the exchange of particles with quantum numbers $J^P = 0^+$ and $J^P = 1^-$, respectively.

The decay width of $B \to Pl\bar{\nu}_l$ can be expressed as a function of the four-momentum transfer (q^2) between initial and final hadrons. The semileptonic decay width [15, 20] for $B \to Pl\bar{\nu}_l$ is given by the formula

$$\Gamma(B \to Pl\bar{\nu}_l) = \frac{G_F^2}{(2\pi^3)} \left|V_{Qq}\right|^2 \int_{m_l^2}^{q^2} dq^2 \left(\frac{q^2 - m_l^2}{q^2} \right)^2 \frac{\sqrt{\lambda(m_B^2, m_P^2, q^2)}}{24 m_B^3} (m_B^2 - m_P^2)^2 \times \left[\frac{3m_l^2}{2q^2} F_0^2(q^2) + \left(1 + \frac{m_l^2}{2q^2} \right) \frac{\lambda(m_B^2, m_P^2, q^2)}{(m_B^2 - m_P^2)^2} F_1^2(q^2) \right], \qquad (2.45)$$

where m_l is the mass of the lepton and $0 \le q^2 \le q_{\max}^2 = (m_B - m_P)^2$ and $\lambda(m_B^2, m_P^2, q^2) = (m_B^2 + m_P^2 - q^2) - 4m_B^2 m_P^2$ is related to the three momentum of the daughter meson in the rest frame of *B* meson by $P_P = \frac{\sqrt{\lambda(m_B^2, m_P^2, q^2)}}{2m_B}$.

2.10 MOMENTUM TRANSFER DEPENDENCE OF FORM FACTORS

In order to calculate the decay modes with a pseudoscalar or vector meson in the final state, we need to estimate the relevant form factors and their q^2 dependence. The maximum q^2 is obtained when $p_D = 0$, that is called the "zero recoil" point. Conversely, $q^2 = 0$ occurs when the D has its maximum possible momentum. It should be realized that the mass of the virtual W is given by the value of q^2. At zero recoil (maximum q^2), the form factors are maximum, because the overlap between the B and D meson wave functions is the largest. The form factors decrease as q^2 decreases [15].

There are several different types of models [21-24], which have been developed to investigate semileptonic weak decays. Quark model calculations estimates meson wave functions and use them to compute matrix elements that appear in the hadronic currents. These integrals are performed by analyzing the decay at a particular value of q^2, either $q^2 = 0$ or $q^2 = q^2_{\max}$. In quark model calculations the form factors with q^2 is determined as a separate step in the calculation in fact, this variation is assumed to have a very simple form. Because the physics being described is nonperturbative, none of these phenomenological forms should be taken too seriously. One approach, used in the KS [22] and BSW models [21], is called "nearest pole dominance," which has its origin in vector-dominance ideas. Here, q^2 dependence of a form factor f_i is assumed to have the form

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2 / m^2_{pole})^n},$$

where n is an integer, usually one for mesons. The pole mass m_{pole} is the mass of the lowest lying meson with the quantum numbers appropriate current. In this work, we employ the BSW model framework [21] for evaluating the meson form factors, which have been quite successful in describing weak mesonic decays. Originally the BSW model assumed the monopole q^2 dependence (n = 1) for all the form factors. However, in the improved version [24] of the BSW model (BSW II), consistency with the heavy quark symmetry seems to require certain form factors such as F_1, A_0, A_2 and V to have dipole q^2 dependence and F_0 and A_1 to have monopole q^2 dependence, i.e.

$$F_0(q^2) = \frac{F_0(0)}{(1 - q^2 / m_S^2)} \text{ and } F_1(q^2) = \frac{F_1(0)}{(1 - q^2 / m_V^2)^2},$$

where m_S and m_V are the pole masses of scalar and vector mesons, respectively. We give the pole masses used for numerical calculations in Table 2.1.

Table 2.1 Pole masses (GeV) used in numerical calculations

Current	$m(0^-)$ A_0	$m(1^-)$ F_1, V	$m(0^+)$ F_0	$m(1^+)$ A_1, A_2
$\bar{d}c$	1.87	2.01	2.47	2.42
$\bar{s}c$	1.97	2.11	2.60	2.53
$\bar{u}c$	5.27	5.32	5.78	5.71
$\bar{s}b$	5.38	5.43	5.89	5.82
$\bar{c}b$	6.30	6.34	6.80	6.73

2.11 BSW MODEL FRAMEWORK

In this framework, the initial and final state mesons are given by the relativistic bound states of a quark q_1 and an antiquark $\bar{q}_2$ in the infinite momentum frame using the relativistic harmonic oscillator potential [21],

$$\left|\mathbf{P}, m, j, j_Z\right\rangle = \sqrt{2}(2\pi)^{3/2} \sum_{S_1 S_2} \int d^3 p_1 d^3 p_2 \delta^3(\mathbf{P} - \mathbf{p_1} - \mathbf{p_2}) \times \psi_m^{j, j_Z}(\mathbf{p_{1T}}, x, s_1, s_2) a_1^{S_1^+}(\mathbf{p_1}) b_2^{S_2^+}(\mathbf{p_2}) \left|0\right\rangle \quad , \tag{2.46}$$

where $P_\mu = (P_0, 0, 0, P)$ with $P \to \infty$, x denotes the fraction of the longitudinal momentum carried by the non-spectator quark q_1, and $\mathbf{p_{1T}}$ denotes its transverse momentum:

$$x = p_{1Z}/P \, , \ \mathbf{p_{1T}} = (p_{1x}, p_{1y}) \, .$$

We calculate all the form factors appearing in the expression (2.41) and (2.43) to later investigate their flavor dependence.

By expressing the current J_μ in terms of the annihilation and creation operators, the form factors are given by the following integrals:

$$F_0^{BP}(0) = F_1^{BP}(0) = \int d^2 p_T \int_0^1 (\psi_P^*(\mathrm{p_T}, x) \psi_B(\mathrm{p_T}, x)) dx \, . \tag{2.47}$$

The meson wave function is given by

$$\psi_m(\mathbf{p_T}, x) = N_m \sqrt{x(1-x)} \exp(-\mathbf{p_T^2}/2\omega^2) \exp(-\frac{m^2}{2\omega^2}(x - \frac{1}{2} - \frac{m_{q_1}^2 - m_{q_2}^2}{2m^2})^2) \, , \tag{2.48}$$

where m denotes the meson mass and m_i denotes the i^{th} quark mass, N_m is the normalization factor and ω is the average transverse quark momentum, $\left\langle \mathbf{p_T^2} \right\rangle = \omega^2$.

2.12 FORM FACTORS AND BRANCHING RATIOS

In the BSW model [21], the form factors are usually calculated by taking ω=0.50 GeV for all the mesons and $m_u = m_d = 0.35$ GeV, $m_s = 0.55$ GeV, $m_c = 1.7$ GeV and $m_b = 4.9$ GeV. The $B \to P$ form factors thus obtained are given in column 3 of Table 2.2.

Table 2.2 Form factors of $B \to P$ transition (ω = 0.50 GeV)

Modes	Transition	$F_0^{BP}(0)$
$\Delta b = 0, \Delta C = 0, \Delta S = 0$	$B \to \pi$	0.39
	$B_s \to K$	0.42
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	$B \to D$	0.70
	$B_s \to D_s$	0.67

Using these form factors, we obtain the branching ratios for various $B \to P l \bar{\nu}_l$ decays, as given in column 2 of Table 2.3. We make the following observations:

i. For $\Delta b = 1$, $\Delta C = 0, \Delta S = 0$ mode, branching ratios of the dominant decays are $B(\bar{B}^0 \to \pi^+ e \bar{\nu}_e)$ = 1.45×10^{-4} and $B(\bar{B}^0 \to \pi^+ \mu \bar{\nu}_\mu)$ = 1.45×10^{-4}, which are slightly higher than the experimental value $(1.36\pm0.09)\times10^{-4}$.

ii. In case of $\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$ mode, branching ratios of dominant decays are $B(B^+ \to D^0 e \bar{\nu}_e)$ = 2.09×10^{-2} and $B(\bar{B}^0 \to D^+ e \bar{\nu}_e)$ = 1.94×10^{-2}, which match well with the experimental value $(2.27\pm0.12)\times10^{-2}$ and $(2.10\pm0.11)\times10^{-2}$.

Table 2.3 Branching ratios of $B \to Pl\bar{\nu}_l$ decays

Decays	ω=0.50 GeV	Expt.
$\Delta b = 1, \Delta C = 0, \Delta S = 0$		
$B^+ \to \pi^0 e\bar{\nu}_e$	7.77×10^{-5}	$(7.7\pm1.2)\times10^{-5}$
$B^+ \to \pi^0 \mu\bar{\nu}_\mu$	7.78×10^{-5}	$(7.7\pm1.2)\times10^{-5}$
$\bar{B}^0 \to \pi^+ e\bar{\nu}_e$	1.45×10^{-4}	$(1.36\pm0.09)\times10^{-4}$
$\bar{B}^0 \to \pi^+ \mu\bar{\nu}_\mu$	1.45×10^{-4}	$(1.36\pm0.09)\times10^{-4}$
$\Delta b = 1, \Delta C = 1, \Delta S = 0$		
$B^+ \to D^0 e\bar{\nu}_e$	2.09×10^{-2}	$(2.27\pm0.12)\times10^{-2}$
$B^+ \to D^0 \mu\bar{\nu}_\mu$	2.09×10^{-2}	$(2.27\pm0.12)\times10^{-2}$
$\bar{B}^0 \to D^+ e\bar{\nu}_e$	1.94×10^{-2}	$(2.17\pm0.11)\times10^{-2}$
$\bar{B}^0 \to D^+ \mu\bar{\nu}_\mu$	1.94×10^{-2}	$(2.17\pm0.11)\times10^{-2}$
$\bar{B}_s^0 \to D_s^+ e\bar{\nu}_e$	1.84×10^{-4}	---
$\bar{B}_s^0 \to D_s^+ \mu\bar{\nu}_\mu$	1.84×10^{-4}	---

2.13 NONLEPTONIC WEAK PROCESSES

In treating exclusive hadronic decay, it is important to recognize from the outset the complex relation between the quark level operators and the actual hadrons; the explicit structure of hadrons is certain to play an important role in understanding exclusive decays. The simplest nonleptonic decays are the one involving two-body final states and their most general form is $\langle M_1 M_2 | H_W^{eff} | B \rangle$ where the QCD modified weak hamiltonian is

$$H_W^{eff} = \frac{G_F}{\sqrt{2}}[c_1(\bar{q}_1 q_2)(\bar{q}_3 q_4) + c_2(\bar{q}_1 q_4)(\bar{q}_3 q_2)],$$

with

$$c_1 = \frac{1}{2}(c_+ + c_-) \text{ and } c_2 = \frac{1}{2}(c_+ - c_-).$$

To calculate the weak decay amplitudes of the weak hamiltonian between the initial one particle and final two particle state, one usually tries to employ a framework in which it is related to the matrix elements of either the currents or of the weak hamiltonian H_W^{eff} between single particle states. Such matrix elements contain all information conscerning the modification of the basic weak interactions by virtual strong interaction effects. In addition, final state interactions [25] are also likely to be important, especially for the charmed mesons, as charm quark masses lie in a region where resonances rescattering due to the indivisual characteristics of particles, like braod width which normally the kinamatics and phase space.

2.14 WEAK HAMILTONIAN FOR BOTTOM CHANGING DECAYS

The weak Hamiltonian generating the *b* quark decays [26] is given by

$$H_w^{\Delta b=1} = \frac{G_F}{\sqrt{2}}[V_{ub}V_{cd}^* \ (\bar{u}b)(\bar{d}c) + V_{ub}V_{cs}^*(\bar{u}b)(\bar{s}c) + V_{ub}V_{ud}^*(\bar{u}b)(\bar{d}u) +$$
$$V_{ub}V_{us}^*(\bar{u}b)(\bar{s}u) + V_{cb}V_{ud}^* \ (\bar{c}b)(\bar{d}u) + V_{cb}V_{us}^* \ (\bar{c}b)(\bar{s}u) + \qquad (2.49)$$
$$V_{cb}V_{cs}^* \ (\bar{c}b)(\bar{s}c) + V_{cb}V_{cd}^*(\bar{c}b)(\bar{d}c)],$$

where

$$(\bar{q}_i q_j) = (\bar{q}_i \gamma_\mu (1-\gamma_5) q_j) \qquad (2.50)$$

denotes the $V-A$ current and the color and space-time structure is omitted. V_{ij} are CKM matrix. Following selection rules for various decay modes generated by the Hamiltonian are given below:

1. CKM-enhanced modes:

   Δb = 1, ΔC = 1, ΔS = 0; Δb = 1, ΔC = 0, ΔS = -1,

2. CKM-suppressed modes:

   Δb = 1, ΔC = 1, ΔS = -1; Δb = 1, ΔC = 0, ΔS = 0,

3. CKM-doubly-suppressed modes:

$$\Delta b = 1, \Delta C = \Delta S = -1; \qquad \Delta b = 1\ \Delta C = -1, \Delta S = 0. \tag{2.51}$$

The weak Hamiltonian defined in (2.49) contains two weak quark currents. As the quarks involved may exchange gluons and the produced quarks in the weak interactions finally form physically observable hadron states, it is not easy to determine the decay matrix element in a straight forward manner. Since the data is measured at the level of hadrons, which are bound states of quarks, one has to match the two levels. Therefore, it is very important to include the QCD modification of the decay amplitudes at various levels, e.g. at quark level and at the hadron level. The effects at the weak vertex are calculated using the perturbative QCD, whereas the long-distance QCD effects like hadronization are a lot more complex. These QCD modifications are explained as follows.

2.15 QCD MODIFICATIONS

Since, the weak Hamiltonian (2.49) contain weak quark currents only, the nonleptonic decays experience significant strong interaction intreference, due to gluon exchange among the quarks. Further, the produced quarks in the weak interactions finally form physically observable hadron states. Therefore, it is essential to investigate the strong interaction effects on the weak decay amplitudes. For this purpose, it is important to recognise the two scales involved in the weak decays of the hadrons. Firstly, the distance scale for *W*-exchange is $R_{weak} \approx 1/M_W \approx 10^{-16} cm$, and secondly the confinement scale given by Λ_{QCD} ($R_{hadron} \approx 1\, fermi$, the typical radius of a hadron). These two different scales allow one to separate the strong interactiona as short and long distance QCD effects. In practice, the short distance gluon exchange effects are calculated around the weak vertex using the perturbative QCD, whereas the long distance strong interactions effects, being non-perturbative in nature, are treated phenomenologically.

2.15.1 HARD GLUON CORRECTIONS

Through the short distance gluon exchange effects, the color structure of the interaction may be distorted by color octet currents. At the bare quark level, in the absence of QCD effects, the general Hamiltonian is given by

$$H_W^{(0)} = \frac{G_F}{\sqrt{2}} (\bar{q}_2 q_1)(\bar{q}_4 q_3) . \tag{2.52}$$

Here, i and j are the flavor indices and color index is omitted. At the weak interaction scale ~ $1/M_W$, the gluons exchanged between the quarks, having a large momenta, are called hard-gluons. The lowest order correction to the basic weak vertex (Figure 2.7), arise from such gluons shown in Figure 2.8. In the leading order at the mass scale μ (i.e. first order in $\alpha_S(\mu)$), the weak Hamiltonian then becomes [27],

$$H_W^{(1)} = H_W^{(0)} - \frac{G_F}{\sqrt{2}} \frac{3\alpha_S}{8\pi} ln \frac{M_W^2}{\mu^2} \ (\bar{q}_2 \lambda^a q_1)(\bar{q}_4 \lambda^a q_3). \tag{2.53}$$

This result reveals that hard gluonic effects induce product of weak color octet currents containing the same chirality and flavor structure as the color singlet current in $H_W^{(0)}$.

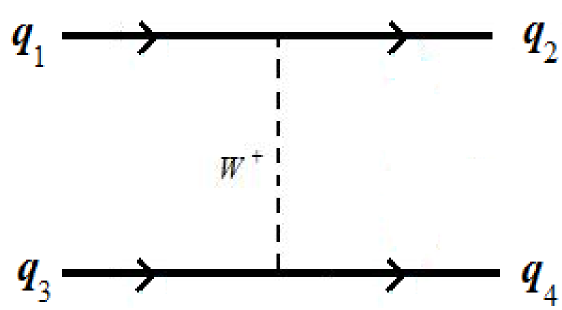


Figure 2.7 Basic quark weak vertex

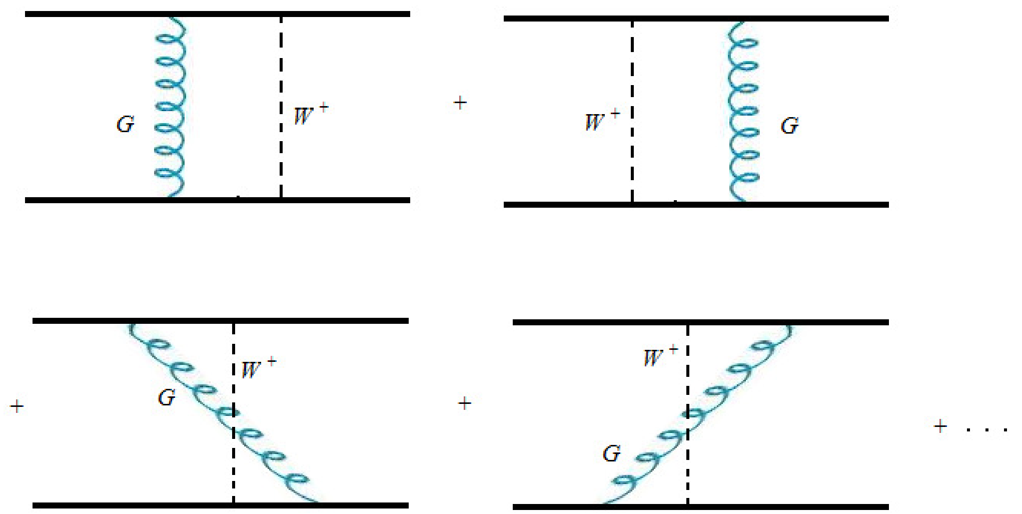


Figure 2.8 Lowest order corrections to the weak Hamiltonian

Using the Fierz identity,

$$\sum_{a=1}^{8}\lambda_{ij}^{a}\lambda_{kl}^{a}=-\frac{2}{3}\delta_{ij}\delta_{kl}+2\delta_{il}\delta_{kj}, \tag{2.54}$$

and Dirac algebra

$$[\gamma_{\mu}(1-\gamma_5)]_{\alpha\beta}[\gamma^{\mu}(1-\gamma_5)]_{\delta\varepsilon}=-[\gamma_{\mu}(1-\gamma_5)]_{\alpha\varepsilon}[\gamma^{\mu}(1-\gamma_5)]_{\delta\beta}, \tag{2.55}$$

$H_W^{(1)}$ can be expressed as

$$H_W^{(1)}=\frac{G_F}{\sqrt{2}}[(1+\frac{\alpha_S}{4\pi}ln\frac{M_W^2}{\mu^2})(\bar{q}_2q_1)(\bar{q}_4q_3)-\frac{3\alpha_S}{4\pi}ln\frac{M_W^2}{\mu^2}(\bar{q}_2q_3)(\bar{q}_4q_1)]. \tag{2.56}$$

The first term represents the renormalized charged current interaction, and the 2nd term describes a new *effective* neutral current interaction shown in Figure 2.9.

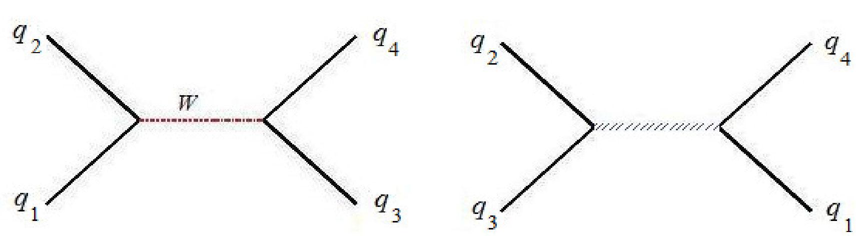


Figure 2.9 Effective charged current and effective neutral current processes

Using operator product expansion (OPE) and renormalization group theory, the short distance correction can be calculated [21, 24, 28] to all orders in α_s. The effective weak Hamiltonian finally[7] becomes

$$H_W^{eff} = \frac{G_F}{\sqrt{2}}[c_1(\bar{q}_1 q_2)(\bar{q}_3 q_4) + c_2(\bar{q}_1 q_4)(\bar{q}_3 q_2)], \tag{2.57}$$

where c_1 and c_2 are known as the scale dependent Wilson QCD coefficients [21]. Defining

$$c_{\pm}(\mu) = c_1(\mu) \pm c_2(\mu). \tag{2.58}$$

The leading-log approximation gives

$$c_{\pm}(\mu) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu)}\right)^{\frac{-6\gamma_\pm}{(33-2n_f)}}, \tag{2.59}$$

where $\gamma_- = -2\gamma_+ = 2$, and n_f is the number of active flavors, which is taken to be five in this case.

The QCD modified weak Hamiltonian describing the B decays is then given by,

A. BOTTOM CHANGING ($\Delta b = 1$) DECAYS

i) The CKM-favored modes ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$; $\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$)

$$\begin{aligned} H_W = \frac{G_F}{\sqrt{2}}\{ & V_{cb}V_{ud}^*[c_1(\bar{c}b)(\bar{d}u) + c_2(\bar{d}b)(\bar{c}u)] + \\ & V_{cb}V_{cs}^*[c_1(\bar{c}b)(\bar{s}c) + c_2(\bar{s}b)(\bar{c}c)] + \\ & V_{cb}V_{us}^*[c_1(\bar{c}b)(\bar{s}u) + c_2(\bar{s}b)(\bar{c}u)] + \\ & V_{cb}V_{cd}^*[c_1(\bar{c}b)(\bar{d}c) + c_2(\bar{d}b)(\bar{c}c)]\}, \end{aligned} \tag{2.60}$$

[7] *In fact, several other terms, with smaller QCD coefficients, arise in the full effective weak Hamiltonian.*

ii) The CKM-suppressed modes ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = -1$; $\Delta b = 1$, $\Delta C = 0$, $\Delta S = 0$, $\Delta b = 1$, $\Delta C = \Delta S = -1$; $\Delta b = 1$ $\Delta C = -1$, $\Delta S = 0$)

$$
\begin{aligned}
H_W = \frac{G_F}{\sqrt{2}}\{&V_{ub}V_{cs}^{*}[c_1(\bar{u}b)(\bar{s}c) + c_2(\bar{s}b)(\bar{u}c)] + \\
&V_{ub}V_{ud}^{*}[c_1(\bar{u}b)(\bar{d}u) + c_2(\bar{d}b)(\bar{u}u)] + \\
&V_{ub}V_{us}^{*}[c_1(\bar{u}b)(\bar{s}u) + c_2(\bar{s}b)(\bar{u}u)] + \\
&V_{ub}V_{cd}^{*}[c_1(\bar{u}b)(\bar{d}c) + c_2(\bar{d}b)(\bar{u}c)]\},
\end{aligned}
\tag{2.61}
$$

2.15.2 LONG DISTANCE EFFECTS

One, usually assumes that the long distance QCD effects arising from confinements of the quarks, can be absorbed into the initial and final state hadron wave functions [21, 28]. Such strong interaction effects manifest themselves in the decay constants of the mesons and the formfactors appearing in the weak currents of meson states. These have already been discussed in the context of leptonic and semileptonic decays. However, for nonleptonic weak decays these may be additional long distance effects due to strong interactions, Final State Interactions (FSIs) [29] among the decay products, broad width resonaces [30], soft gluon exchange giving rise to nonfactorizable contributions [31]. These are essentially non-perturbative phenomena which can not be calculated from the first principles in the QCD. Due to larger characteristic energy transfer in bottom mesons such effects are expected to be small and are ignored in our analysis [25].

2.16 SPECTATOR QUARK MODEL

Now, we move to consider various quark level processes that can contribute to the nonleptonic decays. These processes can be classified as: (a) *W*-external emission (tree level), (b) *W*-internal emission (color-suppressed), (c) *W*-exchange (neutral mesons only),

(d) W-annihilation (charged mesons only), (e) Pure Penguin (internal gluon emission, as shown in Fig. 2.10. Like bottom meson decays, weak nonleptonic decays of B_c meson acquire dominant contributions from spectator diagram involving W-emission processes (a) and (b), which has been quite successful in describing two body decays of heavy flavor mesons. For pseudoscalar mesons, the W-annihilation diagram is disfavored by helicity arguments [21], which yield the following relative ratio of annihilation to spectator graph,

$$\frac{\Gamma_{anni}}{\Gamma_{spect}} \approx |\psi(0)|^2 \, m^3 \approx \left(\alpha_S \frac{m_q}{M_Q} \right)^3 , \tag{2.62}$$

where m_q and M_Q represent masses of the light and heavy quark in heavy flavor mesons. As the mass of heavy quark goes up, the annihilation graph becomes less and less important. The W-exchange diagrams are further suppressed by a factor $1/9$ in comparison to annihilation graph, due to the required color matching. Penguin diagrams are suppressed for bottom conserving and charm changing decay modes due to GIM mechanism. For bottom changing decays, they may generate a small contribution only to ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = 0$) and ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) modes. Following assumptions are made in this approach [28]:

a) The initial hadron is represented by its valance quark configuration. More complex bound state fluctuations, often addressed as the sea of quarks and gluons, are disregarded.
b) Soft gluon interactions accompanying the weak process are neglected.
c) The inclusive sum of hadronic final states is replaced by the final state of "free" quarks emitted in the decay.

In the constituent quark model, the contributions arising from spectator quark processes can be obtained using factorization scheme, which is discussed in next the section.

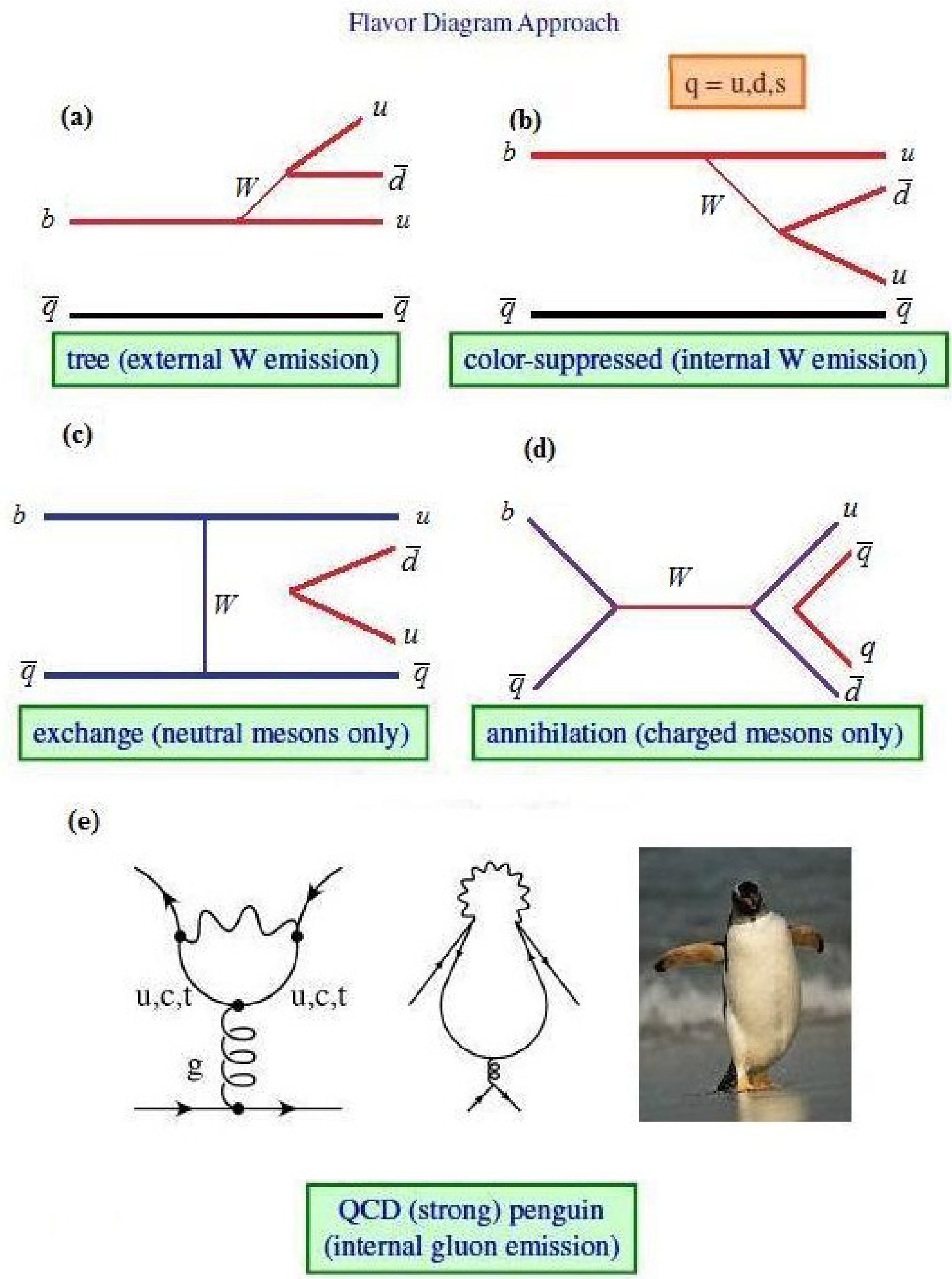


Figure 2.10 Weak quark level processes: (a) *W*-external emission, (b) *W*- internal emission, (c) *W*-exchange, (d) *W*-annihilation and (e) Pure Penguin

2.17 FACTORIZATION SCHEME

Factorization is the assumption that two body hadronic decays of B mesons can be expressed as the product of two independent hadronic currents, one describing the formation of a meson from the converted b quark and the light spectator quark, and the other describing the production of a meson by the hadronization of the virtual W^- [21, 24, 30]. This assumption can be justified more in the decays of heavier hadrons by assuming that the lighter meson couples with vacuum through the vacuum insertion approximation (VIA). Thus three-body matrix elements for the decay $B \to M_1 M_2$ are reduced to the two-body ones,

$$< M_1 M_2 | J_\mu J^{\mu\dagger} | B > \approx < M_1 | J_\mu | 0 >< M_2 | J^{\mu\dagger} | B >, \tag{2.63}$$

As we know that the exclusive decay rates depend on long distance dynamics, in the factorization hypothesis the long distance effects enter through the hadronic matrix elements $< M_1 | J_\mu | 0 >$ and $< M_2 | J^{\mu\dagger} | B >$. To compute $< M_2 | J^{\mu\dagger} | B >$, one takes recourse to some quark model hadronic wave functions. Using the lorentz invariance, these matrix elements are usually expressed in terms of form factors, which are calculated in the chosen quark model. The form factors have certain q^2 dependence, q is the momentum transfer between initial and final meson. It may be noted that these matrix elements also appear in the semileptonic decays $B \to M_2 l \nu_l$, so some of these form factors may be ontained from experimental decay rates, which involve the form factors appearing in the semileptonic decays. To evaluate $< M_1 | J_\mu | 0 >$ part, one may use the current field [21] identities like,

$$(\bar{u}d)_\mu = f_\pi \partial_\mu \pi + f_\rho m_\rho \rho_\mu + f_{A_1} m_{A_1} A_{1_\mu} + \ldots,$$

where the currents are proportional to the interpolating stable or quasi-stable hadron fields carrying the quantum number of the quark currents,

$$(\bar{u}d)_\mu = \left[(\bar{u}\gamma_\mu(1-\gamma_5)d)\right] \rightarrow \left[(\bar{u}\gamma_\mu(1-\gamma_5)d)\right]_H , \tag{2.64}$$

Here, the subscript H now denotes hadron field operators with the appropriate quantum numbers, e.g. the Cabibbo enhanced weak Hamiltonian now can be written as,

$$H_W^{eff} = \frac{G_F}{\sqrt{2}} V_{ud} V_{sc}^* [a_1 (\bar{u}d')_H (\bar{s}'c)_H + a_2 (\bar{u}c)_H (\bar{s}'d')_H], \tag{2.65}$$

where H denotes the effective hadronic field. Note that we have replaced the QCD coefficients $c_{1,2}(\mu)$ by new scale independent parameters a_1 and a_2, which now determine the strength of the charged and neutral current interaction, respectively.

$$a_1 = c_1 + \xi c_2\big|_{\mu=m_c} , \qquad a_2 = c_2 + \xi c_1\big|_{\mu=m_c} . \tag{2.66}$$

These involve new and free parameters ξ of a *priori* unknown size. $\xi = 0$ means color matching is necessary for forming a hadron and $\xi = 1$ means it is not. Hence, the relation of the coefficients a_1 and a_2, to the QCD short-distance coefficients c_+ and c_- of the effective quark Hamiltonian is not straightforward. It must be kept in mind that the origin of various terms in (3.24) is different, while $c_\pm$ are due to hard gluon effects, the parameters ξ is appearing due to the soft gluons [21, 24, 28]. The parameter ξ denotes the relative size of the matrix element in color space. Naively, we could expect $\xi = \frac{1}{N_c}$, (number of colors, $N_c = 3$) arising from color mismatch in forming the color singlets. However, charm meson data seemed to favor ξ = 0. In practice, a_1 and a_2 are treated as a free parameter to be fixed from experiments.

Finally, given a nonleptonic weak decay $B \rightarrow M_1 M_2$, the decay amplitude can be expanded as

$$B \to M_1 + M_2 = \frac{G_F}{\sqrt{2}}(\textit{Cabibbo factors} \times \textit{QCD factors}) \times$$
$$\left\{ \langle M_1 | J_\mu | 0 \rangle \langle M_2 | J_\mu^\dagger | B \rangle + \langle M_2 | J_\mu | 0 \rangle \langle M_1 | J_\mu^\dagger | B \rangle \right\}. \tag{2.67}$$

By factorizing these matrix elements, one can distinguish three classes of decays:

- class I transition <u>caused by color favored diagram</u>: the corresponding decay amplitudes are proportional to a_1, where $a_1(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu)$, and N_c is the number of colors.
- class II transition <u>caused by color suppressed diagram</u>: the corresponding decay amplitudes in this class are proportional to a_2, i.e. for the color suppressed modes $a_2(\mu) = c_2(\mu) + \frac{1}{N_c} c_1(\mu)$.
- class III transition <u>caused by both color favored and color suppressed diagrams</u>: these decays experience the interference of color favored and color suppressed diagrams.

$$c_1(\mu) = 1.26 \,, c_2(\mu) = -0.51 \text{ at } \mu \approx m_c^2 \,,$$

$$c_1(\mu) = 1.12 \,,\; c_2(\mu) = -0.26 \text{ at } \mu \approx m_b^2 \text{ [25].}$$

2.18 BOTTOM MESON DECAYS AND RELATIVE SIGN OF (a_2/a_1)

In the decays of charmed mesons the effect of color supression is obscured by the effects of final state interactions (FSI), and soft gluon effects which enhance *W*-exchange digrams. When the BSW model is used to fit the data on charm decays it gives values $a_1 = 1.26$ and $a_2 = -0.51$ justified in the large N_c limit. The BSW model assumes that values of the coefficients can be extrapolated to $\mu = m_c^2$ to $\mu = m_b^2$ taking into account the evaluation of strong coupling constant (alphas). This extrapolation give the predictions $a_1 = 1.10$ and $a_2 = -0.24$ for B decays. It may be noted that the smaller magnitude of a_2 means that in contrast to the charm sector one expects to find a more consistant pattern of color suppression in B meson decays. However, the experimental results on B meson decays have clearly shown that large N_c limit does not work well.

By comparing B^- and $\overline{B}^0$ decays, $|a_1|$, $|a_2|$ and the relative sign of a_2/a_1 can be determined. CLEO [24, 25] data clearly indicate a constructive interference in charged B decays, $B \to PP$, in sharp contrast to charm decays and hence a positive value of a_2, while the sign of a_1 stays same. Thus, $\overline{B}^0 \to D^+\pi^-$ yield [24, 25]

$$|a_1| = 1.03 \pm 0.04 \pm 0.16, \tag{2.68}$$

$\overline{B}^0 \to \psi X$ decays yield [24]:

$$|a_2| = 0.23 \pm 0.01 \pm 0.01, \tag{2.69}$$

and data on $B^- \to D^0\pi^- / D^0\rho^- / D^{*0}\pi^- / D^{*0}\rho^-$ clearly yield [24, 25]

$$\frac{a_2}{a_1} = 0.25 \pm 0.07 \pm 0.06.$$

By comparing branching ratios of B^- and $\overline{B}^0$ decay modes it is possible to determine the sign of a_2 relative to a_1. The BSW model, predicts the following ratios [24]:

Mode	**Destructive**	**Constructive**	**Experimental**
$\frac{B(B^- \to D^0\pi^-)}{B(B^- \to D^+\pi^-)}$	0.56×10^{-3}	1.73×10^{-3}	$(1.81 \pm 0.10) \times 10^{-3}$
$\frac{B(B^- \to D^0K^-)}{B(B^- \to D^+K^-)}$	0.60×10^{-3}	1.65×10^{-3}	$(2.00 \pm 0.61) \times 10^{-3}$

These ideas have been applied to investigate the weak hadronic decays of the heavy flavor hadrons. Theoretical focus has, so far, been on the weak hadronic decays emitting *s*-wave mesons, $B \to PP / PV / VV$, in the final state [32, 33]. It may be noted that, *B* mesons, being heavy, can also emit *p*-wave mesons [34, 35] like axial-vector (*A*), tensor (*T*) and scalar (*S*) mesons. Experimentally, there exists a reasonable amount of data on branching ratios of axial-vector, tensor and scalar mesons emitting decays of bottom mesons, which has recently attracted the interest of physicists. On the experimental side, we expect numerous experimental observations of heavy flavor decays involving *p*-wave mesons due to the growing experimental facilities at BARBAR, DELPHI, Belle, CLEO, CDF etc. Therefore, we study *p*-wave emitting decays of bottom mesons in the subsequent chapters.

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CHAPTER 3

HADRONIC WEAK DECAYS OF BOTTOM MESONS EMITTING PSEUDOSCALAR AND AXIAL-VECTOR MESONS

3.1 INTRODUCTION

Phenomenological quark model approach has been quite successful in the understanding of semileptonic decays of *B* mesons. For weak hadronic decays emitting *s*-wave mesons, the factorization hypothesis [1, 11] worked reasonably well to explain the available experimental data. Besides these decays, *B* mesons, being heavy, can also emit *p*-wave mesons [12-15], axial-vector (*A*), tensor (*T*) and scalar (*S*) mesons. In this chapter, we investigate axial-vector emitting decays of B^{-}, B^{0} and B_s^{0} mesons, which are the bound state of *b* quark and a light antiquark (*u, d* or *s*), in the CKM-favored and CKM-suppressed modes. Since *B* decays has six selection rules for $b \to c$ and $b \to u$ transitions, large number of such decays are possible to occur. Selection rules for various decay modes generated are given below:

1. CKM-enhanced modes:

$$\Delta b = 1, \Delta C = 1, \Delta S = 0; \quad \Delta b = 1, \Delta C = 0, \Delta S = -1,$$

2. CKM-suppressed modes:

$$\Delta b = 1, \Delta C = 1, \Delta S = -1; \quad \Delta b = 1, \Delta C = 0, \Delta S = 0,$$

3. CKM-doubly-suppressed modes:

$$\Delta b = 1, \Delta C = \Delta S = -1; \quad \Delta b = 1\ \Delta C = -1, \Delta S = 0.$$

On the experimental side, many of these decays have been observed that require theoretical understanding. In the following, experimentally [16], a few $B \to PA$ (where A represents an axial-vector meson) decays have been measured and few upper limits are available:

$$B(B^- \to D^0 a_1^-) = (4 \pm 4) \times 10^{-3},$$

$$B(B^- \to \pi^- D_1^0) = (1.5 \pm 0.6) \times 10^{-3},$$

$$B(B^- \to \pi^- \chi_{c1}) = (2.2 \pm 0.5) \times 10^{-5},$$

$$B(B^- \to K^- \chi_{c1}) = (4.9 \pm 0.5) \times 10^{-4},$$

$$B(B^- \to K^0 a_1^-) = (3.5 \pm 0.7) \times 10^{-5},$$

$$B(B^- \to \pi^0 a_1^-) = (26.0 \pm 7.0) \times 10^{-6},$$

$$B(B^- \to \pi^- a_1^0) = (20.0 \pm 6.0) \times 10^{-6},$$

$$B(\overline{B}^0 \to D^+ a_1^-) = (0.6 \pm 0.3) \times 10^{-2},$$

$$B(\overline{B}^0 \to \overline{K}^0 \chi_{c1}) = (3.9 \pm 0.4) \times 10^{-4},$$

$$B(\overline{B}^0 \to K^- a_1^+) = (16.0 \pm 4.0) \times 10^{-6}$$

$$B(\overline{B}^0 \to \pi^\mp a_1^\pm) = (33.2 \pm 5.0) \times 10^{-6},$$

$$B(B^- \to D_s^- a_1^0) < 1.8 \times 10^{-3},$$

$$B(B^- \to \pi^0 K_1^-(1400)) < 2.6\times 10^{-3},$$

$$B(\overline{B}^0 \to D_s^+ a_1^-) < 2.2\times 10^{-3},$$

$$B(\overline{B}^0 \to \pi^+ K_1^-(1400)) < 1.1\times 10^{-3},$$

$$B(\overline{B}^0 \to \pi^0 a_1^0) < 1.1\times 10^{-3}.$$

Using the factorization scheme to obtain the decay amplitudes, we calculate the branching ratios of these decay modes. Naively, the *p*-wave meson emitting decays of hadrons are expected to be suppressed kinematically due to the large mass of these meson resonances. However, we find some of these decay channels have branching ratios comparable to that of the *s*-wave mesons emitting decay modes and can be within the reach of future experiments.

3.2 AXIAL-VECTOR MESON SPECTROSCOPY

Experimentally [16], two types of the axial-vector mesons exist with different charge conjugations, i.e. ${}^3P_1(J^{PC}=1^{++})$ and ${}^1P_1\,(J^{PC}=1^{+-})$, behave well with respect to the quark model $q\overline{q}$ assignments. Strange and charmed states are most likely a mixture of ${}^3P_1$ and ${}^1P_1$ states, since there is no quantum number forbidding such mixing. In contrast, hidden-flavor diagonal ${}^3P_1$ and ${}^1P_1$ states have opposite C-parity and therefore cannot mix. The following non-strange and uncharmed mesons have been observed:

1. For ${}^3P_1$ multiplet,

i) Three isovector $a_1(1.230)$ with the quark content $u\overline{d}$, $u\overline{u}-d\overline{d}/\sqrt{2}$ and $d\overline{u}$, respectively, are

$$a_1^+,\ a_1^0 \text{ and } a_1^- \tag{3.1}$$

ii) Four isoscalars $f_1(1.285)$, $f_1(1.420)$, $f_1'(1.512)$ and $\chi_{c1}(3.511)$, out of which $f_1(1.420)$ is a multiquark state in the form of a $K\bar{K}\pi$ bound state [17] or a $K\bar{K}^*$ deuteron-state [18].

2. For $^1P_1$ multiplet,

i) isovector $b_1(1.229)$ with flavor content same as given in (3.1) and

ii) three isoscalars $h_1(1.170)$, $h_1'(1.380)$ and $h_{c1}(3.526)$. C-parity of $h_1'(1.380)$ and spin and parity of the $h_{c1}(3.526)$ remains to be confirmed.

Note that the numbers given in the brackets indicate mass (in GeV) of the respective mesons, and hereafter we use the same convention. In the present analysis, mixing of the isoscalar states of (1^{++}) mesons is defined as

$$f_1(1.285) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\phi_A + (s\bar{s})\sin\phi_A,$$

$$f_1'(1.512) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\phi_A - (s\bar{s})\cos\phi_A, \tag{3.2}$$

$$\chi_{c1}(3.511) = (c\bar{c}),$$

where $$\phi_A = \theta(ideal) - \theta_A(physical).$$

Similarly, mixing of isoscalar (1^{+-}) mesons is given by

$$h_1(1.170) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\phi_{A'} + (s\bar{s})\sin\phi_{A'},$$

$$h_1'(1.380) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\phi_{A'} - (s\bar{s})\cos\phi_{A'}, \tag{3.3}$$

$$h_{c1}(3.526) = (c\bar{c}),$$

with $$\phi_{A'} = \theta(ideal) - \theta_{A'}(physical). \tag{3.4}$$

Proximity of $a_1(1.230)$ and $f_1(1.285)$ and to a lesser extent that of $b_1(1.229)$ and $h_1(1.170)$ indicates the ideal mixing for both 1^{++} and 1^{+-} nonets, i.e.

$$\phi_A = \phi_{A'} = 0^\circ. \tag{3.5}$$

This is also supported by their decay patterns. $f_1(1.285)$ decays predominantly to 4π and $\eta\pi\pi$, while $f_1^{'}(1.512)$ decays to $K\overline{K}\pi$. Similarly, $h_1(1.170)$ decays predominantly to $\rho\pi$ and $h_1^{'}(1.380)$ decays to $K\overline{K}^*$ and $\overline{K}K^*$ states.

States involving a strange quark of $A(J^{PC}=1^{++})$ and $A'(J^{PC}=1^{+-})$ mesons ($s\overline{u}$ or $s\overline{d}$) mix to generate the physical states in the following manner:

$$\begin{aligned} K_1(1.270) &= K_{1A}\sin\theta_1 + K_{1A'}\cos\theta_1, \\ \underline{K}_1(1.400) &= K_{1A}\cos\theta_1 - K_{1A'}\sin\theta_1. \end{aligned} \tag{3.6}$$

where K_{1A} and $K_{1A'}$ denote the strange partners of $a_1(1.230)$ and $b_1(1.229)$ respectively. Particle Data Group [16] assumes that the mixing is maximal, i.e. $\theta_1 = 45^0$, whereas $\tau \to K_1(1.270)/K_1(1.400) + \nu_\tau$ data yields $\theta_1 = \pm 37^0$ and $\theta_1 = \pm 58^0$ [19]. However, the study of $D \to K_1(1.270)\pi, K_1(1.400)\pi$ decays rules out positive mixing-angle solutions. As $D \to K_1^-(1.400)\pi^+$ gets largely suppressed for $\theta_1 = -37^0$ the solution $\theta_1 = -58^0$ [13] is experimentally favored, which is used in our analysis.

The mixing of charmed ($c\overline{u}$ or $c\overline{d}$) and strange charmed ($c\overline{s}$) states mesons is similarly given by

$$\begin{aligned} D_1(2.427) &= D_{1A}\sin\theta_{D_1} + D_{1A'}\cos\theta_{D_1}, \\ \underline{D}_1(2.422) &= D_{1A}\cos\theta_{D_1} - D_{1A'}\sin\theta_{D_1}, \end{aligned} \tag{3.7}$$

and

$$D_{s1}(2.460) = D_{s1A}\sin\theta_{D_{s1}} + D_{s1A'}\cos\theta_{D_{s1}},$$
$$\underline{D}_{s1}(2.535) = D_{s1A}\cos\theta_{D_{s1}} - D_{s1A'}\sin\theta_{D_{s1}}, \tag{3.8}$$

However, in the heavy quark limit, the physical mass eigenstates with $J^P = 1^+$ are $P_1^{3/2}$ and $P_1^{1/2}$ rather than $^3P_1$ and $^1P_1$ states as the heavy quark spin S_Q decouples from the other degrees of freedom, so that S_Q and the total angular momentum of the light antiquark are separately good quantum numbers. Therefore, we can write

$$|P_1^{1/2}> = -\sqrt{\frac{1}{3}}\,|^1P_1> + \sqrt{\frac{2}{3}}\,|^3P_1>,$$
$$|P_1^{3/2}> = \sqrt{\frac{2}{3}}\,|^1P_1> + \sqrt{\frac{1}{3}}\,|^3P_1>. \tag{3.9}$$

Hence, the states $D_1(2.427)$ and $\underline{D}_1(2.422)$ can be identified with $P_1^{1/2}$ and $P_1^{3/2}$, respectively. However, beyond the heavy quark limit, there is a mixing between $P_1^{1/2}$ and $P_1^{3/2}$ denoted by

$$D_1(2.427) = D_1^{1/2}\cos\theta_2 + D_1^{3/2}\sin\theta_2,$$
$$\underline{D}_1(2.422) = -D_1^{1/2}\sin\theta_2 + D_1^{3/2}\cos\theta_2. \tag{3.10}$$

Likewise for strange axial-vector charmed mesons,

$$D_{s1}(2.460) = D_{s1}^{1/2}\cos\theta_3 + D_{s1}^{3/2}\sin\theta_3,$$
$$\underline{D}_{s1}(2.535) = -D_{s1}^{1/2}\sin\theta_3 + D_{s1}^{3/2}\cos\theta_3. \tag{3.11}$$

The mixing angle $\theta_2 = (-5.7 \pm 2.4)^\circ$ is obtained by Belle through a detailed $B \to D^*\pi\pi$ analysis [20, 21], while $\theta_3 \approx 7^\circ$ is determined from the quark potential model [13].

3.3 DECAY AMPLITUDES AND RATES

The decay rate formula for $B(0^-) \to P(0^-) + A(1^+)$ decays is given by

$$\Gamma(B \to P\,A) = \frac{p_c^3}{8\pi m_A^2} \left| A(B \to P\,A) \right|^2, \tag{3.12}$$

where p_c is the magnitude of the three-momentum of a final-state particle in the rest frame of B meson and m_A denotes the mass of the axial-vector meson.

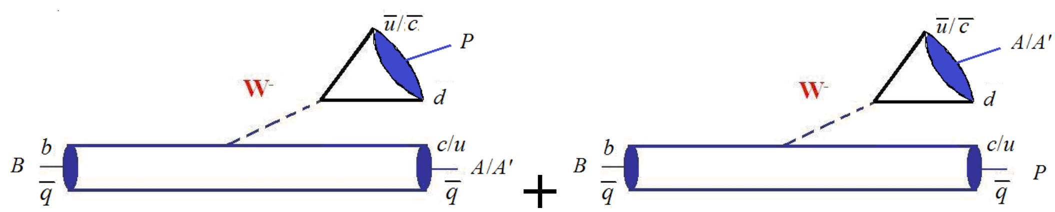


Figure 3.1 $B \to PA$ decay amplitude in factorization scheme

The factorization scheme expresses the decay amplitudes as a product of the matrix elements of weak currents (up to the weak scale factor of $\frac{G_F}{\sqrt{2}} \times$ CKM elements×QCD factor) like

$$\begin{aligned} \langle PA|H_w|B\rangle &\sim \langle P|J^\mu|0\rangle\langle A|J_\mu|B\rangle + \langle A|J^\mu|0\rangle\langle P|J_\mu|B\rangle, \\ \langle PA'|H_w|B\rangle &\sim \langle P|J^\mu|0\rangle\langle A'|J_\mu|B\rangle + \langle A'|J^\mu|0\rangle\langle P|J_\mu|B\rangle. \end{aligned} \tag{3.13}$$

as shown in Figure 3.1.

Using Lorentz invariance, matrix elements of the current between meson states can be expressed [3, 4] as

$$\langle P(k)|A_\mu|0\rangle = -if_P k_\mu ,$$

$$\langle A(k_A,\in)|A_\mu|0\rangle = \in^*_\mu m_A f_A ,$$

$$\langle A'(k_{A'},\in)|A_\mu|0\rangle = \in^*_\mu m_{A'} f_{A'} , \tag{3.14}$$

$$\langle A(k_A,\in)|V_\mu|B(k_B)\rangle = l\in^*_\mu + c_+(\in^*\cdot k_B)(k_B+k_A)_\mu + c_-(\in^*\cdot k_B)(k_B-k_A)_\mu ,$$

$$\langle A'(k_{A'},\in)|V_\mu|B(k_B)\rangle = r\in^*_\mu + s_+(\in^*\cdot k_B)(k_B+k_{A'})_\mu + s_-(\in^*\cdot k_B)(k_B-k_{A'})_\mu .$$

Finally the decay amplitudes can be expressed as

$$\begin{aligned} A(B\to PA) &= (\,2m_A f_A F_1^{B\to P}(m_A^2) + f_P F^{B\to A}(m_P^2)), \\ A(B\to PA') &= (\,2m_{A'} f_{A'} F_1^{B\to P}(m_{A'}^2) + f_P F^{B\to A'}(m_P^2)), \end{aligned} \tag{3.15}$$

where

$$F^{\,B\to A}(m_P^2) = l(m_P^2) + (m_B^2 - m_A^2)\; c_+(m_P^2) + m_P^2\; c_-(m_P^2),$$

$$F^{\,B\to A'}(m_P^2) = r\,(m_P^2) + (m_B^2 - m_{A'}^2)\; s_+(m_P^2) + m_P^2\; s_-(m_P^2). \tag{3.16}$$

Sandwiching the weak Hamiltonian between the initial and final states, we obtain decay amplitudes of B^-, $\overline{B}^0$ and $\overline{B}^0_s$ mesons for the various decay modes as given in the Tables 3.1, 3.2, 3.3 (a) and 3.3 (b).

Table 3.1 Decay Amplitudes for $B \to PA$ decays in CKM-favored mode involving $b \to c$ transition

Decays	Amplitudes
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^*$
$B^- \to \pi^- D_1^0$	$a_1 f_\pi (\sin\theta_2 F^{B\to D_{1A}}(m_\pi^2) + \cos\theta_2 F^{B\to D_{1A'}}(m_\pi^2))$ $+ 2m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 F^{B\to\pi}(m_{D_{1A}}^2) + f_{D_{1A'}} \cos\theta_2 F^{B\to\pi}(m_{D_{1A'}}^2))$
$B^- \to \pi^- \underline{D}_1^0$	$a_1 f_\pi (\cos\theta_2 F^{B\to \underline{D}_{1A}}(m_\pi^2) - \sin\theta_2 F^{B\to \underline{D}_{1A'}}(m_\pi^2))$ $+ 2m_{\underline{D}_1} a_2 (f_{\underline{D}_{1A}} \cos\theta_2 F^{B\to\pi}(m_{\underline{D}_{1A}}^2) - f_{\underline{D}_{1A'}} \sin\theta_2 F^{B\to\pi}(m_{\underline{D}_{1A'}}^2))$
$B^- \to D^0 a_1^-$	$a_2 f_D F^{B\to a_1}(m_D^2) + 2a_1 m_{a_1} f_{a_1} F^{B\to D}(m_{a_1}^2)$
$B^- \to D^0 b_1^-$	$a_2 f_D F^{B\to b_1}(m_D^2) + 2a_1 m_{b_1} f_{b_1} F^{B\to D}(m_{b_1}^2)$
$\overline{B}^0 \to \pi^0 D_1^0$	$\sqrt{2} m_{D_1} a_2 (-f_{D_{1A}} \sin\theta_2 F^{\overline{B}\to\pi}(m_{D_1}^2) - f_{D_{1A'}} \cos\theta_2 F^{\overline{B}\to\pi}(m_{D_1}^2))$
$\overline{B}^0 \to \pi^0 \underline{D}_1^0$	$\sqrt{2} m_{D_1} a_2 (-f_{D_{1A}} \cos\theta_2 F^{\overline{B}\to\pi}(m_{D_1}^2) + f_{D_{1A'}} \sin\theta_2 F^{\overline{B}\to\pi}(m_{D_1}^2))$
$\overline{B}^0 \to \pi^- D_1^+$	$a_1 f_\pi (\sin\theta_2 F^{\overline{B}\to D_{1A}}(m_\pi^2) + \cos\theta_2 F^{\overline{B}\to D_{1A'}}(m_\pi^2))$
$\overline{B}^0 \to \pi^- \underline{D}_1^+$	$a_1 f_\pi (\cos\theta_2 F^{\overline{B}\to D_{1A}}(m_\pi^2) - \sin\theta_2 F^{\overline{B}\to D_{1A'}}(m_\pi^2))$
$\overline{B}^0 \to \eta D_1^0$	$\sqrt{2} m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 \sin\phi_P F^{\overline{B}\to\eta}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \sin\phi_P F^{\overline{B}\to\eta}(m_{D_1}^2))$
$\overline{B}^0 \to \eta \underline{D}_1^0$	$\sqrt{2} m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 \sin\phi_P F^{\overline{B}\to\eta}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \sin\phi_P F^{\overline{B}\to\eta}(m_{\underline{D}_1}^2))$
$\overline{B}^0 \to \eta' D_1^0$	$\sqrt{2} m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 \cos\phi_P F^{\overline{B}\to\eta'}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \cos\phi_P F^{\overline{B}\to\eta'}(m_{D_1}^2))$
$\overline{B}^0 \to \eta' \underline{D}_1^0$	$\sqrt{2} m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 \cos\phi_P F^{\overline{B}\to\eta'}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \cos\phi_P F^{\overline{B}\to\eta'}(m_{\underline{D}_1}^2))$
$\overline{B}^0 \to D^+ a_1^-$	$2a_1 m_{a_1} f_{a_1} F^{\overline{B}\to D}(m_{a_1}^2)$
$\overline{B}^0 \to D^+ b_1^-$	$2a_1 m_{b_1} f_{b_1} F^{\overline{B}\to D}(m_{b_1}^2)$
$\overline{B}^0 \to D^0 a_1^0$	$-\frac{1}{\sqrt{2}} a_2 f_D F^{\overline{B}\to a_1}(m_D^2)$
$\overline{B}^0 \to D^0 f_1$	$\frac{1}{\sqrt{2}} a_2 f_D \cos\phi_A F^{\overline{B}\to f_1}(m_D^2)$
$\overline{B}^0 \to D^0 b_1^0$	$-\frac{1}{\sqrt{2}} a_2 f_D F^{\overline{B}\to b_1}(m_D^2)$
$\overline{B}^0 \to D^0 h_1$	$\frac{1}{\sqrt{2}} a_2 f_D \cos\phi_{A'} F^{\overline{B}\to h_1}(m_D^2)$
$\overline{B}_s^0 \to K^0 D_1^0$	$2m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 F^{B_s\to K}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B_s\to K}(m_{D_1}^2))$
$\overline{B}_s^0 \to K^0 \underline{D}_1^0$	$2m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2))$
$\overline{B}_s^0 \to \pi^- D_{s1}^+$	$a_1 f_\pi (\sin\theta_3 F^{B_s\to D_{s1A}}(m_{D_1}^2) + \cos\theta_3 F^{B_s\to D_{s1A'}}(m_{D_1}^2))$

$\bar{B}_s^0 \to \pi^- \underline{D}_{s1}^+$	$a_1 f_\pi (\cos\theta_3 F^{B_s \to \underline{D}_{s1A}}(m_\pi^2) - \sin\theta_3 F^{B_s \to \underline{D}_{s1A'}}(m_\pi^2))$
$\bar{B}_s^0 \to D^0 K_1^0$	$a_2 f_D (\sin\theta_1 F^{B_s \to K_{1A}}(m_D^2) + \cos\theta_1 F^{B_s \to K_{1A'}}(m_D^2))$
$\bar{B}_s^0 \to D^0 \underline{K}_1^0$	$a_2 f_D (\cos\theta_1 F^{B_s \to \underline{K}_{1A}}(m_D^2) - \sin\theta_1 F^{B_s \to \underline{K}_{1A'}}(m_D^2))$
$\bar{B}_s^0 \to D_s^+ a_1^-$	$2a_1 m_{a_1} f_{a_1} F^{B_s \to D_s}(m_{a_1}^2)$
$\bar{B}_s^0 \to D_s^+ b_1^-$	$2a_1 m_{b_1} f_{b_1} F^{B_s \to D_s}(m_{b_1}^2)$
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^*$
$B^- \to K^- \chi_{c1}$	$2m_{\chi_{c1}} a_2 f_{\chi_{c1}} F^{B \to K}(m_{\chi_{c1}}^2)$
$B^- \to D^0 D_{s1}^-$	$2a_1 m_{D_{s1}} (f_{D_{s1A}} \sin\theta_3 F^{B \to D}(m_{D_{s1}}^2) + f_{D_{s1A'}} \cos\theta_3 F^{B \to D}(m_{D_{s1}}^2))$
$B^- \to D^0 \underline{D}_{s1}^-$	$2a_1 m_{\underline{D}_{s1}} (f_{D_{s1A}} \cos\theta_3 F^{B \to D}(m_{\underline{D}_{s1}}^2) - f_{D_{s1A'}} \sin\theta_3 F^{B \to D}(m_{\underline{D}_{s1}}^2))$
$B^- \to D_s^- D_1^0$	$a_1 f_{D_s} (\sin\theta_2 F^{B \to D_{1A}}(m_{D_s}^2) + \cos\theta_2 F^{B \to D_{1A'}}(m_{D_s}^2))$
$B^- \to D_s^- \underline{D}_1^0$	$a_1 f_{D_s} (\cos\theta_2 F^{B \to D_{1A}}(m_{D_s}^2) - \sin\theta_2 F^{B \to D_{1A'}}(m_{D_s}^2))$
$B^- \to \eta_c K_1^-$	$a_2 f_{\eta_c} (\sin\theta_1 F^{B \to K_{1A}}(m_{\eta_c}^2) + \cos\theta_1 F^{B \to K_{1A'}}(m_{\eta_c}^2))$
$B^- \to \eta_c \underline{K}_1^-$	$a_2 f_{\eta_c} (\cos\theta_1 F^{B \to K_{1A}}(m_{\eta_c}^2) - \sin\theta_1 F^{B \to K_{1A'}}(m_{\eta_c}^2))$
$\bar{B}^0 \to \bar{K}^0 \chi_{c1}$	$2m_{\chi_{c1}} a_2 f_{\chi_{c1}} F^{\bar{B} \to K}(m_{\chi_{c1}}^2)$
$\bar{B}^0 \to D^+ D_{s1}^-$	$2a_1 m_{D_{s1}} (f_{D_{s1A}} \sin\theta_3 F^{\bar{B} \to D}(m_{D_{s1}}^2) + f_{D_{s1A'}} \cos\theta_3 F^{\bar{B} \to D}(m_{D_{s1}}^2))$
$\bar{B}^0 \to D^+ \underline{D}_{s1}^-$	$2a_1 m_{\underline{D}_{s1}} (f_{D_{s1A}} \sin\theta_3 F^{\bar{B} \to D}(m_{\underline{D}_{s1}}^2) + f_{D_{s1A'}} \cos\theta_3 F^{\bar{B} \to D}(m_{\underline{D}_{s1}}^2))$
$\bar{B}^0 \to D_s^- D_1^+$	$a_1 f_{D_s} (\sin\theta_2 F^{B \to D_{1A}}(m_{D_s}^2) + \cos\theta_2 F^{B \to D_{1A'}}(m_{D_s}^2))$
$\bar{B}^0 \to D_s^- \underline{D}_1^+$	$a_1 f_{D_s} (\cos\theta_2 F^{B \to D_{1A}}(m_{D_s}^2) - \sin\theta_2 F^{B \to D_{1A'}}(m_{D_s}^2))$
$\bar{B}^0 \to \eta_c \bar{K}_1^0$	$a_2 f_{\eta_c} (\sin\theta_1 F^{\bar{B} \to K_{1A}}(m_{\eta_c}^2) + \cos\theta_1 F^{\bar{B} \to K_{1A'}}(m_{\eta_c}^2))$
$\bar{B}^0 \to \eta_c \underline{\bar{K}}_1^0$	$a_2 f_{\eta_c} (\cos\theta_1 F^{\bar{B} \to K_{1A}}(m_{\eta_c}^2) - \sin\theta_1 F^{\bar{B} \to K_{1A'}}(m_{\eta_c}^2))$
$\bar{B}_s^0 \to \eta \chi_{c1}$	$2m_{\chi_{c1}} a_2 f_{\chi_{c1}} \cos\phi_P F^{B_s \to \eta}(m_{\chi_{c1}}^2)$
$\bar{B}_s^0 \to \eta' \chi_{c1}$	$2m_{\chi_{c1}} a_2 f_{\chi_{c1}} \sin\phi_P F^{B_s \to \eta'}(m_{\chi_{c1}}^2)$
$\bar{B}_s^0 \to D_s^+ D_{s1}^-$	$2a_1 m_{D_{s1}} (f_{D_{s1A}} \sin\theta_3 F^{B_s \to D}(m_{D_{s1}}^2) + f_{D_{s1A'}} \cos\theta_3 F^{B_s \to D}(m_{D_{s1}}^2))$
$\bar{B}_s^0 \to D_s^+ \underline{D}_{s1}^-$	$2a_1 m_{\underline{D}_{s1}} (f_{D_{s1A}} \cos\theta_3 F^{B_s \to D}(m_{\underline{D}_{s1}}^2) - f_{D_{s1A'}} \sin\theta_3 F^{B_s \to D}(m_{\underline{D}_{s1}}^2))$
$\bar{B}_s^0 \to D_s^- D_{s1}^+$	$a_1 f_{D_s} (\sin\theta_3 F^{B_s \to D_{s1A}}(m_{D_s}^2) + \cos\theta_3 F^{B_s \to D_{s1A'}}(m_{D_s}^2))$
$\bar{B}_s^0 \to D_s^- \underline{D}_{s1}^+$	$a_1 f_{D_s} (\cos\theta_3 F^{B_s \to D_{s1A}}(m_{D_s}^2) - \sin\theta_3 F^{B_s \to D_{s1A'}}(m_{D_s}^2))$
$\bar{B}_s^0 \to \eta_c f_1'$	$-a_2 f_{\eta_c} F^{B_s \to f_1'}(m_{\eta_c}^2)$
$\bar{B}_s^0 \to \eta_c h_1'$	$-a_2 f_{\eta_c} F^{B_s \to h_1'}(m_{\eta_c}^2)$

Table 3.2 Decay Amplitudes for $B \to PA$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Amplitudes
$\Delta b=1, \Delta C=1, \Delta S=-1$	$\times\frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*$
$B^- \to K^- D_1^0$	$a_1 f_K(\sin\theta_2 F^{B\to D_{1A}}(m_K^2)+\cos\theta_2 F^{B\to D_{1A'}}(m_K^2))$ $+2m_{D_1}a_2(f_{D_{1A}}\sin\theta_2 F^{B\to K}(m_{D_1}^2)+f_{D_{1A'}}\cos\theta_2 F^{B\to K}(m_{D_1}^2))$
$B^- \to K^- \underline{D}_1^0$	$a_1 f_K(\cos\theta_2 F^{B\to D_{1A}}(m_K^2)-\sin\theta_2 F^{B\to D_{1A'}}(m_K^2))$ $+2m_{\underline{D}_1}a_2(f_{D_{1A}}\cos\theta_2 F^{B\to K}(m_{\underline{D}_1}^2)-f_{D_{1A'}}\sin\theta_2 F^{B\to K}(m_{\underline{D}_1}^2))$
$B^- \to D^0 K_1^-$	$a_2 f_D(\sin\theta_1 F^{B\to K_{1A}}(m_D^2)+\cos\theta_1 F^{B\to K_{1A'}}(m_D^2))$ $+2m_{K_1}a_1(f_{K_{1A}}\sin\theta_1 F^{B\to D}(m_{K_1}^2)+f_{K_{1A'}}\cos\theta_1 F^{B\to D}(m_{K_1}^2))$
$B^- \to D^0 \underline{K}_1^-$	$a_2 f_D(\cos\theta_1 F^{B\to K_{1A}}(m_D^2)-\sin\theta_1 F^{B\to K_{1A'}}(m_D^2))$ $+2m_{K_1}a_1(f_{K_{1A}}\cos\theta_1 F^{B\to D}(m_{K_1}^2)-f_{K_{1A'}}\sin\theta_1 F^{B\to D}(m_{K_1}^2))$
$\overline{B}^0 \to \overline{K}^0 D_1^0$	$2m_{D_1}a_2(f_{D_{1A}}\sin\theta_2 F^{\overline{B}\to K}(m_{D_1}^2)+f_{D_{1A'}}\cos\theta_2 F^{\overline{B}\to K}(m_{D_1}^2))$
$\overline{B}^0 \to \overline{K}^0 \underline{D}_1^0$	$2m_{\underline{D}_1}a_2(f_{D_{1A}}\cos\theta_2 F^{\overline{B}\to K}(m_{\underline{D}_1}^2)-f_{D_{1A'}}\sin\theta_2 F^{\overline{B}\to K}(m_{\underline{D}_1}^2))$
$\overline{B}^0 \to K^- D_1^+$	$a_1 f_K(\sin\theta_2 F^{\overline{B}\to D_{1A}}(m_K^2)+\cos\theta_2 F^{\overline{B}\to D_{1A'}}(m_K^2))$
$\overline{B}^0 \to K^- \underline{D}_1^+$	$a_1 f_K(\cos\theta_2 F^{\overline{B}\to D_{1A}}(m_K^2)-\sin\theta_2 F^{\overline{B}\to D_{1A'}}(m_K^2))$
$\overline{B}^0 \to D^+ K_1^-$	$2m_{K_1}a_1(f_{K_{1A}}\sin\theta_1 F^{\overline{B}\to D}(m_{K_1}^2)+f_{K_{1A'}}\cos\theta_1 F^{\overline{B}\to D}(m_{K_1}^2))$
$\overline{B}^0 \to D^+ \underline{K}_1^-$	$2m_{\underline{K}_1}a_1(f_{K_{1A}}\cos\theta_1 F^{\overline{B}\to D}(m_{\underline{K}_1}^2)-f_{K_{1A'}}\sin\theta_1 F^{\overline{B}\to D}(m_{\underline{K}_1}^2))$
$\overline{B}^0 \to D^0 \overline{K}_1^0$	$a_2 f_D(\sin\theta_1 F^{\overline{B}\to K_{1A}}(m_D^2)+\cos\theta_1 F^{\overline{B}\to K_{1A'}}(m_D^2))$
$\overline{B}^0 \to D^0 \overline{\underline{K}}_1^0$	$a_2 f_D(\cos\theta_1 F^{\overline{B}\to K_{1A}}(m_D^2)-\sin\theta_1 F^{\overline{B}\to K_{1A'}}(m_D^2))$
$\overline{B}_s^0 \to \eta D_1^0$	$2m_{D_1}a_2(-f_{D_{1A}}\sin\theta_2\cos\phi_P F^{B_s\to\eta}(m_{D_1}^2)-f_{D_{1A'}}\cos\theta_2\cos\phi_P F^{B_s\to\eta}(m_{D_1}^2))$
$\overline{B}_s^0 \to \eta \underline{D}_1^0$	$2m_{\underline{D}_1}a_2(-f_{D_{1A}}\cos\theta_2\cos\phi_P F^{B_s\to\eta}(m_{\underline{D}_1}^2)+f_{D_{1A'}}\sin\theta_2\cos\phi_P F^{B_s\to\eta}(m_{\underline{D}_1}^2))$
$\overline{B}_s^0 \to K^- D_{s1}^+$	$a_1 f_K(\sin\theta_3 F^{B_s\to D_{s1A}}(m_K^2)+\cos\theta_3 F^{B_s\to D_{s1A'}}(m_K^2))$
$\overline{B}_s^0 \to K^- \underline{D}_{s1}^+$	$a_1 f_K(\cos\theta_3 F^{B_s\to D_{s1A}}(m_K^2)-\sin\theta_3 F^{B_s\to D_{s1A'}}(m_K^2))$
$\overline{B}_s^0 \to \eta' D_1^0$	$2m_{D_1}a_2(f_{D_{1A}}\sin\theta_2\sin\phi_P F^{B_s\to\eta'}(m_{D_1}^2)+f_{D_{1A'}}\cos\theta_2\sin\phi_P F^{B_s\to\eta'}(m_{D_1}^2))$
$\overline{B}_s^0 \to \eta' \underline{D}_1^0$	$2m_{\underline{D}_1}a_2(f_{D_{1A}}\cos\theta_2\sin\phi_P F^{B_s\to\eta'}(m_{\underline{D}_1}^2)-f_{D_{1A'}}\sin\theta_2\sin\phi_P F^{B_s\to\eta'}(m_{\underline{D}_1}^2)$
$\overline{B}_s^0 \to D^0 f_1'$	$-a_2 f_D\cos\theta_A F^{B_s\to f_1'}(m_D^2)$
$\overline{B}_s^0 \to D^0 h_1'$	$-a_2 f_D\cos\theta_{A'} F^{B_s\to h_1'}(m_D^2)$
$\overline{B}_s^0 \to D_s^+ K_1^-$	$2m_{K_1}a_1(f_{K_{1A}}\sin\theta_1 F^{B_s\to D_s}(m_{K_1}^2)+f_{K_{1A'}}\cos\theta_1 F^{B_s\to D_s}(m_{K_1}^2)$
$\overline{B}_s^0 \to D_s^+ \underline{K}_1^-$	$2m_{\underline{K}_1}a_1(f_{K_{1A}}\cos\theta_1 F^{B_s\to D_s}(m_{\underline{K}_1}^2)-f_{K_{1A'}}\sin\theta_1 F^{B_s\to D_s}(m_{\underline{K}_1}^2)$

$\Delta b=1, \Delta C=0, \Delta S=0$	$\times\frac{G_F}{\sqrt{2}}V_{cb}V_{cd}^*$
$B^- \to \pi^- \chi_{c1}$	$-2m_{\chi_{c1}} a_2 f_{\chi_{c1}} F^{B\to\pi}(m_{\chi_{c1}}^2)$
$B^- \to D^0 D_1^-$	$-2m_{D_1} a_1 (f_{D_{1A}} \sin\phi_2 F^{B\to D}(m_{D_1}^2) + f_{D_{1A'}} \cos\phi_2 F^{B\to D}(m_{D_1}^2))$
$B^- \to D^0 \underline{D}_1^-$	$-2m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B\to D}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B\to D}(m_{\underline{D}_1}^2))$
$B^- \to D^- D_1^0$	$-a_1 f_D (\sin\theta_2 F^{B\to D_{1A}}(m_D^2) + \cos\theta_2 F^{B\to D_{1A'}}(m_D^2))$
$B^- \to D^- \underline{D}_1^0$	$-a_1 f_D (\cos\theta_2 F^{B\to D_{1A}}(m_D^2) - \sin\theta_2 F^{B\to D_{1A'}}(m_D^2))$
$B^- \to \eta_c a_1^-$	$-a_2 f_{\eta_c} F^{B\to a_1}(m_{\eta_c}^2)$
$B^- \to \eta_c b_1^-$	$-a_2 f_{\eta_c} F^{B\to b_1}(m_{\eta_c}^2)$
$\overline{B}^0 \to \pi^0 \chi_{c1}$	$\sqrt{2} m_{\chi_{c1}} a_2 f_{\chi_{c1}} F^{\overline{B}\to\pi}(m_{\chi_{c1}}^2)$
$\overline{B}^0 \to \eta \chi_{c1}$	$-\sqrt{2} m_{\chi_{c1}} a_2 f_{\chi_{c1}} \sin\phi_P F^{\overline{B}\to\eta}(m_{\chi_{c1}}^2)$
$\overline{B}^0 \to \eta' \chi_{c1}$	$-\sqrt{2} m_{\chi_{c1}} a_2 f_{\chi_{c1}} \cos\phi_P F^{\overline{B}\to\eta'}(m_{\chi_{c1}}^2)$
$\overline{B}^0 \to D^+ D_1^-$	$-2m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{\overline{B}\to D}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{\overline{B}\to D}(m_{D_1}^2))$
$\overline{B}^0 \to D^+ \underline{D}_1^-$	$-2m_{D_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{\overline{B}\to D}(m_{D_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{\overline{B}\to D}(m_{D_1}^2))$
$\overline{B}^0 \to D^- D_1^+$	$-a_1 f_D (\sin\theta_2 F^{\overline{B}\to D_{1A}}(m_D^2) + \cos\theta_2 F^{\overline{B}\to D_{1A'}}(m_D^2))$
$\overline{B}^0 \to D^- \underline{D}_1^+$	$-a_1 f_D (\cos\theta_2 F^{B\to D_{1A}}(m_D^2) - \sin\theta_2 F^{B\to D_{1A'}}(m_D^2))$
$\overline{B}^0 \to \eta_c a_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta_c} F^{\overline{B}\to a_1}(m_{\eta_c}^2)$
$\overline{B}^0 \to \eta_c f_1$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta_c} \cos\phi_A F^{\overline{B}\to f_1}(m_{\eta_c}^2)$
$\overline{B}^0 \to \eta_c b_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta_c} F^{\overline{B}\to b_1}(m_{\eta_c}^2)$
$\overline{B}^0 \to \eta_c h_1$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta_c} \cos\phi_{A'} F^{\overline{B}\to h_1}(m_{\eta_c}^2)$
$\overline{B}_s^0 \to K^0 \chi_{c1}$	$-2m_{\chi_{c1}} a_2 f_{\chi_{c1}} F^{B_s\to K}(m_{\chi_{c1}}^2)$
$\overline{B}_s^0 \to D_s^+ D_1^-$	$-2m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{B_s\to D_s}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B_s\to D_s}(m_{D_1}^2))$
$\overline{B}_s^0 \to D_s^+ \underline{D}_1^-$	$-2m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B_s\to D_s}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B_s\to D_s}(m_{\underline{D}_1}^2))$
$\overline{B}_s^0 \to D^- D_{s1}^+$	$-a_1 f_D (\sin\theta_3 F^{B_s\to D_{s1A}}(m_D^2) + \cos\theta_3 F^{B_s\to D_{s1A'}}(m_D^2))$
$\overline{B}_s^0 \to D^- \underline{D}_{s1}^+$	$-a_1 f_D (\cos\theta_3 F^{B_s\to D_{s1A}}(m_D^2) - \sin\theta_3 F^{B_s\to D_{s1A'}}(m_D^2))$
$\overline{B}_s^0 \to \eta_c K_1^0$	$-a_2 f_{\eta_c} (\sin\theta_1 F^{B_s\to K_{1A}}(m_{\eta_c}^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_{\eta_c}^2))$
$\overline{B}_s^0 \to \eta_c \underline{K}_1^0$	$-a_2 f_{\eta_c} (\cos\theta_1 F^{B_s\to K_{1A}}(m_{\eta_c}^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_{\eta_c}^2))$

Table 3.3 (a) Decay Amplitudes for $B \to PA$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1, \Delta C = -1, \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cs}$
$B^- \to \pi^0 D^-_{s1}$	$\sqrt{2} m_{D_{s1}} a_1 (f_{D_{1sA}} \sin\theta_3 F^{B\to\pi}(m^2_{D_{s1}}) + f_{D_{s1A'}} \cos\theta_3 F^{B\to\pi}(m^2_{D_{s1}}))$
$B^- \to \pi^0 \underline{D}^-_{s1}$	$\sqrt{2} m_{\underline{D}_{s1}} a_1 (f_{D_{1sA}} \cos\theta_3 F^{B\to\pi}(m^2_{\underline{D}_{s1}}) - f_{D_{s1A'}} \sin\theta_3 F^{B\to\pi}(m^2_{\underline{D}_{s1}}))$
$B^- \to \eta D^-_{s1}$	$\sqrt{2} m_{D_{s1}} a_1 (f_{D_{1sA}} \sin\phi_P \sin\theta_3 F^{B\to\eta}(m^2_{D_{s1}}) + f_{D_{s1A'}} \cos\phi_P \cos\theta_3 F^{B\to\eta}(m^2_{D_{s1}}))$
$B^- \to \eta \underline{D}^-_{s1}$	$\sqrt{2} m_{\underline{D}_{s1}} a_1 (f_{D_{1sA}} \sin\phi_P \cos\theta_3 F^{B\to\eta}(m^2_{\underline{D}_{s1}}) - f_{D_{s1A'}} \cos\phi_P \sin\theta_3 F^{B\to\eta}(m^2_{\underline{D}_{s1}}))$
$B^- \to K^- \bar{D}^0_1$	$2 m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 F^{B\to K}(m^2_{D_1}) + f_{D_{1A'}} \cos\theta_2 F^{B\to K}(m^2_{D_1}))$
$B^- \to K^- \underline{\bar{D}}^0_1$	$2 m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 F^{B\to K}(m^2_{\underline{D}_1}) - f_{D_{1A'}} \sin\theta_2 F^{B\to K}(m^2_{\underline{D}_1}))$
$B^- \to \eta' D^-_{s1}$	$\sqrt{2} m_{D_{s1}} a_1 (f_{D_{1sA}} \cos\phi_P \sin\theta_3 F^{B\to\eta'}(m^2_{D_{s1}}) + f_{D_{s1A'}} \cos\phi_P \cos\theta_3 F^{B\to\eta'}(m^2_{D_{s1}}))$
$B^- \to \eta' \underline{D}^-_{s1}$	$\sqrt{2} m_{\underline{D}_{s1}} a_1 (f_{D_{1sA}} \cos\phi_P \cos\theta_3 F^{B\to\eta'}(m^2_{\underline{D}_{s1}}) - f_{D_{s1A'}} \cos\phi_P \sin\theta_3 F^{B\to\eta'}(m^2_{\underline{D}_{s1}}))$
$B^- \to \bar{D}^0 K^-_1$	$a_2 f_D (\sin\theta_1 F^{B\to K_{1A}}(m^2_D) + \cos\theta_1 F^{B\to K_{1A'}}(m^2_D))$
$B^- \to \bar{D}^0 \underline{K}^-_1$	$a_2 f_D (\cos\theta_1 F^{B\to K_{1A}}(m^2_D) - \sin\theta_1 F^{B\to K_{1A'}}(m^2_D))$
$B^- \to D^-_s a^0_1$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} F^{B\to a_1}(m^2_{D_s})$
$B^- \to D^-_s f_1$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} \cos\phi_A F^{B\to f_1}(m^2_{D_s})$
$B^- \to D^-_s b^0_1$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} F^{B\to b_1}(m^2_{D_s})$
$B^- \to D^-_s h_1$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} \cos\phi_{A'} F^{B\to h_1}(m^2_{D_s})$
$\bar{B}^0 \to \pi^+ D^-_{s1}$	$2 m_{D_{s1}} a_1 (f_{D_{1sA}} \sin\theta_3 F^{\bar{B}\to\pi}(m^2_{D_{s1}}) + f_{D_{s1A'}} \cos\theta_3 F^{\bar{B}\to\pi}(m^2_{D_{s1}}))$
$\bar{B}^0 \to \pi^+ \underline{D}^-_{s1}$	$2 m_{\underline{D}_{s1}} a_1 (f_{D_{1sA}} \cos\theta_3 F^{\bar{B}\to\pi}(m^2_{\underline{D}_{s1}}) - f_{D_{s1A'}} \sin\theta_3 F^{\bar{B}\to\pi}(m^2_{\underline{D}_{s1}}))$
$\bar{B}^0 \to \bar{K}^0 \bar{D}^0_1$	$2 m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 F^{\bar{B}\to K}(m^2_{D_1}) + f_{D_{1A'}} \cos\theta_2 F^{\bar{B}\to K}(m^2_{D_1}))$
$\bar{B}^0 \to \bar{K}^0 \underline{\bar{D}}^0_1$	$2 m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 F^{\bar{B}\to K}(m^2_{\underline{D}_1}) - f_{D_{1A'}} \sin\theta_2 F^{\bar{B}\to K}(m^2_{\underline{D}_1}))$
$\bar{B}^0 \to \bar{D}^0 \bar{K}^0_1$	$a_2 f_D (\sin\theta_1 F^{\bar{B}\to K_{1A}}(m^2_D) + \cos\theta_1 F^{\bar{B}\to K_{1A'}}(m^2_D))$
$\bar{B}^0 \to \bar{D}^0 \underline{\bar{K}}^0_1$	$a_2 f_D (\cos\theta_1 F^{\bar{B}\to K_{1A}}(m^2_D) - \sin\theta_1 F^{\bar{B}\to K_{1A'}}(m^2_D))$
$\bar{B}^0 \to D^-_s a^+_1$	$a_1 f_{D_s} F^{B\to a_1}(m^2_{D_s})$
$\bar{B}^0 \to D^-_s b^+_1$	$a_1 f_{D_s} F^{B\to b_1}(m^2_{D_s})$
$\bar{B}^0_s \to K^+ D^-_{s1}$	$2 m_{D_{s1}} a_1 (f_{D_{1sA}} \sin\theta_3 F^{B_s\to K}(m^2_{D_{s1}}) + f_{D_{s1A'}} \cos\theta_3 F^{B_s\to K}(m^2_{D_{s1}}))$
$\bar{B}^0_s \to K^+ \underline{D}^-_{s1}$	$2 m_{\underline{D}_{s1}} a_1 (f_{D_{1sA}} \cos\theta_3 F^{B_s\to K}(m^2_{\underline{D}_{s1}}) - f_{D_{s1A'}} \sin\theta_3 F^{B_s\to K}(m^2_{\underline{D}_{s1}}))$

$\bar{B}_s^0 \to \eta \bar{D}_1^0$	$2m_{D_1} a_2(-f_{D_{1A}} \sin\theta_2 \cos\phi_P F^{B_s\to\eta}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \cos\phi_P F^{B_s\to\eta}(m_{D_1}^2))$
$\bar{B}_s^0 \to \eta \bar{\underline{D}}_1^0$	$2m_{\underline{D}_1} a_2(-f_{D_{1A}} \cos\theta_2 \cos\phi_P F^{B_s\to\eta}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \cos\phi_P F^{B_s\to\eta}(m_{\underline{D}_1}^2))$
$\bar{B}_s^0 \to \eta' \bar{D}_1^0$	$2m_{D_1} a_2(f_{D_{1A}} \sin\theta_2 \sin\phi_P F^{B_s\to\eta'}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \sin\phi_P F^{B_s\to\eta'}(m_{D_1}^2))$
$\bar{B}_s^0 \to \eta' \bar{\underline{D}}_1^0$	$2m_{\underline{D}_1} a_2(f_{D_{1A}} \cos\theta_2 \sin\phi_P F^{B_s\to\eta'}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \sin\phi_P F^{B_s\to\eta'}(m_{\underline{D}_1}^2))$
$\bar{B}_s^0 \to \bar{D}^0 f_1'$	$-a_2 f_D \cos\phi_A F^{B_s\to f_1'}(m_D^2)$
$\bar{B}_s^0 \to \bar{D}^0 h_1'$	$-a_2 f_D \cos\phi_{A'} F^{B_s\to h_1'}(m_D^2)$
$\bar{B}_s^0 \to D_s^- K_1^+$	$a_1 f_{D_s}(\sin\theta_1 F^{B_s\to K_{1A}}(m_{D_s}^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_{D_s}^2))$
$\bar{B}_s^0 \to D_s^- \underline{K}_1^+$	$a_1 f_{D_s}(\cos\theta_1 F^{B_s\to K_{1A}}(m_{D_s}^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_{D_s}^2))$
$\Delta b = 1, \Delta C = 0, \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*$
$B^- \to \pi^0 a_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B\to a_1}(m_\pi^2) + \sqrt{2} m_{a_1} a_1 f_{a_1} F^{B\to\pi}(m_{a_1}^2)$
$B^- \to \pi^0 b_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B\to b_1}(m_\pi^2) + \sqrt{2} m_{b_1} a_1 f_{b_1} F^{B\to\pi}(m_{b_1}^2)$
$B^- \to \pi^- a_1^0$	$\frac{1}{\sqrt{2}} a_1 f_\pi F^{B\to a_1}(m_\pi^2) + \sqrt{2} m_{a_1} a_2 f_{a_1} F^{B\to\pi}(m_{a_1}^2)$
$B^- \to \pi^- b_1^0$	$\frac{1}{\sqrt{2}} a_1 f_\pi F^{B\to b_1}(m_\pi^2)$
$B^- \to \pi^- f_1$	$\frac{1}{\sqrt{2}} a_1 \cos\phi_A f_\pi F^{B\to f_1}(m_\pi^2) + \sqrt{2} m_{f_1} a_2 \cos\phi_A f_{f_1} F^{B\to\pi}(m_{f_1}^2)$
$B^- \to \pi^- h_1$	$\frac{1}{\sqrt{2}} a_1 \cos\phi_{A'} f_\pi F^{B\to h_1}(m_\pi^2) + \sqrt{2} m_{h_1} a_2 \cos\phi_{A'} f_{h_1} F^{B\to\pi}(m_{h_1}^2)$
$B^- \to \eta a_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to a_1}(m_\eta^2) + \sqrt{2} m_{a_1} a_1 f_{a_1} \sin\phi_P F^{B\to\eta}(m_{a_1}^2)$
$B^- \to \eta b_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to b_1}(m_\eta^2) + \sqrt{2} m_{b_1} a_1 f_{b_1} \sin\phi_P F^{B\to\eta}(m_{b_1}^2)$
$B^- \to \eta' a_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B\to a_1}(m_{\eta'}^2) + \sqrt{2} m_{a_1} a_1 f_{a_1} \cos\phi_P F^{B\to\eta'}(m_{a_1}^2)$
$B^- \to \eta' b_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B\to b_1}(m_{\eta'}^2) + \sqrt{2} m_{b_1} a_1 f_{b_1} \cos\phi_P F^{B\to\eta'}(m_{b_1}^2)$
$\bar{B}^0 \to \pi^+ a_1^-$	$2m_{a_1} a_1 f_{a_1} F^{\bar{B}\to\pi}(m_{a_1}^2)$
$\bar{B}^0 \to \pi^+ b_1^-$	$2m_{b_1} a_1 f_{b_1} F^{\bar{B}\to\pi}(m_{b_1}^2)$
$\bar{B}^0 \to \pi^0 a_1^0$	$a_2(-\frac{1}{2} f_\pi F^{\bar{B}\to a_1}(m_\pi^2) - m_{a_1} f_{a_1} F^{\bar{B}\to\pi}(m_{a_1}^2))$
$\bar{B}^0 \to \pi^0 f_1$	$a_2(\frac{1}{2} f_\pi \cos\phi_A F^{\bar{B}\to f_1}(m_\pi^2) - m_{f_1} f_{f_1} \cos\phi_A F^{\bar{B}\to\pi}(m_{f_1}^2))$

$\bar{B}^0 \to \pi^0 b_1^0$	$-\frac{1}{2} a_2 f_\pi F^{\bar{B}\to b_1}(m_\pi^2)$
$\bar{B}^0 \to \pi^0 h_1$	$\frac{1}{2} a_2 f_\pi \cos\phi_{A'} F^{\bar{B}\to h_1}(m_\pi^2)$
$\bar{B}^0 \to \pi^- a_1^+$	$a_1 f_\pi F^{\bar{B}\to a_1}(m_\pi^2)$
$\bar{B}^0 \to \pi^- b_1^+$	$a_1 f_\pi F^{\bar{B}\to b_1}(m_\pi^2)$
$\bar{B}^0 \to \eta a_1^0$	$a_2(-\frac{1}{2} f_\eta \sin\phi_P F^{\bar{B}\to a_1}(m_\eta^2) + m_{a_1} f_{a_1} \sin\phi_P F^{\bar{B}\to\eta}(m_{a_1}^2))$
$\bar{B}^0 \to \eta f_1$	$a_2(\frac{1}{2} f_\eta \cos\phi_A \sin\phi_P F^{B\to f_1}(m_\eta^2) + m_{f_1} f_{f_1} \cos\phi_A \sin\phi_P F^{B\to\eta}(m_{f_1}^2))$
$\bar{B}^0 \to \eta b_1^0$	$-\frac{1}{2} a_2 f_\eta \sin\phi_P F^{\bar{B}\to b_1}(m_\eta^2)$
$\bar{B}^0 \to \eta h_1$	$\frac{1}{2} a_2 f_\eta \cos\phi_A \sin\phi_P F^{B\to h_1}(m_\eta^2)$
$\bar{B}^0 \to \eta' a_1^0$	$a_2(-\frac{1}{2} f_{\eta'} \cos\phi_P F^{\bar{B}\to a_1}(m_{\eta'}^2) + m_{a_1} f_{a_1} \cos\phi_P F^{\bar{B}\to\eta'}(m_{a_1}^2))$
$\bar{B}^0 \to \eta' f_1$	$a_2(\frac{1}{2} f_{\eta'} \cos\phi_A \cos\phi_P F^{\bar{B}\to f_1}(m_{\eta'}^2) + m_{f_1} f_{f_1} \cos\phi_A \cos\phi_P F^{\bar{B}\to\eta'}(m_{f_1}^2))$
$\bar{B}^0 \to \eta' b_1^0$	$-\frac{1}{2} a_2 f_{\eta'} \cos\phi_P F^{\bar{B}\to b_1}(m_{\eta'}^2)$
$\bar{B}^0 \to \eta' h_1$	$\frac{1}{2} a_2 f_{\eta'} \cos\phi_A \cos\phi_P F^{\bar{B}\to h_1}(m_{\eta'}^2)$
$\bar{B}_s^0 \to K^+ a_1^-$	$2 m_{a_1} a_1 f_{a_1} F^{B_s\to K}(m_{a_1}^2)$
$\bar{B}_s^0 \to K^+ b_1^-$	$2 m_{b_1} a_1 f_{b_1} F^{B_s\to K}(m_{b_1}^2)$
$\bar{B}_s^0 \to K^0 a_1^0$	$\sqrt{2} m_{a_1} a_2 f_{a_1} F^{B_s\to K}(m_{a_1}^2)$
$\bar{B}_s^0 \to K^0 f_1$	$\sqrt{2} m_{f_1} a_2 f_{f_1} \cos\phi_A F^{B_s\to K}(m_{f_1}^2)$
$\bar{B}_s^0 \to \pi^0 K_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\sin\theta_1 F^{B_s\to K_{1A}}(m_\pi^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_\pi^2))$
$\bar{B}_s^0 \to \pi^0 \underline{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\cos\theta_1 F^{B_s\to K_{1A}}(m_\pi^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_\pi^2))$
$\bar{B}_s^0 \to \pi^- K_1^+$	$a_1 f_\pi (\sin\theta_1 F^{B_s\to K_{1A}}(m_\pi^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_\pi^2))$
$\bar{B}_s^0 \to \pi^- \underline{K}_1^+$	$a_1 f_\pi (\cos\theta_1 F^{B_s\to K_{1A}}(m_\pi^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_\pi^2))$
$\bar{B}_s^0 \to \eta K_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\theta_1 \sin\phi_P F^{B_s\to K_{1A}}(m_\eta^2) + \cos\theta_1 \sin\phi_P F^{B_s\to K_{1A'}}(m_\eta^2))$
$\bar{B}_s^0 \to \eta \underline{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\phi_P \cos\theta_1 F^{B_s\to K_{1A}}(m_\eta^2) - \sin\theta_1 \sin\phi_P F^{B_s\to K_{1A'}}(m_\eta^2))$
$\bar{B}_s^0 \to \eta' K_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\sin\theta_1 \cos\phi_P F^{B_s\to K_{1A}}(m_{\eta'}^2) + \cos\theta_1 \cos\phi_P F^{B_s\to K_{1A'}}(m_{\eta'}^2))$
$\bar{B}_s^0 \to \eta' \underline{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\cos\theta_1 \cos\phi_P F^{B_s\to \underline{K}_{1A}}(m_{\eta'}^2) - \sin\theta_1 \cos\phi_P F^{B_s\to K_{1A'}}(m_{\eta'}^2))$

Table 3.3 (b) Decay Amplitudes for $B \to PA$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^*$
$B^- \to \pi^0 K_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\sin\theta_1 F^{B\to K_{1A}}(m_\pi^2) + \cos\theta_1 F^{B\to K_{1A'}}(m_\pi^2))$ $+\sqrt{2} m_{K_1} a_1 (f_{K_{1A}} \sin\theta_1 F^{B\to\pi}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1 F^{B\to\pi}(m_{K_1}^2))$
$B^- \to \pi^0 \underline{K}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\cos\theta_1 F^{B\to K_{1A}}(m_\pi^2) - \sin\theta_1 F^{B\to K_{1A'}}(m_\pi^2))$ $+\sqrt{2} m_{\underline{K}_1} a_1 (f_{K_{1A}} \cos\theta_1 F^{B\to\pi}(m_{\underline{K}_1}^2) - f_{K_{1A'}} \sin\theta_1 F^{B\to\pi}(m_{\underline{K}_1}^2))$
$B^- \to \eta K_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\theta_1 \sin\phi_P F^{B\to K_{1A}}(m_\eta^2) + \cos\theta_1 \sin\phi_P F^{B\to K_{1A'}}(m_\eta^2))$ $+\sqrt{2} m_{K_1} a_1 (f_{K_{1A}} \sin\theta_1 \sin\phi_P F^{B\to\eta}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1 \sin\phi_P F^{B\to\eta}(m_{K_1}^2))$
$B^- \to \eta \underline{K}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\cos\theta_1 \sin\phi_P F^{B\to K_{1A}}(m_\eta^2) - \sin\theta_1 \sin\phi_P F^{B\to K_{1A'}}(m_\eta^2))$ $+\sqrt{2} m_{\underline{K}_1} a_1 (f_{K_{1A}} \cos\theta_1 \sin\phi_P F^{B\to\eta}(m_{\underline{K}_1}^2) - f_{K_{1A'}} \sin\theta_1 \sin\phi_P F^{B\to\eta}(m_{\underline{K}_1}^2))$
$B^- \to K^- a_1^0$	$(\frac{1}{\sqrt{2}} a_1 f_K F^{B\to a_1}(m_K^2) + \sqrt{2} m_{a_1} a_2 f_{a_1} F^{B\to K}(m_{a_1}^2))$
$B^- \to K^- f_1$	$(\frac{1}{\sqrt{2}} a_1 f_K \cos\phi_A F^{B\to f_1}(m_K^2) + \sqrt{2} m_{f_1} a_2 f_{f_1} \cos\phi_A F^{B\to K}(m_{f_1}^2))$
$B^- \to K^- b_1^0$	$\frac{1}{\sqrt{2}} a_1 f_K F^{B\to b_1}(m_K^2)$
$B^- \to K^- h_1$	$\frac{1}{\sqrt{2}} a_1 f_K \cos\phi_{A'} F^{B\to h_1}(m_K^2)$
$B^- \to \eta' K_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\sin\theta_1 \cos\phi_P F^{B\to K_{1A}}(m_{\eta'}^2) + \cos\theta_1 \cos\phi_P F^{B\to K_{1A'}}(m_{\eta'}^2))$ $+\sqrt{2} m_{K_1} a_1 (f_{K_{1A}} \sin\theta_1 \cos\phi_P F^{B\to\eta}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1 \cos\phi_P F^{B\to\eta}(m_{K_1}^2))$
$B^- \to \eta' \underline{K}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\cos\theta_1 \cos\phi_P F^{B\to K_{1A}}(m_{\eta'}^2) - \sin\theta_1 \cos\phi_P F^{B\to K_{1A'}}(m_{\eta'}^2))$ $+\sqrt{2} m_{\underline{K}_1} a_1 (f_{K_{1A}} \cos\theta_1 \cos\phi_P F^{B\to\eta'}(m_{\underline{K}_1}^2) - f_{K_{1A'}} \sin\theta_1 \cos\phi_P F^{B\to\eta'}(m_{\underline{K}_1}^2))$
$\bar{B}^0 \to \pi^+ K_1^-$	$2 m_{K_1} a_1 (f_{K_{1A}} \sin\theta_1 F^{B\to\pi}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1 F^{B\to\pi}(m_{K_1}^2))$
$\bar{B}^0 \to \pi^+ \underline{K}_1^-$	$2 m_{\underline{K}_1} a_1 (f_{K_{1A}} \cos\theta_1 F^{B\to\pi}(m_{\underline{K}_1}^2) - f_{K_{1B}} \sin\theta_1 F^{B\to\pi}(m_{\underline{K}_1}^2))$
$\bar{B}^0 \to \pi^0 \bar{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\sin\theta_1 F^{\bar{B}\to K_{1A}}(m_\pi^2) + \cos\theta_1 F^{\bar{B}\to K_{1A'}}(m_\pi^2))$

$\overline{B}^0 \to \pi^0 \underline{\overline{K}}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\cos\theta_1 F^{\overline{B}\to K_{1A}}(m_\pi^2) - \sin\theta_1 F^{\overline{B}\to K_{1A'}}(m_\pi^2))$
$\overline{B}^0 \to \eta \overline{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\phi_P \sin\theta_1 F^{\overline{B}\to K_{1A}}(m_\eta^2) + \sin\phi_P \cos\theta_1 F^{\overline{B}\to K_{1A'}}(m_\eta^2))$
$\overline{B}^0 \to \eta \underline{\overline{K}}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\phi_P \cos\theta_1 F^{\overline{B}\to K_{1A}}(m_\eta^2) - \sin\phi_P \sin\theta_1 F^{\overline{B}\to K_{1A'}}(m_\eta^2))$
$\overline{B}^0 \to \overline{K}^0 a_1^0$	$\sqrt{2} m_{a_1} a_2 f_{a_1} F^{\overline{B}\to K}(m_{a_1}^2)$
$\overline{B}^0 \to \overline{K}^0 f_1$	$\sqrt{2} m_{f_1} a_2 f_{f_1} F^{\overline{B}\to K}(m_{f_1}^2)$
$\overline{B}^0 \to K^- a_1^+$	$a_1 f_K F^{\overline{B}\to a_1}(m_K^2)$
$\overline{B}^0 \to K^- b_1^+$	$a_1 f_K F^{\overline{B}\to b_1}(m_K^2)$
$\overline{B}^0 \to \eta' \overline{K}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\cos\phi_P \sin\theta_1 F^{\overline{B}\to K_{1A}}(m_{\eta'}^2) - \cos\theta_P \cos\theta_1 F^{\overline{B}\to K_{1A'}}(m_{\eta'}^2))$
$\overline{B}^0 \to \eta' \underline{\overline{K}}_1^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} (\cos\phi_P \cos\theta_1 F^{\overline{B}\to \underline{K}_{1A}}(m_{\eta'}^2) - \cos\phi_P \sin\theta_1 F^{\overline{B}\to \underline{K}_{1A'}}(m_{\eta'}^2))$
$\overline{B}_s^0 \to K^+ K_1^-$	$2 m_{K_1} a_1 (f_{K_{1A}} \sin\theta_1 F^{B_s\to K}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1 F^{B_s\to K}(m_{K_1}^2))$
$\overline{B}_s^0 \to K^+ \underline{K}_1^-$	$2 m_{\underline{K}_1} a_1 (f_{K_{1A}} \cos\theta_1 F^{B_s\to K}(m_{\underline{K}_1}^2) - f_{K_{1A'}} \sin\theta_1 F^{B_s\to K}(m_{\underline{K}_1}^2))$
$\overline{B}_s^0 \to \pi^0 f_1'$	$-\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_s\to f_1'}(m_\pi^2)$
$\overline{B}_s^0 \to \pi^0 h_1'$	$-\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_s\to h_1'}(m_\pi^2)$
$\overline{B}_s^0 \to \eta a_1^0$	$-\sqrt{2} m_{a_1} a_2 f_{a_1} \cos\phi_P F^{B_s\to \eta}(m_{f_1}^2)$
$\overline{B}_s^0 \to \eta f_1$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_A \cos\phi_P F^{B_s\to f_1}(m_\eta^2)$
$\overline{B}_s^0 \to \eta f_1'$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \cos\phi_A \sin\phi_P F^{B_s\to f_1'}(m_\eta^2)$
$\overline{B}_s^0 \to \eta h_1'$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \cos\phi_{A'} \sin\phi_P F^{B_s\to h'}(m_\eta^2)$
$\overline{B}_s^0 \to K^- K_1^+$	$2 a_1 f_K (\sin\theta_1 F^{B_s\to K_{1A}}(m_K^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_K^2))$
$\overline{B}_s^0 \to K^- \underline{K}_1^+$	$2 a_1 f_K (\cos\theta_1 F^{B_s\to K_{1A}}(m_K^2) - \cos\theta_1 F^{B_s\to K_{1A'}}(m_K^2))$
$\overline{B}_s^0 \to \eta' a_1^0$	$\sqrt{2} m_{a_1} a_2 f_{a_1} \sin\phi_P F^{B_s\to \eta'}(m_{a_1}^2))$
$\overline{B}_s^0 \to \eta' f_1$	$\sqrt{2} m_{f_1} a_2 f_{f_1} \cos\phi_A \sin\phi_P F^{B_s\to \eta'}(m_{f_1}^2))$
$\overline{B}_s^0 \to \eta' f_1'$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \cos\phi_A \cos\phi_P F^{B_s\to f_1'}(m_{\eta'}^2)$
$\overline{B}_s^0 \to \eta' h_1'$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_{A'} \cos\phi_P F^{B_s\to h_1}(m_{\eta'}^2)$

$\Delta b=1, \Delta C=-1, \Delta S=0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^*$
$B^- \to \pi^0 D_1^-$	$-\sqrt{2} m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{B\to\pi}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B\to\pi}(m_{D_1}^2))$
$B^- \to \pi^0 \underline{D}_1^-$	$-\sqrt{2} m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2))$
$B^- \to \pi^- \bar{D}_1^0$	$-2 m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2))$
$B^- \to \pi^- \bar{\underline{D}}_1^0$	$-2 m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2))$
$B^- \to \eta D_1^-$	$-\sqrt{2} m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 \sin\phi_P F^{B\to\eta}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \sin\phi_P F^{B\to\eta}(m_{D_1}^2))$
$B^- \to \eta \underline{D}_1^-$	$-\sqrt{2} m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 \sin\phi_P F^{B\to\eta}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \sin\phi_P F^{B\to\eta}(m_{\underline{D}_1}^2))$
$B^- \to \eta' D_1^-$	$-\sqrt{2} m_{D_1} a_1 (f_{D_{1A}} \cos\theta_2 \sin\phi_P F^{B\to\eta'}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \cos\phi_P F^{B\to\eta'}(m_{D_1}^2))$
$B^- \to \eta' \underline{D}_1^-$	$-\sqrt{2} m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 \cos\phi_P F^{B\to\eta'}(m_{\underline{D}_1}^2) - f_{D_{1B}} \sin\theta_2 \cos\phi_P F^{B\to\eta'}(m_{\underline{D}_1}^2))$
$B^- \to D^- a_1^0$	$-\frac{1}{\sqrt{2}} a_1 f_D F^{B\to a_1}(m_D^2)$
$B^- \to D^- f_1$	$-\frac{1}{\sqrt{2}} a_1 f_D \cos\phi_A F^{B\to f_1}(m_D^2)$
$B^- \to D^- b_1^0$	$-\frac{1}{\sqrt{2}} a_1 f_D F^{B\to b_1}(m_D^2)$
$B^- \to D^- h_1$	$-\frac{1}{\sqrt{2}} a_1 f_D \cos\phi_{A'} F^{B\to h_1}(m_D^2)$
$B^- \to \bar{D}^0 a_1^-$	$-a_2 f_D F^{B\to a_1}(m_D^2)$
$B^- \to \bar{D}^0 b_1^-$	$-a_2 f_D F^{B\to b_1}(m_D^2)$
$\bar{B}^0 \to \pi^+ D_1^-$	$-2 m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{\bar{B}\to\pi}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{\bar{B}\to\pi}(m_{D_1}^2))$
$\bar{B}^0 \to \pi^+ \underline{D}_1^-$	$-2 m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B\to\pi}(m_{D_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B\to\pi}(m_{\underline{D}_1}^2))$
$\bar{B}^0 \to \pi^0 \bar{D}_1^0$	$-\sqrt{2} m_{D_1} a_2 (-f_{D_{1A}} \sin\theta_2 F^{\bar{B}\to\pi}(m_{D_1}^2) - f_{D_{1A'}} \cos\theta_2 F^{\bar{B}\to\pi}(m_{D_1}^2))$
$\bar{B}^0 \to \pi^0 \bar{\underline{D}}_1^0$	$-\sqrt{2} m_{\underline{D}_1} a_2 (-f_{D_{1A}} \cos\theta_2 F^{\bar{B}\to\pi}(m_{D_1}^2) + f_{D_{1A'}} \sin\theta_2 F^{\bar{B}\to\pi}(m_{\underline{D}_1}^2))$
$\bar{B}^0 \to \eta \bar{D}_1^0$	$-\sqrt{2} m_{D_1} a_2 (f_{\underline{D}_{1A}} \sin\theta_2 \sin\phi_P F^{\bar{B}\to\eta}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \sin\phi_P F^{\bar{B}\to\eta}(m_{D_1}^2))$
$\bar{B}^0 \to \eta \bar{\underline{D}}_1^0$	$-\sqrt{2} m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 \sin\phi_P F^{\bar{B}\to\eta}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \sin\phi_P F^{\bar{B}\to\eta}(m_{\underline{D}_1}^2))$
$\bar{B}^0 \to \eta' \bar{D}_1^0$	$-\sqrt{2} m_{D_1} a_2 (f_{D_{1A}} \sin\theta_2 \cos\phi_P F^{\bar{B}\to\eta'}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 \cos\phi_P F^{\bar{B}\to\eta'}(m_{D_1}^2))$
$\bar{B}^0 \to \eta' \bar{\underline{D}}_1^0$	$-\sqrt{2} m_{\underline{D}_1} a_2 (f_{D_{1A}} \cos\theta_2 \cos\phi_P F^{\bar{B}\to\eta'}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 \cos\phi_P F^{\bar{B}\to\eta'}(m_{\underline{D}_1}^2))$
$\bar{B}^0 \to D^- a_1^+$	$-a_1 f_D F^{\bar{B}\to a_1}(m_D^2)$
$\bar{B}^0 \to D^- b_1^+$	$-a_1 f_D F^{\bar{B}\to b_1}(m_D^2)$
$\bar{B}^0 \to \bar{D}^0 a_1^0$	$\frac{1}{\sqrt{2}} a_2 f_D F^{\bar{B}\to a_1}(m_D^2)$

$\bar{B}^0 \to \bar{D}^0 f_1$	$-\frac{1}{\sqrt{2}} a_2 f_D \cos\phi_A F^{\bar{B}\to f_1}(m_D^2)$
$\bar{B}^0 \to \bar{D}^0 b_1^0$	$\frac{1}{\sqrt{2}} a_2 f_D F^{\bar{B}\to b_1}(m_D^2)$
$\bar{B}^0 \to \bar{D}^0 h_1$	$-\frac{1}{\sqrt{2}} a_2 f_D \cos\phi_{A'} F^{\bar{B}\to h_1}(m_D^2)$
$\bar{B}_s^0 \to K^+ D_1^-$	$-2m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{B_s\to K}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B_s\to K}(m_{D_1}^2))$
$\bar{B}_s^0 \to K^+ \underline{D}_1^-$	$-2m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2))$
$\bar{B}_s^0 \to K^0 \bar{D}_1^0$	$-2m_{D_1} a_1 (f_{D_{1A}} \sin\theta_2 F^{B_s\to K}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2 F^{B_s\to K}(m_{D_1}^2))$
$\bar{B}_s^0 \to K^0 \underline{\bar{D}}_1^0$	$-2m_{\underline{D}_1} a_1 (f_{D_{1A}} \cos\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2) - f_{D_{1A'}} \sin\theta_2 F^{B_s\to K}(m_{\underline{D}_1}^2))$
$\bar{B}_s^0 \to D^- K_1^+$	$-2a_1 f_D (\sin\theta_1 F^{B_s\to K_{1A}}(m_D^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_D^2))$
$\bar{B}_s^0 \to D^- \underline{K}_1^+$	$-2a_1 f_D (\cos\theta_1 F^{B_s\to K_{1A}}(m_D^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_D^2))$
$\bar{B}_s^0 \to \bar{D}^0 K_1^0$	$-2a_2 f_D (\sin\theta_1 F^{B_s\to K_{1A}}(m_D^2) + \cos\theta_1 F^{B_s\to K_{1A'}}(m_D^2))$
$\bar{B}_s^0 \to \bar{D}^0 \underline{K}_1^0$	$-2a_2 f_D (\cos\theta_1 F^{B_s\to K_{1A}}(m_D^2) - \sin\theta_1 F^{B_s\to K_{1A'}}(m_D^2))$

In order to calculate the decay amplitudes given in the Tables 3.1, 3.2, 3.3 (a) and 3.3 (b), one has to calculate the form factors given in (3.16). In the next section, we obtained the form factors in ISGW II model.

3.4 MAIN FEATURES OF ISGW II MODEL

The basic idea of the ISGW model is to make a correspondence between the Lorentz-invariant form factors which occur as the coefficient of the various vectors X_i^μ that one can form from available kinematic variables in the expansion of the matrix element $\langle X(p_X, s_X) | j^\mu(0) | B(p_B) \rangle$ of the physical B and X mesons, and those which appear in the quark-model calculation of $\langle \tilde{X}(\tilde{p}_X, \tilde{s}_X) | j^\mu(0) | \tilde{B}(\tilde{p}_B) \rangle$ where, for example, $| \tilde{X}(\tilde{p}_X, \tilde{s}_X) \rangle$ is the quark-model state vector in the weak binding, nonrelativistic limit and is given by

$$\left|\tilde{X}(\tilde{p}_X,\tilde{s}_X)\right\rangle = \sqrt{2\tilde{m}_X}\int d^3p\Sigma C^{S_X LS}_{m_L m_S}\phi_X(\vec{p})_{Lm_L}\chi^{Sm_S}_{S\overline{S}}$$
$$\left| q\left(\frac{m_q}{\tilde{m}_X}\vec{p}_X+\vec{p},s\right)\overline{q}\left(\frac{m_{\overline{q}}}{\tilde{m}_X}\vec{p}_X-\vec{p},s\right)\right\rangle,$$

here $\tilde{m}_X$ is defined as the mock mass in $\tilde{f}_i$, $\chi^{Sm_S}_{S\overline{S}}$ couples the spin s and $\overline{s}$ to the total spin *S*, $\phi_X(\vec{p})$ is the $q\overline{q}$ relative momentum wave function and the *C* factors couple *L* and *S* to the total angular momentum s_X. This so called 'mock meson method' is based on the observation that in this limit the quark model state vectors form good representations of the Lorentz group. ISGW model expresses the properly normalized meson state vectors in the nonrelativistic limit and normalize the form factors at maximal q^2, where both mesons are at rest. They obtain an exponential q^2 dependence of the form factors using wave functions which are variational solutions of the Schrödinger equation based on the harmonic oscillator wave functions, with the coulomb and linear potential.

In general, the form factors evaluated are reliable only at $q^2 = q_m^2$, the maximum momentum transfer $(m_B - m_X)^2$. The reason is that the form-factor q^2 dependence in the ISGW model is proportional to $\exp[-(q_m^2 - q^2)]$ and hence the form factor decreases exponentially as a function of $(q_m^2 - q^2)$. This has been improved in the ISGW II model in which the form factor has a more realistic behavior at large $(q_m^2 - q^2)$ which is expressed in terms of a certain polynomial term. In addition to the form factor momentum dependence, the ISGW II model incorporates a number of improvements, such as the constraints imposed by heavy quark symmetry, hyperfine distortions of wave functions, etc [4]. The new version of the ISGW model is called the ISGW II model [4] which includes the following features:

(1) heavy quark symmetry constraints on the relations between form factors away from zero recoil are respected,

(2) heavy quark symmetry constraints on the slopes of form factors near zero recoil are built in,

(3) the naive currents of the quark model are related to the full weak currents via the matching conditions of heavy quark effective theory (HQET),

(4) heavy-quark-symmetry-breaking color magnetic interaction are included, whereas ISGW only included the symmetry breaking due to the heavy quark kinetic energy,

(5) the ISGW prescription for connecting its quark model form factors is modified to be consistent with the constraints of heavy quark symmetry breaking at order $1/m_Q$,

(6) relativistic corrections to the axial vector coupling constants are taken into account, and

(7) more realistic form factors shapes, based on the measured pion form factors, are employed.

The form factor expressions have drastically changed in the new version of ISGW model in the light of HQS, so in the present thesis we use ISGW II model to calculate the form factors.

3.4.1 $B \to A / A'$ TRANSITION FORM FACTORS

The form factors have the following expressions in the ISGW II model [4].

$$l = -\tilde{m}_B \beta_B [\frac{1}{\mu_-} + \frac{m_d \tilde{m}_A (\tilde{\omega}-1)}{\beta_B^2} (\frac{5+\tilde{\omega}}{6m_q} - \frac{m_d \beta_B^2}{2\mu_- \beta_{BA}^2})] F_5^{(l)},$$

$$c_+ + c_- = -\frac{m_d \tilde{m}_A}{2m_q \tilde{m}_B \beta_B} \left(1 - \frac{m_d m_q \beta_B^2}{2\tilde{m}_A \mu_- \beta_{BA}^2}\right) F^{(c_+ + c_-)},$$

(3.17)

$$c_+ - c_- = -\frac{m_d \tilde{m}_A}{2m_q \tilde{m}_B \beta_B} \left(\frac{\tilde{\omega}+2}{3} - \frac{m_d m_q \beta_B^2}{2\tilde{m}_A \mu_- \beta_{BA}^2}\right) F^{(c_+ - c_-)},$$

$$r = \frac{\tilde{m}_B \beta_B}{\sqrt{2}} [\frac{1}{\mu_+} + \frac{m_d \tilde{m}_A}{3m_q \beta_B^2} (\tilde{\omega} - 1)^2] F_5^{(r)},$$

$$s_+ + s_- = -\frac{m_d}{2\tilde{m}_B \beta_B} \left(1 - \frac{m_d}{m_q} + \frac{m_d \beta_B^2}{2\mu_+ \beta_{BA}^2} \right) F^{(s_+ + s_-)},$$

$$s_+ - s_- = -\frac{m_d}{2m_q \beta_B} \left(\frac{4 - \tilde{\omega}}{3} - \frac{m_q m_d \beta_B^2}{2\tilde{m}_A \mu_+ \beta_{BA}^2} \right) F^{(s_+ - s_-)}, \tag{3.18}$$

where

$$F_5^{(l)} = F_5^{(r)} = F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{1/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{1/2},$$

$$F_5^{(c_+ + c_-)} = F_5^{(s_+ + s_-)} = F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{-3/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{1/2}, \tag{3.19}$$

$$F_5^{(c_+ - c_-)} = F_5^{(s_+ - s_-)} = F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{-1/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{-1/2}.$$

The $t(\equiv q^2)$ dependence is given by

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_B \bar{m}_X},$$

and

$$F_5 = \left(\frac{\tilde{m}_X}{\tilde{m}_B} \right)^{1/2} \left(\frac{\beta_B \beta_X}{B_{BX}} \right)^{5/2} \left[1 + \frac{1}{18} h^2 (t_m - t) \right]^{-3}, \tag{3.20}$$

where

$$h^2 = \frac{3}{4m_b m_q} + \frac{3m_d^2}{2\bar{m}_B \bar{m}_X \beta_{BX}^2} + \frac{1}{\bar{m}_B \bar{m}_X} (\frac{16}{33 - 2n_f}) \ln[\frac{\alpha_S(\mu_{QM})}{\alpha_S(m_q)}], \tag{3.21}$$

with

$$\beta_{BX}^2 = \frac{1}{2} \left(\beta_B^2 + \beta_X^2 \right), \tag{3.22}$$

and

$$\mu_\pm = \left(\frac{1}{m_q} \pm \frac{1}{m_b} \right)^{-1},$$

which is used throughout the analysis for transition $B \to X$, where $X = (q\bar{b})$ in the states $1^1S_0(J^{PC} = 0^{-+})$, $1^3P_1(J^{PC} = 1^{++})$, $1^1P_1(J^{PC} = 1^{+-})$. $\tilde{m}$ is the sum of the mesons constituent quarks masses, $\overline{m}$ is the hyperfine averaged physical masses, n_f is the number of active flavors, which is taken to be five in the present case, $t_m = (m_B - m_X)^2$ is the maximum momentum transfer and μ_{QM} is the quark model scale. The subscript in the q depends upon the quark currents $\bar{q}\gamma_\mu b$ and $\bar{q}\gamma_\mu\gamma_5 b$ appearing in different transitions. For $B_s \to X$ transition, m_q is replaced with m_s. The values of parameter β for different s-wave and p-wave mesons are given in the Table 3.4 [4].

Table 3.4 The parameter β for s-wave and p-wave mesons in the ISGW II model

Quark Content	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$	$u\bar{b}$	$s\bar{b}$
β_S (GeV)	0.41	0.44	0.53	0.45	0.56	0.43	0.54
β_P (GeV)	0.28	0.30	0.33	0.33	0.38	0.35	0.41

In the original version of the ISGW I model [4], the function F_n has a different expression in its $t_m - t$ dependence.

$$F_n = \left(\frac{\tilde{m}_A}{\tilde{m}_B}\right)^{1/2}\left(\frac{\beta_B\beta_A}{\beta_{BA}}\right)^n \exp\{-\frac{m_2}{4\tilde{m}_B\tilde{m}_A}\frac{(t_m - t)}{k^2\beta_{BA}^2}\},$$

$k = 0.7$ the relativistic correction factor. The form factors are given by

$$l = -\tilde{m}_B \beta_B [\frac{1}{\mu_-} + \frac{m_2(t_m - t)}{2\tilde{m}_B k^2 \beta_B^2}(\frac{1}{m_1} - \frac{m_2 \beta_B^2}{2\mu_- \tilde{m}_A \beta_{BA}^2})] F_5,$$

$$c_+ = -\frac{m_2 m_c}{4\tilde{m}_B \mu_- \beta_B}(1 - \frac{m_1 m_2 \beta_B^2}{2\mu_- \tilde{m}_A \beta_{BA}^2})] F_5.$$

The form factors l, c_+, c_-, r, s_+ and s_- are calculated in improved version, the ISGW II model [4]. Note that the results for the form factor c_+ are quite different in the ISGW and ISGW II models [4]: c_+ is positive in the former model while it becomes negative in the latter. Expressions in (3.17) and (3.18), the ISGW II model allows one to determine the form factors c_- and s_-, which vanish in the ISGW I model [3]. The obtained form factors are given in Tables 3.5 and 3.6.

Table 3.5 Form factors of $B(0^-) \to A(1^+)$ transition at $q^2 = t_m$ in the ISGW II quark Model

Transition	l	c_+	c_-
$B \to a_1$	-2.385	-0.032	-0.0091
$B \to f_1$	-2.378	-0.032	-0.0090
$B \to K_1$	-1.619	-0.035	-0.0074
$B \to D_1$	-0.546	-0.049	-0.0041
$B_s \to f_1'$	-1.847	-0.043	-0.0067
$B_s \to K_1$	-2.623	-0.038	-0.0085
$B_s \to D_{s1}$	-0.661	-0.062	-0.0040

Table 3.6 Form factors of $B(0^-) \to A'(1^+)$ transition at $q^2 = t_m$ in the ISGW II quark Model

Transition	***r***	s_+	s_-
$B \to b_1$	1.945	0.126	-0.094
$B \to h_1$	1.908	0.128	-0.096
$B \to \underline{K}_1$	1.423	0.125	-0.085
$B \to \underline{D}_1$	0.796	0.108	-0.043
$B_s \to h_1'$	2.388	0.143	-0.107
$B_s \to \underline{K}_1$	2.124	0.128	-0.096
$B_s \to \underline{D}_{s1}$	0.965	0.124	-0.048

For $B \to P$ transition form factors, we use the BSW model results which have already been described in detail in section 2.11 of chapter 2.

3.5 DECAY CONSTANTS OF AXIAL-VECTOR MESONS

For axial-vector meson, decay constants for $J^{PC} = 1^{+-}$ mesons may vanish due to the C-parity behavior. Under charge conjunction, the two types of axial-vector mesons transform as

$$M_b^a\ (1^{++}) \to +M_a^b(1^{++})$$
$$(a,\, b = 1,\, 2,\, 3)$$
$$M_b^a\ (1^{+-}) \to -M_a^b(1^{+-})$$

where M_b^a denotes meson 3×3 matrix elements in SU(3) flavor symmetry. Since the weak axial-vector current transforms as $(A_\mu)_b^a \to +(A_\mu)_a^b$ under charge conjunction, only the (1^{++}) state can be produced through the axial-vector current in the SU(3) symmetry limit [19]. Particle Data Group [16] assumes that the mixing is maximal, i.e. $\theta = 45^0$, whereas

$\tau \to K_1(1.270)/K_1(1.400) + \nu_\tau$ data yields $\theta = \pm 37^0$ and $\theta = \pm 58^0$. To determine the decay constant of $K_1(1.270)$, we use the following formula:

$$\Gamma(\tau \to K_1 \nu_\tau) = \frac{G_F^2}{16\pi} |V_{us}|^2 \, f_{K_1}^2 \, \frac{(m_\tau^2 + 2m_{K_1}^2)(m_\tau^2 - m_{K_1}^2)^2}{m_\tau^3}, \tag{3.23}$$

which gives $f_{K_1(1270)} = 0.175 \pm 0.019$ GeV. The decay constant of $K_1(1.400)$ can be obtained from $f_{K_1(1.400)} / f_{K_1(1.270)} = \cot\theta$. A small value around 0.011 GeV for the decay constant of K_{1B} may arise through SU(3) breaking, which yields

$$\begin{aligned} f_{\underline{K}_1(1.400)} &= f_{K_{1A}} \cos\theta_1 - f_{K_{1A'}} \sin\theta_1 \\ &= -0.087\, GeV, \end{aligned} \tag{3.24}$$

for $\theta_1 = -58^0$ [13]. Similarly, decay constant of $a_1(1.260)$ can be obtained from $B(\tau \to a_1 \nu_\tau)$. However, this branching ratio is not given in Particle Data Group [16], although the data on $\tau \to a_1 \nu_\tau \to \rho\pi\nu_\tau$ have been reported by various experiments. We take $f_{a_1} = 0.203 \pm 0.018$ GeV from the analysis given by J.C.R. Bloch *et al.* [22]. For the decay constant of $f_1(1.285)$, we assume $f_{f_1} \approx f_{a_1}$. The decay constants

$$f_{D_{1A}} = -0.127 \text{ GeV}, \; f_{D_{1A'}} = 0.045 \text{ GeV}, \; f_{D_{s1A}} = -0.121 \text{ GeV}, \; f_{D_{s1A'}} = 0.038 \text{ GeV},$$

$$f_{B_{1A}} = -0.115 \text{ GeV}, \; f_{B_{1A'}} = 0.064 \text{ GeV}, \; f_{B_{s1A}} = -0.101 \text{ GeV},$$

$$f_{B_{s1A'}} = 0.054 \text{ GeV and } f_{\chi_{c1}} \approx -0.160 \text{ GeV}, \tag{3.25}$$

have been taken from [13].

In case of pseudoscalar mesons, to evaluate the factorization amplitudes, we use the following decay constants [23-25]:

$$f_\pi = 0.131 \text{ GeV}, \; f_K = 0.160 \text{ GeV}, \; f_D = 0.208 \text{ GeV}, \; f_{D_s} = 0.273 \text{ GeV},$$

$$f_\eta = 0.133 \text{ GeV}, \; f_{\eta'} = 0.126 \text{ GeV} \text{ and } f_{\eta_c} = 0.400 \text{ GeV}. \tag{3.26}$$

3.6 NUMERICAL RESULTS AND DISCUSSIONS

Finally, we calculate branching ratios of B meson decays in CKM-favored and CKM-suppressed modes involving $b \to c$ and $b \to u$ transitions. Using the decay rate formula (3.12), the form factors calculated in section 3.4 and the numerical values of decay constants given in section 3.5, we predict the branching ratios as given in column 2nd of the Tables 3.7, 3.8, 3.9 (a) and 3.9 (b) for various possible modes.

Table 3.7 Branching ratios for $B \to PA$ decays in CKM-favored mode involving $b \to c$ transition

Decays	Branching ratios
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	
$B^- \to \pi^- D_1^0$	1.8×10^{-3}
$B^- \to \pi^- \underline{D}_1^0$	3.9×10^{-4}
$B^- \to D^0 a_1^-$	5.6×10^{-3}
$B^- \to D^0 b_1^-$	6.5×10^{-4}
$\overline{B}^0 \to \pi^0 D_1^0$	5.8×10^{-5}
$\overline{B}^0 \to \pi^0 \underline{D}_1^0$	1.1×10^{-6}
$\overline{B}^0 \to \pi^- D_1^+$	2.7×10^{-3}
$\overline{B}^0 \to \pi^- \underline{D}_1^+$	3.1×10^{-4}
$\overline{B}^0 \to \eta D_1^0$	3.0×10^{-5}
$\overline{B}^0 \to \eta \underline{D}_1^0$	5.8×10^{-7}
$\overline{B}^0 \to \eta' D_1^0$	1.4×10^{-5}
$\overline{B}^0 \to \eta' \underline{D}_1^0$	2.8×10^{-7}
$\overline{B}^0 \to D^+ a_1^-$	1.1×10^{-2}
$\overline{B}^0 \to D^+ b_1^-$	9.9×10^{-8}
$\overline{B}^0 \to D^0 a_1^0$	5.7×10^{-4}
$\overline{B}^0 \to D^0 f_1$	5.2×10^{-4}
$\overline{B}^0 \to D^0 b_1^0$	3.0×10^{-4}

$\overline{B}^0 \to D^0 h_1$	3.1×10^{-4}
$\overline{B}_s^0 \to K^0 D_1^0$	9.1×10^{-5}
$\overline{B}_s^0 \to K^0 \underline{D}_1^0$	1.8×10^{-6}
$\overline{B}_s^0 \to \pi^- D_{s1}^+$	3.6×10^{-3}
$\overline{B}_s^0 \to \pi^- \underline{D}_{s1}^+$	3.0×10^{-4}
$\overline{B}_s^0 \to D^0 K_1^0$	1.4×10^{-3}
$\overline{B}_s^0 \to D^0 \underline{K}_1^0$	7.3×10^{-7}
$\overline{B}_s^0 \to D_s^+ a_1^-$	9.9×10^{-3}
$\overline{B}_s^0 \to D_s^+ b_1^-$	8.9×10^{-8}
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	
$B^- \to K^- \chi_{c1}$	1.3×10^{-4}
$B^- \to D^0 D_{s1}^-$	2.5×10^{-3}
$B^- \to D^0 \underline{D}_{s1}^-$	7.6×10^{-5}
$B^- \to D_s^- D_1^0$	5.1×10^{-3}
$B^- \to D_s^- \underline{D}_1^0$	9.6×10^{-4}
$B^- \to \eta_c K_1^-$	2.5×10^{-3}
$B^- \to \eta_c \underline{K}_1^-$	7.0×10^{-5}
$\overline{B}^0 \to \overline{K}^0 \chi_{c1}$	1.2×10^{-4}
$\overline{B}^0 \to D^+ D_{s1}^-$	2.3×10^{-3}
$\overline{B}^0 \to D^+ \underline{D}_{s1}^-$	7.0×10^{-5}
$\overline{B}^0 \to D_s^- D_1^+$	4.7×10^{-3}
$\overline{B}^0 \to D_s^- \underline{D}_1^+$	9.0×10^{-4}
$\overline{B}^0 \to \eta_c \overline{K}_1^0$	2.3×10^{-3}
$\overline{B}^0 \to \eta_c \underline{\overline{K}}_1^0$	6.5×10^{-5}
$\overline{B}_s^0 \to \eta \chi_{c1}$	4.3×10^{-5}
$\overline{B}_s^0 \to \eta' \chi_{c1}$	3.8×10^{-5}
$\overline{B}_s^0 \to D_s^+ D_{s1}^-$	2.1×10^{-3}
$\overline{B}_s^0 \to D_s^+ \underline{D}_{s1}^-$	6.3×10^{-5}
$\overline{B}_s^0 \to D_s^- D_{s1}^+$	6.9×10^{-3}
$\overline{B}_s^0 \to D_s^- \underline{D}_{s1}^+$	9.3×10^{-4}
$\overline{B}_s^0 \to \eta_c f_1'$	1.1×10^{-3}
$\overline{B}_s^0 \to \eta_c h_1'$	9.4×10^{-4}

Table 3.8 Branching ratios for $B \to PA$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Branching ratios
$\Delta b = 1, \Delta C = 1, \Delta S = -1$	
$B^- \to K^- D_1^0$	1.5×10^{-4}
$B^- \to K^- \underline{D}_1^0$	3.0×10^{-5}
$B^- \to D^0 K_1^-$	1.6×10^{-4}
$B^- \to D^0 \underline{K}_1^-$	1.7×10^{-4}
$\overline{B}^0 \to \overline{K}^0 D_1^0$	6.8×10^{-6}
$\overline{B}^0 \to \overline{K}^0 \underline{D}_1^0$	1.4×10^{-7}
$\overline{B}^0 \to K^- D_1^+$	2.0×10^{-4}
$\overline{B}^0 \to K^- \underline{D}_1^+$	2.4×10^{-5}
$\overline{B}^0 \to D^+ K_1^-$	4.6×10^{-4}
$\overline{B}^0 \to D^+ \underline{K}_1^-$	1.3×10^{-4}
$\overline{B}^0 \to D^0 \overline{K}_1^0$	8.1×10^{-5}
$\overline{B}^0 \to D^0 \underline{\overline{K}}_1^0$	7.3×10^{-7}
$\overline{B}_s^0 \to \eta D_1^0$	2.4×10^{-6}
$\overline{B}_s^0 \to \eta \underline{D}_1^0$	5.0×10^{-8}
$\overline{B}_s^0 \to K^- D_{s1}^+$	2.7×10^{-4}
$\overline{B}_s^0 \to K^- \underline{D}_{s1}^+$	2.3×10^{-5}
$\overline{B}_s^0 \to \eta' D_1^0$	2.6×10^{-6}
$\overline{B}_s^0 \to \eta' \underline{D}_1^0$	5.4×10^{-8}
$\overline{B}_s^0 \to D^0 f_1'$	3.9×10^{-5}
$\overline{B}_s^0 \to D^0 h_1'$	2.3×10^{-5}
$\overline{B}_s^0 \to D_s^+ K_1^-$	4.1×10^{-4}
$\overline{B}_s^0 \to D_s^+ \underline{K}_1^-$	1.2×10^{-4}
$\Delta b = 1, \Delta C = 0, \Delta S = 0$	
$B^- \to \pi^- \chi_{c1}$	6.3×10^{-6}
$B^- \to D^0 D_1^-$	1.6×10^{-2}
$B^- \to D^0 \underline{D}_1^-$	3.0×10^{-6}
$B^- \to D^- D_1^0$	1.7×10^{-4}

$B^- \to D^- \underline{D}_1^0$	3.2×10^{-5}
$B^- \to \eta_c a_1^-$	9.4×10^{-5}
$B^- \to \eta_c b_1^-$	7.0×10^{-5}
$\overline{B}^0 \to \pi^0 \chi_{c1}$	2.9×10^{-6}
$\overline{B}^0 \to \eta \chi_{c1}$	1.4×10^{-6}
$\overline{B}^0 \to \eta' \chi_{c1}$	5.2×10^{-7}
$\overline{B}^0 \to D^+ D_1^-$	1.4×10^{-4}
$\overline{B}^0 \to D^+ \underline{D}_1^-$	2.8×10^{-6}
$\overline{B}^0 \to \eta \chi_{c1}$	1.4×10^{-6}
$\overline{B}^0 \to \eta' \chi_{c1}$	5.2×10^{-7}
$\overline{B}^0 \to D^+ D_1^-$	1.4×10^{-4}
$\overline{B}^0 \to D^+ \underline{D}_1^-$	2.8×10^{-6}
$\overline{B}^0 \to D^- D_1^+$	1.6×10^{-4}
$\overline{B}^0 \to D^- \underline{D}_1^+$	2.9×10^{-5}
$\overline{B}^0 \to \eta_c a_1^0$	4.4×10^{-5}
$\overline{B}^0 \to \eta_c f_1$	3.9×10^{-5}
$\overline{B}^0 \to \eta_c b_1^0$	3.3×10^{-5}
$\overline{B}^0 \to \eta_c h_1$	3.5×10^{-5}
$\overline{B}_s^0 \to K^0 \chi_{c1}$	4.5×10^{-6}
$\overline{B}_s^0 \to D_s^+ D_1^-$	1.3×10^{-4}
$\overline{B}_s^0 \to D_s^+ \underline{D}_1^-$	2.5×10^{-6}
$\overline{B}_s^0 \to D^- D_{s1}^+$	2.4×10^{-4}
$\overline{B}_s^0 \to D^- \underline{D}_{s1}^+$	3.0×10^{-5}
$\overline{B}_s^0 \to \eta_c K_1^0$	1.4×10^{-4}
$\overline{B}_s^0 \to \eta_c \underline{K}_1^0$	1.2×10^{-6}

We make the following observations:

3.6.1 $B \to PA$ DECAYS INVOLVING $b \to c$ TRANSITION

1. $\Delta b = 1, \Delta C = 1, \Delta S = 0$ mode:

a) Branching ratios for dominant mode is $B(\bar{B}^0 \to D^+ a_1^-)$ having branching ratio 1.1×10^{-2}. We hope that this value is within the reach of the current experiments.

b) For $\bar{B} \to D a_1$ decay mode the calculated branching ratios are

$B(B^- \to D^0 a_1^-) = 5.6\times10^{-3}$ $(4 \pm 4)\times10^{-3}$ $(Expt)$

$B(\bar{B}^0 \to D^+ a_1^-) = 1.1\times10^{-2}$ $(0.6 \pm 0.3)\times10^{-2}$ $(Expt)$

$B(\bar{B}^0 \to D^0 a_1^0) = 5.7\times10^{-4}$

Similarly, for $\bar{B} \to \pi D_1$ decay mode the, we calculate

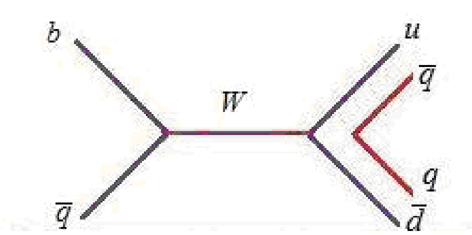
$B(B^- \to \pi^- D_1^0) = 1.8\times10^{-3}$ $(1.5 \pm 0.6)\times10^{-3}$ $(Expt)$

$B(\bar{B}^0 \to \pi^- D_1^+) = 2.7\times10^{-3}$

$B(\bar{B}^0 \to \pi^0 D_1^0) = 5.8\times10^{-5}$

Theoretical estimates are consistent with the observed modes.

c) The next order branching ratios in this mode are $B(B^- \to D^0 a_1^-) =$ 5.6×10^{-3}, $B(\bar{B}_s^0 \to D_s^+ a_1^-) =$ 9.9×10^{-3}, $B(\bar{B}_s^0 \to \pi^- D_{s1}^+) =$ 3.6×10^{-3} and $B(\bar{B}_s^0 \to D^0 K_1^0)$ $=$ 1.4×10^{-3}.



Annihilation Diagram

d) Decays $\bar{B}^0 \to D_s^+ K_1^- / D_s^+ \underline{K}_1^- / K^- D_{s1}^+ / K^- \underline{D}_{s1}^+$ are forbidden in the spectator model. These decays may be generated through quark annihilation diagrams. However, these annihilation contributions involve creation of $(s\bar{s})$ pair which is relatively suppressed.

e) $\bar{B}^0 \to D^0 f_1' / D^0 h_1'$ are forbidden in the limit of ideal mixing for $f_1 - f_1'$ and $h_1 - h_1'$ mesons. Any deviation from the ideal mixing may generate these decays.

f) It may be noted that no penguin or single quark transition contribute to this decay mode. However, $\bar{B}^0$ meson decays of this mode may have contribution from annihilation diagrams.

2. $\underline{\Delta b = 1, \Delta C = 0, \Delta S = -1 \text{ mode}:}$

a) For $K\chi_{c1}$ mode, we obtain

$$B(B^- \to K^- \chi_{c1}) = 1.3\times10^{-4},$$

$$B(\bar{B}^0 \to \bar{K}^0 \chi_{c1}) = 1.2\times10^{-4},$$

which are smaller than the measured branching ratios, i.e $B(B^- \to K^- \chi_{c1})$ = $(4.9 \pm 0.5)\times10^{-4}$ and $B(\bar{B}^0 \to \bar{K}^0 \chi_{c1}) =$ $(3.9 \pm 0.4)\times10^{-4}$, roughly by a factor of 4. Though, it may be remarked that penguin and annihilation diagrams do not contribute to these decays.

b) We obtain $B(B^- \to D_s^- D_1^0)$ = 5.1×10^{-3}, $B(B^- \to D^0 D_{s1}^-)$ = 2.5×10^{-3}, $B(B^- \to D_s^- \underline{D}_1^0)$ = 9.6×10^{-4}, $B(B^- \to D^0 \underline{D}_{s1}^-)$ = 7.6×10^{-5}, $B(\bar{B}^0 \to D_s^- D_1^+)$ = 4.7×10^{-3}, $B(\bar{B}^0 \to D^+ D_{s1}^-)$ = 2.3×10^{-3}, $B(\bar{B}^0 \to D_s^- \underline{D}_1^+)$ = 9.0×10^{-4} and $B(\bar{B}^0 \to D^+ \underline{D}_{s1}^-)$ = 7.0×10^{-5}. Inspite of the kinematic suppression, these modes acquire large branching ratios as these involve color-favored quark diagram and large value of decay constants of the charmed mesons.

c) Due to the vanishing decay constant $(f_{A'})$, decays $B^- \to K^- h_{c1}$ and $\bar{B}^0 \to \bar{K}^0 h_{c1}$ are forbidden in the present analysis.

d) Also, $\bar{B}_s^0 \to \pi^0 \chi_{c1} / \eta \chi_{c1} / \eta' \chi_{c1} / D^+ D_1^- / D^+ \underline{D}_1^- / D^0 \bar{D}_1^0 / D^0 \bar{\underline{D}}_1^0 / D^- D_1^+ / D^- \underline{D}_1^+ / \bar{D}^0 D_1^0 / \bar{D}^0 \underline{D}_1^0 / \eta_c a_1^0 / \eta_c f_1 / \eta_c f_1' / \eta_c b_1^0 / \eta_c h_1'$ are forbidden in present framework. However these are likely to remain suppressed as these decays require $c\bar{c}$ pair to be created.

e) Annihilation diagrams do not contribute to this decay mode. However, $\bar{B}^0 \to DD_{s1} / D\underline{D}_{s1} / D_s D_1 / D_s \underline{D}_1$ decay modes may have suppressed contribution from penguin diagrams which include $(c\bar{c})$ pair.

3. $\underline{\Delta b = 1, \Delta C = 0, \Delta S = 0 \text{ mode}:}$

a) For dominant decay, we predict branching ratios as $B(B^- \to D^0 D_1^-)$ = 1.6×10^{-2}.

b) In the present analysis, we obtain $B(B^- \to \pi^- \chi_{c1})$ = 0.63×10^{-5} which is smaller than the experimental branching ratio $(2.2 \pm 0.6)\times10^{-5}$.

c) Decays $\bar{B}^0 \to \pi^0 h_{c1} / \eta' h_{c1} / D^0 \bar{D}_1^0 / D^0 \bar{\underline{D}}_1^0 / D_s^+ D_{s1}^- / D_s^+ \underline{D}_{s1}^- / \bar{D}^0 D_1^0 / \bar{D}^0 \underline{D}_1^0 / D_s^- D_{s1}^+ / D_s^- \underline{D}_{s1}^+ / \eta_c f_1' / \eta_c h_1'$ and $\bar{B}_s^0 \to K^0 h_{c1}$ are forbidden in the present analysis. Annihilation diagrams, elastic FSI and penguin diagrams may generate these decays to the naked charm mesons. However, decays emitting charmonium h_{c1} remains forbidden in the ideal mixing limit.

4. $\underline{\Delta b = 1, \Delta C = 1, \Delta S = -1 \text{ mode}:}$

a) Branching ratios of the dominant decays in the present mode are $B(B^- \to D^0 \underline{K}_1^-)$ = 1.7×10^{-4}, $B(B^- \to D^0 K_1^-)$ = 1.6×10^{-4}, $B(B^- \to K^- D_1^0)$ = 1.5×10^{-4}, $B(\bar{B}^0 \to D^+ K_1^-)$= 4.6×10^{-4}, $B(\bar{B}^0 \to K^- D_1^+)$ = 2.0×10^{-4}, $B(\bar{B}^0 \to D^+ \underline{K}_1^-)$ = 1.3×10^{-4}, $B(\bar{B}_s^0 \to D_s^+ K_1^-)$ = 4.1×10^{-4}, $B(\bar{B}_s^0 \to K^- D_{s1}^+)$ = 2.7×10^{-4} and $B(\bar{B}_s^0 \to D_s^+ \underline{K}_1^-)$ = 1.2×10^{-4}.

b) Decays $\overline{B}_s^0 \to \pi^- D_1^+ / \pi^- \underline{D}_1^+ / \pi^0 D_1^0 / \pi^0 \underline{D}_1^0 / D^+ a_1^- / D^+ b_1^- / D^0 a_1^0 / D^0 f_1 / D^0 b_1^0 / D^0 h_1$ are forbidden in our analysis. Annihilation diagrams also do not contribute to these decays. However, these may acquire non zero branching ratios through elastic FSI.

Table 3.9 (a) Branching ratios for $B \to PA$ decays involving $b \to u$ transition

Decays	Branching ratios
$\Delta b = 1, \Delta C = -1, \Delta S = -1$	
$B^- \to \pi^0 D_{s1}^-$	9.9×10^{-6}
$B^- \to \pi^0 \underline{D}_{s1}^-$	5.2×10^{-7}
$B^- \to \eta D_{s1}^-$	5.1×10^{-6}
$B^- \to \eta \underline{D}_{s1}^-$	2.6×10^{-7}
$B^- \to K^- \overline{D}_1^0$	1.4×10^{-6}
$B^- \to K^- \overline{\underline{D}}_1^0$	4.9×10^{-8}
$B^- \to \eta' D_{s1}^-$	2.4×10^{-6}
$B^- \to \eta' \underline{D}_{s1}^-$	1.2×10^{-7}
$B^- \to \overline{D}^0 K_1^-$	1.3×10^{-5}
$B^- \to \overline{D}^0 \underline{K}_1^-$	1.2×10^{-7}
$B^- \to D_s^- a_1^0$	1.4×10^{-4}
$B^- \to D_s^- f_1$	1.3×10^{-4}
$B^- \to D_s^- b_1^0$	7.7×10^{-5}

$B^- \to D_s^- h_1$	8.1×10^{-5}
$\bar{B}^0 \to \pi^+ D_{s1}^-$	1.9×10^{-5}
$\bar{B}^0 \to \pi^+ \underline{D}_{s1}^-$	9.6×10^{-7}
$\bar{B}^0 \to \bar{K}^0 \bar{D}_1^0$	1.3×10^{-6}
$\bar{B}^0 \to \bar{K}^0 \underline{\bar{D}}_1^0$	4.6×10^{-8}
$\bar{B}^0 \to \bar{D}^0 \bar{K}_1^0$	1.2×10^{-5}
$\bar{B}^0 \to \bar{D}^0 \underline{\bar{K}}_1^0$	1.1×10^{-7}
$\bar{B}^0 \to D_s^- a_1^+$	2.7×10^{-4}
$\bar{B}^0 \to D_s^- b_1^+$	1.4×10^{-4}
$\bar{B}_s^0 \to K^+ D_{s1}^-$	1.5×10^{-5}
$\bar{B}_s^0 \to K^+ \underline{D}_{s1}^-$	7.6×10^{-7}
$\bar{B}_s^0 \to \eta \bar{D}_1^0$	4.5×10^{-7}
$\bar{B}_s^0 \to \eta \underline{\bar{D}}_1^0$	1.6×10^{-8}
$\bar{B}_s^0 \to \eta' \bar{D}_1^0$	4.9×10^{-7}
$\bar{B}_s^0 \to \eta' \underline{\bar{D}}_1^0$	1.8×10^{-8}
$\bar{B}_s^0 \to \bar{D}^0 f_1'$	5.9×10^{-6}
$\bar{B}_s^0 \to \bar{D}^0 h_1'$	3.5×10^{-6}
$\bar{B}_s^0 \to D_s^- K_1^+$	3.3×10^{-4}
$\bar{B}_s^0 \to D_s^- \underline{K}_1^+$	2.6×10^{-7}

Decays	Branching ratios		
	This Work	**CMV [14]**	**CHENG [10]**
$\Delta b = 1, \Delta C = 0, \Delta S = 0$			
$B^- \to \pi^0 a_1^-$	12.0×10^{-6}	13.6×10^{-6}	$14.4^{-1.4}_{+1.3}\times10^{-6}$
$B^- \to \pi^0 b_1^-$	1.03×10^{-6}	4.2×10^{-6}	$0.4^{-0.0}_{+0.0}\times10^{-6}$
$B^- \to \pi^- a_1^0$	27.8×10^{-6}	43.2×10^{-6}	$7.6^{-0.3}_{+0.3}\times10^{-6}$
$B^- \to \pi^- b_1^0$	17.8×10^{-6}	18.6×10^{-6}	$9.6^{-0.3}_{+0.3}\times10^{-6}$
$B^- \to \pi^- f_1$	55.2×10^{-6}	34.1×10^{-6}	-
$B^- \to \pi^- h_1$	18.5×10^{-6}	18.6×10^{-6}	-
$B^- \to \eta a_1^-$	6.0×10^{-6}	13.4×10^{-6}	-
$B^- \to \eta b_1^-$	0.63×10^{-6}	0.06×10^{-6}	-
$B^- \to \eta' a_1^-$	2.8×10^{-6}	13.6×10^{-6}	-
$B^- \to \eta' b_1^-$	0.37×10^{-6}	0.58×10^{-6}	-
$\overline{B}^0 \to \pi^+ a_1^-$	46.5×10^{-6}	36.7×10^{-6}	$23.4^{-2.3}_{+2.2}\times10^{-6}$
$\overline{B}^0 \to \pi^+ b_1^-$	3.92×10^{-10}	4.4×10^{-6}	0.3×10^{-6}
$\overline{B}^0 \to \pi^0 a_1^0$	3.4×10^{-6}	0.27×10^{-6}	$0.9^{-0.1}_{+0.1}\times10^{-6}$
$\overline{B}^0 \to \pi^0 f_1$	3.5×10^{-6}	0.47×10^{-6}	-
$\overline{B}^0 \to \pi^0 b_1^0$	0.47×10^{-6}	0.15×10^{-6}	$1.1^{-0.2}_{+0.2}\times10^{-6}$
$\overline{B}^0 \to \pi^0 h_1$	0.5×10^{-6}	0.16×10^{-6}	-
$\overline{B}^0 \to \pi^- a_1^+$	77.2×10^{-6}	74.3×10^{-6}	$9.1^{-0.2}_{+0.2}\times10^{-6}$
$\overline{B}^0 \to \pi^- b_1^+$	33.2×10^{-6}	36.2×10^{-6}	$11.2^{-0.3}_{+0.3}\times10^{-6}$
$\overline{B}^0 \to \eta a_1^0$	5.5×10^{-8}	0.54×10^{-6}	-
$\overline{B}^0 \to \eta f_1$	2.3×10^{-8}	37.1×10^{-6}	-
$\overline{B}^0 \to \eta b_1^0$	0.29×10^{-6}	0.17×10^{-6}	-
$\overline{B}^0 \to \eta h_1$	0.30×10^{-6}	18.2×10^{-6}	-
$\overline{B}^0 \to \eta' a_1^0$	0.04×10^{-8}	0.09×10^{-6}	-
$\overline{B}^0 \to \eta' f_1$	1.98×10^{-8}	22.1×10^{-6}	-
$\overline{B}^0 \to \eta' b_1^0$	0.17×10^{-6}	0.02×10^{-6}	-
$\overline{B}^0 \to \eta' h_1$	1.8×10^{-7}	11.2×10^{-6}	-
$\overline{B}_s^0 \to K^+ a_1^-$	36.4×10^{-6}	19.2×10^{-6}	-
$\overline{B}_s^0 \to K^+ b_1^-$	3.1×10^{-10}	-	-
$\overline{B}_s^0 \to K^0 a_1^0$	9.8×10^{-9}	0.09×10^{-6}	-

$\overline{B}_s^0 \to K^0 f_1$	1.2×10^{-6}	0.03×10^{-6}	-
$\overline{B}_s^0 \to \pi^0 K_1^0$	2.3×10^{-6}	-	-
$\overline{B}_s^0 \to \pi^0 \underline{K}_1^0$	8.6×10^{-10}	-	-
$\overline{B}_s^0 \to \pi^- K_1^+$	81.9×10^{-6}	-	-
$\overline{B}_s^0 \to \pi^- \underline{K}_1^+$	3.1×10^{-8}	-	-
$\overline{B}_s^0 \to \eta K_1^0$	1.4×10^{-6}	-	-
$\overline{B}_s^0 \to \eta \underline{K}_1^0$	3.6×10^{-10}	-	-
$\overline{B}_s^0 \to \eta' K_1^0$	8.1×10^{-7}	-	-
$\overline{B}_s^0 \to \eta' \underline{K}_1^0$	6.1×10^{-11}	-	-

Table 3.9 (b) Branching ratios for $B \to PA$ decays involving $b \to u$ transition

Decays	Branching ratios		
	This Work	CMV [14]	CHENG [10]
$\Delta b = 1, \Delta C = 0, \Delta S = -1$			
$B^- \to \pi^0 K_1^-$	0.37×10^{-6}	2.5×10^{-6}	$2.7^{+0.1}_{-0.1}$×10^{-6}
$B^- \to \pi^0 \underline{K}_1^-$	0.34×10^{-6}	0.7×10^{-6}	$3.0^{+0.4}_{-0.4}$×10^{-6}
$B^- \to \eta K_1^-$	0.18×10^{-6}	0.95×10^{-6}	-
$B^- \to \eta \underline{K}_1^-$	0.18×10^{-6}	95.1×10^{-6}	-
$B^- \to K^- a_1^0$	2.2×10^{-6}	43.4×10^{-6}	$13.9^{+0.9}_{-0.9}$×10^{-6}
$B^- \to K^- f_1$	4.1×10^{-6}	31.1×10^{-6}	-
$B^- \to K^- b_1^0$	1.4×10^{-6}	18.1×10^{-6}	$6.5^{+0.5}_{-0.5}$×10^{-6}
$B^- \to K^- h_1$	1.4×10^{-6}	19.0×10^{-6}	-
$B^- \to \eta \underline{K}_1^-$	8.1×10^{-8}	0.53×10^{-6}	-
$B^- \to \eta' K_1^-$	9.3×10^{-8}	80.0×10^{-6}	-
$\overline{B}^0 \to \pi^+ K_1^-$	1.9×10^{-6}	4.3×10^{-6}	$3.0^{+0.8}_{-0.6}$×10^{-6}
$\overline{B}^0 \to \pi^+ \underline{K}_1^-$	5.9×10^{-7}	2.3×10^{-6}	$5.4^{+1.1}_{-1.0}$×10^{-6}
$\overline{B}^0 \to \pi^0 \overline{K}_1^0$	1.6×10^{-7}	2.3×10^{-6}	$1.0^{+0.0}_{-0.0}$×10^{-6}
$\overline{B}^0 \to \pi^0 \overline{\underline{K}}_1^0$	3.9×10^{-10}	1.7 ×10^{-6}	$2.9^{+0.3}_{-0.3}$×10^{-6}
$\overline{B}^0 \to \eta \overline{K}_1^0$	9.4×10^{-8}	-	-
$\overline{B}^0 \to \eta \overline{\underline{K}}_1^0$	2.7×10^{-10}	-	-
$\overline{B}^0 \to \overline{K}^0 a_1^0$	7.5×10^{-8}	42.3×10^{-6}	$6.9^{+0.3}_{-0.3}$×10^{-6}
$\overline{B}^0 \to \overline{K}^0 f_1$	8.9×10^{-8}	34.7×10^{-6}	-
$\overline{B}^0 \to K^- a_1^+$	5.9×10^{-6}	72.2×10^{-6}	$18.3^{+1.0}_{-1.0}$×10^{-6}

$\overline{B}^0 \to K^- b_1^+$	2.6×10^-6	35.7×10^-6	$12.1^{+1.0}_{-0.9}\times10^{-6}$
$\overline{B}^0 \to \eta' \overline{K}_1^0$	5.3×10^{-8}	1.1×10^{-6}	-
$\overline{B}^0 \to \eta' \underline{\overline{K}}_1^0$	2.0×10^{-10}	51.4×10^{-6}	-
$\overline{B}_s^0 \to K^+ K_1^-$	1.5×10^{-6}	3.3×10^{-6}	-
$\overline{B}_s^0 \to K^+ \underline{K}_1^-$	0.46×10^{-6}	1.8×10^{-6}	-
$\overline{B}_s^0 \to \pi^0 f_1'$	7.5×10^{-8}	-	-
$\overline{B}_s^0 \to \pi^0 h_1'$	3.5×10^{-8}	-	-
$\overline{B}_s^0 \to \eta a_1^0$	2.6×10^{-8}	0.14×10^{-6}	-
$\overline{B}_s^0 \to \eta f_1$	3.1×10^{-8}	0.19×10^{-6}	-
$\overline{B}_s^0 \to \eta f_1'$	4.6×10^{-8}	-	-
$\overline{B}_s^0 \to \eta h_1'$	2.2×10^{-8}	-	-
$\overline{B}_s^0 \to K^- K_1^+$	6.3×10^{-3}	-	-
$\overline{B}_s^0 \to K^- \underline{K}_1^+$	1.8×10^{-9}	-	-
$\overline{B}_s^0 \to \eta' a_1^0$	3.0×10^{-8}	0.14×10^{-6}	-
$\overline{B}_s^0 \to \eta' f_1$	3.5×10^{-8}	0.18×10^{-6}	-
$\overline{B}_s^0 \to \eta' f_1'$	2.6×10^{-8}	-	-
$\overline{B}_s^0 \to \eta' h_1'$	1.3×10^{-8}	-	-
$\Delta b = 1, \Delta C = -1, \Delta S = 0$			
$B^- \to \pi^0 D_1^-$	5.9×10^{-7}	-	-
$B^- \to \pi^0 \underline{D}_1^-$	2.0×10^{-8}	-	-
$B^- \to \pi^- \overline{D}_1^0$	6.4×10^{-8}	-	-
$B^- \to \pi^- \underline{\overline{D}}_1^0$	2.2×10^{-9}	-	-
$B^- \to \eta D_1^-$	3.1×10^{-7}	-	-
$B^- \to \eta \underline{D}_1^-$	1.1×10^{-8}	-	-
$B^- \to \eta' D_1^-$	1.5×10^{-9}	-	-
$B^- \to \eta' \underline{D}_1^-$	5.0×10^{-9}	-	-
$B^- \to D^- a_1^0$	4.5×10^{-6}	-	-
$B^- \to D^- f_1$	4.1×10^{-6}	-	-
$B^- \to D^- b_1^0$	2.4×10^{-6}	-	-
$B^- \to D^- h_1$	2.5×10^{-6}	-	-
$B^- \to \overline{D}^0 a_1^-$	5.0×10^{-7}	-	-
$B^- \to \overline{D}^0 b_1^-$	2.9×10^{-7}	-	-
$\overline{B}^0 \to \pi^+ D_1^-$	1.1×10^{-6}	-	-

$\overline{B}^0 \to \pi^+ \underline{D}_1^-$	3.8×10^{-8}	-	-
$\overline{B}^0 \to \pi^0 \overline{D}_1^0$	3.0×10^{-8}	-	-
$\overline{B}^0 \to \pi^0 \underline{\overline{D}}_1^0$	1.0×10^{-9}	-	-
$\overline{B}^0 \to \eta \overline{D}_1^0$	1.5×10^{-8}	-	-
$\overline{B}^0 \to \eta \underline{\overline{D}}_1^0$	5.3×10^{-10}	-	-
$\overline{B}^0 \to \eta' \overline{D}_1^0$	7.3×10^{-9}	-	-
$\overline{B}^0 \to \eta' \underline{\overline{D}}_1^0$	2.5×10^{-10}	-	-
$\overline{B}^0 \to D^- a_1^+$	8.4×10^{-6}	-	-
$\overline{B}^0 \to D^- b_1^+$	4.4×10^{-6}	-	-
$\overline{B}^0 \to \overline{D}^0 a_1^0$	2.3×10^{-7}	-	-
$\overline{B}^0 \to \overline{D}^0 f_1$	2.1×10^{-7}	-	-
$\overline{B}^0 \to \overline{D}^0 b_1^0$	1.2×10^{-7}	-	-
$\overline{B}^0 \to \overline{D}^0 h_1$	1.3×10^{-7}	-	-
$\overline{B}_s^0 \to K^+ D_1^-$	8.7×10^{-7}	-	-
$\overline{B}_s^0 \to K^+ \underline{D}_1^-$	3.0×10^{-8}	-	-
$\overline{B}_s^0 \to K^0 \overline{D}_1^0$	4.7×10^{-8}	-	-
$\overline{B}_s^0 \to K^0 \underline{\overline{D}}_1^0$	1.6×10^{-9}	-	-
$\overline{B}_s^0 \to D^- K_1^+$	1.0×10^{-5}	-	-
$\overline{B}_s^0 \to D^- \underline{K}_1^+$	5.4×10^{-9}	-	-
$\overline{B}_s^0 \to \overline{D}^0 K_1^0$	5.7×10^{-7}	-	-
$\overline{B}_s^0 \to \overline{D}^0 \underline{K}_1^0$	3.0×10^{-10}	-	-

3.6.2 $B \to PA$ DECAYS INVOLVING $b \to u$ TRANSITION

1. $\underline{\Delta b = 1, \Delta C = 0, \Delta S = 0 \text{ mode}:}$

a) For $\overline{B} \to \pi a_1$ decay mode, we calculate

$$B(B^- \to \pi^0 a_1^-) = 12.0\times10^{-6} \qquad (26.0 \pm 7.0)\times10^{-6} \quad (Expt),$$

which is smaller by a factor of 2 with experimental value.

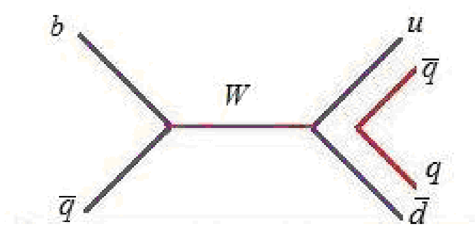
$$B(B^- \to \pi^- a_1^0) = 27.8\times10^{-6} \qquad (20.0 \pm 6.0)\times10^{-6} \quad (Expt)$$

$$B(\bar{B}^0 \to \pi^+ a_1^-) = 46.5\times10^{-6} \qquad (33.2\pm5.0)\times10^{-6} \quad (Expt)$$

agrees with the experimental value.

$$B(\bar{B}^0 \to \pi^0 a_1^0) = 3.4\times10^{-6} \qquad <1.1\times10^{-3} \qquad (Expt)$$

is well below the experimental upper limit. Annihilation diagram may contribute to these decays which may improve the branching ratios.



Annihilation Diagram

b) $B^- \to K^0 K_1^- / K^0 \underline{K}_1^- / K^- K_1^0 / K^- \underline{K}_1^0 / \pi^- f_1' / \pi^- h_1' / \pi^- h_{c1}, \bar{B}^0 \to K^+ K_1^- / K^+ \underline{K}_1^- / K^0 \bar{K}_1^0 /$ $K^0 \underline{\bar{K}}_1^0 / \pi^0 f_1' / \pi^0 h_1' / \eta f_1' / \eta h_1' / \bar{K}^0 K_1^0 / \bar{K}^0 \underline{K}_1^0 / K^- K_1^+ / K^- \underline{K}_1^+ / \eta' f_1' / \eta' h_1'$ and $\bar{B}_s^0 \to$ $K^0 f_1' / K^0 b_1^0 / K^0 h_1 / K^0 h_1'$ are forbidden in the present analysis. Annihilation process and FSIs may generate these decays.

c) $B^- \to \pi^- \bar{K}_1^0 / \pi^- \underline{\bar{K}}_1^0$ and $\bar{B}^0 \to \pi^+ K_1^- / \pi^+ \underline{K}_1^-$ are also forbidden in the present analysis which may be generated through annihilation diagram or elastic FSI.

2. $\underline{\Delta b = 1, \Delta C = -1, \Delta S = -1 \text{ mode:}}$

a) Dominant decay in the present mode are $B(B^- \to D_s^- a_1^0)$ = 1.4×10^{-4}, $B(B^- \to D_s^- f_1)$ = 1.3×10^{-4}, $B(\bar{B}^0 \to D_s^- a_1^+)$ = 2.7×10^{-4}, $B(\bar{B}^0 \to D_s^- b_1^+)$ = 1.4×10^{-4} and $B(\bar{B}_s^0 \to D_s^- K_1^+)$ = 3.3×10^{-4}.

b) Calculated branching ratios $B(B^- \to D_s^- a_1^0)$ = 1.4×10^{-4} and $B(\bar{B}^0 \to D_s^- a_1^+)$ = 2.7×10^{-4} are consistent with the experimental upper limits $<1.8\times10^{-3}$ and $<2.2\times10^{-3}$.

c) Decays $B^- \to \bar{K}^0 D_1^- / \bar{K}^0 \underline{D}_1^- / D^- \bar{K}_1^0 / D^- \underline{\bar{K}}_1^0 / D_s^- f_1' / D_s^- h_1'$ and $\bar{B}_s^0 \to \pi^+ D_1^- / \pi^+ \underline{D}_1^- / \pi^0 \bar{D}_1^0 / \pi^0 \underline{\bar{D}}_1^0 / D^- a_1^+ / D^- b_1^+ / \bar{D}^0 a_1^0 / \bar{D}^0 f_1 / \bar{D}^0 b_1^0 / \bar{D}^0 h_1$ are forbidden in the present analysis. Annihilation and FSIs may generate these decays.

3. $\underline{\Delta b = 1, \Delta C = -1, \Delta S = 0 \text{ mode}:}$

a) Branching ratios of $B(\bar{B}^0 \to D^- a_1^+) =$ 8.4×10^{-6}, which is very small as compare to experimental value $(6.0 \pm 3.3) \times 10^{-3}$. The disagreement with the experiment may be attributed due to the reason that annihilation diagram may generate this decay, which are neglected in the present work.

b) Decays $B^- \to K^0 D_{s1}^- / K^0 \underline{D}_{s1}^- / D^- f_1' / D^- h_1' / D_s^- K_1^0 / D_s^- \underline{K}_1^0$ and $\bar{B}^0 \to K^+ D_{s1}^- / K^+ \underline{D}_{s1}^- / \bar{D}^0 f_1' / \bar{D}^0 h_1' / D_s^- K_1^+ / D_s^- \underline{K}_1^+$ are forbidden in the present analysis. Annihilation diagrams may generate these decays.

4. $\underline{\Delta b = 1, \Delta C = 0, \Delta S = -1 \text{ mode}:}$

a) For $\bar{B} \to \pi K_1$ decay mode,

$$B(B^- \to \pi^0 \underline{K}_1^-) = 3.4\times10^{-7} \qquad < 2.6\times10^{-3} \quad (Expt)$$

$$B(\bar{B}^0 \to \pi^+ \underline{K}_1^-) = 5.9\times10^{-7} \qquad < 1.1\times10^{-3} \quad (Expt)$$

$$B(\bar{B}^0 \to \pi^0 \underline{K}_1^0) = 3.9\times10^{-10}$$

which are well below the experimental upper limits and

$$B(\bar{B}^0 \to K^- a_1^+) = 5.9\times10^{-6} \qquad (16.0 \pm 4.0)\times10^{-6} \quad (Expt)$$

b) Decays $B^- \to \pi^- \bar{K}_1^0 / \pi^- \underline{\bar{K}}_1^0 / \bar{K}^0 a_1^- / \bar{K}^0 b_1^- / K^- f_1' / K^- h_1'$, $\bar{B}^0 \to \bar{K}^0 f_1' / \bar{K}^0 b_1^0 / \bar{K}^0 h_1 / \bar{K}^0 h_1'$ and $\bar{B}_s^0 \to K^0 \bar{K}_1^0 / K^0 \underline{\bar{K}}_1^0 / \pi^+ a_1^- / \pi^+ b_1^- / \pi^0 a_1^0 / \pi^0 f_1 / \pi^0 b_1^0 / \pi^0 h_1 / \pi^- a_1^+ / \pi^- b_1^+ / \eta b_1^0 / \eta h_1\ \bar{K}^0 K_1^0 / \bar{K}^0 \underline{K}_1^0 / \eta' b_1^0 / \eta' h_1$ are forbidden in the present analysis. Annihilation and FSIs may generate these decays.

3.6.3 COMPARISON WITH OTHER WORKS

Also, we compare our results with branching ratios calculated in the other models [13, 14, 15]. The predicted branching ratios in CMV [13] shown in 3rd column of tables VI, VII, VIII(a) and VIII(b) are generally larger as compared to the present branching ratios because of the difference in the form factors have been used in the two works, particularly for $B \to \eta^{(\prime)} K_1$ **.** In CMV [13] the large value of branching ratio $B \to \eta^{(\prime)} K_1$ as a combination of effects of the penguin contribution in the effective Hamiltonian and the two mixing $K_{1A} - K_{1A'}$ and $\eta - \eta'$. Branching ratios have also been calculated by Cheng [14, 15]. His predictions for hadronic charmed [14] decays $B(B^- \to \pi^- D_1^0)$ = 3.7×10^{-4}, $B(B^- \to \pi^- \underline{D}_1^0)$ = 1.1×10^{-3}, $B(B^- \to D_s^- D_1^0)$ = 9.6×10^{-4}, $B(B^- \to D_s^- \underline{D}_1^0)$ = 1.3×10^{-3}, $B(B^- \to D^0 D_{s1}^-)$ = 4.3×10^{-3}, $B(B^- \to D^0 \underline{D}_{s1}^-)$ = 3.1×10^{-4}, $B(\overline{B}^0 \to \pi^- D_1^+)$ = 6.8×10^{-4}, $B(\overline{B}^0 \to \pi^- \underline{D}_1^+)$ = 1.0×10^{-3}, $B(\overline{B}^0 \to D_s^- D_1^+)$ = 8.8×10^{-4}, $B(\overline{B}^0 \to D_s^- \underline{D}_1^+)$ = 1.2×10^{-3}, $B(\overline{B}^0 \to D^+ D_{s1}^-)$ = 3.9×10^{-3}, $B(\overline{B}^0 \to D^+ \underline{D}_{s1}^-)$ = 2.8×10^{-4}, $B(\overline{B}_s^0 \to \pi^- D_{s1}^+)$ = 5.2×10^{-4} and $B(\overline{B}_s^0 \to \pi^- \underline{D}_{s1}^+)$ = 1.5×10^{-3} are different from our results owing to the different values used for the decay constants and different form factor values used. Branching ratios for hadronic charmless [15] decays are generally smaller than our numerical values of branching ratios. The disagreement with their predictions may be attributed to the difference in the form factors obtained in the covariant light-front approach (CLF).

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CHAPTER 4

HADRONIC WEAK DECAYS OF BOTTOM MESONS EMITTING PSEUDOSCALAR AND TENSOR MESONS

4.1 INTRODUCTION

In this chapter, we analyze two-body weak hadronic decays of B^-, $\overline{B}^0$ and $\overline{B}_s^0$ mesons to pseudoscalar meson and tensor meson, for which the experiments have provided the following branching ratios [1]:

$$B(B^- \to \pi^- D_2^0) = (7.8 \pm 1.4) \times 10^{-4} ,$$

$$B(B^- \to \pi^- f_2) = (8.2 \pm 2.5) \times 10^{-6} ,$$

$$B(B^- \to K^- f_2) = (1.3^{+0.4}_{-0.5}) \times 10^{-6} ,$$

$$B(B^- \to \eta K_2^-) = (9.1 \pm 3.0) \times 10^{-6} ,$$

$$B(\overline{B}^0 \to \eta \overline{K}_2^0) = (9.6 \pm 2.1) \times 10^{-6} ,$$

$$B(\overline{B}^0 \to \overline{D}^0 f_2) = (1.2 \pm 0.4) \times 10^{-4} ,$$

$$B(\overline{B}^0 \to \pi^{\mp} a_2^{\pm}) < 3.0 \times 10^{-4} ,$$

$$B(B^- \to \pi^- \overline{K}_2^0) < 6.9\times10^{-6},$$

$$B(\overline{B}^0 \to D_s^- a_2^+) < 1.9\times10^{-4},$$

$$B(\overline{B}^0 \to \pi^+ K_2^-) < 1.8\times10^{-5},$$

$$B(\overline{B}^0 \to \pi^- D_2^+) < 2.2\times10^{-3}.$$

Employing the factorization scheme, we calculate the decay amplitudes for CKM-favored and CKM-suppressed modes involving $b \to c$ and $b \to u$ transitions in the Isgur, Scora, Grinstein and Wise (ISGW II) model [2, 3]. In general, *W*-annihilation and *W*-exchange diagrams [4, 5] may also contribute to these decays under consideration. Normally, such contributions are expected to be suppressed due to the helicity and color arguments and are neglected in this work. We also compare our predictions with the recent works [6, 7].

4.2 TENSOR MESON SPECTROSCOPY

Experimentally [1], the tensor meson $(J^P = 2^+)$ sixteen-plet comprises of an isovector $a_2(1.318)$, strange isospinor $K_2^*(1.429)$, charm SU(3) triplet $D_2^*(2.457)$, $D_{s2}^*(2.573)$ and three isoscalars $f_2(1.275)$, $f_2'(1.525)$ and $\chi_{c2}(3.555)$. These states behave well with respect to the quark model assignments, though the spin and parity of the charm isosinglet $D_{s2}^*(2.573)$ remain to be confirmed. The numbers given within parentheses indicate mass (in GeV units) of the respective mesons. $\chi_{c2}(3.555)$ is assumed to be pure $(c\overline{c})$ state, and mixing of the isoscalar states is defined as:

$$f_2(1.275) = \frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d})\cos\phi_T + (s\overline{s})\sin\phi_T,$$

$$f_2^{'}(1.525)\frac{1}{\sqrt{2}}(u\overline{u} + d\overline{d})\sin\phi_T - (s\overline{s})\cos\phi_T, \tag{4.1}$$

where $\phi_T = \theta(ideal) - \theta_T(physical)$ and $\theta_T(physical) = 27^\circ$ [1].

4.3 METHODOLOGY

The effective weak Hamiltonian generating the b-quark decays involving $b \to c$ and $b \to u$ transitions has given already been in earlier chapter 2 for CKM-favored and CKM-suppressed modes, respectively.

4.3.1 DECAY AMPLITUDES AND RATES

The decay rate formula for $B(0^-) \to P(0^-) + T(2^+)$ decay is given by

$$\Gamma(B \to PT) = \left(\frac{m_B}{m_T}\right)^2 \frac{p_c^5}{12\pi m_T^2} \left|A(B \to PT)\right|^2, \tag{4.2}$$

where p_c is the magnitude of the three-momentum of the final-state particle in the rest frame of B meson and m_B and m_T denote masses of the B meson and tensor meson, respectively.

The factorization scheme in general expresses the weak decay amplitude as the product of matrix elements of weak currents (up to the weak scale factor of $\frac{G_F}{\sqrt{2}} \times CKM$ elements $\times$ QCD factor),

$$A(B \to PT) = \langle P | J^\mu | 0 \rangle \langle T | J_\mu | B \rangle + \langle T | J^\mu | 0 \rangle \langle P | J_\mu | B \rangle. \tag{4.3}$$

The matrix elements $\langle P | J^\mu | 0 \rangle$ and $\langle P | J_\mu | B \rangle$ are already given in the chapter 2, in (2.68). However, the matrix elements $\langle T(q_\mu) | J_\mu | 0 \rangle$ vanish due to the tracelessness of the polarization tensor $\in_{\mu\upsilon}$ of spin 2 meson and the auxiliary condition $q^\mu \in_{\mu\upsilon} = 0$ [8]. Remaining matrix element is expressed as:

$$\begin{aligned} \langle T(P_T) | J_\mu | B(P_B) \rangle = ih \in_{\mu\upsilon\lambda\rho} \in^{*\upsilon\alpha} P_{B\alpha} (P_B + P_T)^\lambda (P_B - P_T)^\rho + k \in^*_{\mu\upsilon} P_B^\upsilon \\ + b_+ (\in^*_{\alpha\beta} P_B^\alpha P_B^\beta)[(P_B + P_T)_\mu + b_- (P_B - P_T)_\mu], \end{aligned} \tag{4.4}$$

in the ISGW model [3] which yields

$$\langle PT|H_W|B\rangle = -if_P(\in^{*}_{\mu\upsilon}\, P^{\mu}_{B}P^{\upsilon}_{B})F^{B\to T}(m_P^2), \tag{4.5}$$

where

$$F^{B\to T}(m_P^2) = k(m_P^2) + (m_{\rm B}^2 - m_T^2)b_+(m_P^2) + m_p^2 b_-(m_P^2). \tag{4.6}$$

Thus

$$A(B\to PT) = \frac{G_F}{\sqrt{2}}\times(CKM\ \ factors\times QCD\ factors\times CG\ factors)\times f_P F^{B\to T}(m_P^2)\,. \tag{4.7}$$

Sandwiching the weak Hamiltonian between the initial and final states, we obtain decay amplitudes of B^-, $\overline{B}^0$ and $\overline{B}^0_s$ mesons for various decay modes as given in the Tables 4.1, 4.2, 4.3 (a) and 4.3 (b).

Table 4.1 Decay amplitudes of $B\to PT$ decays in CKM-favored mode involving $b\to c$ transition

Decays	Amplitudes
$\Delta b=1,\ \Delta C=1,\ \Delta S=0$	$\times\frac{G_F}{\sqrt{2}}V_{cb}V^{*}_{ud}$
$B^-\to\pi^- D_2^0$	$a_1 f_\pi F^{B\to D_2}(m_\pi^2)$
$B^-\to D^0 a_2^-$	$a_2 f_D F^{B\to a_2}(m_D^2)$
$\overline{B}^0\to\pi^- D_2^+$	$a_1 f_\pi F^{B\to D_2}(m_\pi^2)$
$\overline{B}^0\to D^0 a_2^0$	$-\frac{1}{\sqrt{2}}a_2 f_D F^{B\to a_2}(m_D^2)$
$\overline{B}^0\to D^0 f_2$	$\frac{1}{\sqrt{2}}a_2 f_D\cos\phi_T F^{B\to f_2}(m_D^2)$
$\overline{B}^0\to D^0 f_2'$	$\frac{1}{\sqrt{2}}a_2 f_D\sin\phi_T F^{B\to f_2'}(m_D^2)$
$\overline{B}^0_s\to\pi^- D_{s2}^+$	$a_1 f_\pi F^{B_s\to D_{s2}}(m_\pi^2)$
$\overline{B}^0_s\to D^0 K_2^0$	$a_2 f_D F^{B_s\to K_2}(m_D^2)$
$\Delta b=1,\ \Delta C=0,\ \Delta S=-1$	$\times\frac{G_F}{\sqrt{2}}V_{cb}V^{*}_{cs}$
$B^-\to D_s^- D_2^0$	$a_1 f_{D_s} F^{B\to D_2}(m_{D_s}^2)$
$B^-\to\eta_c K_2^-$	$a_2 f_{\eta_c} F^{B\to K_2}(m_{\eta_c}^2)$

$\overline{B}^0 \to D_s^- D_2^+$	$a_1 f_{D_s} F^{B\to D_2}(m_{D_s}^2)$
$\overline{B}^0 \to \eta_c K_2^0$	$a_2 f_{\eta_c} F^{B\to K_2}(m_{\eta_c}^2)$
$\overline{B}_s^0 \to D_s^- D_{s2}^+$	$a_1 f_{D_s} F^{B_s\to D_{s2}}(m_{D_s}^2)$
$\overline{B}_s^0 \to \eta_c f_2$	$a_2 f_{\eta_c} \sin\phi_T F^{B_s\to f_2}(m_{\eta_c}^2)$
$\overline{B}_s^0 \to \eta_c f_2'$	$-a_2 f_{\eta_c} \cos\phi_T F^{B_s\to f_2'}(m_{\eta_c}^2)$

Table 4.2 Decay amplitudes of $B \to PT$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^*$
$B^- \to K^- D_2^0$	$a_1 f_K F^{B\to D_2}(m_K^2)$
$B^- \to D^0 K_2^-$	$a_2 f_D F^{B\to K_2}(m_D^2)$
$\overline{B}^0 \to K^- D_2^+$	$a_1 f_K F^{B\to D_2}(m_K^2)$
$\overline{B}^0 \to D^0 \overline{K}_2^0$	$a_2 f_D F^{B\to K_2}(m_D^2)$
$\overline{B}_s^0 \to K^- D_{s2}^+$	$a_1 f_K F^{B_s\to D_{s2}}(m_K^2)$
$\overline{B}_s^0 \to D^0 f_2$	$a_2 f_D \sin\phi_T F^{B_s\to f_2}(m_D^2)$
$\overline{B}_s^0 \to D^0 f_2'$	$-a_2 f_D \cos\phi_T F^{B_s\to f_2'}(m_D^2)$
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^*$
$B^- \to D^- D_2^0$	$a_1 f_D F^{B\to D_2}(m_D^2)$
$B^- \to \eta_c a_2^-$	$a_2 f_{\eta_c} F^{B\to a_2}(m_{\eta_c}^2)$
$\overline{B}^0 \to D^- D_2^+$	$a_1 f_D F^{B\to D_2}(m_D^2)$
$\overline{B}^0 \to \eta_c a_2^0$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta_c} F^{B\to a_2}(m_{\eta_c}^2)$
$\overline{B}^0 \to \eta_c f_2$	$\frac{1}{\sqrt{2}} a_2 f_{\eta_c} \cos\phi_T F^{B\to f_2}(m_{\eta_c}^2)$
$\overline{B}^0 \to \eta_c f_2'$	$\frac{1}{\sqrt{2}} a_2 f_{\eta_c} \sin\phi_T F^{B\to f_2'}(m_{\eta_c}^2)$
$\overline{B}_s^0 \to D^- D_{s2}^+$	$a_1 f_D F^{B_s\to D_{s2}}(m_D^2)$
$\overline{B}_s^0 \to \eta_c K_2^0$	$a_2 f_{\eta_c} F^{B_s\to K_2}(m_{\eta_c}^2)$

Table 4.3 (a) Decay amplitudes of $B \to PT$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = -1,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^*$
$B^- \to \overline{D}^0 K_2^-$	$a_2 f_D F^{B\to K_2}(m_D^2)$
$B^- \to D_s^- a_2^0$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} F^{B\to a_2}(m_{D_s}^2)$
$B^- \to D_s^- f_2$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} \cos\phi_T F^{B\to f_2}(m_{D_s}^2)$
$B^- \to D_s^- f_2'$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} \sin\phi_T F^{B\to f_2'}(m_{D_s}^2)$
$\overline{B}^0 \to \overline{D}^0 \overline{K}_2^0$	$a_2 f_D F^{B\to K_2}(m_D^2)$
$\overline{B}^0 \to D_s^- a_2^+$	$a_1 f_{D_s} F^{B\to a_2}(m_{D_s}^2)$
$\overline{B}_s^0 \to \overline{D}^0 f_2$	$a_2 f_D \sin\phi_T F^{B_s\to f_2}(m_D^2)$
$\overline{B}_s^0 \to \overline{D}^0 f_2'$	$-a_2 f_D \cos\phi_T F^{B_s\to f_2}(m_D^2)$
$\overline{B}_s^0 \to D_s^- K_2^+$	$a_1 f_{D_s} F^{B_s\to K_2}(m_{D_s}^2)$
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*$
$B^- \to \pi^0 a_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B\to a_2}(m_\pi^2)$
$B^- \to \eta a_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to a_2}(m_\eta^2)$
$B^- \to \eta' a_2^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B\to a_2}(m_{\eta'}^2)$
$B^- \to \pi^- a_2^0$	$\frac{1}{\sqrt{2}} a_1 f_\pi F^{B\to a_2}(m_\pi^2)$
$B^- \to \pi^- f_2$	$\frac{1}{\sqrt{2}} a_1 f_\pi \cos\phi_T F^{B\to f_2}(m_\pi^2)$
$B^- \to \pi^- f_2'$	$\frac{1}{\sqrt{2}} a_1 f_\pi \sin\phi_T F^{B\to f_2'}(m_\pi^2)$
$\overline{B}^0 \to \pi^- a_2^+$	$a_1 f_\pi F^{B\to a_2}(m_\pi^2)$
$\overline{B}^0 \to \pi^0 a_2^0$	$-\frac{1}{2} a_2 f_\pi F^{B\to a_2}(m_\pi^2)$
$\overline{B}^0 \to \pi^0 f_2$	$\frac{1}{2} a_2 f_\pi \cos\phi_T F^{B\to f_2}(m_\pi^2)$
$\overline{B}^0 \to \pi^0 f_2'$	$\frac{1}{2} a_2 f_\pi \sin\phi_T F^{B\to f_2'}(m_\pi^2)$

$\overline{B}^0 \to \eta a_2^0$	$-\frac{1}{2} a_2 f_\eta \sin\phi_P F^{B\to a_2}(m_\eta^2)$
$\overline{B}^0 \to \eta f_2$	$\frac{1}{2} a_2 f_\eta \sin\phi_P \cos\phi_T F^{B\to f_2}(m_\eta^2)$
$\overline{B}^0 \to \eta f_2'$	$\frac{1}{2} a_2 f_\eta \sin\phi_P \sin\phi_T F^{B\to f_2'}(m_\eta^2)$
$\overline{B}^0 \to \eta' a_2^0$	$-\frac{1}{2} a_2 f_{\eta'} \cos\phi_P F^{B\to a_2}(m_{\eta'}^2)$
$\overline{B}^0 \to \eta' f_2$	$\frac{1}{2} a_2 f_{\eta'} \cos\phi_P \cos\phi_T F^{B\to f_2}(m_{\eta'}^2)$
$\overline{B}^0 \to \eta' f_2'$	$\frac{1}{2} a_2 f_{\eta'} \cos\phi_P \sin\phi_T F^{B\to f_2'}(m_{\eta'}^2)$
$\overline{B}_s^0 \to \pi^0 K_2^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_s\to K_2}(m_\pi^2)$
$\overline{B}_s^0 \to \pi^- K_2^+$	$a_1 f_\pi F^{B_s\to K_2}(m_\pi^2)$
$\overline{B}_s^0 \to \eta K_2^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_s\to K_2}(m_\eta^2)$
$\overline{B}_s^0 \to \eta' K_2^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_s\to K_2}(m_{\eta'}^2)$

Table 4.3 (b) Decay amplitudes of $B \to PT$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$\times\frac{G_F}{\sqrt{2}} V_{ub} V_{us}^*$
$B^- \to K^- a_2^0$	$\frac{1}{\sqrt{2}} a_1 f_K F^{B\to a_2}(m_K^2)$
$B^- \to K^- f_2$	$\frac{1}{\sqrt{2}} a_1 f_K \cos\phi_T F^{B\to f_2}(m_K^2)$
$B^- \to K^- f_2'$	$\frac{1}{\sqrt{2}} a_1 f_K \sin\phi_T F^{B\to f_2'}(m_K^2)$
$B^- \to \pi^0 K_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B\to K_2}(m_\pi^2)$
$B^- \to \eta K_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to K_2}(m_\eta^2)$
$B^- \to \eta' K_2^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B\to K_2}(m_{\eta'}^2)$

$\overline{B}^0 \to K^- a_2^+$	$a_1 f_K F^{B\to a_2}(m_K^2)$
$\overline{B}^0 \to \pi^0 \overline{K}_2^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B\to K_2}(m_\pi^2)$
$\overline{B}^0 \to \eta \overline{K}_2^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to K_2}(m_\eta^2)$
$\overline{B}^0 \to \eta' \overline{K}_2^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B\to K_2}(m_{\eta'}^2)$
$\overline{B}_s^0 \to \pi^0 f_2$	$\frac{1}{\sqrt{2}} a_2 f_\pi \sin\phi_T F^{B_s\to f_2}(m_\pi^2)$
$\overline{B}_s^0 \to \pi^0 f_2'$	$-\frac{1}{\sqrt{2}} a_2 f_\pi \cos\phi_T F^{B_s\to f_2'}(m_\pi^2)$
$\overline{B}_s^0 \to \eta f_2$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P \sin\phi_T F^{B_s\to f_2}(m_\eta^2)$
$\overline{B}_s^0 \to K^- K_2^+$	$a_1 f_K F^{B_s\to K_2}(m_K^2)$
$\overline{B}_s^0 \to \eta f_2'$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P \cos\phi_T F^{B_s\to f_2'}(m_\eta^2)$
$\overline{B}_s^0 \to \eta' f_2$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P \sin\phi_T F^{B_s\to f_2}(m_{\eta'}^2)$
$\overline{B}_s^0 \to \eta' f_2'$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P \cos\phi_T F^{B_s\to f_2'}(m_{\eta'}^2)$
$\Delta b = 1,\ \Delta C = -1,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^*$
$B^- \to D^- a_2^0$	$\frac{1}{\sqrt{2}} a_1 f_D F^{B\to a_2}(m_D^2)$
$B^- \to D^- f_2$	$\frac{1}{\sqrt{2}} a_1 f_D \cos\phi_T F^{B\to f_2}(m_D^2)$
$B^- \to D^- f_2'$	$\frac{1}{\sqrt{2}} a_1 f_D \sin\phi_T F^{B\to f_2'}(m_D^2)$
$B^- \to \overline{D}^0 a_2^-$	$a_2 f_D F^{B\to a_2}(m_D^2)$
$\overline{B}^0 \to \overline{D}^0 a_2^0$	$-\frac{1}{\sqrt{2}} a_2 f_D F^{B\to a_2}(m_D^2)$
$\overline{B}^0 \to \overline{D}^0 f_2$	$\frac{1}{\sqrt{2}} a_2 f_D \cos\phi_T F^{B\to f_2}(m_D^2)$
$\overline{B}^0 \to \overline{D}^0 f_2'$	$\frac{1}{\sqrt{2}} a_2 f_D \sin\phi_T F^{B\to f_2'}(m_D^2)$
$\overline{B}^0 \to D^- a_2^+$	$a_1 f_D F^{B\to a_2}(m_D^2)$
$\overline{B}_s^0 \to D^- K_2^+$	$a_1 f_D F^{B_s\to K_2}(m_D^2)$
$\overline{B}_s^0 \to \overline{D}^0 K_2^0$	$a_2 f_D F^{B_s\to K_2}(m_D^2)$

4.4 CALCULATION OF THE $B \to T$ TRANSITION FORM FACTORS IN ISGW II MODEL

The form factors have the following expressions in the ISGW II quark model, (whose salient features are already described in section 3.4 of chapter 3) for $B \to T$ transitions [3]:

$$k = \frac{m_d}{\sqrt{2\beta_B}}(1+\tilde{\omega})\ F_5^{(k)},$$

$$b_+ + b_- = \frac{m_d}{4\sqrt{2}m_d m_b \tilde{m}_B \beta_B}\frac{\beta_T^2}{\beta_{BT}^2}\left(1-\frac{m_d}{2\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right)F_5^{(b_+ + b_-)},$$

$$b_+ - b_- = -\frac{m_d}{\sqrt{2}m_b \tilde{m}_T \beta_B}\left(1-\frac{m_d m_b}{2\mu_+ \tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}+\frac{\beta_T^2}{4\beta_{BT}^2}\left(1-\frac{m_d}{2\tilde{m}_B}\frac{\beta_T^2}{\beta_{BT}^2}\right)\right)F_5^{(b_+ - b_-)}, \quad (4.8)$$

where

$$F_5^{(k)} = F_5\left(\frac{\overline{m}_B}{\tilde{m}_B}\right)^{-1/2}\left(\frac{\overline{m}_T}{\tilde{m}_T}\right)^{1/2},$$

$$F_5^{(b_+ + b_-)} = F_5\left(\frac{\overline{m}_B}{\tilde{m}_B}\right)^{-5/2}\left(\frac{\overline{m}_T}{\tilde{m}_T}\right)^{1/2}, \quad (4.9)$$

$$F_5^{(b_+ - b_-)} = F_5\left(\frac{\overline{m}_B}{\tilde{m}_B}\right)^{-3/2}\left(\frac{\overline{m}_T}{\tilde{m}_T}\right)^{-1/2},$$

Here, m_d is the spectator quark mass in the decaying particle. For $B_s \to T$ transitions, m_d is replaced with m_s. Values of the parameter β for different *s*-wave and *p*-wave mesons are given in Table 3.4 of chapter 3. We obtain the following form factors describing $B \to T$ transitions which are given in Table 4.4 at $q^2 = t_m$.

Table 4.4 Form factors of $B \to T$ transition at $q^2 = t_m$ in the ISGW II quark model

Transition	k	b_+	b_-
$B \to a_2$	0.432	-0.013	0.015
$B \to f_2$	0.425	-0.014	0.014
$B \to K_2$	0.480	-0.015	0.015
$B \to D_2$	0.677	-0.013	0.013
$B_s \to f_2'$	0.572	-0.016	0.017
$B_s \to K_2$	0.492	-0.013	0.015
$B_s \to D_{s2}$	0.854	-0.015	0.016

4.5 NUMERICAL RESULTS AND DISCUSSIONS

We use the decay rate formula given in (4.2), to evaluate the numerical values of the branching ratios of B meson emitting pseudoscalar and tensor mesons in CKM-favored and CKM-suppressed modes involving $b \to c$ and $b \to u$ transitions. Here we have used the pseudoscalar decay constants [9, 10] given in (3.26) and the form factors calculated in section 4.4. Obtained results are given in column 2nd of the Tables 4.5, 4.6, 4.7 (a) and 4.8 (b) for various possible modes.

Table 4.5 Branching ratios of $B \to PT$ decays in CKM-favored mode involving $b \to c$ transition

Decays	Branching ratios	
	This Work	KLO [6]
$\Delta b = 1, \Delta C = 1, \Delta S = 0$		
$B^- \to \pi^- D_2^0$	6.7×10^{-4}	3.5×10^{-4}
$B^- \to D^0 a_2^-$	1.8×10^{-4}	1.0×10^{-4}
$\overline{B}^0 \to \pi^- D_2^+$	6.1×10^{-4}	3.3×10^{-4}
$\overline{B}^0 \to D^0 a_2^0$	8.2×10^{-5}	4.8×10^{-4}
$\overline{B}^0 \to D^0 f_2$	8.8×10^{-5}	5.3×10^{-5}
$\overline{B}^0 \to D^0 f_2'$	1.7×10^{-6}	0.62×10^{-6}
$\overline{B}_s^0 \to \pi^- D_{s2}^+$	7.1×10^{-4}	-
$\overline{B}_s^0 \to D^0 K_2^0$	1.1×10^{-4}	-
$\Delta b = 1, \Delta C = 0, \Delta S = -1$		
$B^- \to D_s^- D_2^0$	6.8×10^{-4}	4.9×10^{-4}
$B^- \to \eta_c K_2^-$	1.4×10^{-4}	1.1×10^{-4}
$\overline{B}^0 \to D_s^- D_2^+$	6.4×10^{-4}	4.6×10^{-4}
$\overline{B}^0 \to \eta_c \overline{K}_2^0$	1.3×10^{-4}	9.6×10^{-5}
$\overline{B}_s^0 \to D_s^- D_{s2}^-$	7.7×10^{-4}	-
$\overline{B}_s^0 \to \eta_c f_2$	2.7×10^{-6}	-
$\overline{B}_s^0 \to \eta_c f_2'$	1.3×10^{-4}	-

Table 4.6 Branching ratios of $B \to PT$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Branching ratios	
	This Work	**KLO [6]**
$\Delta b = 1, \Delta C = 1, \Delta S = -1$		
$B^- \to K^- D_2^0$	4.8×10^{-5}	2.5×10^{-5}
$B^- \to D^0 K_2^-$	8.7×10^{-6}	7.3×10^{-6}
$\overline{B}^0 \to K^- D_2^+$	4.5×10^{-5}	2.4×10^{-5}
$\overline{B}^0 \to D^0 \overline{K}_2^0$	8.1×10^{-6}	6.8×10^{-6}
$\overline{B}_s^0 \to K^- D_{s2}^+$	5.2×10^{-5}	-
$\overline{B}_s^0 \to D^0 f_2$	9.9×10^{-8}	-
$\overline{B}_s^0 \to D^0 f_2'$	6.7×10^{-6}	-
$\Delta b = 1, \Delta C = 0, \Delta S = 0$		
$B^- \to D^- D_2^0$	2.5×10^{-5}	2.2×10^{-5}
$B^- \to \eta_c a_2^-$	9.2×10^{-6}	4.9×10^{-6}
$\overline{B}^0 \to D^- D_2^+$	2.4×10^{-5}	2.1×10^{-5}
$\overline{B}^0 \to \eta_c a_2^0$	4.3×10^{-6}	2.3×10^{-6}
$\overline{B}^0 \to \eta_c f_2$	4.8×10^{-6}	2.7×10^{-6}
$\overline{B}^0 \to \eta_c f_2'$	6.7×10^{-8}	0.02×10^{-6}
$\overline{B}_s^0 \to D^- D_{s2}^+$	2.9×10^{-5}	-
$\overline{B}_s^0 \to \eta_c K_2^0$	6.9×10^{-6}	-

Table 4.7 (a) Branching ratios of $B \to PT$ decays involving $b \to u$ transition

Decays	Branching ratios		
	This Work	**KLO [6]**	**MQ [7]**
$\Delta b = 1, \Delta C = -1, \Delta S = -1$			
$B^- \to \overline{D}^0 K_2^-$	1.3×10^{-6}	1.2×10^{-6}	-
$B^- \to D_s^- a_2^0$	2.0×10^{-5}	9.4×10^{-6}	-
$B^- \to D_s^- f_2$	2.2×10^{-5}	1.0×10^{-5}	-
$B^- \to D_s^- f_2'$	4.3×10^{-7}	0.12×10^{-6}	-
$\overline{B}^0 \to \overline{D}^0 \overline{K}_2^0$	1.2×10^{-6}	1.1×10^{-6}	-
$\overline{B}^0 \to D_s^- a_2^+$	3.8×10^{-5}	1.8×10^{-5}	-
$\overline{B}_s^0 \to D_s^- K_2^+$	2.6×10^{-5}	-	-
$\overline{B}_s^0 \to \overline{D}^0 f_2$	1.5×10^{-8}	-	-
$\overline{B}_s^0 \to \overline{D}^0 f_2'$	1.0×10^{-6}	-	-
$\Delta b = 1, \Delta C = 0, \Delta S = 0$			
$B^- \to \pi^- a_2^0$	6.7×10^{-6}	2.6×10^{-6}	4.38×10^{-6}
$B^- \to \pi^- f_2$	7.1×10^{-6}	-	-
$B^- \to \pi^- f_2'$	1.5×10^{-7}	-	-
$B^- \to \pi^0 a_2^-$	0.38×10^{-6}	0.001×10^{-6}	0.015×10^{-6}
$B^- \to \eta a_2^-$	0.23×10^{-6}	0.29×10^{-6}	45.8×10^{-6}
$B^- \to \eta' a_2^-$	0.13×10^{-6}	1.31×10^{-6}	71.3×10^{-6}
$\overline{B}^0 \to \pi^- a_2^+$	13.0×10^{-6}	4.88×10^{-6}	8.19×10^{-6}
$\overline{B}^0 \to \pi^0 a_2^0$	0.18×10^{-6}	0.0003×10^{-6}	0.007×10^{-6}
$\overline{B}^0 \to \pi^0 f_2$	1.9×10^{-7}	-	-
$\overline{B}^0 \to \pi^0 f_2'$	3.9×10^{-9}	-	-
$\overline{B}^0 \to \eta a_2^0$	0.11×10^{-6}	0.14×10^{-6}	25.2×10^{-6}
$\overline{B}^0 \to \eta f_2$	1.1×10^{-7}	-	-
$\overline{B}^0 \to \eta f_2'$	2.4×10^{-9}	-	-
$\overline{B}^0 \to \eta' a_2^0$	0.06×10^{-6}	0.62×10^{-6}	43.3×10^{-6}
$\overline{B}^0 \to \eta' f_2$	6.3×10^{-8}	-	-
$\overline{B}^0 \to \eta' f_2'$	1.3×10^{-9}	-	-
$\overline{B}_s^0 \to \pi^- K_2^+$	7.8×10^{-6}	-	-
$\overline{B}_s^0 \to \pi^0 K_2^0$	2.2×10^{-7}	-	-
$\overline{B}_s^0 \to \eta K_2^0$	1.3×10^{-7}	-	-
$\overline{B}_s^0 \to \eta' K_2^0$	7.5×10^{-8}	-	-

Table 4.7 (b) Branching ratios of $B \to PT$ decays involving $b \to u$ transition

Decays	Branching ratios		
	This Work	**KLO [6]**	**MQ [7]**
$\Delta b = 1, \Delta C = 0, \Delta S = -1$			
$B^- \to K^- a_2^0$	0.51×10^{-6}	0.31×10^{-6}	0.39×10^{-6}
$B^- \to K^- f_2$	5.4×10^{-7}	-	-
$B^- \to K^- f_2'$	1.5×10^{-8}	-	-
$B^- \to \pi^0 K_2^-$	0.02×10^{-6}	0.09×10^{-6}	0.15×10^{-6}
$B^- \to \eta K_2^-$	0.01×10^{-6}	0.03×10^{-6}	1.19×10^{-6}
$B^- \to \eta' K_2^-$	0.007×10^{-6}	1.40×10^{-6}	2.70×10^{-6}
$\overline{B}^0 \to K^- a_2^+$	0.95×10^{-6}	0.58×10^{-6}	0.73×10^{-6}
$\overline{B}^0 \to \pi^0 \overline{K}_2^0$	0.02×10^{-6}	0.08×10^{-6}	0.13×10^{-6}
$\overline{B}^0 \to \eta \overline{K}_2^0$	0.01×10^{-6}	0.03×10^{-6}	1.09×10^{-6}
$\overline{B}^0 \to \eta' \overline{K}_2^0$	0.006×10^{-6}	1.3×10^{-6}	2.46×10^{-6}
$\overline{B}_s^0 \to K^- K_2^+$	5.9×10^{-7}	-	-
$\overline{B}_s^0 \to \pi^0 f_2$	1.9×10^{-10}	-	-
$\overline{B}_s^0 \to \pi^0 f_2'$	1.4×10^{-8}	-	-
$\overline{B}_s^0 \to \eta f_2$	1.1×10^{-10}	-	-
$\overline{B}_s^0 \to \eta f_2'$	8.3×10^{-9}	-	-
$\overline{B}_s^0 \to \eta' f_2$	6.5×10^{-11}	-	-
$\overline{B}_s^0 \to \eta' f_2'$	4.7×10^{-9}	-	-
$\Delta b = 1, \Delta C = -1, \Delta S = 0$			
$B^- \to D^- a_2^0$	6.5×10^{-7}	-	-
$B^- \to D^- f_2$	6.9×10^{-7}	-	-
$B^- \to D^- f_2'$	1.4×10^{-7}	-	-
$B^- \to \overline{D}^0 a_2^-$	7.3×10^{-8}	-	-
$\overline{B}^0 \to D^- a_2^+$	1.2×10^{-6}	-	-
$\overline{B}^0 \to \overline{D}^0 a_2^0$	3.4×10^{-8}	-	-
$\overline{B}^0 \to \overline{D}^0 f_2$	3.6×10^{-8}	-	-
$\overline{B}^0 \to \overline{D}^0 f_2'$	7.1×10^{-10}	-	-
$\overline{B}_s^0 \to D^- K_2^+$	8.3×10^{-7}	-	-
$\overline{B}_s^0 \to \overline{D}^0 K_2^0$	4.6×10^{-8}	-	-

We make the following observations:

4.5.1 $B \to PT$ DECAYS INVOLVING $b \to c$ TRANSITION

1. $\Delta b = 1, \Delta C = 1, \Delta S = 0$ mode:

a) Calculated branching ratio $B(B^- \to \pi^- D_2^0)$ = 6.7×10^{-4} agrees well with the experiment value [1] $(7.8 \pm 1.4)\times 10^{-4}$, and $B(\overline{B}^0 \to \pi^- D_2^+)$ = 6.1×10^{-4}, is well below the experimental upper limit $< 2.2\times 10^{-3}$.

b) Branching ratios of other dominant modes are $B(B^- \to D^0 a_2^-)$ = 1.8×10^{-4}, $B(\overline{B}_s^0 \to \pi^- D_{s2}^+)$ = 7.1×10^{-4}, and $B(\overline{B}_s^0 \to D^0 K_2^0)$ = 1.1×10^{-4}. We hope that these values are within the reach of the future experiments.

c) Decays $\overline{B}^0 \to D^0 a_2^0$ and $\overline{B}^0 \to D^0 f_2$ have branching ratios of the order of 10^{-5}, since these involve color-suppressed spectator process. The branching ratio of $\overline{B}^0 \to D^0 f_2'$ decay is further suppressed due to the $f_2 - f_2'$ mixing being close to the ideal mixing.

d) Decays $\overline{B}^0 \to \pi^0 D_2^0 / \eta D_2^0 / \eta' D_2^0 / D^+ a_2^- / D_s^+ K_2^- / K^- D_{s2}^+$ and $\overline{B}_s^0 \to K^0 D_2^0 / D_s^+ a_2^-$ are forbidden in the present analysis due to the vanishing matrix element between the vacuum and tensor meson. However, these may occur through an annihilation mechanism. The decays $\overline{B}^0 \to \pi^0 D_2^0 / D^+ a_2^-$ may also occur through elastic final state interactions (FSIs).

2. $\Delta b = 1, \Delta C = 0, \Delta S = -1$ mode:

a) Dominant modes are found to have branching ratios: $B(B^- \to D_s^- D_2^0)$ = 6.8×10^{-4}, $B(B^- \to \eta_c K_2^-)$ = 1.4×10^{-4}, $B(\overline{B}^0 \to D_s^- D_2^+)$ = 6.4×10^{-4}, $B(\overline{B}^0 \to \eta_c \overline{K}_2^0)$ = 1.3×10^{-4}, $B(\overline{B}_s^0 \to D_s^- D_{s2}^-)$ = 7.7×10^{-4} and $B(\overline{B}_s^0 \to \eta_c f_2')$ = 1.3×10^{-4}.

b) Decays $B^- \to D^0 D_{s2}^- / D_s^- D_2^0 \; / K^- \chi_{c2}(1P)$, $\overline{B}^0 \to D^+ D_{s2}^- / D_s^- D_2^+ / K^- \chi_{c2}(1P)$ and $\overline{B}_s^0 \to \pi^0 \chi_{c2} / \eta \chi_{c2} / \eta' \chi_{c2} / D^+ D_2^- / D^0 \overline{D}_2^0 / D_s^+ D_{s2}^- / D^- D_2^+ / \overline{D}^0 D_2^0 / \eta_c a_2^0$ are forbidden in our work. Penguin diagrams may cause $B^- \to D^0 D_{s2}^- \; / D_s^- D_2^0$ and $\overline{B}^0 \to D^+ D_{s2}^- / D_s^- D_2^+$ decays, however these are likely to remain suppressed as these decays require $c\overline{c}$ pair to be created.

3. $\Delta b = 1, \Delta C = 0, \Delta S = 0$ mode:

a) For dominant decays, we predict $B(B^- \to D^- D_2^0)$ = 2.5×10^{-5}, $B(\overline{B}^0 \to D^- D_2^+)$ = 2.4×10^{-5} and $B(\overline{B}_s^0 \to D^- D_{s2}^+)$ = 2.9×10^{-5}.

b) Decays $B^- \to D^0 D_2^- / \pi^- \chi_{c2}(1P)$, $\overline{B}^0 \to D^0 \overline{D}_2^0 / D_s^- D_{s2}^+ \; / D^+ D_2^- \; / \overline{D}^0 D_2^0 / D_s^+ D_{s2}^- / \pi^0 \chi_{c2}(1P) / \eta \chi_{c2}(1P) / \eta' \chi_{c2}(1P)$ and $\overline{B}_s^0 \to K^0 \chi_{c2} / D_s^+ D_2^-$ are forbidden in our analysis. Annihilation diagrams, elastic FSI and penguin diagrams may generate these decays to the naked charm mesons. However, decays emitting charmonium $\chi_{c2}(1P)$ remains forbidden in the ideal mixing limit.

4. $\Delta b = 1, \Delta C = 1, \Delta S = -1$ mode:

a) Branching ratios of the dominant decays are $B(B^- \to K^- D_2^0)$ = 4.8×10^{-5}, $B(\overline{B}^0 \to K^- D_2^+)$ = 4.5×10^{-5} and $B(\overline{B}_s^0 \to K^- D_{s2}^+)$ = 5.2×10^{-5}.

b) Decays $\overline{B}^0 \to \overline{K}^0 D_2^0 / D^+ K_2^-$ and $\overline{B}_s^0 \to \pi^- D_2^+ / \pi^0 D_2^0 / \eta D_2^0 / \eta' D_2^0 / \; D^+ a_2^- / \; D^0 a_2^0 / D_s^+ K_2^-$ are forbidden in our analysis. Annihilation diagrams do not contribute to these decays. However, these may acquire nonzero branching ratios through elastic FSI.

4.5.2 $B \to PT$ DECAYS INVOLVING $b \to u$ TRANSITION

1. $\Delta b = 1, \Delta C = 0, \Delta S = 0$ mode:

a) $B(B^- \to \pi^- f_2)$ = 7.1×10^{-6} is in good agreement with the experimental value (8.2±2.5)×10^{-6} and $B(\overline{B}^0 \to \pi^- a_2^+)$ = 1.3×10^{-5} is well below the experimental upper limit $<3.0\times10^{-4}$.

b) $B^- \to K^0 K_2^-$ / $K^- K_2^0$, $\overline{B}^0 \to K^+ K_2^- / K^0 \overline{K}_2^0 / K^0 \overline{K}_2^0 / \overline{K}^0 K_2^0 /$ $K^- K_2^+$ $/ \pi^+ a_2^-$ and $\overline{B}_s^0 \to K^+ a_2^- / K^0 a_2^0 / K^0 f_2 / K^0 f_2'$ are forbidden in the present analysis. Annihilation process and FSIs may generate these decays.

c) $B^- \to \pi^- \overline{K}_2^0$ and $\overline{B}^0 \to \pi^+ K_2^-$ are also forbidden in the present analysis which may be generated through annihilation diagram or elastic FSI.

2. $\Delta b = 1, \Delta C = -1, \Delta S = -1$ mode:

a) Branching ratios $B(B^- \to D_s^- a_2^0)$ = 2.0×10^{-5}, $B(B^- \to D_s^- f_2')$ = 2.2×10^{-5}, $B(\overline{B}^0 \to D_s^- a_2^+)$ = 3.8×10^{-5} and $B(\overline{B}_s^0 \to D_s^- K_2^+)$ = 2.6×10^{-5} have relatively large branching ratios.

b) Decays $B^- \to \pi^0 D_{s2}^- / \eta D_{s2}^- / \eta' D_{s2}^- / \overline{K}^0 D_2^- / K^- D_2^0 / D^- \overline{K}_2^0$, $\overline{B}^0 \to \overline{K}^0 D_2^0 / \pi^+ D_{s2}^-$ and $\overline{B}_s^0 \to K^+ D_{s2}^- / \pi^+ D_2^- / \pi^0 \overline{D}_2^0$ $/ \eta \overline{D}_2^0$ $/ \eta' \overline{D}_2^0$ $/ \overline{D}^0 a_2^0 / D^- a_2^+$ are forbidden in the present analysis. Annihilation and FSIs may generate these decays.

3. $\Delta b = 1, \Delta C = -1, \Delta S = 0$ mode:

a) Branching ratios of $B(\overline{B}^0 \to \overline{D}^0 f_2)$ = 3.6×10^{-8} is smaller than the experimental value $(1.2 \pm 0.4)\times10^{-4}$. It may be noted that *W*-annihilation and *W*-exchange diagrams may also contribute to the *B* decays under consideration. Normally, such contributions are expected to be suppressed due to the helicity and color arguments. Including the factorizable contribution of such diagrams, the decay amplitude of $\overline{B}^0 \to \overline{D}^0 f_2$ get modified to (leaving aside the scale factor $\frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^*$)

$$A(\overline{B}^0 \to \overline{D}^0 f_2) = \frac{1}{\sqrt{2}} a_2 f_D \cos\phi_T F^{B\to f_2}(m_D^2) + \frac{1}{\sqrt{2}} a_2 f_B \cos\phi_T F^{f_2 \to D}(m_B^2). \quad (4.10)$$

Using f_B = 0.176 GeV, we find that the experimental branching ratio $B(\overline{B}^0 \to \overline{D}^0 f_2)$ requires $F^{f_2 \to D}(m_B^2)$ = -9.99 GeV. This in turn enhances the branching ratio for $B^- \to D^- f_2$ to 1.2×10^{-4}.

b) Dominant decay is $B(\overline{B}^0 \to D^- a_2^+)$ = 1.2×10^{-6} and next order dominant decays are $B(B^- \to D^- f_2)$ = 6.9×10^{-7} $B(B^- \to D^- a_2^0)$ = 6.5×10^{-7} and $B(\overline{B}_s^0 \to D^- K_2^+)$ = 8.3×10^{-7}.

c) Decays $B^- \to K^0 D_{s2}^- / \pi^0 D_2^- / \pi^- \overline{D}_2^0 / \eta D_2^- / \eta' D_2^- / D_s^- K_2^0 / \eta_c D_2^-$, $\overline{B}^0 \to K^+ D_{s2}^- / \pi^+ D_2^- / \pi^0 \overline{D}_2^0 / \eta \overline{D}_2^0 / \eta' \overline{D}_2^0 / D_s^- K_2^+ / \eta_c D_2^-$ and $\overline{B}_s^0 \to K^0 \overline{D}_2^0$ are forbidden in the present analysis. Annihilation diagrams may generate these decays.

4. $\Delta b = 1, \Delta C = 0, \Delta S = -1$ mode:

a) $B(B^- \to K^- f_2)$ = 0.54×10^{-6} is smaller than the experimental value $(1.3^{+0.4}_{-0.5}) \times 10^{-6}$. This decay mode is also likely to have contribution from the *W*-annihilation and *W*-exchange processes. Including the factorizable contribution of such diagrams, the decay amplitudes of $B^- \to K^- f_2$ get modified to (putting aside the scale factor $\frac{G_F}{\sqrt{2}} V_{ub} V_{us}^*$

$$A(B^- \to K^- f_2) = \frac{1}{\sqrt{2}} a_1 f_K \cos\phi_T F^{B\to f_2}(m_K^2) +$$

$$\frac{1}{\sqrt{2}} a_1 f_B \cos\phi_T F^{f_2 \to K}(m_B^2)\,. \qquad (4.11)$$

As it is not possible to evaluate the form factor $F^{f_2\to K}$ at m_B^2 even in the phenomenological models, it is treated as a free parameter. Taking f_B = 0.176 GeV, we find that the experimental branching ratio $B(B^- \to K^- f_2)$ $= (1.3^{+0.4}_{-0.5})\times 10^{-6}$ requires $F^{f_2\to K}(m_B^2)$ = - 0.083 GeV. This value in turn enhances the branching ratio for $B^- \to K^- f_2$ through the *W*-annihilation contribution to 1.3×10^{-6}.

b) Branching ratios of $B(B^- \to \eta K_2^-)$ = 1.2×10^{-8} is small than the experimental value $(9.1 \pm 3.0)\times 10^{-6}$. Similar to $B^- \to K^- f_2$ decay, this decay mode is also likely to have contribution from the *W*-annihilation and *W*-exchange processes. Including the factorizable contribution of such diagrams, the decay amplitudes of $B \to \eta K_2$ get modified to (leaving aside the scale factor $\frac{G_F}{\sqrt{2}} V_{ub} V_{us}^*$)

$$A(B^- \to \eta K_2^-) = \frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to K_2}(m_\eta^2) +$$

$$\frac{1}{\sqrt{2}} a_2 f_B \sin\phi_P F^{K_2\to\eta}(m_B^2)\,,$$

$$A(\overline{B}^0 \to \eta \overline{K}_2^0) = \frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B\to K_2}(m_\eta^2) +$$

$$\frac{1}{\sqrt{2}} a_2 f_B \sin\phi_P F^{K_2\to\eta}(m_B^2)\,. \qquad (4.12)$$

For f_B = 0.176 GeV, we find that the experimental branching ratio $B(B^- \to \eta K_2^-)$ = $(9.1 \pm 3.0)\times 10^{-6}$ requires $F^{K_2\to\eta}(m_B^2)$ = - 3.03 GeV. This in turn enhances the branching ratio for $\overline{B}^0 \to \eta \overline{K}_2^0$ to 8.1×10^{-6}, which is consistent with the experimental value $(9.6 \pm 2.1) \times 10^{-6}$.

c) Decays $B^- \to \pi^- \overline{K}_2^0 / \overline{K}^0 a_2^-$, $\overline{B}^0 \to \pi^+ K_2^- / \overline{K}^0 a_2^0 / \overline{K}^0 f_2 / \overline{K}^0 f_2'$ and $\overline{B}_s^0 \to K^+ K_2^- / K^0 \overline{K}_2^0 / \pi^+ a_2^- / \pi^0 a_2^0 / \pi^- a_2^+ / \eta a_2^0 / \overline{K}^0 K_2^0 / \eta' a_2^0$ are forbidden in the present analysis. Annihilation and FSIs may generate these decays.

4.5.3 COMPARISON WITH OTHER WORKS

We also compare our results with branching ratios calculated in the other models [6, 7, 11]. The predicted branching ratios in KLO [6] shown in 3rd column of Tables 4.5, 4.6, 4.7 (a) and 4.7 (b) are generally smaller as compared to the present branching ratios because of the difference in the form factors since different quark masses have been used in the two works. Branching ratios have also been calculated by Cheng [11]. His predictions $B(B^- \to \pi^- D_2^0)$ = 6.7×10^{-4} and $B(\overline{B}^0 \to \pi^- D_2^+)$ = 6.1×10^{-4} match well with the numerical branching ratios obtained in the present work. However, the other branching ratios $B(B^- \to D_s^- D_2^0)$ = 4.2×10^{-4}, $B(\overline{B}^0 \to D_s^- D_2^+)$ = 3.8×10^{-4} and $B(\overline{B}_s^0 \to \pi^- D_{s2}^+)$ = 3.8×10^{-4} are different from our results owing to the different values used for the decay constant f_{D_s}. MQ [7] have recently studied few charmless decays of $B \to PT$ mode. Some of the branching ratios are smaller than our numerical value of branching ratios, while the others are large as compared to the present predictions, particularly for η or η' emitting decays. The disagreement with their predictions may be attributed to the difference in the form factors obtained in the covariant light-front approach (CLF) and inclusion of the non-factorizable contributions in their results. It may be noted that the form factors at small q^2 obtained in the CLF and ISGW II quark model agrees within 40% [3]. However, when q^2 increases

$h(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ increases more rapidly in the covariant light front model than in the ISGW II model. Another important fact is that the behavior of the form factor k in both models is different.

The Belle collaboration is currently searching for some $B \to PT$ modes and their preliminary results indicate that the branching ratios for these may not be very small compared to $B \to PP$ modes. We hope our predictions would be within the reach of the current experiments. Observation of these decays in the *B* experiments such as Belle, Babar, BTeV, LHC and so on will be crucial in testing the ISGW II and other quark models as well as validity of the factorization scheme.

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CHAPTER 5

HADRONIC WEAK DECAYS OF BOTTOM MESONS EMITTING PSEUDOSCALAR AND SCALAR MESONS

5.1 INTRODUCTION

In this chapter, we study B decays emitting a pseudoscalar and a scalar mesons in CKM-favored and CKM-suppressed modes, for which the experiments have reported three measured branching ratios and one upper limit as follows [1]:

$$B(B^- \to \eta K_0^-) = (1.8 \pm 0.4) \times 10^{-5},$$

$$B(B^- \to \pi^- K_0^0) = (4.7 \pm 0.5) \times 10^{-5},$$

$$B(\overline{B}^0 \to \eta \overline{K}_0^0) = (1.10 \pm 0.22) \times 10^{-5},$$

$$B(\overline{B}^0 \to D_s^+ a_0^-) < 1.9 \times 10^{-5}.$$

These modes give additional and complementary information about exclusive nonleptonic weak decays of B mesons. Here also, we obtain the decay amplitudes using the factorization hypothesis and consequently, predict branching ratios of these decays based on the spectator quark model. It is expected that some of these decay channels have relatively large branching ratios and can be measured within the reach of future experiments.

5.2 SCALAR MESON SPECTROSCOPY

The heavy scalar meson $(J^P = 0^+)$ comprises of the isovector $a_0(1.474)$, isodoublet $K_0(1.412)$ and isoscalars $f_0(1.370)$, $f_0(1.500)/\ f_0(1.710)$ and one isoscalar $\chi_{c0}(1P)(3.145)$, charm triplet $D_0(2.308)$, $D_{s0}(2.317)$ [1], behave well with respects to quark model assignments.

In the present analysis, mixing of the isoscalar states of mesons can also be expressed as

$$f_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\theta + s\bar{s}\cos\theta,$$
$$f_0' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\theta - s\bar{s}\sin\theta, \tag{5.1}$$

where $\theta = \pi + (\theta_{ideal} - \theta_S)$ and $\theta_S = 68^\circ$.

5.3 METHODOLOGY

The effective weak Hamiltonian generating the bottom meson decays involving $b \to c$ and $b \to u$ transitions has already been given in earlier chapter 2 for CKM-favored and CKM-suppressed modes, respectively.

5.3.1 DECAY AMPLITUDES AND RATES

The decay rate formula for $B(0^-) \to P(0^-) + S(0^+)$ decays is given by

$$\Gamma(B \to P\,S) = \frac{p_c}{8\pi m_B^2}\left|A(B \to P\,S)\right|^2, \tag{5.2}$$

where p_c is the magnitude of the three-momentum of a final-state particle in the rest frame of *B* meson and m_B denotes the mass of the *B* meson.

The factorization scheme expresses the decay amplitude as the product of matrix elements of weak currents (up to the weak scale factor of $\frac{G_F}{\sqrt{2}}$×CKM elements×QCD factors) as

$$A(B \to PS) \sim \langle P | J^{\mu} | 0 \rangle \langle S | J_{\mu} | B \rangle + \langle S | J^{\mu} | 0 \rangle \langle P | J_{\mu} | B \rangle, \tag{5.3}$$

Using the Lorentz invariance, matrix element of the current $\langle P | J_{\mu} | 0 \rangle$ and $\langle P | J_{\mu} | B \rangle$ are already given in chapter 2. Remaining matrix element of the current between meson states can be expressed [2-4] as

$$\langle S(k_{S\mu}) | J_{\mu} | 0 \rangle = f_S k_{S\mu},$$

$$\langle S(k_S) | A_{\mu} | B(k_B) \rangle = i(u_+(k_B + k_S)_{\mu} + u_-(k_B - k_S)_{\mu}), \tag{5.4}$$

Thus, the decay amplitude becomes

$$A(B \to PS) = (\frac{G_F}{\sqrt{2}} \times CKM\ factors \times QCD\ factors)(f_P\, F^{B \to S}(m_P^2) + f_S\, F^{B \to P}(m_S^2)), \tag{5.5}$$

where [2, 3]

$$F^{B \to S}(m_P^2) = (m_B^2 - m_S^2)\, u_+(m_P^2) + m_P^2\, u_-(m_P^2). \tag{5.6}$$

Sandwiching the weak Hamiltonian between the initial and the final states, the decay amplitudes for CKM-favored and CKM-suppressed $B \to PS$ decay modes are obtained as shown in Tables 5.1, 5.2, 5.3 (a) and 5.3 (b).

Table 5.1 Decay amplitudes of $B \to PS$ decays in CKM-favored mode involving $b \to c$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^{*}$
$B^- \to \pi^- D_0^0$	$a_1 f_\pi F^{B\to D_0}(m_B^2 - m_{D_0}^2) + a_2 f_{D_0} F^{B\to\pi}(m_B^2 - m_\pi^2)$
$B^- \to D^0 a_0^-$	$a_2 f_D F^{B\to a_0}(m_B^2 - m_{a_0}^2) + a_1 f_{a_0} F^{B\to D}(m_B^2 - m_D^2)$
$\bar{B}^0 \to \pi^0 D_0^0$	$-\frac{1}{\sqrt{2}} a_2 f_{D_0} F^{B\to\pi}(m_B^2 - m_\pi^2)$
$\bar{B}^0 \to \pi^- D_0^+$	$a_1 f_\pi F^{B\to D_0}(m_B^2 - m_{D_0}^2)$
$\bar{B}^0 \to \eta D_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} \sin\phi_P F^{B\to\eta}(m_B^2 - m_\eta^2)$
$\bar{B}^0 \to \eta' D_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} \cos\phi_P F^{B\to\eta'}(m_B^2 - m_{\eta'}^2)$
$\bar{B}^0 \to D^+ a_0^-$	$a_1 f_{a_0} F^{B\to D}(m_B^2 - m_D^2)$
$\bar{B}^0 \to D^0 a_0^0$	$-\frac{1}{\sqrt{2}} a_2 f_D F^{B\to a_0}(m_B^2 - m_{a_0}^2)$
$\bar{B}_s^0 \to K^0 D_0^0$	$a_2 f_{D_0} F^{B_s\to K}(m_B^2 - m_K^2)$
$\bar{B}_s^0 \to \pi^- D_{s0}^+$	$a_1 f_\pi F^{B_s\to D_{s0}}(m_B^2 - m_{D_{s0}}^2)$
$\bar{B}_s^0 \to D^0 K_0^0$	$a_2 f_D F^{B_s\to K_0}(m_B^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to D_s^+ a_0^-$	$a_1 f_{a_0} F^{B_s\to D_s}(m_B^2 - m_{D_s}^2)$
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^{*}$
$B^- \to D^0 D_{s0}^-$	$a_1 f_{D_{s0}} F^{B\to D}(m_B^2 - m_D^2)$
$B^- \to D_s^- D_0^0$	$a_1 f_{D_s} F^{B\to D_0}(m_B^2 - m_{D_0}^2)$
$B^- \to \eta_c K_0^-$	$a_2 f_{\eta_c} F^{B\to K_0}(m_B^2 - m_{K_0}^2)$
$\bar{B}^0 \to D^+ D_{s0}^-$	$a_1 f_{D_{s0}} F^{\bar{B}\to D}(m_{\bar{B}}^2 - m_D^2)$
$\bar{B}^0 \to D_s^- D_0^+$	$a_1 f_{D_s} F^{\bar{B}\to D_0}(m_{\bar{B}}^2 - m_{D_0}^2)$
$\bar{B}^0 \to \eta_c \bar{K}_0^0$	$a_2 f_{\eta_c} F^{\bar{B}\to K_0}(m_{\bar{B}}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to D_s^+ D_{s0}^-$	$a_1 f_{D_{s0}} F^{B_s\to D_s}(m_{B_s}^2 - m_{D_s}^2)$
$\bar{B}_s^0 \to D_s^- D_{s0}^+$	$a_1 f_{D_s} F^{B_s\to D_{s0}}(m_{B_s}^2 - m_{D_{s0}}^2)$
$\bar{B}_s^0 \to \eta_c f_0$	$-a_2 f_{\eta_c} F^{B_s\to f_0}(m_{B_s}^2 - m_{f_0}^2)$

Table 5.2 Decay amplitudes of $B \to PS$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^*$
$B^- \to K^- D_0^0$	$a_1 f_K F^{B \to D_0} (m_B^2 - m_{D_0}^2)$
$B^- \to D^0 K_0^-$	$a_2 f_D F^{B \to K_0} (m_B^2 - m_{K_0}^2)$
$\bar{B}^0 \to \bar{K}^0 D_0^0$	$a_2 f_{D_0} F^{B \to K} (m_B^2 - m_K^2)$
$\bar{B}^0 \to K^- D_0^+$	$a_1 f_K F^{B \to D_0} (m_B^2 - m_{D_0}^2)$
$\bar{B}^0 \to D^+ K_0^-$	$a_1 f_{K_0} F^{B \to D} (m_B^2 - m_D^2)$
$\bar{B}^0 \to D^0 \bar{K}_0^0$	$a_2 f_D F^{B \to K_0} (m_B^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to \eta D_0^0$	$-\frac{1}{\sqrt{2}} a_2 \sin\phi_P f_{D_0} F^{B_s \to \eta} (m_B^2 - m_\eta^2)$
$\bar{B}_s^0 \to K^- D_{s0}^+$	$a_1 f_K F^{B_s \to D_{s0}} (m_B^2 - m_{D_{s0}}^2)$
$\bar{B}_s^0 \to \eta' D_0^0$	$-\frac{1}{\sqrt{2}} a_2 \cos\phi_P f_{D_0} F^{B_s \to \eta'} (m_B^2 - m_{\eta'}^2)$
$\bar{B}_s^0 \to D^0 f_0$	$a_2 f_D F^{B_s \to f_0} (m_B^2 - m_{f_0}^2)$
$\bar{B}_s^0 \to D_s^+ K_0^-$	$a_1 f_{K_0} F^{B_s \to D_s} (m_B^2 - m_{D_s}^2)$
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^*$
$B^- \to D^0 D_0^-$	$a_1 f_{D_0} F^{B \to D} (m_B^2 - m_D^2)$
$B^- \to D^- D_0^0$	$a_1 f_D F^{B \to D_0} (m_B^2 - m_{D_0}^2)$
$B^- \to \eta_c a_0^-$	$a_2 f_{\eta_c} F^{B \to a_0} (m_B^2 - m_{a_0}^2)$
$\bar{B}^0 \to D^+ D_0^-$	$a_1 f_{D_0} F^{\bar{B} \to D} (m_{\bar{B}}^2 - m_D^2)$
$\bar{B}^0 \to D^- D_0^+$	$a_1 f_D F^{\bar{B} \to D_0} (m_{\bar{B}}^2 - m_{D_0}^2)$
$\bar{B}^0 \to \eta_c a_0^0$	$-\frac{1}{\sqrt{2}} a_2 f_{\eta_c} F^{B \to a_0} (m_B^2 - m_{a_0}^2)$
$\bar{B}_s^0 \to D_s^+ D_0^-$	$a_1 f_{D_0} F^{B_s \to D_s} (m_B^2 - m_{D_s}^2)$
$\bar{B}_s^0 \to D^- D_{s0}^+$	$a_1 f_D F^{B_s \to D_{s0}} (m_B^2 - m_{D_{s0}}^2)$
$\bar{B}_s^0 \to \eta_c K_0^0$	$a_2 f_{\eta_c} F^{B_s \to K_0} (m_B^2 - m_{K_0}^2)$

Table 5.3 (a) Decay amplitudes of $B \to PS$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = -1,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^*$
$B^- \to \pi^0 D_{s0}^-$	$\frac{1}{\sqrt{2}} a_1 f_{D_{s0}} F^{B\to\pi}(m_B^2 - m_\pi^2)$
$B^- \to \eta D_{s0}^-$	$\frac{1}{\sqrt{2}} a_1 f_{D_{s0}} \sin\phi_P F^{B\to\eta}(m_B^2 - m_\eta^2)$
$B^- \to K^- \bar{D}_0^0$	$a_2 f_{D_0} F^{B\to K}(m_B^2 - m_K^2)$
$B^- \to \eta' D_{s0}^-$	$\frac{1}{\sqrt{2}} a_1 f_{D_{s0}} \cos\phi_P F^{B\to\eta'}(m_B^2 - m_{\eta'}^2)$
$B^- \to \bar{D}^0 K_0^-$	$a_2 f_D F^{B\to K_0}(m_B^2 - m_{K_0}^2)$
$B^- \to D_s^- a_0^0$	$\frac{1}{\sqrt{2}} a_1 f_{D_s} F^{B\to a_0}(m_B^2 - m_{a_0}^2)$
$\bar{B}^0 \to \pi^+ D_{s0}^-$	$a_1 f_{D_{s0}} F^{\bar{B}\to\pi}(m_{\bar{B}}^2 - m_\pi^2)$
$\bar{B}^0 \to \bar{K}^0 \bar{D}_0^0$	$a_2 f_{D_0} F^{\bar{B}\to K}(m_{\bar{B}}^2 - m_K^2)$
$\bar{B}^0 \to \bar{D}^0 \bar{K}_0^0$	$a_2 f_D F^{\bar{B}\to K_0}(m_{\bar{B}}^2 - m_{K_0}^2)$
$\bar{B}^0 \to D_s^- a_0^+$	$a_1 f_{D_s} F^{B\to a_0}(m_B^2 - m_{a_0}^2)$
$\bar{B}_s^0 \to K^+ D_{s0}^-$	$a_1 f_{D_{s0}} F^{B_s\to K}(m_{B_s}^2 - m_K^2)$
$\bar{B}_s^0 \to \eta \bar{D}_0^0$	$-\frac{1}{\sqrt{2}} a_2 f_{D_0} \sin\phi_P F^{B_s\to\eta}(m_{B_s}^2 - m_K^2)$
$\bar{B}_s^0 \to \eta' \bar{D}_0^0$	$-\frac{1}{\sqrt{2}} a_2 f_D \sin\phi_P F^{B_s\to f_0}(m_{B_s}^2 - m_{f_0}^2)$
$\bar{B}_s^0 \to \bar{D}^0 f_0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} \cos\phi_P F^{B_s\to\eta'}(m_{B_s}^2 - m_K^2)$
$\bar{B}_s^0 \to D_s^- K_0^+$	$a_1 f_{D_s} F^{B_s\to K_0}(m_{B_s}^2 - m_{K_0}^2)$
$\Delta b = 1, \Delta C = 0, \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*$
$B^- \to \pi^0 a_0^-$	$\frac{1}{\sqrt{2}}(a_2 f_\pi F^{B\to a_0}(m_B^2 - m_{a_0}^2) + a_1 f_{a_0} F^{B\to\pi}(m_B^2 - m_\pi^2))$
$B^- \to \eta a_0^-$	$\frac{1}{\sqrt{2}}(a_2 f_\eta \sin\phi_P F^{B\to a_0}(m_B^2 - m_{a_0}^2) + a_1 f_{a_0} \sin\phi_P F^{B\to\eta}(m_B^2 - m_\eta^2))$
$B^- \to \eta' a_0^-$	$\frac{1}{\sqrt{2}}(a_2 f_{\eta'} \sin\phi_P F^{B\to a_0}(m_B^2 - m_{a_0}^2) + a_1 f_{a_0} \sin\phi_P F^{B\to\eta'}(m_B^2 - m_{\eta'}^2))$

$B^- \to \pi^- a_0^0$	$\frac{1}{\sqrt{2}}(a_1 f_\pi F^{B\to a_0}(m_B^2 - m_{a_0}^2) + a_2 f_{a_0} F^{B\to\pi}(m_B^2 - m_\pi^2))$
$\bar{B}^0 \to \pi^+ a_0^-$	$a_1 f_{a_0} F^{\bar{B}^0\to\pi}(m_{\bar{B}^0}^2 - m_\pi^2)$
$\bar{B}^0 \to \pi^0 a_0^0$	$-\frac{1}{2} a_2 f_\pi F^{B\to a_0}(m_{B^-}^2 - m_{a_0}^2)$
$\bar{B}^0 \to \pi^- a_0^+$	$a_1 f_\pi F^{\bar{B}^0\to a_0}(m_{\bar{B}^0}^2 - m_{a_0}^2)$
$\bar{B}^0 \to \eta a_0^0$	$-\frac{1}{2} a_2 f_\eta \sin\phi_P F^{B\to a_0}(m_{B^-}^2 - m_{a_0}^2)$
$\bar{B}^0 \to \eta' a_0^0$	$-\frac{1}{2} a_2 f_{\eta'} \cos\phi_P F^{\bar{B}\to a_0}(m_{\bar{B}}^2 - m_{a_0}^2)$
$\bar{B}_s^0 \to K^+ a_0^-$	$a_1 f_{a_0} F^{B_s\to K}(m_{B_s}^2 - m_K^2)$
$\bar{B}_s^0 \to \pi^0 K_0^0$	$a_2 f_\pi F^{B_s\to K_0}(m_{B_s}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to \pi^- K_0^+$	$a_1 f_\pi F^{B_s\to K_0}(m_{B_s}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to \eta K_0^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_s\to K_0}(m_{B_s}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to \eta' K_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_s\to K_0}(m_{B_s}^2 - m_{K_0}^2)$

Table 5.3 (b) Decay amplitudes of $B \to PS$ decays involving $b \to u$ transition

Decays	Amplitudes
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^*$
$B^- \to K^- a_0^0$	$\frac{1}{\sqrt{2}} a_2 f_K F^{B \to a_0} (m_B^2 - m_{a_0}^2)$
$B^- \to \pi^0 K_0^-$	$\frac{1}{\sqrt{2}} (a_2 f_\pi F^{B \to K_0} (m_B^2 - m_{K_0}^2) + a_1 f_{K_0} F^{B^- \pi} (m_{B^-}^2 - m_\pi^2))$
$B^- \to \eta K_0^-$	$\frac{1}{\sqrt{2}} (a_2 f_\eta \sin\phi_P F^{B \to K_0} (m_B^2 - m_{K_0}^2) + a_1 f_{K_0} \sin\phi_P F^{B \to \eta} (m_{B^-}^2 - m_\eta^2))$
$B^- \to \eta' K_0^-$	$\frac{1}{\sqrt{2}} (a_2 f_{\eta'} \cos\phi_P F^{B \to K_0} (m_B^2 - m_{K_0}^2) + a_1 f_{K_0} \cos\phi_P F^{B \to \eta'} (m_{B^-}^2 - m_{\eta'}^2))$
$\overline{B}^0 \to \pi^+ K_0^-$	$a_1 f_{K_0} F^{\overline{B}^0 \to \pi} (m_{\overline{B}^0}^2 - m_\pi^2)$
$\overline{B}^0 \to \pi^0 \overline{K}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{\overline{B}^0 \to \overline{K}_0} (m_{\overline{B}^0}^2 - m_{\overline{K}_0}^2)$
$\overline{B}^0 \to \eta \overline{K}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{\overline{B} \to K_0} (m_{\overline{B}}^2 - m_{K_0}^2)$
$\overline{B}^0 \to \eta' \overline{K}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{\overline{B} \to K_0} (m_{\overline{B}}^2 - m_{K_0}^2)$
$\overline{B}^0 \to K^- a_0^+$	$a_1 f_K F^{\overline{B}^0 \to a_0} (m_{\overline{B}^0}^2 - m_{a_0}^2)$
$\overline{B}_s^0 \to K^+ K_0^-$	$a_1 f_{K_0} F^{B_s \to K} (m_{B_s}^2 - m_K^2)$
$\overline{B}_s^0 \to \pi^0 f_0$	$-\frac{1}{2} a_2 f_\pi \sin\phi_s F^{B_s \to f_0} (m_{B_s}^2 - m_{f_0}^2)$
$\overline{B}_s^0 \to \eta f_0$	$-\frac{1}{2} a_2 f_\eta \sin\phi_P \sin\phi_s F^{B_s \to f_0} (m_{B_s}^2 - m_{f_0}^2)$
$\overline{B}_s^0 \to K^- K_0^+$	$a_1 f_K F^{B_s \to K_0} (m_{B_s}^2 - m_{K_0}^2)$
$\overline{B}_s^0 \to \eta' f_0$	$-\frac{1}{2} a_2 f_{\eta'} \cos\phi_P \sin\phi_s F^{B_s \to f_0} (m_{B_s}^2 - m_{f_0}^2)$

$\Delta b = 1,\ \Delta C = -1,\ \Delta S = 0$	$\times \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^*$
$B^- \to \pi^0 D_0^-$	$-\frac{1}{\sqrt{2}} a_1 f_{D_0} F^{B\to\pi} (m_B^2 - m_\pi^2)$
$B^- \to \pi^- \bar{D}_0^0$	$-a_2 f_{D_0} F^{B\to\pi} (m_B^2 - m_\pi^2)$
$B^- \to \eta D_0^-$	$-\frac{1}{\sqrt{2}} a_1 f_{D_0} \sin\phi_P F^{B\to\eta} (m_B^2 - m_\eta^2)$
$B^- \to \eta' D_0^-$	$-\frac{1}{\sqrt{2}} a_1 f_{D_0} \cos\phi_P F^{B\to\eta'} (m_B^2 - m_{\eta'}^2)$
$B^- \to D^- a_0^0$	$-\frac{1}{\sqrt{2}} a_1 f_D F^{B\to a_0} (m_B^2 - m_{a_0}^2)$
$B^- \to \bar{D}^0 a_0^-$	$-a_2 f_{D_0} F^{B\to a_0} (m_B^2 - m_{a_0}^2)$
$\bar{B}^0 \to \pi^+ D_0^-$	$-a_1 f_{D_0} F^{\bar{B}\to\pi} (m_{\bar{B}}^2 - m_\pi^2)$
$\bar{B}^0 \to \pi^0 \bar{D}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} F^{\bar{B}\to\pi} (m_{\bar{B}}^2 - m_\pi^2)$
$\bar{B}^0 \to \eta \bar{D}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} \sin\phi_P F^{\bar{B}\to\eta} (m_{\bar{B}}^2 - m_\eta^2)$
$\bar{B}^0 \to \eta' \bar{D}_0^0$	$\frac{1}{\sqrt{2}} a_2 f_{D_0} \cos\phi_P F^{\bar{B}\to\eta'} (m_{\bar{B}}^2 - m_{\eta'}^2)$
$\bar{B}^0 \to D^- a_0^+$	$-a_1 f_D F^{\bar{B}\to a_0} (m_{\bar{B}}^2 - m_{a_0}^2)$
$\bar{B}^0 \to \bar{D}^0 a_0^0$	$\frac{1}{\sqrt{2}} a_2 f_D F^{\bar{B}\to a_0} (m_{\bar{B}}^2 - m_{a_0}^2)$
$\bar{B}_s^0 \to K^+ D_0^-$	$-a_2 f_{D_0} F^{B_s\to K} (m_{B_s}^2 - m_K^2)$
$\bar{B}_s^0 \to K^0 \bar{D}_0^0$	$-a_1 f_D F^{B_s\to K_0} (m_{B_s}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to D^- K_0^+$	$-a_1 f_D F^{B_s\to K_0} (m_{B_s}^2 - m_{K_0}^2)$
$\bar{B}_s^0 \to \bar{D}^0 K_0^0$	$-a_2 f_D F^{B_s\to K_0} (m_{B_s}^2 - m_{K_0}^2)$

5.3.2 DECAY CONSTANTS OF SCALAR MESON

At present, the decay constants of the scalar mesons are poorly known. Since, the diagonal-scalar resonances $(a_0^0, f_0$ and $f_0')$ cannot be produced via the vector current owing to the C-invariance or conservation of the vector current, their respective decay constant values vanish, i.e.

$$f_{a_0^0} = f_{f_0} = f_{f_0'} = 0. \tag{5.7}$$

However, the decay constants of off-diagonal states may not vanish due to SU(2) and SU(3) breaking. Maltman using finite energy sum rules [5] has obtained

$$f_{a_0^\pm(1.450)} = 1.1 \text{ MeV and } f_{K_0} = 42 \text{ MeV}, \tag{5.8}$$

consistent with the range estimated by Narison on the basis of QCD spectral rules [6]

$$f_{K_0} = 33 \text{ to } 46 \text{ MeV}. \tag{5.9}$$

Chernyak [7] has calculated $f_{K_0} = (70 \pm 10)$ MeV, indicating quite strong SU(3) breaking for the scalar mesons.

Another calculation of the scalar meson decay constants based on the generalized NLJ model [8] yields

$$f_{a_0^\pm(1.450)} = 0.4 \text{ MeV and } f_{K_0} = 31 \text{ MeV}. \tag{5.10}$$

The value of the scalar decay constants

$$f_{a_0^\pm} = 0.0011,\ f_{K_0} = 0.021 \text{ GeV},\ f_{D_0} = 0.088 \text{ GeV},$$

$$f_{D_{s0}} = 0.073 \text{ GeV},\ f_{B_0} = 0.112 \text{ GeV and } f_{B_{s0}} = 0.112 \text{ GeV}, \tag{5.11}$$

have been taken from [4, 9].

5.4 CALCULATION OF THE $B \to S$ TRANSITION FORM FACTORS IN ISGW II MODEL

The form factors have the following expressions in the improved ISGW II quark model [3]:

$$u_+ + u_- = -\sqrt{\frac{2}{3}} \frac{m_d}{\beta_B} F_5^{(u_+ + u_-)},$$
$$u_+ - u_- = \sqrt{\frac{2}{3}} \frac{m_d \tilde{m}_B}{\tilde{m}_S \beta_B} F_5^{(u_+ - u_-)}, \tag{5.12}$$

where

$$F_5^{(u_+ + u_-)} = F_5 (\frac{\bar{m}_B}{\tilde{m}_B})^{-1/2} (\frac{\bar{m}_S}{\tilde{m}_S})^{1/2},$$
$$F_5^{(u_+ - u_-)} = F_5 (\frac{\bar{m}_B}{\tilde{m}_B})^{1/2} (\frac{\bar{m}_S}{\tilde{m}_S})^{-1/2}, \tag{5.13}$$

We obtain the required form factors for $B \to S$ transition in the ISGW II model which present in Tables 5.4 at $q^2 = t_m$.

Table 5.4 Form factors of $B \to S$ transition at $q^2 = t_m$ in the ISGW II quark model

Transition	u_+	u_-
$B \to a_0$	0.408	-0.670
$B \to f_0$	0.434	-0.748
$B \to K_0$	0.431	-0.712
$B \to D_0$	0.249	-0.658
$B_s \to f_0'$	0.449	-0.766
$B_s \to K_0$	0.426	-0.697
$B_s \to D_0$	0.294	-0.773

5.5 NUMERICAL RESULTS AND DISCUSSIONS

For the numerical calculation, we use the $B \to P$ transition form factors [10] given in chapter 2 and $B \to S$ transition form factors calculated in last section 5.4. Using the decay constants given in section 5.3.2 and (3.26), we obtain the numerical values of the branching ratios for $B \to PS$ decays, which are given in the Tables 5.5, 5.6, 5.7 (a) and 5.7 (b).

Table 5.5 Branching ratios of $B \to PS$ decays in CKM-favored mode involving $b \to c$ transition

Decays	Branching ratios
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$	
$B^- \to \pi^- D_0^0$	1.0×10^{-4}
$B^- \to D^0 a_0^-$	4.0×10^{-5}
$\bar{B}^0 \to \pi^0 D_0^0$	2.8×10^{-3}
$\bar{B}^0 \to \pi^- D_0^+$	2.8×10^{-4}
$\bar{B}^0 \to \eta D_0^0$	1.5×10^{-5}
$\bar{B}^0 \to \eta' D_0^0$	7.3×10^{-6}
$\bar{B}^0 \to D^+ a_0^-$	3.2×10^{-7}
$\bar{B}^0 \to D^0 a_0^0$	1.5×10^{-5}
$\bar{B}_s^0 \to K^0 D_0^0$	4.3×10^{-5}
$\bar{B}_s^0 \to \pi^- D_{s0}^+$	3.2×10^{-4}
$\bar{B}_s^0 \to D^0 K_0^0$	2.1×10^{-5}
$\bar{B}_s^0 \to D_s^+ a_0^-$	2.9×10^{-7}
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$	
$B^- \to D^0 D_{s0}^-$	1.4×10^{-3}
$B^- \to D_s^- D_0^0$	4.2×10^{-4}
$B^- \to \eta_c K_0^-$	5.6×10^{-5}
$\bar{B}^0 \to D^+ D_{s0}^-$	1.3×10^{-3}
$\bar{B}^0 \to D_s^- D_0^+$	3.9×10^{-4}
$\bar{B}^0 \to \eta_c \bar{K}_0^0$	5.3×10^{-5}
$\bar{B}_s^0 \to D_s^+ D_{s0}^-$	1.2×10^{-3}
$\bar{B}_s^0 \to D_s^- D_{s0}^+$	5.4×10^{-4}
$\bar{B}_s^0 \to \eta_c f_0$	4.5×10^{-5}

Table 5.6 Branching ratios of $B \to PS$ decays in CKM-suppressed mode involving $b \to c$ transition

Decays	Branching ratios
$\Delta b = 1,\ \Delta C = 1,\ \Delta S = -1$	
$B^- \to K^- D_0^0$	7.8×10^{-6}
$B^- \to D^0 K_0^-$	1.3×10^{-6}
$\bar{B}^0 \to \bar{K}^0 D_0^0$	3.3×10^{-6}
$\bar{B}^0 \to K^- D_0^+$	2.0×10^{-5}
$\bar{B}^0 \to D^+ K_0^-$	6.3×10^{-6}
$\bar{B}^0 \to D^0 \bar{K}_0^0$	2.0×10^{-6}
$\bar{B}_s^0 \to \eta D_0^0$	1.1×10^{-6}
$\bar{B}_s^0 \to K^- D_{s0}^+$	2.4×10^{-5}
$\bar{B}_s^0 \to \eta' D_0^0$	1.3×10^{-6}
$\bar{B}_s^0 \to D^0 f_0$	1.5×10^{-6}
$\bar{B}_s^0 \to D_s^+ K_0^-$	5.6×10^{-6}
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0$	
$B^- \to D^0 D_0^-$	1.1×10^{-4}
$B^- \to D^- D_0^0$	1.5×10^{-5}
$B^- \to \eta_c a_0^-$	2.4×10^{-6}
$\bar{B}^0 \to D^+ D_0^-$	1.0×10^{-4}
$\bar{B}^0 \to D^- D_0^+$	1.4×10^{-5}
$\bar{B}^0 \to \eta_c a_0^0$	1.1×10^{-6}
$\bar{B}_s^0 \to D_s^+ D_0^-$	8.9×10^{-5}
$\bar{B}_s^0 \to D^- D_{s0}^+$	1.9×10^{-5}
$\bar{B}_s^0 \to \eta_c K_0^0$	2.0×10^{-6}

Table 5.7 (a) Branching ratios of $B \to PS$ decays involving $b \to u$ transition

Decays	Branching ratios	
	This work	Cheng [10]
$\Delta b = 1,\ \Delta C = -1,\ \Delta S = -1$		
$B^- \to \pi^0 D_{s0}^-$	3.2×10^{-6}	-
$B^- \to \eta D_{s0}^-$	1.7×10^{-6}	-
$B^- \to K^- \bar{D}_0^0$	5.8×10^{-7}	-
$B^- \to \eta' D_{s0}^-$	8.5×10^{-7}	-
$B^- \to \bar{D}^0 K_0^-$	3.3×10^{-7}	-
$B^- \to D_s^- a_0^0$	3.9×10^{-6}	-
$\bar{B}^0 \to \pi^+ D_{s0}^-$	6.0×10^{-6}	-
$\bar{B}^0 \to \bar{K}^0 \bar{D}_0^0$	5.4×10^{-7}	-
$\bar{B}^0 \to \bar{D}^0 \bar{K}_0^0$	3.1×10^{-7}	-
$\bar{B}^0 \to D_s^- a_0^+$	7.3×10^{-6}	-
$\bar{B}_s^0 \to K^+ D_{s0}^-$	4.7×10^{-6}	-
$\bar{B}_s^0 \to \eta \bar{D}_0^0$	1.9×10^{-7}	-
$\bar{B}_s^0 \to \eta' \bar{D}_0^0$	2.1×10^{-7}	-
$\bar{B}_s^0 \to \bar{D}^0 f_0$	2.3×10^{-7}	-
$\bar{B}_s^0 \to D_s^- K_0^+$	5.0×10^{-6}	-
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0$		
$B^- \to \pi^0 a_0^-$	6.9×10^{-8}	0.6×10^{-6}
$B^- \to \eta a_0^-$	4.2×10^{-8}	-
$B^- \to \eta' a_0^-$	2.4×10^{-8}	-
$B^- \to \pi^- a_0^0$	1.0×10^{-6}	4.1×10^{-6}
$\bar{B}^0 \to \pi^+ a_0^-$	1.2×10^{-9}	0.1×10^{-6}
$\bar{B}^0 \to \pi^0 a_0^0$	2.6×10^{-8}	0.3×10^{-6}
$\bar{B}^0 \to \pi^- a_0^+$	1.9×10^{-6}	12.9×10^{-6}
$\bar{B}^0 \to \eta a_0^0$	1.6×10^{-8}	-
$\bar{B}^0 \to \eta' a_0^0$	9.4×10^{-9}	-
$\bar{B}_s^0 \to K^+ a_0^-$	9.6×10^{-10}	-
$\bar{B}_s^0 \to \pi^0 K_0^0$	3.2×10^{-8}	-
$\bar{B}_s^0 \to \pi^- K_0^+$	1.2×10^{-6}	-
$\bar{B}_s^0 \to \eta K_0^0$	2.0×10^{-8}	-
$\bar{B}_s^0 \to \eta' K_0^0$	1.1×10^{-8}	-

Table 5.7 (b) Branching ratios of $B \to PS$ decays involving $b \to u$ transition

Decays	Branching ratios	
	This work	Cheng [10]
$\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$		
$B^- \to K^- a_0^0$	7.8×10^{-8}	5.6×10^{-6}
$B^- \to \pi^0 K_0^-$	2.5×10^{-9}	0.3×10^{-6}
$B^- \to \eta K_0^-$	1.1×10^{-9}	-
$B^- \to \eta' K_0^-$	4.6×10^{-10}	-
$\bar{B}^0 \to \pi^+ K_0^-$	2.4×10^{-8}	1.1×10^{-6}
$\bar{B}^0 \to \pi^0 \bar{K}_0^0$	3.7×10^{-9}	0.6×10^{-6}
$\bar{B}^0 \to \eta \bar{K}_0^0$	2.2×10^{-9}	-
$\bar{B}^0 \to \eta' \bar{K}_0^0$	1.3×10^{-9}	-
$\bar{B}^0 \to K^- a_0^+$	1.5×10^{-7}	11.6×10^{-6}
$\bar{B}_s^0 \to K^+ K_0^-$	1.9×10^{-8}	-
$\bar{B}_s^0 \to \pi^0 f_0$	2.5×10^{-9}	-
$\bar{B}_s^0 \to \eta f_0$	1.5×10^{-9}	-
$\bar{B}_s^0 \to K^- K_0^+$	9.0×10^{-8}	-
$\bar{B}_s^0 \to \eta' f_0$	8.9×10^{-10}	-
$\Delta b = 1,\ \Delta C = -1,\ \Delta S = 0$		
$B^- \to \pi^0 D_0^-$	2.4×10^{-7}	-
$B^- \to \pi^- \bar{D}_0^0$	2.6×10^{-8}	-
$B^- \to \eta D_0^-$	1.3×10^{-7}	-
$B^- \to \eta' D_0^-$	6.4×10^{-8}	-
$B^- \to D^- a_0^0$	1.2×10^{-7}	-
$B^- \to \bar{D}^0 a_0^-$	1.4×10^{-8}	-
$\bar{B}^0 \to \pi^+ D_0^-$	4.5×10^{-7}	-
$\bar{B}^0 \to \pi^0 \bar{D}_0^0$	1.2×10^{-8}	-
$\bar{B}^0 \to \eta \bar{D}_0^0$	6.4×10^{-9}	-
$\bar{B}^0 \to \eta' \bar{D}_0^0$	3.2×10^{-9}	-
$\bar{B}^0 \to D^- a_0^+$	2.3×10^{-7}	-
$\bar{B}^0 \to \bar{D}^0 a_0^0$	6.3×10^{-9}	-
$\bar{B}_s^0 \to K^+ D_0^-$	3.5×10^{-7}	-
$\bar{B}_s^0 \to K^0 \bar{D}_0^0$	1.9×10^{-8}	-
$\bar{B}_s^0 \to D^- K_0^+$	1.5×10^{-7}	-
$\bar{B}_s^0 \to \bar{D}^0 K_0^0$	8.5×10^{-9}	-

The following observations are made:

5.5.1 $B \to PS$ DECAYS INVOLVING $b \to c$ TRANSITION

1. $\Delta b = 1,\ \Delta C = 1,\ \Delta S = 0$ mode:

a) In the present mode, dominant decays are $B(\overline{B}_s^0 \to \pi^- D_{s0}^+)$ = 3.2×10^{-4}, $B(\overline{B}^0 \to \pi^- D_0^+)$ = 2.8×10^{-4} and $B(B^- \to \pi^- D_0^0)$ = 3.6×10^{-2}. These may also be generated through annihilation mechanism and seem to be the best candidates for experimental observation.

b) Decay $\overline{B}^0 \to \pi^0 D_0^0$ may also be get contribution through elastic final state interactions (FSI).

c) Decays $\overline{B}^0 \to K^- D_{s0}^+ / D^0 f_0 / D_s^+ K_0^-$ are forbidden in the present analysis. Annihilation diagram may generate these decays.

2. $\Delta b = 1,\ \Delta C = 0,\ \Delta S = -1$ mode:

a) Branching ratios of the dominant decays are, $B(B^- \to D^0 D_{s0}^-)$ = 1.4×10^{-3}, $B(\overline{B}^0 \to D^+ D_{s0}^-)$ = 1.3×10^{-3} and $B(\overline{B}_s^0 \to D_s^- D_{s0}^+)$ = 1.2×10^{-3} . Next order dominant decays are $B(\overline{B}_s^0 \to D_s^- D_{s0}^+)$ = 5.4×10^{-4}, $B(B^- \to D_s^- D_0^0)$ = 4.2×10^{-4} and $B(\overline{B}^0 \to D_s^- D_0^+)$ = 3.9×10^{-4}. However, $B^- \to D^0 D_{s0}^-$ and $\overline{B}^0 \to D^+ D_{s0}^-$ decays may appear through penguin diagram.

b) Decays $B^- \to K^- \chi_{c0}(1P)$, $\overline{B}^0 \to \overline{K}^0 \chi_{c0}(1P)$ and $\overline{B}_s^0 \to$ $\pi^0 \chi_{c1}(1P) /$ $\eta \chi_{c1}(1P) / \eta' \chi_{c1}(1P) / D^+ D_0^- / D^0 \overline{D}_0^0 / / D^- D_0^+ / \overline{D}^0 D_0^0 / \eta_c a_0^0$ are forbidden in present analysis.

3. $\Delta b = 1,\ \Delta C = 1,\ \Delta S = -1$ mode:

a) $B(\overline{B}_s^0 \to K^- D_{s0}^+)$ = 2.4×10^{-5} and $B(\overline{B}^0 \to K^- D_0^+)$ = 2.0×10^{-5} decays are dominant.

b) In the present analysis $\bar{B}_s^0 \to \pi^0 D_0^0 / \pi^- D_0^+ / D^+ a_0^- / D^0 a_0^0 / D^0 \chi_{c0}$ decays are forbidden. Moreover, these decays also get contribution through the annihilation process.

4. $\underline{\Delta b = 1,\ \Delta C = 0,\ \Delta S = 0 \text{ mode}:}$

a) In this mode, branching ratios of the dominant decay are $B(B^- \to D^0 D_0^-)$ = 1.1×10^{-4} and $B(\bar{B}^0 \to D^+ D_0^-)$ = 1.0×10^{-4}.

b) Decays $B^- \to \pi^- \chi_{c0}(1P)$, $\bar{B}^0 \to D^0 \bar{D}_0^0 / D_s^- D_{s0}^+ / \bar{D}^0 D_0^0 / D_s^+ D_{s0}^- / \pi^0 \chi_{c0}(1P)$ $/ \eta \chi_{c0}(1P) / \eta' \chi_{c0}(1P) / \eta_c f_0$ and $\bar{B}_s^0 \to K_0 \chi_{c0}$ are forbidden in this framework. Decays involving naked charm mesons may be generated through annihilation diagrams, elastic FSI and penguin diagrams. However, decays emitting charmonium $\chi_{c0}(1P)$ remains forbidden.

In case of $b \to u$ transitions, the branching ratios of all the decays are highly suppressed due to the small values of the CKM factor as well as the decay constants of scalar mesons. However, these may get contributions from *W*-annihilation and penguin diagrams.

5.5.3 COMPARISON WITH OTHER WORKS

We compare our results with branching ratios calculated in the other models [10, 11]. The predicted branching ratios in Cheng [10] shown in 3rd column of Tables 5.7 (a) and 5.7 (b) are generally smaller as compared to the present branching ratios because of the difference in the form factors obtained in the covariant light-front approach (CLF) and different quark masses have been used in the two works. Branching ratios for the hadronic weak charmed mesons have also been calculated by Cheng [11]. His predictions $B(\bar{B}^0 \to \pi^- D_0^+)$ = 2.6×10^{-4} and $B(\bar{B}_s^0 \to \pi^- D_{s0}^+)$ = 3.3×10^{-4} match well with our values of the branching ratios and $B(B^- \to \pi^- D_0^0)$ = 7.7×10^{-4} , $B(B^- \to D_s^- D_0^0)$ = 8.4×10^{-4}, $B(B^- \to D^0 D_{s0}^-)$ = 5.1×10^{-3}, $B(\bar{B}^0 \to D_s^- D_0^+)$ = 7.3×10^{-4} and $B(\bar{B}^0 \to D^+ D_{s0}^-)$ = 4.7×10^{-3} are different from our results. The disagreement with their predictions may be attributed to the difference in the form factors values and different values used for the decay constant.

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CHAPTER 6

HADRONIC WEAK DECAYS OF NAKED BOTTOM-CHARM MESON TO PSEUDOSCALAR AND *P*-WAVE MESONS

6.1 INTRODUCTION

The B_c meson discovered at Fermilab [1] is a unique quark-antiquark bound state $(\bar{b}c)$ composed of two heavy quarks (b, c) with different flavors and are thus flavor asymmetric. Recently, CDF Collaboration [2] announced an accurate determination of the B_c meson mass and its life time, which is in good agreement with their theoretical estimates. The investigation of the B_c meson properties (mass spectrum, decay rates, etc.) is therefore of special interest compared to symmetric heavy quarkonium $(\bar{b}b, \bar{c}c)$ states. The difference of quark flavors forbids the annihilation of B_c meson into gluons. As a result, the pseudoscalar $(\bar{b}c)$ state is much more stable than the heavy quarkonium states, and decays only weakly. The decay processes of the B_c meson can be broadly divided into three classes:

i) involving the decay of b quark with c being spectator,

ii) involving the decay of c quark with b being spectator and

iii) the two component annihilate, b and $\bar{c}$, weakly.

Processes i) and ii), as mentioned above, can contribute to semileptonic and nonleptonic weak decays, while the process iii) can only contribute to leptonic decays. Experimental study [3] of the B_c mesons are in plan for *B*-Physics both at the TEVATRON and Large Hadron Collider (LHC). These experimental efforts have opened up new investigation concerning the structure of strong and weak interactions for heavy flavor sector. Also, B_c meson attracts the interest of experimentalists for testing the predictions of various theoretical efforts in the laboratory Theoretically, there exists an extensive study concerning semileptonic and nonleptonic decays of B_c to *s*-wave mesons in different models [4-21]. Their estimates of B_c decay rates indicate that the *c*-quark give dominant contribution as compared to *b*-quark decays. However, a little attention is being paid to the decays of B_c meson to a *p*-wave meson final state including an axial-vector (A), a tensor (T) or scalar (S) mesons [6, 8, 12, 17-21].

In this chapter, we extend the formalism developed in earlier chapters to study the weak hadronic decays of B_c meson involving one *p*-wave meson in the final state in CKM-favored and CKM-suppressed modes:

$$B_c \to P + A / A',$$

$$B_c \to P + T,$$

and $$B_c \to P + S.$$

Using factorization scheme and employing the Isgur, Scora, Grienstein and Wise (ISGW II) quark model [22, 23] to obtain the form factors involved in the decay amplitude and consequently, we predicted their branching ratios. The study of B_c meson is of special interest as a lot of data is expected on its weak decays in the near future.

6.2 WEAK HAMILTONIAN

We have already stated that B_c meson can decay to the final states either via *b*-quark or *c*-quark decay. In addition to the QCD modified weak Hamiltonian for the Bottom changing ($\Delta b = 1$) decays as given in chapter 2, we also need weak Hamiltonian for Bottom conserving and charm changing $(\Delta b = 0)$ decays as given by

6.2.1 BOTTOM CHANGING DECAYS

i) The CKM favored $b \to c$ transition

$$H_W = \frac{G_F}{\sqrt{2}}\{V_{cb}V_{ud}^*[c_1(\bar{c}b)(\bar{d}u) + c_2(\bar{d}b)(\bar{c}u)] +$$

$$V_{cb}V_{cs}^*[c_1(\bar{c}b)(\bar{s}c) + c_2(\bar{s}b)(\bar{c}c)] +$$

$$V_{cb}V_{us}^*[c_1(\bar{c}b)(\bar{s}u) + c_2(\bar{s}b)(\bar{c}u)] +$$

$$V_{cb}V_{cd}^*[c_1(\bar{c}b)(\bar{d}c) + c_2(\bar{d}b)(\bar{c}c)]\},$$

iii) The CKM suppressed $b \to u$ transition,

$$H_W = \frac{G_F}{\sqrt{2}}\{V_{ub}V_{cs}^*[c_1(\bar{u}b)(\bar{s}c) + c_2(\bar{s}b)(\bar{u}c)] +$$

$$V_{ub}V_{ud}^*[c_1(\bar{u}b)(\bar{d}u) + c_2(\bar{d}b)(\bar{u}u)] +$$

$$V_{ub}V_{us}^*[c_1(\bar{u}b)(\bar{s}u) + c_2(\bar{s}b)(\bar{u}u)] +$$

$$V_{ub}V_{cd}^*[c_1(\bar{u}b)(\bar{d}c) + c_2(\bar{d}b)(\bar{u}c)]\}.$$

6.2.2 CHARM CHANGING AND BOTTOM CONSERVING DECAYS

i) CKM favored $(\Delta C = -1,\ \Delta S = -1)$ decays

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [c_1 (\bar{u}d)(\bar{s}c) + c_2 (\bar{s}d)(\bar{u}c)];$$

Equation Chapter 6 Section 6(6.1)

ii) CKM suppressed $(\Delta C = 1,\ \Delta S = 0)$ decays

$$\begin{aligned} H_W = \frac{G_F}{\sqrt{2}} \{ & V_{cd} V_{ud}^* [c_1 (\bar{c}d)(\bar{d}u) + c_2 (\bar{c}u)(\bar{d}d)] \\ & + V_{cs} V_{ud}^* [c_1 (\bar{c}s)(\bar{d}u) + c_2 (\bar{c}u)(\bar{d}s)] \}, \end{aligned} \quad (6.2)$$

iii) CKM doubly suppressed $(\Delta C = -\Delta S = -1)$ decays

$$H_W = \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* [c_1 (\bar{u}s)(\bar{d}c) + c_2 (\bar{d}s)(\bar{u}c)]. \quad (6.3)$$

By factorizing matrix elements of the four-quark operator contained in the effective Hamiltonian, here also we can divide the decays in three classes as stated in chapter 2.

6.3 B_c DECAYS INTO PSEUDOSCALAR AND AXIAL-VECTOR MESONS

In this section, we study the CKM-favored and CKM-suppressed $B_c \to P(0^-) + A/A'(1^+)$ decays in analogy to the methodology given chapter 3.

In earlier chapters, we have studied decays of bottom mesons involving b-quark as a decaying particle. However, B_c meson, being heavy, can also emit bottom mesons in the final state; therefore, we need spectroscopy of these mesons.

6.3.1 SPECTROSCOPY OF BOTTOM AXIAL-VECTOR MESONS

In addition to the axial-vector meson spectroscopy upto the charm sector, which are already given in chapter 3 upto charm level, we also need the bottom axial-vector meson spectroscopy for the B_c decays.

Similar to the mixing scheme applied for the strange and charm level, we use the following mixing scheme in case of the bottom $(b\bar{u})$ and bottom-strange $(b\bar{s})$ mesons,

$$\begin{aligned} B_1(5.670) &= B_{1A}\sin\theta_4 + B_{1A'}\cos\theta_4, \\ \underline{B}_1(5.721) &= B_{1A}\cos\theta_4 - B_{1A'}\sin\theta_4, \end{aligned} \tag{6.4}$$

and

$$\begin{aligned} B_{s1}(5.762) &= B_{s1A}\sin\theta_5 + B_{s1A'}\cos\theta_5, \\ \underline{B}_{s1}(5.830) &= B_{s1A}\cos\theta_5 - B_{s1A'}\sin\theta_5, \end{aligned} \tag{6.5}$$

Recent work of Colangelo, De Fazio and Ferrandes [24] shows that, like the charm mesons, the mixing angle for the beauty sector is also small, i.e. $(-1.60\pm 0.69)^\circ$, which is used for θ_4 as well as θ_5 in this work.

6.3.2 DECAY RATE FORMULA

Following the discussion and formalism for the $B(0^-) \to P(0^-) + A(1^+)$ decays given in the section 3.3 of chapter 3, we obtain the decay rate formula for $B_c(0^-) \to P(0^-) + A(1^+)$ as:

$$\Gamma(B_c \to P\,A) = \frac{p_c^3}{8\pi m_A^2}\left|A(B_c \to P\,A)\right|^2,$$

where p_c is the magnitude of the three-momentum of a final-state particle in the rest frame of B_c meson and m_A denotes the mass of the axial-vector meson.

Sandwiching the weak Hamiltonian between the initial and the final states, the decay amplitudes for various $B_c \to PA$ decay modes are obtained, which are given in Tables 6.1 and 6.2.

Table 6.1 Decay amplitudes of CKM-favored mode of $B_c \to PA$ decays for bottom conserving and charm changing modes

Decays	Amplitudes
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	
$B_c^+ \to \pi^+ B_{s1}^0$	$a_1 f_\pi (\sin\theta_5\, F^{B_c\to B_{s1A}}(m_\pi^2) + \cos\theta_5\, F^{B_c\to B_{s1A'}}(m_\pi^2))$
$B_c^+ \to \pi^+ \underline{B}_{s1}^0$	$a_1 f_\pi (\cos\theta_5\, F^{B_c\to B_{s1A}}(m_\pi^2) - \sin\theta_5\, F^{B_c\to B_{s1A'}}(m_\pi^2))$
$B_c^+ \to \overline{K}^0 B_1^+$	$a_2 f_K (\sin\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) + \cos\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \overline{K}^0 \underline{B}_1^+$	$a_2 f_K (\cos\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) - \sin\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$\Delta b = 0, \Delta C = -1, \Delta S = 1$	
$B_c^+ \to K^+ B_1^0$	$-a_1 f_K (\sin\theta_4\, F^{B_c\to B_{1A}}(m_K^2) + \cos\theta_4\, F^{B_c\to B_{1A'}}(m_K^2))$
$B_c^+ \to K^+ \underline{B}_1^0$	$-a_1 f_K (\cos\theta_4\, F^{B_c\to B_{1A}}(m_K^2) - \sin\theta_4\, F^{B_c\to B_{1A'}}(m_K^2))$
$B_c^+ \to K^0 B_1^+$	$-a_2 f_K (\sin\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) + \cos\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to K^0 \underline{B}_1^+$	$-a_2 f_K (\cos\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) - \sin\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$\Delta b = 0, \Delta C = -1, \Delta S = 0$	
$B_c^+ \to K^+ B_{s1}^0$	$a_1 f_K (\sin\theta_5\, F^{B_c\to B_{s1A}}(m_K^2) + \cos\theta_5\, F^{B_c\to B_{s1A'}}(m_K^2))$
$B_c^+ \to K^+ \underline{B}_{s1}^0$	$a_1 f_K (\cos\theta_5\, F^{B_c\to B_{s1A}}(m_K^2) - \sin\theta_5\, F^{B_c\to B_{s1A'}}(m_K^2))$
$B_c^+ \to \pi^+ B_1^0$	$-a_1 f_\pi (\sin\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) + \cos\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \pi^+ \underline{B}_1^0$	$-a_1 f_\pi (\cos\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) + \sin\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \pi^0 B_1^+$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\sin\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) + \cos\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \pi^0 \underline{B}_1^+$	$\frac{1}{\sqrt{2}} a_2 f_\pi (\cos\theta_4\, F^{B_c\to B_{1A}}(m_\pi^2) - \sin\theta_4\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \eta B_1^+$	$-\frac{1}{\sqrt{2}} a_2 f_\eta (\sin\theta_4 \sin\theta_5\, F^{B_c\to B_{1A}}(m_\pi^2) + \cos\theta_4 \cos\theta_5\, F^{B_c\to B_{1A'}}(m_\pi^2))$
$B_c^+ \to \eta \underline{B}_1^+$	$-\frac{1}{\sqrt{2}} a_2 f_\eta (\cos\theta_4 \sin\theta_5\, F^{B_c\to B_{1A}}(m_\pi^2) - \sin\theta_4 \cos\theta_5\, F^{B_c\to B_{1A'}}(m_\pi^2))$

Table 6.2 Decay amplitudes of CKM-favored modes of $B_c \to PA$ decays for bottom changing modes

Decays	Amplitudes
$\Delta b=1, \Delta C=1, \Delta S=0$	
$B_c^- \to D^- D_1^0$	$2a_2 m_{D_1}(f_{D_{1A}} \sin\theta_2\, F^{B_c\to D}(m_{D_1}^2) + f_{D_{1A'}} \cos\theta_2\, F^{B_c\to D}(m_{D_1}^2))$
$B_c^- \to D^- \underline{D}_1^0$	$2a_2 m_{\underline{D}_1}(f_{D_{1A}} \sin\theta_2\, F^{B_c\to D}(m_{\underline{D}_1}^2) + f_{D_{1A'}} \cos\theta_2\, F^{B_c\to D}(m_{\underline{D}_1}^2))$
$B_c^- \to \eta_c a_1^-$	$2a_1 m_{a_1} f_{a_1} F^{B_c\to\eta_c}(m_{a_1}^2)$
$B_c^- \to \eta_c b_1^-$	$2a_1 m_{b_1} f_{b_1} F^{B_c\to\eta_c}(m_{b_1}^2)$
$B_c^- \to \pi^- \chi_{c1}$	$a_1 f_\pi F^{B_c\to\chi_{c1}}(m_\pi^2)$
$B_c^- \to \pi^- h_{c1}$	$a_1 f_\pi F^{B_c\to h_{c1}}(m_\pi^2)$
$B_c^- \to D^0 D_1^-$	$a_2 f_D(\sin\theta_2\, F^{B_c\to D_{1A}}(m_D^2) + \cos\theta_2\, F^{B_c\to D_{1A'}}(m_D^2))$
$B_c^- \to D^0 \underline{D}_1^-$	$a_2 f_D(\cos\theta_2\, F^{B_c\to D_{1A}}(m_D^2) - \sin\theta_2\, F^{B_c\to D_{1A'}}(m_D^2))$
$\Delta b=1, \Delta C=0, \Delta S=-1$	
$B_c^- \to \overline{D}^0 K_1^-$	$2a_1 m_{K_1}(f_{K_{1A}} \sin\theta_1\, F^{B_c\to D}(m_{K_1}^2) + f_{K_{1A'}} \cos\theta_1\, F^{B_c\to D}(m_{K_1}^2))$
$B_c^- \to \overline{D}^0 \underline{K}_1^-$	$2a_1 m_{\underline{K}_1}(f_{K_{1A}} \cos\theta_1\, F^{B_c\to D}(m_{\underline{K}_1}^2) - f_{K_{1A'}} \sin\theta_1\, F^{B_c\to D}(m_{\underline{K}_1}^2))$
$B_c^- \to D_s^- a_1^0$	$\sqrt{2} a_2 m_{a_1} f_{a_1} F^{B_c\to D_s}(m_{a_1}^2)$
$B_c^- \to D_s^- f_1$	$\sqrt{2} a_2 m_{f_1} f_{f_1} \cos\theta\, F^{B_c\to D_s}(m_{f_1}^2)$
$B_c^- \to \pi^0 D_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi(\sin\theta_3 F^{B_c\to D_{s1A}}(m_\pi^2) + \cos\theta_3 F^{B_c\to D_{s1A'}}(m_\pi^2))$
$B_c^- \to \pi^0 \underline{D}_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi(\cos\theta_3 F^{B_c\to D_{s1A}}(m_\pi^2) - \sin\theta_3 F^{B_c\to D_{s1A'}}(m_\pi^2))$
$B_c^- \to \eta D_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta(\cos\theta_3 \sin\phi_P F^{B_c\to D_{s1A}}(m_\eta^2) - \sin\theta_3 \cos\phi_P F^{B_c\to D_{s1A'}}(m_\eta^2))$
$B_c^- \to \eta D_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta(\cos\theta_3 \sin\phi_P F^{B_c\to D_{s1A}}(m_\eta^2) - \sin\theta_3 \cos\phi_P F^{B_c\to D_{s1A'}}(m_\eta^2))$
$B_c^- \to K^- \overline{D}_1^0$	$a_1 f_K(\sin\theta_2 F^{B_c\to D_{1A}}(m_K^2) + \cos\theta_2 F^{B_c\to D_{1A'}}(m_K^2))$
$B_c^- \to K^- \underline{D}_1^0$	$a_1 f_K(\cos\theta_2 F^{B_c\to D_{1A}}(m_K^2) - a_1 f_K \sin\theta_2 F^{B_c\to D_{1A'}}(m_K^2))$
$B_c^- \to \eta' D_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'}(\sin\theta_3 \cos\phi_P F^{B_c\to D_{s1A}}(m_{\eta'}^2) + \cos\theta_3 \cos\phi_P F^{B_c\to D_{s1A'}}(m_{\eta'}^2))$
$B_c^- \to \eta' \underline{D}_{s1}^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'}(\cos\theta_3 \cos\phi_P F^{B_c\to D_{s1A}}(m_{\eta'}^2) - \sin\theta_3 \cos\phi_P F^{B_c\to D_{s1A'}}(m_{\eta'}^2))$
$B_c^- \to D_s^- h_{c1}$	$a_1 f_{D_s} F^{B_c\to h_{c1}}(m_{D_s}^2)$

$B_c^- \to D_s^- \chi_{c1}$	$2a_2 m_{\chi_{c1}} f_{\chi_{c1}} F^{B_c \to D_s}(m_{\chi_{c1}}^2) + a_1 f_{D_s} F^{B_c \to \chi_{c1}}(m_{D_s}^2)$
$B_c^- \to \eta_c D_{s1}^-$	$a_2 f_{\eta_c}(\sin\theta_3 F^{B_c \to D_{s1A}}(m_{\eta_c}^2) + \cos\theta_3 F^{B_c \to D_{s1A'}}(m_{\eta_c}^2))$ $+2a_1 m_{D_{s1}}(f_{D_{s1A}} \sin\theta_3 F^{B_c \to \eta_c}(m_{D_{s1}}^2) + f_{D_{s1A'}} \cos\theta_3 F^{B_c \to \eta_c}(m_{D_{s1}}^2))$
$B_c^- \to \eta_c \underline{D}_{s1}^-$	$a_2 f_{\eta_c}(\cos\theta_3 F^{B_c \to D_{s1A}}(m_{\eta_c}^2) - \sin\theta_3 F^{B_c \to D_{s1A'}}(m_{\eta_c}^2))$ $+2a_1 m_{\underline{D}_{s1}}(f_{D_{s1A}} \cos\theta_3 F^{B_c \to \eta_c}(m_{\underline{D}_{s1}}^2) - f_{D_{s1A'}} \sin\theta_3 F^{B_c \to \eta_c}(m_{\underline{D}_{s1}}^2))$
$\Delta b = 0, \Delta C = 1, \Delta S = -1$	
$B_c^- \to K^- \chi_{c1}$	$a_1 f_K F^{B_c \to \chi_{c1}}(m_K^2)$
$B_c^- \to K^- h_{c1}$	$a_1 f_K F^{B_c \to h_{c1}}(m_K^2)$
$B_c^- \to D^0 D_{s1}^-$	$a_2 f_D(\sin\theta_3 F^{B_c \to D_{s1A}}(m_D^2) + \cos\theta_3 F^{B_c \to D_{s1A'}}(m_D^2))$
$B_c^- \to D^0 \underline{D}_{s1}^-$	$a_2 f_D(\cos\theta_3 F^{B_c \to D_{s1A}}(m_D^2) - \sin\theta_3 F^{B_c \to D_{s1A'}}(m_D^2))$
$B_c^- \to D_s^- D_1^0$	$2m_{D_1} a_2 f_{D_1}(\sin\theta_2 F^{B_c \to D_s}(m_{D_1}^2) + \cos\theta_2 F^{B_c \to D_s}(m_{D_1}^2))$
$B_c^- \to D_s^- \underline{D}_1^0$	$2m_{\underline{D}_1} a_2 f_{\underline{D}_1}(\cos\theta_2 F^{B_c \to D_s}(m_{\underline{D}_1}^2) - \sin\theta_2 F^{B_c \to D_s}(m_{\underline{D}_1}^2))$
$B_c^- \to \eta_c K_1^-$	$2m_{K_1} a_1 f_{K_1}(\sin\theta_1 F^{B_c \to \eta_c}(m_{K_1}^2) + \cos\theta_1 F^{B_c \to \eta_c}(m_{K_1}^2))$
$B_c^- \to \eta_c \underline{K}_1^-$	$2m_{\underline{K}_1} a_1 f_{\underline{K}_1}(\cos\theta_1 F^{B_c \to \eta_c}(m_{\underline{K}_1}^2) - \sin\theta_1 F^{B_c \to \eta_c}(m_{\underline{K}_1}^2))$
$\Delta b = 0, \Delta C = 0, \Delta S = 0$	
$B_c^- \to \pi^0 D_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi(\sin\theta_2 F^{B_c \to D_{1A}}(m_\pi^2) + \cos\theta_2 F^{B_c \to D_{1A'}}(m_\pi^2))$
$B_c^- \to \pi^0 \underline{D}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi(\cos\theta_2 F^{B_c \to D_{1A}}(m_\pi^2) - \sin\theta_2 F^{B_c \to D_{1A'}}(m_\pi^2))$
$B_c^- \to \pi^- \overline{D}_1^0$	$a_1 f_\pi(\sin\theta_2 F^{B_c \to D_{1A}}(m_\pi^2) + \cos\theta_2 F^{B_c \to D_{1A'}}(m_\pi^2))$
$B_c^- \to \pi^- \overline{\underline{D}}_1^0$	$a_1 f_\pi(\cos\theta_2 F^{B_c \to D_{1A}}(m_\pi^2) - \sin\theta_2 F^{B_c \to D_{1A'}}(m_\pi^2))$
$B_c^- \to \eta D_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta(\sin\theta_2 F^{B_c \to D_{1A}}(m_\eta^2) + \cos\theta_2 F^{B_c \to D_{1A'}}(m_\eta^2))$
$B_c^- \to \eta \underline{D}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta(\cos\theta_2 F^{B_c \to D_{1A}}(m_\eta^2) - \sin\theta_2 F^{B_c \to D_{1A'}}(m_\eta^2))$
$B_c^- \to \eta' D_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'}(\sin\theta_2 F^{B_c \to D_{1A}}(m_{\eta'}^2) + \cos\theta_2 F^{B_c \to D_{1A'}}(m_{\eta'}^2))$
$B_c^- \to \eta' \underline{D}_1^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'}(\cos\theta_2 F^{B_c \to D_{1A}}(m_{\eta'}^2) - \sin\theta_2 F^{B_c \to D_{1A'}}(m_{\eta'}^2))$
$B_c^- \to D^- a_1^0$	$\sqrt{2} a_2 m_{a_1} f_{a_1} F^{B_c \to D}(m_{a_1}^2)$
$B_c^- \to D^- f_1$	$\sqrt{2} a_2 m_{f_1} f_{f_1} \cos\phi_A F^{B_c \to D}(m_{f_1}^2)$
$B_c^- \to \eta_c \underline{D}_1^-$	6.79×10^{-5}
$B_c^- \to D^- \chi_{c1}$	$-(2a_2 m_{\chi_{c1}} f_{\chi_{c1}} F^{B_c \to D}(m_{\chi_{c1}}^2) + a_1 f_D F^{B_c \to \chi_{c1}}(m_D^2))$

$B_c^- \to D^- h_{c1}$	$-a_1 f_D F^{B_c \to h_{c1}}(m_D^2)$
$B_c^- \to \overline{D}^0 a_1^-$	$2a_1 f_{a_1} m_{a_1} F^{B_c \to D}(m_{a_1}^2)$
$B_c^- \to \overline{D}^0 b_1^-$	$2a_1 f_{b_1} m_{b_1} F^{B_c \to D}(m_{b_1}^2)$
$B_c^- \to \eta_c D_1^-$	$-(2a_1 m_{D_1} f_{D_1}(\sin\theta_2 F^{B_c \to \eta_c}(m_{D_1}^2) + \cos\theta_2 F^{B_c \to \eta_c}(m_{D_1}^2))$ $+ a_2 f_{\eta_c}(\sin\theta_2 F^{B_c \to D_1}(m_{\eta_c}^2) + \cos\theta_2 F^{B_c \to D_1}(m_{\eta_c}^2)))$
$\Delta b = 0, \Delta C = -1, \Delta S = 0$	
$B_c^- \to D^- \overline{D}_1^0$	$-(2a_2 m_{D_1} f_{D_1}(\sin\theta_2 F^{B_c \to D}(m_{D_1}^2) + \cos\theta_2 F^{B_c \to D}(m_{D_1}^2))$ $+ a_1 f_D(\sin\theta_2 F^{B_c \to D_1}(m_D^2) + \cos\theta_2 F^{B_c \to D_1}(m_D^2)))$
$B_c^- \to D^- \overline{\underline{D}}_1^0$	$-(2a_2 m_{\underline{D}_1} f_{\underline{D}_1}(\cos\theta_2 F^{B_c \to D}(m_{\underline{D}_1}^2) - \sin\theta_2 F^{B_c \to D}(m_{\underline{D}_1}^2))$ $+ a_1 f_D(\cos\theta_2 F^{B_c \to \underline{D}_1}(m_D^2) - \sin\theta_2 F^{B_c \to \underline{D}_1}(m_D^2)))$
$B_c^- \to \overline{D}^0 D_1^-$	$-(2a_1 m_{D_1} f_{D_1}(\sin\theta_2 F^{B_c \to D}(m_{D_1}^2) + \cos\theta_2 F^{B_c \to D}(m_{D_1}^2))$ $+ a_2 f_D(\sin\theta_2 F^{B_c \to D_1}(m_D^2) + \cos\theta_2 F^{B_c \to D_1}(m_D^2)))$
$B_c^- \to \overline{D}^0 \underline{D}_1^-$	$-(2a_1 m_{\underline{D}_1} f_{\underline{D}_1}(\cos\theta_2 F^{B_c \to D}(m_{\underline{D}_1}^2) - \sin\theta_2 F^{B_c \to D}(m_{\underline{D}_1}^2))$ $+ a_2 f_D(\cos\theta_2 F^{B_c \to \underline{D}_1}(m_D^2) - \sin\theta_2 F^{B_c \to \underline{D}_1}(m_D^2)))$
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	
$B_c^- \to \overline{D}^0 D_{s1}^-$	$(2a_1 m_{D_{s1}} f_{D_{s1}}(\sin\theta_3 F^{B_c \to D}(m_{D_{s1}}^2) + \cos\theta_3 F^{B_c \to D}(m_{D_{s1}}^2))$ $+ a_2 f_D(\sin\theta_3 F^{B_c \to D_{s1A}}(m_D^2) + \cos\theta_3 F^{B_c \to D_{s1A'}}(m_D^2)))$
$B_c^- \to \overline{D}^0 \underline{D}_{s1}^-$	$(2a_1 m_{\underline{D}_{s1}} f_{\underline{D}_{s1}}(\cos\theta_3 F^{B_c \to D}(m_{\underline{D}_{s1}}^2) \sin\theta_3 F^{B_c \to D}(m_{\underline{D}_{s1}}^2))$ $+ a_2 f_D(\cos\theta_3 F^{B_c \to \underline{D}_{s1A}}(m_D^2) - \sin\theta_3 F^{B_c \to \underline{D}_{s1A'}}(m_D^2)))$
$B_c^- \to D_s^- \overline{D}_1^0$	$2a_2 m_{D_1} f_{D_1}(\sin\theta_2 F^{B_c \to D_s}(m_{D_1}^2) + \cos\theta_2 F^{B_c \to D_s}(m_{D_1}^2))$ $+ a_1 f_{D_s}(\sin\theta_2 F^{B_c \to D_1}(m_{D_s}^2) + \cos\theta_2 F^{B_c \to D_1}(m_{D_s}^2))$
$B_c^- \to D_s^- \overline{\underline{D}}_1^0$	$2a_2 m_{\underline{D}_1} f_{\underline{D}_1}(\cos\theta_2 F^{B_c \to D_s}(m_{\underline{D}_1}^2) - \sin\theta_2 F^{B_c \to D_s}(m_{\underline{D}_1}^2))$ $+ a_1 f_{D_s}(\cos\theta_2 F^{B_c \to \underline{D}_1}(m_{D_s}^2) - \sin\theta_2 F^{B_c \to \underline{D}_1}(m_{D_s}^2))$

6.3.3 CALCULATION OF THE FORM FACTORS IN ISGW II MODEL

In this section, we extend ISGW II model framework [22] to calculate $B_c \to A/A'$ and $B_c \to P$ transition form factors.

A. $B_c \to A/A'$ TRANSITION FORM FACTORS

The form factors have the following simplified expressions in the ISGW II model for $B_c \to A/A'$ transitions caused by $b \to c$ quark transition [22]:

$$l = -\tilde{m}_{B_c}\beta_{B_c}[\frac{1}{\mu_-} + \frac{m_c\tilde{m}_A(\tilde{\omega}-1)}{\beta_{B_c}^2}(\frac{5+\tilde{\omega}}{6m_q} - \frac{m_c\beta_{B_c}^2}{2\mu_-\beta_{B_cA}^2})]F_5^{(l)}, \tag{6.6}$$

$$c_+ + c_- = -\frac{\tilde{m}_A}{2\tilde{m}_{B_c}\beta_{B_c}}\left(1 - \frac{m_c^2\beta_{B_c}^2}{2\tilde{m}_A\mu_-\beta_{B_cA}^2}\right)F^{(c_+ + c_-)}, \tag{6.7}$$

$$c_+ - c_- = -\frac{\tilde{m}_A}{2\tilde{m}_{B_c}\beta_{B_c}}\left(\frac{\tilde{\omega}+2}{3} - \frac{m_c^2\beta_{B_c}^2}{2\tilde{m}_A\mu_-\beta_{B_cA}^2}\right)F^{(c_+ - c_-)}, \tag{6.8}$$

$$r = \frac{\tilde{m}_{B_c}\beta_{B_c}}{\sqrt{2}}[\frac{1}{\mu_+} + \frac{\tilde{m}_A}{3\beta_{B_c}^2}(\tilde{\omega}-1)^2]F_5^{(r)}, \tag{6.9}$$

$$s_+ + s_- = \frac{m_c}{\sqrt{2}\tilde{m}_{B_c}\beta_{B_c}}\left(\frac{m_c\beta_{B_c}^2}{2\mu_+\beta_{B_cA}^2}\right)F^{(s_+ + s_-)}, \tag{6.10}$$

$$s_+ - s_- = \frac{1}{\sqrt{2}\beta_{B_c}}\left(\frac{4-\tilde{\omega}}{3} - \frac{m_c^2\beta_{B_c}^2}{2\tilde{m}_A\mu_+\beta_{B_cA}^2}\right)F^{(s_+ - s_-)}, \tag{6.11}$$

where

$$\mu_{\pm} = (\frac{1}{m_c} + \frac{1}{m_b})^{-1}. \tag{6.12}$$

and

$$\begin{aligned}
F_5^{(l)} = F_5^{(r)} &= F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{1/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{1/2}, \\
F_5^{(c_+ + c_-)} = F_5^{(s_+ + s_-)} &= F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{-3/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{1/2}, \\
F_5^{(c_+ - c_-)} = F_5^{(s_+ - s_-)} &= F_5 (\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}})^{-1/2} (\frac{\bar{m}_A}{\tilde{m}_A})^{-1/2}.
\end{aligned} \tag{6.13}$$

The $t(\equiv q^2)$ dependence is given by

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_{B_c}\bar{m}_A}, \tag{6.14}$$

and

$$F_5 = \left(\frac{\tilde{m}_A}{\tilde{m}_{B_c}}\right)^{1/2} \left(\frac{\beta_{B_c}\beta_A}{\beta_{B_cA}}\right)^{5/2} \left[1 + \frac{1}{18}\chi^2(t_m - t)\right]^{-3}, \tag{6.15}$$

with

$$\chi^2 = \frac{3}{4m_b m_c} + \frac{3m_c^2}{2\bar{m}_{B_c}\bar{m}_A\beta_{B_cA}^2} + \frac{1}{\bar{m}_{B_c}\bar{m}_A}\left(\frac{16}{33 - 2n_f}\right)\ln[\frac{\alpha_S(\mu_{QM})}{\alpha_S(m_c)}], \tag{6.16}$$

and

$$\beta_{B_cA}^2 = \frac{1}{2}\left(\beta_{B_c}^2 + \beta_A^2\right). \tag{6.17}$$

$\tilde{m}$ is the sum of the mesons constituent quarks masses, $\bar{m}$ is the hyperfine averaged physical masses, n_f is the number of active flavors, which is taken to be five in the present case, $t_m = (m_{B_c} - m_A)^2$ is the maximum momentum transfer and μ_{QM} is the quark model scale. The values of parameter β for different *s*-wave and *p*-wave mesons are given in the Table 6.3 [22]. Using a similar method, $B_c \to A/A'$ transition form factors for $c \to s$ channel can also be obtained [22]. These are given in Tables 6.4 and 6.5.

Table 6.3 The values of parameter β for s-wave and p-wave mesons in the ISGW II quark model

Quark content	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$	$u\bar{b}$	$s\bar{b}$	$c\bar{c}$	$b\bar{c}$
β_s (GeV)	0.41	0.44	0.53	0.45	0.56	0.43	0.54	0.88	0.92
β_P (GeV)	0.28	0.30	0.33	0.33	0.38	0.35	0.41	0.52	0.60

Table 6.4 Form factors of $B_c \to A$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	Transition	l	c_+	c_-
$\Delta b = 0$, ΔC = -1, ΔS = -1	$B_c \to B_{s1}$	-15.816	1.710	0.177
	$B_c \to B_1$	-2.838	0.453	0.065
Δb =1, ΔC = 0, ΔS = -1	$B \to D_1$	-2.129	-0.030	-0.001
	$B_c \to D_{s1}$	-1.982	-0.043	-0.001
Δb =1, ΔC = 1, ΔS = 0	$B_c \to \chi_{c1}(c\bar{c})$	-0.491	-0.148	-0.006

Table 6.5 Form factors of $B_c \to A'$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	Transition	r	s_+	s_-
$\Delta b = 0$, ΔC = -1, ΔS = -1	$B_c \to \underline{B}_{s1}$	-10.424	-0.701	-0.279
	$B_c \to \underline{B}_1$	-3.947	-0.201	0.0003
Δb =1, ΔC = 0, ΔS = -1	$B \to \underline{D}_1$	1.451	0.038	-0.023
	$B_c \to \underline{D}_{s1}$	1.424	0.062	-0.032
Δb =1, ΔC = 1, ΔS = 0	$B_c \to h_{c1}(c\bar{c})$	2.129	0.212	-0.062

B. $B_c \to P$ TRANSITION FORM FACTORS

For $B_c \to P$ transition form factors also, we the use ISGW II quark model [22] which provides the following formulae for $b \to c$ quark transition:

$$f_+ + f_- = \left[1 - \frac{\tilde{m}_P}{m_c}\left(1 - \frac{m_c^2 \beta_{B_c}^2}{2\tilde{m}_P \mu_+ \beta_{B_c P}^2}\right)\right] F_3^{(f_+ + f_-)} R^{(f_+ + f_-)},$$

$$f_+ - f_- = \frac{\tilde{m}_{B_c}}{m_c}\left(1 - \frac{m_c^2 \beta_{B_c}^2}{2\tilde{m}_P \mu_+ \beta_{B_c P}^2}\right) F_3^{(f_+ - f_-)} R^{(f_+ - f_-)}, \tag{6.18}$$

For more details of the form factors and correction factors one can refer to the original work [22]. The obtained form factors, f_+ and f_-, are given in column 3rd and 4th of Table 6.6, respectively.

Tables 6.6 Form factors of $B_c \to P$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	**Transition**	f_+	f_-
$\Delta b = 0$, ΔC = -1, ΔS = -1	$B_c \to B_s$	0.926	-0.374
	$B_c \to B$	1.103	-0.652
Δb =1, ΔC = 0, ΔS = -1	$B_c \to D$	2.110	-1.975
	$B_c^- \to D_s$	1.543	-1.356
Δb =1, ΔC = 1, ΔS = 0	$B_c \to \eta_c (c\bar{c})$	1.193	-0.716

Decay constants for the pseudoscalar and axial-vector mesons are already been discussed in chapter 3.

6.3.4 NUMERICAL RESULTS AND DISCSSIONS

Using $B_c \to A/A'/P$ form factors obtained in the previous section and the decay rate formula given in section 6.3.2, we finally predict branching ratios of various $B_c \to PA$ decays as shown in Tables 6.7 and 6.8. The numerical values for the decay constants of pseudoscalar mesons used here are taken from (3.26) while for the axial-vector meson one can refer to section 3.5.

Table 6.7 Branching ratios of CKM-favored mode of $B_c \to PA$ decays for bottom conserving and charm changing modes

Decays	Branching ratios
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	
$B_c^+ \to \pi^+ B_{s1}^0$	2.9×10^{-2}
$B_c^+ \to \pi^+ \underline{B}_{s1}^0$	0.47×10^{-2}
$B_c^+ \to \overline{K}^0 B_1^+$	0.54×10^{-2}
$B_c^+ \to \overline{K}^0 \underline{B}_1^+$	0.10×10^{-2}
$\Delta b = 0, \Delta C = -1, \Delta S = 1$	
$B_c^+ \to K^+ B_1^0$	9.1×10^{-5}
$B_c^+ \to K^+ \underline{B}_1^0$	1.6×10^{-5}
$B_c^+ \to K^0 B_1^+$	1.5×10^{-5}
$B_c^+ \to K^0 \underline{B}_1^+$	2.6×10^{-6}
$\Delta b = 0, \Delta C = -1, \Delta S = 0$	
$B_c^+ \to K^+ B_{s1}^0$	7.5×10^{-4}
$B_c^+ \to K^+ \underline{B}_{s1}^0$	1.1×10^{-4}
$B_c^+ \to \pi^+ B_1^0$	0.22×10^{-2}
$B_c^+ \to \pi^+ \underline{B}_1^0$	4.1×10^{-4}
$B_c^+ \to \pi^0 B_1^+$	1.8×10^{-4}
$B_c^+ \to \pi^0 \underline{B}_1^+$	3.3×10^{-5}
$B_c^+ \to \eta B_1^+$	2.1×10^{-4}
$B_c^+ \to \eta \underline{B}_1^+$	3.6×10^{-5}

Table 6.8 Branching ratios of CKM-favored modes of $B_c \to PA$ decays for bottom changing modes

Decays	Branching ratios
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	
$B_c^- \to D^- D_1^0$	3.1×10^{-5}
$B_c^- \to D^- \underline{D}_1^0$	9.8×10^{-6}
$B_c^- \to \eta_c a_1^-$	0.31×10^{-2}
$B_c^- \to \eta_c b_1^-$	2.6×10^{-8}
$B_c^- \to \pi^- \chi_{c1}$	0.07×10^{-2}
$B_c^- \to \pi^- h_{c1}$	0.06×10^{-2}
$B_c^- \to D^0 D_1^-$	6.1×10^{-5}
$B_c^- \to D^0 \underline{D}_1^-$	6.5×10^{-6}
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	
$B_c^- \to \overline{D}^0 K_1^-$	4.9×10^{-7}
$B_c^- \to \overline{D}^0 \underline{K}_1^-$	1.5×10^{-7}
$B_c^- \to D_s^- a_1^0$	1.1×10^{-8}
$B_c^- \to D_s^- f_1$	1.3×10^{-8}
$B_c^- \to \pi^0 D_{s1}^-$	7.9×10^{-9}
$B_c^- \to \pi^0 \underline{D}_{s1}^-$	7.2×10^{-10}
$B_c^- \to \eta D_{s1}^-$	4.9×10^{-9}
$B_c^- \to \eta D_{s1}^-$	4.3×10^{-10}
$B_c^- \to K^- \overline{D}_1^0$	2.5×10^{-7}
$B_c^- \to K^- \underline{D}_1^0$	3.7×10^{-8}
$B_c^- \to \eta' D_{s1}^-$	3.0×10^{-9}
$B_c^- \to \eta' \underline{D}_{s1}^-$	2.5×10^{-10}
$B_c^- \to D_s^- h_{c1}$	0.15×10^{-2}
$B_c^- \to D_s^- \chi_{c1}$	0.10×10^{-2}
$B_c^- \to \eta_c D_{s1}^-$	3.3×10^{-5}
$B_c^- \to \eta_c \underline{D}_{s1}^-$	1.3×10^{-4}
$\Delta b = 0, \Delta C = 1, \Delta S = -1$	
$B_c^- \to K^- \chi_{c1}$	5.1×10^{-5}
$B_c^- \to K^- h_{c1}$	4.4×10^{-5}
$B_c^- \to D^0 D_{s1}^-$	6.0×10^{-6}
$B_c^- \to D^0 \underline{D}_{s1}^-$	3.3×10^{-7}
$B_c^- \to D_s^- D_1^0$	8.6×10^{-7}

$B_c^- \to D_s^- \underline{D}_1^0$	2.7×10^{-7}
$B_c^- \to \eta_c K_1^-$	1.4×10^{-4}
$B_c^- \to \eta_c \underline{K}_1^-$	4.0×10^{-5}
$\Delta b = 0, \Delta C = 0, \Delta S = 0$	
$B_c^- \to K^- \overline{D}_1^0$	2.5×10^{-7}
$B_c^- \to K^- \underline{D}_1^0$	3.6×10^{-8}
$B_c^- \to \pi^0 D_1^-$	1.0×10^{-7}
$B_c^- \to \pi^0 \underline{D}_1^-$	1.4×10^{-8}
$B_c^- \to \pi^- \overline{D}_1^0$	3.8×10^{-6}
$B_c^- \to \pi^- \overline{\underline{D}}_1^0$	5.0×10^{-6}
$B_c^- \to \eta D_1^-$	6.5×10^{-8}
$B_c^- \to \eta \underline{D}_1^-$	8.4×10^{-9}
$B_c^- \to \eta' D_1^-$	4.0×10^{-8}
$B_c^- \to \eta' \underline{D}_1^-$	4.9×10^{-9}
$B_c^- \to D^- a_1^0$	2.6×10^{-6}
$B_c^- \to D^- f_1$	3.1×10^{-6}
$B_c^- \to D^- \chi_{c1}$	2.5×10^{-5}
$B_c^- \to D^- h_{c1}$	4.8×10^{-5}
$B_c^- \to \overline{D}^0 a_1^-$	8.6×10^{-5}
$B_c^- \to \overline{D}^0 b_1^-$	7.3×10^{-11}
$B_c^- \to \eta_c D_1^-$	5.9×10^{-5}
$B_c^- \to \eta_c \underline{D}_1^-$	6.8×10^{-5}
$\Delta b = 0, \Delta C = -1, \Delta S = 0$	
$B_c^- \to D^- \overline{D}_1^0$	4.0×10^{-7}
$B_c^- \to D^- \overline{\underline{D}}_1^0$	2.9×10^{-8}
$B_c^- \to \overline{D}^0 D_1^-$	6.2×10^{-8}
$B_c^- \to \overline{D}^0 \underline{D}_1^-$	3.3×10^{-8}
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	
$B_c^- \to \overline{D}^0 D_{s1}^-$	5.7×10^{-7}
$B_c^- \to \overline{D}^0 \underline{D}_{s1}^-$	6.7×10^{-7}
$B_c^- \to D_s^- \overline{D}_1^0$	1.5×10^{-5}
$B_c^- \to D_s^- \overline{\underline{D}}_1^0$	1.2×10^{-6}

We observe the following:

i) Naively, the $c \rightarrow d, s$ (charm changing and bottom conserving) decay channels are expected to be kinematically suppressed, however, the large value of the CKM matrix elements along with the large value of $c \rightarrow d, s$ transition form factors overcome this suppression. As a result, branching ratios of the charm changing mode are enhanced as compare to the bottom changing modes.

ii) The dominant decay for charm changing and bottom conserving are: $B(B_c^+ \rightarrow \pi^+ B_{s1}^0)$ = 2.9×10^{-2}, $B(B_c^+ \rightarrow \overline{K}^0 B_1^+)$ = 0.54×10^{-2}, $B(B_c^+ \rightarrow \pi^+ \underline{B}_{s1}^0)$ = 0.47×10^{-2}, $B(B_c^+ \rightarrow \pi^+ B_1^0)$ = 0.24×10^{-2} and $B\ (B_c^+ \rightarrow \overline{K}^0 \underline{B}_1^+)$ = 0.10×10^{-2}.

iii) For bottom changing transitions the dominating decays are: $B(B_c^- \rightarrow \eta_c a_1^-)$ = 0.31×10^{-2}, $B(B_c^- \rightarrow D_s^- h_{c1})$ = 0.15×10^{-2} and $B(B_c^- \rightarrow D_s^- \chi_{c1})$ = 0.10×10^{-2} . The rest of the decay modes remain highly suppressed partly due to the small values of the CKM matrix elements and the small values of the form factors.

iv) In contrast to the charm meson sector, the experimental data of B meson decays favor the constructive interference between color favored and color suppressed diagrams [25], giving $a_1 = 1.10 \pm 0.08$ and $a_2 = 0.20 \pm 0.02$. Taking $a_1 = 1.10$ and $a_2 = 0.20$ for the constructive interference case, we obtain larger value for $B(B_c^- \rightarrow D_s^- \chi_{c1})$ = 0.12×10^{-2} in comparison to 0.10×10^{-2} (for destructive interference).

6.3.5 COMPARISON WITH OTHER WORKS

We also give the comparison of the present results with other predictions, obtained only for $B_c^- \to \pi^- h_{c1} / \pi^- \chi_{c1}$, using the nonrelativistic constituent quark model [6], sum rules of QCD [8], relativistic constituent quark model [12] and the instantaneous nonrelativistic approach to the Bethe-Salpeter equation [17]. Prediction for the branching ratio of $B_c^- \to \pi^- h_{c1}$ is 1.60×10^{-2} [8] and 0.11×10^{-2} [12] match well with our result 0.06×10^{-2} which, however, smaller than the values 1.60×10^{-2} and 0.11×10^{-2} given by [8] and [12], respectively. The branching ratio for $B_c^- \to \pi^- \chi_{c1}$ predicted by [6], [8], [12] and [17] are 0.00014×10^{-2}, 0.0089×10^{-2}, 0.0068×10^{-2} and 0.0070×10^{-2}, respectively, which are smaller in comparison with our prediction 0.07×10^{-2}. The disagreement in the predictions may be attributed due to the different values of the form factors used in these approaches.

6.4 B_c DECAYS INTO PSEUDOSCALAR AND TENSOR MESONS

In analogy to the framework given for $B \to PT$ meson decays in chapter 4, here also, we extend the same formalism to investigate the B_c meson decaying to pseudoscalar and tensor mesons [20]. Using the effective weak Hamiltonian for bottom changing modes as given in (2.61) and (2.62) in chapter 2 and for bottom conserving and charm changing modes given in (6.1), (6.2) and (6.3), we obtain the decay amplitudes for CKM-favored and CKM-suppressed modes listed in Tables 6.9 and 6.10. It may be noted that $B_c \to P$ form factors do not appear in these decay amplitudes.

In earlier chapters, we have studied decays of bottom mesons involving *b*-quark as a decaying particle. However, B_c meson, being heavy, can also emit bottom mesons in the final state; therefore, we need spectroscopy of these mesons.

6.4.1 SPECTROSCOPY OF BOTTOM TENSOR MESONS

In addition to the tensor meson spectroscopy upto the charm level, which are already given in chapter 4, we also need the bottom tensor meson spectroscopy for the B_c decays, i.e, $B_2(5.747)$, and $B_{s0}(5.840)$ [1].

Table 6.9 Decay amplitudes of $B_c \to PT$ decays for bottom conserving and charm changing modes

Decays	Amplitudes
Δb =0, ΔC = -1, ΔS = -1	
$B_c^+ \to \pi^+ B_{s2}^0$	$a_1 f_\pi F^{B_c \to B_{s2}}(m_\pi^2) V_{cs} V_{ud}^*$
$B_c^+ \to \overline{K}^0 B_2^+$	$a_2 f_K F^{B_c \to B_2}(m_K^2) V_{cs} V_{ud}^*$
Δb =0, ΔC = -1, ΔS = 1	
$B_c^+ \to K^+ B_2^0$	$a_1 f_K F^{B_c \to B_2}(m_K^2) V_{cd} V_{us}^*$
$B_c^+ \to K^0 B_2^+$	$a_2 f_K F^{B_c \to B_2}(m_K^2) V_{cd} V_{us}^*$
Δb =0, ΔC = -1, ΔS = 0	
$B_c^+ \to K^+ B_{s2}^0$	$a_1 f_K F^{B_c \to B_{s2}}(m_K^2) V_{cs} V_{us}^*$
$B_c^+ \to \pi^+ B_2^0$	$a_1 f_\pi F^{B_c \to B_2}(m_\pi^2) V_{cd} V_{ud}^*$
$B_c^+ \to \pi^0 B_2^+$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_c \to B_2}(m_\pi^2) V_{cd} V_{ud}^*$
$B_c^+ \to \eta B_2^+$	$-\frac{1}{\sqrt{2}} a_2 f_\eta \cos\phi_P F^{B_c \to B_2}(m_\eta^2) V_{cs} V_{us}^*$

Table 6.10 Decay amplitudes of $B_c \to PT$ decays for bottom changing modes

Decays	Amplitudes
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	
$B_c^- \to \pi^- \chi_{c2}$	$a_1 f_\pi F^{B_c \to \chi_{c2}}(m_\pi^2) V_{cb} V_{ud}^*$
$B_c^- \to D^0 D_2^-$	$a_2 f_D F^{B_c \to D_2}(m_D^2) V_{cb} V_{ud}^*$
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	
$B_c^- \to \pi^0 D_{s2}^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_c \to D_{s2}}(m_\pi^2) V_{ub} V_{us}^*$
$B_c^- \to \eta D_{s2}^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_c \to D_{s2}}(m_\eta^2) V_{ub} V_{us}^*$
$B_c^- \to K^- \overline{D}_2^0$	$a_1 f_K F^{B_c \to D_2}(m_K^2) V_{ub} V_{us}^*$
$B_c^- \to \eta' D_{s2}^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_c \to D_{s2}}(m_{\eta'}^2) V_{ub} V_{us}^*$
$B_c^- \to D_s^- \chi_{c2}$	$a_1 f_{D_s} F^{B_c \to \chi_{c2}}(m_{D_s}^2) V_{cb} V_{cs}^*$
$B_c^- \to \eta_c D_{s2}^-$	$a_2 f_{\eta_c} F^{B_c \to D_{s2}}(m_{\eta_c}^2) V_{cb} V_{cs}^*$
$\Delta b = 0, \Delta C = 1, \Delta S = -1$	
$B_c^- \to K^- \chi_{c2}$	$a_1 f_K F^{B_c \to \chi_{c2}}(m_K^2) V_{cb} V_{us}^*$
$B_c^- \to D^0 D_{s2}^-$	$a_2 f_D F^{B_c \to D_{s2}}(m_D^2) V_{cb} V_{us}^*$
$\Delta b = 0, \Delta C = 0, \Delta S = 0$	
$B_c^- \to \pi^0 D_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_c \to D_2}(m_\pi^2) V_{ub} V_{ud}^*$
$B_c^- \to \pi^- \overline{D}_2^0$	$a_1 f_\pi F^{B_c \to D_2}(m_\pi^2) V_{ub} V_{ud}^*$
$B_c^- \to \eta D_2^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_c \to D_2}(m_\eta^2) V_{ub} V_{ud}^*$
$B_c^- \to \eta' D_2^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_c \to D_2}(m_{\eta'}^2) V_{ub} V_{ud}^*$
$B_c^- \to D^- \chi_{c2}$	$a_1 f_D F^{B_c \to \chi_{c2}}(m_D^2) V_{cb} V_{cd}^*$
$B_c^- \to \eta_c D_2^-$	$a_2 f_{\eta_c} F^{B_c \to D_2}(m_{\eta_c}^2) V_{cb} V_{cd}^*$
$\Delta b = 0, \Delta C = -1, \Delta S = 0$	
$B_c^- \to D^- \overline{D}_2^0$	$a_1 f_D F^{B_c \to D_2}(m_D^2) V_{ub} V_{cd}^*$
$B_c^- \to \overline{D}^0 D_2^-$	$a_2 f_D F^{B_c \to D_2}(m_D^2) V_{ub} V_{cd}^*$
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	
$B_c^- \to \overline{D}^0 D_{s2}^-$	$a_2 f_D F^{B_c \to D_{s2}}(m_D^2) V_{ub} V_{cs}^*$
$B_c^- \to D_s^- \overline{D}_2^0$	$a_1 f_{D_s} F^{B_c \to D_2}(m_{D_s}^2) V_{ub} V_{cs}^*$

6.4.2 FORM FACTORS INVOLVING $B_c \to T$ TRANSITION

Here also, we use the ISGW II model [22] to calculate the required form factors h, k, b_+ and b_- using the following expressions:

$$h = \frac{m_d}{2\sqrt{2\tilde{m}_{B_c}\beta_{B_c}}}\left(\frac{1}{m_q} - \frac{m_d\beta_{B_c}^2}{2\mu_-\tilde{m}_T\beta_{B_cT}^2}\right)F_5^{(h)},$$

$$k = \frac{m_d}{\sqrt{2\beta_{B_c}}}(1+\tilde{\omega})\ F_5^{(k)}, \tag{6.19}$$

$$b_+ + b_- = \frac{m_d^2}{4\sqrt{2}m_q m_b \tilde{m}_{B_c}\beta_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2}\left(1 - \frac{m_d}{2\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2}\right)F_5^{(b_+ + b_-)},$$

$$b_+ - b_- = -\frac{m_d}{\sqrt{2}m_b\tilde{m}_T\beta_{B_c}}\left(1 - \frac{m_d m_b}{2\mu_+\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2} + \frac{m_d}{4m_q}\frac{\beta_T^2}{\beta_{B_cT}^2}\left(1 - \frac{m_d}{2\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2}\right)\right)F_5^{(b_+ - b_-)},$$

where

$$F_5^{(h)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2},$$

$$F_5^{(k)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-1/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2},$$

$$F_5^{(b_+ + b_-)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-5/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2}, \tag{6.20}$$

$$F_5^{(b_+ - b_-)} = F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2},$$

The obtained the form factors describing $B_c \to T$ transitions are given in Table 6.11 at $q^2 = t_m$.

Table 6.11 Form factors of $B_c \to T$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	Transition	*h*	*k*	b_+	b_-
$\Delta b = 0, \Delta C = -1, \Delta S = -1$	$B_c \to B_{s2}$	0.119	3.632	-0.049	0.165
	$B_c \to B_2$	0.100	2.722	-0.034	0.148
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	$B_c \to D_2$	0.017	0.556	-0.008	0.011
	$B_c \to D_{s2}$	0.019	0.739	-0.011	0.014
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	$B_c \to \chi_{c2}$	0.023	1.411	-0.017	0.019

6.4.3 NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculations, we have taken the values of pseudoscalar mesons decay constants (given in GeV units) from Chapter 2. Finally, the branching ratios of $B_c \to PT$ meson decays in charm changing and in bottom changing decay modes are calculated. The measurement of these decays would provide an additional test of the quark models used to compute the hadronic matrix elements. The results are given in Tables 6.12 and 6.13 for the various possible CKM-favored and CKM-suppressed decay modes.

Table 6.12 Branching ratios of $B_c \rightarrow PT$ decays for bottom conserving and charm changing modes

Decays	Branching ratios	
	This work	CMM [18]
Δb =0, ΔC = -1, ΔS = -1		
$B_c^+ \rightarrow \pi^+ B_{s2}^0$	3.0×10^{-4}	2.0×10^{-4}
$B_c^+ \rightarrow \overline{K}^0 B_2^+$	1.0×10^{-5}	4.2×10^{-6}
Δb =0, ΔC = -1, ΔS = 1		
$B_c^+ \rightarrow K^+ B_2^0$	1.8×10^{-7}	1.9×10^{-7}
$B_c^+ \rightarrow K^0 B_2^+$	2.7×10^{-8}	1.2×10^{-8}
Δb =0, ΔC = -1, ΔS = 0		
$B_c^+ \rightarrow K^+ B_{s2}^0$	3.7×10^{-7}	5.0×10^{-7}
$B_c^+ \rightarrow \pi^+ B_2^0$	1.6×10^{-5}	1.2×10^{-5}
$B_c^+ \rightarrow \pi^0 B_2^+$	1.4×10^{-6}	3.9×10^{-7}
$B_c^+ \rightarrow \eta B_2^+$	2.0×10^{-7}	6.5×10^{-8}

Table 6.13 Branching ratios of $B_c \to PT$ decays for bottom changing modes

Decays	Branching ratios		
	This work	**Chang [17]**	**CMM [18]**
Δb =1, ΔC = 1, ΔS = 0			
$B_c^- \to \pi^- \chi_{c2}$	2.0×10^{-4}	2.5×10^{-4}	7.5×10^{-5}
$B_c^- \to D^0 D_2^-$	4.0×10^{-6}	-	6.3×10^{-8}
Δb =1, ΔC = 0, ΔS = -1			
$B_c^- \to \pi^0 D_{s2}^-$	6.8×10^{-10}	-	1.9×10^{-11}
$B_c^- \to \eta D_{s2}^-$	3.6×10^{-10}	-	2.5×10^{-12}
$B_c^- \to K^- \overline{D}_2^0$	1.6×10^{-8}	-	1.4×10^{-10}
$B_c^- \to \eta' D_{s2}^-$	3.1×10^{-10}	-	1.7×10^{-11}
$B_c^- \to D_s^- \chi_{c2}$	3.2×10^{-4}	4.5×10^{-4}	1.54×10^{-4}
$B_c^- \to \eta_c D_{s2}^-$	1.4×10^{-5}	-	1.4×10^{-6}
Δb =1, ΔC = 1, ΔS = -1			
$B_c^- \to K^- \chi_{c2}$	1.5×10^{-5}	-	5.5×10^{-6}
$B_c^- \to D^0 D_{s2}^-$	4.4×10^{-7}	-	1.9×10^{-8}
Δb =1, ΔC = 0, ΔS = 0			
$B_c^- \to \pi^0 D_2^-$	5.7×10^{-9}	-	5.8×10^{-11}
$B_c^- \to \pi^- \overline{D}_2^0$	2.1×10^{-7}	-	1.8×10^{-9}
$B_c^- \to \eta D_2^-$	3.0×10^{-9}	-	7.5×10^{-12}
$B_c^- \to \eta' D_2^-$	2.6×10^{-9}	-	5.4×10^{-11}
$B_c^- \to D^- \chi_{c2}$	1.2×10^{-5}	-	7.6×10^{-6}
$B_c^- \to \eta_c D_2^-$	4.5×10^{-7}	-	1.9×10^{-8}
Δb =1, ΔC = -1, ΔS = 0			
$B_c^- \to D^- \overline{D}_2^0$	3.0×10^{-8}	-	8.6×10^{-10}
$B_c^- \to \overline{D}^0 D_2^-$	1.6×10^{-9}	-	5.6×10^{-11}
Δb =1, ΔC = -1, ΔS = -1			
$B_c^- \to \overline{D}^0 D_{s2}^-$	6.7×10^{-8}	-	5.7×10^{-9}
$B_c^- \to D_s^- \overline{D}_2^0$	9.9×10^{-7}	-	2.2×10^{-8}

The observations are listed as follows:

i) Dominant decays for bottom changing decay modes are, $B(B_c^- \to D_s^- \chi_{c2})$ = 3.2 × 10^{-4} and $B(B_c^- \to \pi^- \chi_{c2})$ = 2.0×10^{-4}, which seems to be at the reach of future experiments. The next order dominant decays are $B(B_c^- \to K^- \chi_{c2})$ = 1.5×10^{-5} , $B(B_c^- \to \eta_c D_{s2}^-)$ = 1.4×10^{-5} and $B(B_c^- \to D^- \chi_{c2})$ = 1.2×10^{-5}.

ii) Branching ratio of decay, $B(B_c^- \to \pi^- \chi_{c2})$ = 2.0×10^{-4} , are comparable with the numerical value of the recent work [17].

iii) Branching ratio of dominant decay for charm changing decay mode is, $B(B_c^+ \to \pi^+ B_{s2}^0)$ = 3.0×10^{-4}, which proceeds via *b*-quark as an spectator, has a similar order of branching ratio than *c* quark spectator decays, i.e. $B_c^- \to D_s^- \chi_{c2} / \pi^- \chi_{c2}$, although it is suppressed by phase space but favored by the CKM factor.

iv) Among $\Delta b = 1, \Delta C = 1, \Delta S = 0$ mode, $B_c^- \to K^0 K_2^- / \pi^0 a_2^- / \pi^- a_2^0 / \pi^- f_2$ $/ \pi^- f_2' \; / \eta a_2^- / K^- K_2^0 / \eta' a_2^- / D^- D_2^0 / \eta_c a_2^-$ are forbidden in our analysis. However, these decays occur through the annihilation mechanism. Decay $B_c^- \to D^- D_2^0$ may also be generated through elastic final state interactions (FSIs).

v) In case of $\Delta b = 1, \Delta C = 0, \Delta S = -1$ decay mode, $B_c^- \to \bar{K}^0 D_2^- / D^- \bar{K}_2^0$ $/ \bar{D}^0 K_2^- / D_s^- a_2^0 / D_s^- f_2 \; / D_s^- f_2'$ are forbidden. However, these decays occur through the annihilation mechanism. Decay $B_c^- \to \bar{K}^0 D_2^-$ may also be generated through elastic final state interactions (FSIs).

6.4.4 COMPARISON WITH OTHER WORKS

For the sake of comparison, the results of other works [17, 18] are given in the Table V. C.H. Chang *et al.* [17] has calculated only the *c* spectator decay modes using generalized instantaneous approximation. In general, the present branching ratios of few decays are of the same order of magnitude as observed in [17, 18] and in other cases branching ratios are larger as compared to [18]. Ivanov *et al.* [12] studied exclusive nonleptonic and semileptonic decays of the B_c meson within a relativistic constituent quark model developed by them. In their recent work [12], they have calculated the nonleptonic decays with one of the final state being pure $c\bar{c}$. They predict $B(B_c^- \to \pi^- \chi_{c2})$ as 4.6×10^{-4}, which are large as compare to present results. Similarly, in another recent work [6] the same decays have been quoted with the branching ratio ($B(B_c^- \to \pi^- \chi_{c2}) =$ 2.2×10^{-4}) that is of the same order of magnitude as the compared to present work. It has also been observed that the largest numerical values of branching ratios $B_c \to PT$ are of the same order as those of some $B_c \to PP/PV/VV$ decay modes [6-18]. In *B* meson decays, the experimental data favors constructive interference, in contrast to the charm meson sector, between the color favored and color suppressed diagrams, thereby yielding $a_1 = 1.10 \pm 0.08$ and $a_2 = 0.20 \pm 0.02$. Our results remain unaffected from interference of a_1 (color favored) and a_2 (color suppressed).

6.5 B_c DECAYS INTO PSEUDOSCALAR AND SCALAR MESONS

In this section, we extend our analysis to study the two-body hadronic weak decays of B_c meson to pseudoscalar (P) meson and scalar (S) meson. Here also, we use the same methodology given in chapter 5 for $B \to PS$. In the factorization hypothesis, we calculate the decay amplitude for $B_c \to PS$ decays for bottom changing and bottom conserving-charm changing modes which are given in Tables 6.14 and 6.15.

6.5.1 SPECTROSCOPY OF BOTTOM SCALAR MESONS

B_c meson, being heavy, can also emit bottom mesons in the final state. In addition to the scalar meson spectroscopy upto the charm level, which are already given in chapter 5, we also need the bottom scalar meson spectroscopy for the B_c decays, i.e, $B_0(5.670)$, and $B_{s0}(5.767)$ [1].

Table 6.14 Decay amplitudes of $B_c \to PS$ decays for bottom conserving and charm changing modes

Decays	Amplitudes
a) CKM-favored mode	
$\Delta b = 0$, $\Delta C = -1$, $\Delta S = -1$	
$B_c^+ \to \pi^+ B_{s0}^0$	$a_1 f_\pi F^{B_c \to B_{s0}} (m_{B_c}^2 - m_{B_{s0}}^2) V_{cs} V_{ud}^*$
$B_c^+ \to \bar{K}^0 B_0^+$	$a_2 f_{K_0} F^{B_c \to B_0} (m_{B_c}^2 - m_{B_0}^2) V_{cs} V_{ud}^*$
$B_c^+ \to B^+ \bar{K}_0^0$	$a_2 f_{K_0} F^{B_c \to B} (m_{B_c}^2 - m_B^2) V_{cs} V_{ud}^*$
$B_c^+ \to B_s^0 a_0^+$	$a_1 f_{a_0} F^{B_c \to B_s} (m_{B_c}^2 - m_{B_s}^2) V_{cs} V_{ud}^*$
b) CKM-suppressed mode	
$\Delta b = 0$, $\Delta C = -1$, $\Delta S = 0$	
$B_c^+ \to K^+ B_{s0}^0$	$a_1 f_K F^{B_c \to B_{s0}} (m_{B_c}^2 - m_{B_{s0}}^2) V_{cd} V_{ud}^*$
$B_c^+ \to \pi^+ B_0^0$	$-a_1 f_\pi F^{B_c \to B_0} (m_{B_c}^2 - m_{B_0}^2) V_{cd} V_{ud}^*$
$B_c^+ \to \pi^0 B_0^+$	$a_2 f_\pi F^{B_c \to B_0} (m_{B_c}^2 - m_{B_0}^2) V_{cd} V_{ud}^*$

$B_c^+ \to \eta B_0^+$	$\frac{1}{\sqrt{2}} a_2 f_\eta F^{B_c \to B_0}(m_{B_c}^2 - m_{B_0}^2) V_{cd} V_{ud}^*$
$B_c^+ \to B^0 a_0^+$	$-a_1 f_{a_0} F^{B_c \to B}(m_{B_c}^2 - m_B^2) V_{cd} V_{ud}^*$
$B_c^+ \to B_s^0 K_0^+$	$a_1 f_{K_0} F^{B_c \to B_s}(m_{B_c}^2 - m_{B_s}^2) V_{cs} V_{us}^*$
Δb =0, ΔC = -1, ΔS = 1	
$B_c^+ \to K^+ B_0^0$	$-a_1 f_K F^{B_c \to B_0}(m_{B_c}^2 - m_{B_0}^2) V_{cd} V_{us}^*$
$B_c^+ \to K^0 B_0^+$	$-a_2 f_K F^{B_c \to B_0}(m_{B_c}^2 - m_{B_0}^2) V_{cd} V_{us}^*$
$B_c^+ \to B^+ K_0^0$	$-a_2 f_{K_0} F^{B_c \to B}(m_{B_c}^2 - m_B^2) V_{cd} V_{us}^*$
$B_c^+ \to B^0 K_0^+$	$-a_1 f_{K_0} F^{B_c \to B}(m_{B_c}^2 - m_B^2) V_{cd} V_{us}^*$

Table 6.15 Decay amplitudes of $B_c \to PS$ decays for bottom changing modes

Decays	Amplitudes
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	
$B_c^- \to D^- D_0^0$	$a_2 f_{D_0} F^{B_c \to D}(m_{B_c}^2 - m_D^2) V_{cb} V_{ud}^*$
$B_c^- \to \eta_c a_0^-$	$a_1 f_{a_0} F^{B_c \to \eta_c}(m_{B_c}^2 - m_{\eta_c}^2) V_{cb} V_{ud}^*$
$B_c^- \to \pi^- \chi_{c0}$	$a_1 f_\pi F^{B_c \to \chi_{c0}}(m_{B_c}^2 - m_{\chi_{c0}}^2) V_{cb} V_{ud}^*$
$B_c^- \to D^0 D_0^-$	$a_2 f_D F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{cb} V_{ud}^*$
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	
$B_c^- \to K^- \overline{D}_0^0$	$a_1 f_K F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{us}^*$
$B_c^- \to \overline{D}^0 K_0^-$	$a_1 f_K F^{B_c \to D}(m_{B_c}^2 - m_D^2) V_{ub} V_{us}^*$
$B_c^- \to \pi^0 D_{s0}^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2) V_{ub} V_{us}^*$
$B_c^- \to \eta D_{s0}^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2) V_{ub} V_{us}^*$
$B_c^- \to K^- \overline{D}_0^0$	$a_1 f_K F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{us}^*$
$B_c^- \to \eta' D_{s0}^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2) V_{ub} V_{us}^*$
$B_c^- \to D_s^- \chi_{c0}$	$a_1 f_{D_s} F^{B_c \to \chi_{c0}}(m_{B_c}^2 - m_{\chi_{c0}}^2) V_{cb} V_{cs}^*$
$B_c^- \to \eta_c D_{s0}^-$	$(a_2 f_{\eta_c} F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2) + a_1 f_{D_{s0}} F^{B_c \to \eta_c}(m_{B_c}^2 - m_{\eta_c}^2)) V_{cb} V_{cs}^*$

$\Delta b=1, \Delta C=1, \Delta S=-1$	
$B_c^- \to D_s^- D_0^0$	$a_2 f_{D_0} F^{B_c \to D_s}(m_{B_c}^2 - m_{D_s}^2) V_{cb} V_{us}^*$
$B_c^- \to \eta_c K_0^-$	$a_1 f_{K_0} F^{B_c \to \eta_c}(m_{B_c}^2 - m_{\eta_c}^2) V_{cb} V_{ud}^*$
$B_c^- \to K^- \chi_{c0}$	$a_1 f_K F^{B_c \to \chi_{c0}}(m_{B_c}^2 - m_{\chi_{c0}}^2) V_{cb} V_{us}^*$
$B_c^- \to D^0 D_{s0}^-$	$a_2 f_D F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2) V_{cb} V_{us}^*$
$\Delta b=1, \Delta C=0, \Delta S=0$	
$B_c^- \to \bar{D}^0 a_0^-$	$a_1 f_{a_0} F^{B_c \to D}(m_{B_c}^2 - m_D^2) V_{ub} V_{ud}^*$
$B_c^- \to \pi^0 D_0^-$	$\frac{1}{\sqrt{2}} a_2 f_\pi F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{ud}^*$
$B_c^- \to \pi^- \overline{D}_0^0$	$a_1 f_\pi F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{ud}^*$
$B_c^- \to \eta D_0^-$	$\frac{1}{\sqrt{2}} a_2 f_\eta \sin\phi_P F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{ud}^*$
$B_c^- \to \eta' D_0^-$	$\frac{1}{\sqrt{2}} a_2 f_{\eta'} \cos\phi_P F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) V_{ub} V_{ud}^*$
$B_c^- \to D^- \chi_{c0}$	$-a_1 f_D F^{B_c \to \chi_{c0}}(m_{B_c}^2 - m_{\chi_{c0}}^2) V_{cb} V_{cd}^*$
$B_c^- \to \eta_c D_0^-$	$-(a_2 f_{\eta_c} F^{B_c \to D_s}(m_{B_c}^2 - m_{D_s}^2) + a_1 f_{D_s} F^{B_c \to \eta_c}(m_{B_c}^2 - m_{\eta_c}^2)) V_{cb} V_{cd}^*$
Δb =1, ΔC = -1, ΔS = 0	
$B_c^- \to D^- \bar{D}_0^0$	$-(a_2 f_{D_0} F^{B_c \to D}(m_{B_c}^2 - m_D^2) + a_1 f_D F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2)) V_{ub} V_{cd}^*$
$B_c^- \to \overline{D}^0 D_0^-$	$-(a_2 f_D F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) + a_1 f_{D_0} F^{B_c \to D}(m_{B_c}^2 - m_D^2)) V_{ub} V_{cd}^*$
Δb =1, ΔC = -1, ΔS = -1	
$B_c^- \to \overline{D}^0 D_{s0}^-$	$(a_1 f_{D_{s0}} F^{B_c \to D}(m_{B_c}^2 - m_D^2) + a_2 f_D F^{B_c \to D_{s0}}(m_{B_c}^2 - m_{D_{s0}}^2)) V_{ub} V_{cs}^*$
$B_c^- \to D_s^- \bar{D}_0^0$	$(a_1 f_{D_s} F^{B_c \to D_0}(m_{B_c}^2 - m_{D_0}^2) + a_2 f_{D_0} F^{B_c \to D_s}(m_{B_c}^2 - m_{D_s}^2)) V_{ub} V_{cs}^*$

6.5.2 CALCULATION OF THE $B_c \to S$ TRANSITION FORM FACTORS IN ISGW II MODEL

The effective weak Hamiltonian generating the bottom meson decays involving $b \to c$ and $b \to u$ transitions is given in earlier chapter 3 in section 3.3 for CKM-favored and CKM-suppressed modes, respectively. Scalar meson spectroscopy has already been discussed in the section 5.2, chapter 5.

The required form factors for $B_c \to S$, u_+ and u_-, are calculated from the following expressions taken from ISGW II model [22]:

$$u_+ + u_- = -\sqrt{\frac{2}{3}} \frac{m_d}{\beta_{B_c}} F_5^{(u_+ + u_-)},$$
$$u_+ - u_- = \sqrt{\frac{2}{3}} \frac{m_d \tilde{m}_{B_c}}{\tilde{m}_S \beta_{B_c}} F_5^{(u_+ - u_-)}, \tag{6.21}$$

where

$$F_5^{(u_+ + u_-)} = F_5 \left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-1/2} \left(\frac{\bar{m}_S}{\tilde{m}_S}\right)^{1/2},$$
$$F_5^{(u_+ - u_-)} = F_5 \left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{1/2} \left(\frac{\bar{m}_S}{\tilde{m}_S}\right)^{-1/2}, \tag{6.22}$$

We obtain the form factors describing $B_c \to S$ transitions which are given in Table 6.16 at $q^2 = t_m$.

Table 6.16 Form factors for $B_c \to S$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	Transition	u_+	u_-
$\Delta b = 0$, $\Delta C = -1$, $\Delta S = -1$	$B_c \to B_{s0}$	-0.227	4.012
	$B_c \to B_0$	-0.066	1.018
$\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$	$B_c \to D_0$	0.124	-0.271
	$B_c \to D_{s0}$	0.177	-0.389
$\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$	$B_c \to \chi_{c0}$	0.437	-1.462

For $B_c \to P$ transition, the obtained form factors are given in Table 6.17.

Tables 6.17 Form factors for $B_c \to P$ transition at $q^2 = t_m$ in the ISGW II quark model

Modes	Transition	f_+	f_-
$\Delta b = 0$, ΔC = -1, ΔS = -1	$B_c \to B_s$	0.926	-0.374
	$B_c \to B$	1.103	-0.652
Δb =1, ΔC = 0, ΔS = -1	$B_c \to D$	2.139	-2.003
	$B_c^- \to D_s$	1.543	-1.356
Δb =1, ΔC = 1, ΔS = 0	$B_c \to \eta_c(c\bar{c})$	1.193	-0.716

6.5.3 NUMERICAL RESULTS AND DISCUSSIONS

Using the same formalism given in chapter 5, we obtain the branching ratios for the bottom charm meson emitting pseudoscalar and scalar mesons as given in Tables 6.18 and 6.19. Here also, we use the pseudoscalar (0^-) decay constants given in (3.26) and decay constants of the scalar meson [26-30] decay constants given in chapter 5 for numerical calculations.

Table 6.18 Branching ratios for $B_c \to PS$ decays for bottom conserving and charm changing modes

Decays	Branching ratios
Δb =0, ΔC = -1, ΔS = -1	
$B_c^+ \to \pi^+ B_{s0}^0$	3.9×10^{-4}
$B_c^+ \to \bar{K}^0 B_0^+$	2.9×10^{-5}
Δb =0, ΔC = -1, ΔS = 0	
$B_c^+ \to K^+ B_{s0}^0$	5.1×10^{-6}
$B_c^+ \to \pi^+ B_0^0$	1.9×10^{-5}
$B_c^+ \to \pi^0 B_0^+$	1.6×10^{-6}
$B_c^+ \to \eta B_0^+$	1.0×10^{-6}
Δb =0, ΔC = -1, ΔS = 1	
$B_c^+ \to K^+ B_0^0$	5.0×10^{-7}
$B_c^+ \to K^0 B_0^+$	7.9×10^{-8}

Table 6.19 Branching ratios of $B_c \to PS$ decays for Bottom changing modes

Decays	Branching ratios
$\Delta b = 1, \Delta C = 1, \Delta S = 0$	
$B_c^- \to D^- D_0^0$	2.6×10^{-5}
$B_c^- \to \eta_c a_0^-$	1.1×10^{-7}
$B_c^- \to \pi^- \chi_{c0}$	9.7×10^{-5}
$B_c^- \to D^0 D_0^-$	1.1×10^{-6}
$\Delta b = 1, \Delta C = 0, \Delta S = -1$	
$B_c^- \to \bar{D}^0 K_0^-$	7.6×10^{-9}
$B_c^- \to \pi^0 D_{s0}^-$	1.9×10^{-10}
$B_c^- \to \eta D_{s0}^-$	1.2×10^{-10}
$B_c^- \to K^- \bar{D}_0^0$	4.2×10^{-9}
$B_c^- \to \eta' D_{s0}^-$	7.2×10^{-11}
$B_c^- \to D_s^- \chi_{c0}$	1.9×10^{-4}
$B_c^- \to \eta_c D_{s0}^-$	4.4×10^{-4} (6.7×10^{-4})
$\Delta b = 1, \Delta C = 1, \Delta S = -1$	
$B_c^- \to D_s^- D_0^0$	8.6×10^{-7}
$B_c^- \to \eta_c K_0^-$	2.1×10^{-6}
$B_c^- \to K^- \chi_{c0}$	7.3×10^{-6}
$B_c^- \to D^0 D_{s0}^-$	1.4×10^{-7}
$\Delta b = 1, \Delta C = 0, \Delta S = 0$	
$B_c^- \to \bar{D}^0 a_0^-$	4.1×10^{-10}
$B_c^- \to \pi^0 D_0^-$	1.5×10^{-9}
$B_c^- \to \pi^- \overline{D}_0^0$	5.3×10^{-8}
$B_c^- \to \eta D_0^-$	9.1×10^{-10}
$B_c^- \to \eta' D_0^-$	5.5×10^{-8}
$B_c^- \to D^- \chi_{c0}$	6.6×10^{-6}
$B_c^- \to \eta_c D_0^-$	3.7×10^{-5} (4.7×10^{-5})
$\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$	
$B_c^- \to D^- \bar{D}_0^0$	1.1×10^{-10} (3.8×10^{-8})
$B_c^- \to \overline{D}^0 D_0^-$	1.6×10^{-7} (2.0×10^{-7})
$\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$	
$B_c^- \to \overline{D}^0 D_{s0}^-$	2.0×10^{-6} (2.8×10^{-6})
$B_c^- \to D_s^- \bar{D}_0^0$	5.7×10^{-8} (7.9×10^{-7})

Values given in parentheses are for constructive interference.

We observe the following:

i) Dominant decay for charm changing and bottom conserving is $B(B_c^+ \to \pi^+ B_{s0}^0)$ = 3.9×10^{-4}. $B(B_c^+ \to \bar{K}^0 B_0^+)$ = 2.9×10^{-5} and $B(B_c^+ \to \pi^+ B_0^0)$ = 1.9×10^{-5} are next order dominant decays.

ii) For bottom changing transitions the dominating decays are $B(B_c^- \to \eta_c D_{s0}^-)$ = 6.7×10^{-4} and $B(B_c^- \to D_s^- \chi_{c0})$ = 1.9×10^{-4}. The next order branching ratios are $B(B_c^- \to \pi^- \chi_{c0})$ = 9.7×10^{-5}, $B(B_c^- \to \eta_c D_0^-)$ = 4.7×10^{-5} and $B(B_c^- \to D^- D_0^0)$ = 2.6×10^{-5}. The rest of the decay modes remain highly suppressed partly due to the small values of the CKM matrix elements, the small values of the form factors and vanishingly small decay constants of the scalar mesons.

iii) Among $\Delta b = 1,\ \Delta C = 1, \Delta S = 0$, $\Delta b = 1,\ \Delta C = 0, \Delta S = -1$, $\Delta b = 1,\ \Delta C = 1,$ $\Delta S = -1$ and $\Delta b = 1,\ \Delta C = 0, \Delta S = 0$ modes, several other decays are permitted through the annihilation mechanism, but are forbidden in our analysis. Few decays may also be generated through elastic final state interactions (FSI).

iv) In sharp contrast to the charm meson decays, the experimental data show constructive interference for B meson decays involving both the color favored and color suppressed diagrams [25]. It may be noted that except few decays, all the other decays of B_c meson involve either the color favored or the color suppressed diagram. Therefore, their branching ratios remain unaffected due to the sign of a_2. However, the numerical branching ratios correspond to these decays get enhanced as, $B(B_c^- \to \eta_c D_{s0}^-)$ = 6.7×10^{-4},

$B(B_c^- \to \eta_c D_0^-)$ = 4.7×10^{-5}, $B(B_c^- \to \overline{D}^0 D_{s0}^-)$ = 2.8×10^{-6}, $B(B_c^- \to D_s^- \overline{D}_0^0)$ = 7.9×10^{-7}, $B(B_c^- \to \overline{D}^0 D_0^-)$ = 2.0×10^{-7} and $B(B_c^- \to D^- \overline{D}_0^0)$ = 3.8×10^{-8}. Because of the constructive interference the branching ratio increases by a factor of 2 to 20, as shown in Table 6.19.

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CHAPTER 7

SUMMARY AND CONCLUSIONS

In this thesis, we have investigated the two-body weak hadronic decays of heavy flavor mesons. It has been found experimentally that two-body decays dominate the decay spectrum. Theoretical focus has also, so far, been on the *s*-wave meson (i.e. Pseudoscalar and Vector mesons) emitting decays. However, charm and bottom mesons, being heavy, can also emit *p*-wave mesons, i.e. axial-vector (*A*), tensor (*T*) and scalar (*S*) mesons. Naively, the *p*-wave mesons emitting decays of the hadrons are expected to be suppressed kinematically due to the large mass of these meson resonances. However, there now exist reasonable amount of experimental data on branching ratios of *p*-wave emitting decays of charm and bottom mesons which requires theoretical understanding. In our research work, we have studied such weak decays of bottom mesons (B^-, $\overline{B}^0$ and $\overline{B}_s^0$), which are the bound state of *b* quark a light anti quark and of a uniquely observed bottom-charm (B_c) meson made up of both the heavy quarks.

In chapter 2, we lay down the physical and mathematical preliminaries which have been applied for the study of weak decays of mesons emitting the *s*-wave mesons. To start with we present the hadron spectroscopy upto the bottom level and classification of the weak decays into leptonic, semileptonic and nonleptonic decays. In general, these weak decays proceed through exchange of virtual *W*-boson between the charged weak (*V*-*A*) currents. We have discussed the semileptonic decays of the bottom mesons, which proceed via the so-called spectator quark diagrams. Their decay amplitudes can easily be expressed in terms of decay constants of meson or the form factors appearing in the matrix elements of weak hadronic current between the initial and the final states. This forms the basis for the '*factorization approach*' later applied to the weak nonleptonic decays. These form factors are usually calculated from the phenomenological approaches. We have used $B \to P$ form factors based on the BSW quark model framework which match well with the experimental information. In the following chapters, we have extended the factorization approach to study *p*-wave meson emitting decays, i.e. $B \to PA / PT / PS$ decays.

In chapter 3, we have studied hadronic weak decays of bottom mesons emitting pseudoscalar and an axial-vector mesons. After describing the axial-vector meson spectroscopy of the two kinds, i.e. $A\,(J^{PC} = 1^{++})$ and $A'\,(J^{PC} = 1^{+-})$, we have obtained the decay amplitudes in terms of appropriate meson decay constants and meson to meson transition form factors for the color-favored and color-suppressed diagrams. We have obtained the $B \to A / A'$ transition form factors using the ISGW II model which provides a more realistic description. Consequently, we have predicted the branching ratios of $B \to PA$ decays involving $b \to c$ and $b \to u$ transitions in the CKM-favored and CKM-suppressed modes. Experimentally, branching ratios of eleven decays have been measured and upper limits are also available for five other decays. Branching ratios predicted in our model reasonably match well with the available experimental data. We found that the decays involving $b \to c$ transition can have branching ratios of the order of 10^{-3} to 10^{-8}, whereas the decays occurring through $b \to u$ transition acquire branching ratios of the order of 10^{-5} to 10^{-11}. We have shown that the predicted branching ratios are comparable to that of the *s*-wave meson emitting weak decays. Specifically, the dominant decay modes

$B^- \to D^0 D_1^-$ and $\overline{B}^0 \to D^+ a_1^-$ have branching ratios 1.6×10^{-2} and 1.1×10^{-2} respectively. We have also compared our predictions with other theoretical results.

In chapter 4, we have studied hadronic weak decays of bottom mesons emitting pseudoscalar and tensor mesons. Because of the tracelessness of the polarization tensor of spin 2 meson and the auxiliary condition the tensor meson does not materialize from the weak currents. Therefore, either color-favored or color-suppressed diagrams contribute to these decays. We employ ISGW II model to determine the $B \to T$ transition form factors appearing in the decay matrix element of weak currents involving $b \to c$ and $b \to u$ transitions. Consequently, we have obtained the decay amplitudes and predicted the branching ratios of $B \to PT$ decays in CKM-favored and CKM-suppressed modes. Experimentally, there exist branching ratios of only six decay modes, while the upper limits are available for five other decays. We found that the decays involving $b \to c$ transition have branching ratios of the order of 10^{-4} to 10^{-8} and decays involving $b \to u$ transition have branching ratios of the order of 10^{-5} to 10^{-11}. Dominant decay modes are $B^- \to D_s^- D_2^0$, $B^- \to \pi^- D_2^0$, $B^- \to D^0 a_2^-$, $\overline{B}^0 \to D_s^- D_2^+$, $\overline{B}^0 \to \pi^- D_2^+$, $\overline{B}_s^0 \to D_s^- D_{s2}^-$, $\overline{B}_s^0 \to \pi^- D_{s2}^+$ and $\overline{B}_s^0 \to D^0 K_2^0$. Here also, we have compared the predicted branching ratios with the experimental measurements and also with other theoretical calculations. We have noticed that the calculated branching ratios $B(B^- \to \pi^- D_2^0) = 6.7\times10^{-4}$ ($(7.8 \pm 1.4)\times10^{-4}$ *Expt*) and $B(B^- \to \pi^- f_2) = 7.1\times10^{-6}$ ($(8.2\pm2.5)\times10^{-6}$ *Expt*) are in good agreement with the experimental value, whereas the remaining decays seems to acquire contribution from *W*-annihilation diagram to bridge the gap between theoretical and experimental value and the experimental upper limits honored the predicted branching ratios..

In chapter 5, we have studied hadronic weak decays of bottom mesons emitting pseudoscalar and scalar mesons involving $b \to c$ and $b \to u$ transitions. To determine the form factors appearing in the decay matrix element of weak currents of $B \to S$ transition, we use the ISGW II model. Consequently, we obtain the decay amplitude and calculated the branching ratios in the CKM-favored and CKM-suppressed modes. Though, for these decays both kinds of the spectator diagrams can contribute, usually one of these is suppressed due to the small values of the scalar meson decay constants. Therefore, these decays are not seriously affected by the nature of interference of the color-favored and color-suppressed processes. On experimental side, branching ratios of only three decay modes are measured and upper limit is available for one other decay. The main conclusion is that the dominant decays are $B^- \to \pi^- D_0^0$, $B^- \to D^0 a_0^-$, $B^- \to D_s^- D_0^0$, $B^- \to D^- D_0^0$, $\bar{B}^0 \to D_s^- D_0^+$, $\bar{B}^0 \to D^- D_0^+$, $\bar{B}^0 \to \pi^- D_0^+$, $\bar{B}^0 \to D^0 a_0^0$, $\bar{B}_s^0 \to \pi^- D_{s0}^+$, $\bar{B}_s^0 \to D^0 K_0^0$, $\bar{B}_s^0 \to D_s^- D_{s0}^+$ and $\bar{B}_s^0 \to D^- D_{s0}^+$. We hope these decays would be the best candidates from experimental point of view. Here also, we have compared our predicted branching ratios with other theoretical calculations.

In chapter 6, we have studied hadronic weak decays of uniquely observed bottom-charm (B_c) meson, which is the only quark-antiquark, bound system composed of the heavy quarks (b, c) with different flavors. Investigation of the B_c meson decay rates is therefore of special interest compared to the symmetric heavy quarkonium $(\bar{b}b, \bar{c}c)$ states. Heavy quarkonium states decay through quark-antiquark annihilation processes, while for B_c meson *W*-annihilation diagram is relatively suppressed in comparison to the *W*-emission from either *b* quark, or *c* quark. The decay processes of the B_c meson can thus be broadly divided into two classes: bottom changing and bottom conserving (but charm changing) decay modes. Already, there exists an extensive literature for the semileptonic and nonleptonic decays of B_c emitting *s*-wave mesons, pseudoscalar (*P*) and vector (*V*) mesons. However, relatively less work has been done on its kinematically allowed *p*-wave meson emitting weak decays. Therefore, we have extended our work to predict B_c decays emitting axial-vector (*A*), tensor (*T*) or scalar (*S*) mesons in the CKM-favored channels and CKM-suppressed channels. Since, there is no experimental information available on these decay modes, we have compared our predictions with other theoretical results. One naively expects the bottom conserving modes to be kinematically suppressed in comparison to the bottom changing modes. However, we have shown that the branching ratios of the bottom conserving are relatively larger than that of the bottom changing mode due to the large difference in the corresponding values of the CKM matrix elements. Particularly, we have found that $B_c^+ \to \pi^+ B_{s1}^0$, $B_c^+ \to \pi^+ B_{s2}^0$ and $B_c^+ \to \pi^+ B_{s0}^0$ are dominant. These observations would help the experimentalists to identify the *p*-wave meson emitting decays of the heaviest bottom meson.

The continued operation and upgrade of the high energy accelerators and the facilities at various labs all over the world ensure that the knowledge and database of High Energy Physics will continue to expand. We hope that the results obtained in the present thesis would act as guide to these experimental searches and help in deciphering the relative strengths of various competing weak decay mechanisms in the heavy flavor sector.

APPENDIX A

Table 1. Properties of Leptons

Leptons	**Mass (MeV)**	**Spin**	**Lepton numbers**		
			L_e	L_μ	L_τ
electron(e^-)	0.51	½	1	0	0
neutrino(ν_e)	17×10^{-6}	½	1	0	0
muon(μ)	105.66	½	0	1	0
mu-neutrino(ν_μ)	< 0.27	½	0	1	0
tau(τ)	1784.1	½	0	0	1
tau-neutrino(ν_τ)	< 35	½	0	0	1

Table 2. Properties of Quarks

Quark	**Mass (GeV)**	**Baryon number**	**Spin (ħ/2)**	**Charge (Q)**	**Isospin I, I_3**	**Strange (S)**	**Charm (C)**	**Bottom (B)**	**Top (T)**
u	0.34	1/3	½	2/3	1/2,1/2	0	0	0	0
d	0.34	1/3	½	-1/3	1/2,-1/2	0	0	0	0
s	0.51	1/3	½	-1/3	0,0	-1	0	0	0
c	1.6	1/3	½	2/3	0,0	0	1	0	0
b	5.0	1/3	½	-1/3	0,0	0	0	-1	0
t	174	1/3	½	2/3	0,0	0	0	0	1