PHOTON PAIRING IN QUANTUM ELECTRODYNAMICS

T. Inagaki

Department of Physics, Faculty of Science and Technology,
Keio University, Yokohama 223, Japan

Abstract

In this talk, we discuss photon pairing phenomenon in the strong coupling phase of massive Quantum Electrodynamics (QED) through the analysis of the Cooper equation. Using the well known low energy effective Lagrangian for photon, it is shown that when the coupling constant exceeds some finite value, the normal vacuum of QED becomes unstable with respect to the formation of the photon pair. It is also found that the pairing instability is *enhanced* and the critical coupling has a tendency to become smaller in the presence of the weak constant electric field. This may give a theoretical basis for the anomalous GSI e^+e^- events.

1. Introduction

A number of observations based on the computer simulations using the lattice gauge theory and on the Schwinger-Dyson equation suggest that there exists the strong coupling phase in Quantum Electrodynamics (QED) when the coupling constant $\alpha = (e^2/4\pi)$ becomes larger than some critical value.[1)2)3)] Much effort has been paid to investigate the characteristics of this new phase of QED, because it is hoped that it may give a theoretical basis for explaining the GSI peak[4)] and may resolve the Flavour-Changing Neutral Current (FCNC) problem in the technicolour theory.[1)] The purpose of this talk is to give some discussions on the photon pairing phenomenon in the strong coupling phase of massive QED with the help of the Cooper equation that is well known in the theory of the superconductivity. The talk is based on the paper listed ref.5).

2. The low energy effective Lagrangian and the Hamiltonian for photon

Let us begin with the Euler Heisenberg effective Lagrangian for photon $\mathcal{L}_{eff}$ [6)] ;

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + a(F^{\mu\nu}F_{\mu\nu})^2 + b(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \quad , \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ and $a = \frac{4}{7}b = (\alpha^2/90m^4)$. Here $\varepsilon^{\mu\nu\rho\sigma}$ is the completely anti-symmetric tensor with $\varepsilon_{0123} = +1$, A^μ the renormalized photon field, m the electron mass and $\alpha = (e^2/4\pi)$ the renormalized fine structure constant. Here and in the following the indices μ, ν, ... run from 0 to 3 and the indices i, j, ... run from 1 to 3 .

The effective Lagrangian in the presence of the external field A_c^μ can be obtained the deviding the photon field A^μ in (1) into two parts,

$$A^\mu \rightarrow A_c^\mu + A^\mu \ . \tag{2}$$

(We use the same notation A^μ as in (1), since there may be no confusions.) Here we define the A_c^μ by $[\delta S/\delta A_\mu]_{A=A_c} = 0$, where S is the action given by $S = \int d^4x \mathcal{L}_{eff}$. Then we get $\mathcal{L}_{eff}$ as the sum of three terms, i.e. $\mathcal{L}_{eff} = \mathcal{L}_c + \mathcal{L} + \mathcal{L}_G$, where

$$\begin{aligned}
\mathcal{L}_c &= -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + a(G^{\mu\nu}G_{\mu\nu})^2 + b(G^{\mu\nu}\tilde{G}_{\mu\nu})^2 \ , \\
\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + a(F^{\mu\nu}F_{\mu\nu})^2 + b(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \ , \\
\mathcal{L}_G &= 2a(G^{\mu\nu}G_{\mu\nu})(F^{\rho\sigma}F_{\rho\sigma}) + 2b(G^{\mu\nu}\tilde{G}_{\mu\nu})(F^{\rho\sigma}\tilde{F}_{\rho\sigma}) \\
&\quad +4a(G^{\mu\nu}F_{\mu\nu})^2 + 4b(G^{\mu\nu}\tilde{F}_{\mu\nu})^2 \\
&\quad +2a(G^{\mu\nu}F_{\mu\nu})(F^{\rho\sigma}F_{\rho\sigma}) + 2b(G^{\mu\nu}\tilde{F}_{\mu\nu})(F^{\rho\sigma}\tilde{F}_{\rho\sigma}) \ ,
\end{aligned} \tag{3}$$

and $G^{\mu\nu} = \partial^\mu A_c^\nu - \partial^\nu A_c^\mu$, $\tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$. We can neglect $\mathcal{L}_c$ since it has no effect on our problem.

From now on, we choose the Feynman gauge for convenience by adding $\mathcal{L}_{GF} = -\frac{1}{2}(\partial_\mu A^\mu)^2$ to $\mathcal{L}_{eff}$ but our arguments are gauge invariant, of cource.

The canonical momentum Π^μ defined by $\Pi^\mu = (\partial \mathcal{L}'_{eff}/\partial \dot{A}_\mu)$ ($\mathcal{L}'_{eff} \equiv \mathcal{L}_{eff} + \mathcal{L}_{GF}$) is given by $\Pi^0 = -(\partial_\mu A^\mu)$, $\Pi^k = \Pi^{(1)k} + \Pi^{(2)k}$, where

$$\begin{aligned}
\Pi^{(1)k} &= -F^{0k} + 8a(F^{\mu\nu}F_{\mu\nu})F^{0k} + 8b(F^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{F}^{0k} \ , \\
\Pi^{(2)k} &= 8a(G^{\mu\nu}G_{\mu\nu})F^{0k} + 8b(G^{\mu\nu}\tilde{G}_{\mu\nu})\tilde{F}^{0k} + 16a(G^{\mu\nu}F_{\mu\nu})G^{0k} \\
&\quad +16b(G^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{G}^{0k} + 16a(G^{\mu\nu}F_{\mu\nu})F^{0k} + 16b(G^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{F}^{0k} \\
&\quad +8a(F^{\mu\nu}F_{\mu\nu})G^{0k} + 8b(F^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{G}^{0k} \ .
\end{aligned} \tag{4}$$

Therefore, the Hamiltonian density $\mathcal{H}_{eff} = \Pi_\mu \dot{A}^\mu - \mathcal{L}'_{eff} = \mathcal{H}_0 + \mathcal{H}_I + \mathcal{H}_G$ is obtained up to $O(\alpha^2)$ as follows;

$$\begin{aligned}
\mathcal{H}_{eff} &= \mathcal{H}_0 + \mathcal{H}_I + \mathcal{H}_G \ , \\
\mathcal{H}_0 &= -\frac{1}{2}\Pi_\mu\Pi^\mu + \frac{1}{4}(F^{ij}F_{ij}) - \Pi_0(\partial_k A^k) + \Pi_k(\partial^k A_0) \ ,
\end{aligned}$$

$$\mathcal{H}_I = -a(2\Pi^k\Pi_k - F^{ij}F_{ij})^2 - b(2\varepsilon_{0ijk}\Pi^i F^{jk})^2 \quad ,$$
$$\begin{aligned}\mathcal{H}_G = & -8a(G^{\mu\nu}G_{\mu\nu})(2\Pi^k\Pi_k - F^{ij}F_{ij}) - 8b(G^{\mu\nu}\tilde{G}_{\mu\nu})(2\varepsilon_{0ijk}\Pi^i F^{jk})^2 \\ & -4a(G^{ij}F_{ij} - 2G^{0j}F_{0j})^2 - 4b[\varepsilon_{0ijk}(G^{0i}F^{jk} - \Pi^i G^{jk})]^2 \\ & -4a(2\Pi^k\Pi_k - F^{ij}F_{ij})(G^{ij}F_{ij} - 2G^{0j}F_{0j}) \\ & -4b(2\varepsilon_{0ijk}\Pi^i F^{jk})[\varepsilon_{0ijk}(G^{0i}F^{jk} - \Pi^i G^{jk})] \quad . \end{aligned} \tag{5}$$

3. The Cooper equation

Now if we introduce $a^{(\Lambda)\dagger}(\mathrm{k})$ as the creation operator for photon with momentum k and helicity Λ , the Cooper state we use in the following is given as

$$|C\rangle = \sum_{\Lambda=R,L} \int \frac{d^3k}{(2\pi)^3 2k_0} f(\mathrm{k})\, a^{(\Lambda)\dagger}(\mathrm{k})\, a^{(\Lambda)\dagger}(-\mathrm{k})\, |0\rangle [2\delta^{(3)}(0)]^{-1/2} \quad , \tag{6}$$

where $k_0 = |\mathrm{k}|$, $\delta^{(3)}(0) = (2\pi)^{-3} \int d^3x = (2\pi)^{-3}V$ (V is the volume of this system.), $|0\rangle$ is the normal vacuum of QED and $f(\mathrm{k})$ the weight function which is determined by the variational principle.

Then, the expectation value of the Hamiltonian $H(= \int d^3x \mathcal{H}_{eff})$ under the constant electric field only $(G_{0i} = E_i = const, G_{ij} = 0)$, can be written as the sum of three terms;

$$\begin{aligned} \langle C| : H : |C\rangle = & \langle C| : H_0 : |C\rangle + \langle C| : H_I : |C\rangle + \langle C| : H_G : |C\rangle \quad , \\ \langle C| : H_0 : |C\rangle = & 2\int d^3k\, k_0\, |f(\mathrm{k})|^2 \quad , \\ \langle C| : H_I : |C\rangle = & -\frac{8}{\pi^3} \int d^3k d^3k'\, k_0 k'_0 f^*(\mathrm{k}) f(\mathrm{k'}) \{a[4 + (1+\cos\theta)^2] - b(1+\cos\theta)^2\} \quad , \\ \langle C| : H_G : |C\rangle = & \int d^3k\, k_0^{-1} \chi(\mathrm{k}) |f(\mathrm{k})|^2 \quad , \end{aligned} \tag{7}$$

where ": ... :" stands for the normal ordering and $H_0 = \int d^3x \mathcal{H}_0$, $H_I = \int d^3x \mathcal{H}_I$, $H_G = \int d^3x \mathcal{H}_G$ and θ is the angle between k and k$'$. If we denote the angle between k and the electric field vector E as φ, the function $\chi(\mathrm{k})$ in (7) can be described as follows,

$$\chi(\mathrm{k}) = -16(a+b)k_0^2 |\mathrm{E}|^2 \sin^2\varphi \quad . \tag{8}$$

In order to minimize the expectation value of the normal ordered Hamiltonian under the normalization condition $\langle C|C\rangle = \int d^3k |f(\mathrm{k})|^2 = 1$, we take the variation of $\langle C| : H : |C\rangle - \mathcal{E}\langle C|C\rangle$ with respect to $f^*(\mathrm{k})$. Thus we get the Cooper equation;

$$\begin{aligned} (2k_0 + \frac{\chi(\mathrm{k})}{k_0} - \mathcal{E}) f(\mathrm{k}) = & \frac{8}{\pi^3} k_0 \int d^3k d^3k' k'_0 f(\mathrm{k'}) \\ & \times \{a[4 + (1+\cos\theta)^2] - b(1+\cos\theta)^2\} \quad , \end{aligned} \tag{9}$$

where $\mathcal{E}$ is the energy of our Cooper state.

In the following discussion, we analyze this Cooper equation in two cases separately; without or with the external electric field.

(i) Photon pairing in the absence of the external field.

In this case, we denote the solution of the Cooper equation as $f_0(\mathbf{k})$ and the energy of the Cooper state as $\mathcal{E}_0$. Then we obtain the following integral equation. We assume here that the solution of this Cooper state depends only on k_0 , because the system has the rotational symmetry. Therefore, the Cooper equation takes the form,

$$(2k_0 - \mathcal{E}_0)f_0(k_0) = \frac{2^7}{3\pi^2}(4a-b)k_0 \int_0^\Lambda dk'_0\, k'^3_0 f_0(k'_0) \ , \tag{10}$$

where Λ is the ultraviolet cut off. Since $b=(7/4)a>0$, photon-photon interaction in massive QED is attractive in the low energy region.

The solution of (10) has obviously the form $f_0(k_0) \propto k_0(2k_0-\mathcal{E}_0)^{-1}$ and the energy eigenvalue of the Cooper state is determined by the following equation,

$$1 = \frac{g}{m^4}\int_0^\Lambda \frac{k_0^4 dk_0}{2k_0 - \mathcal{E}_0} \ , \tag{11}$$

where $g=(16/15)\pi^2\alpha^2$. Equation (11) shows that there exists negative energy eigenvalue state $(\mathcal{E}<0)$ when

$$g > g_0 \equiv 2m^4 \left[\int_0^\Lambda dk_0 k_0^3\right]^{-1} = 8(m/\Lambda)^4 \ . \tag{12}$$

This means that when $\alpha > \alpha_c = \sqrt{15/2}\,\pi\,(m/\Lambda)^2$, the normal vacuum of massive QED becomes unstable with respect to the formation of the photon pair (Cooper instability) and the new condensed vacuum is realized after the condensation of these pairs.

The above results agree qualitatively with those obtained in ref.3) using the B-S equation.

(ii) Photon pairing under the electric field.

By assuming that $\mathbf{E}$ is small, we solve (9) perturbatively in $\mathbf{E}$. We expand the energy $\mathcal{E}$ of the Cooper state and the weight function $f(\mathbf{k})$ with respect to $\mathbf{E}$,

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_0 + \mathcal{E}_1|\mathbf{E}|^2 + \dots \ , \\ f(\mathbf{k}) &= f_0(k_0) + f_1(k_0)(\mathbf{k}\cdot\mathbf{E})^2 + f_2(k_0)|\mathbf{E}|^2 + \dots \ . \end{aligned} \tag{13}$$

$\mathcal{E}_0$ and $f_0(k_0)$ satisfy (10), of cource. The energy shift $\mathcal{E}_1|\mathbf{E}|^2$ caused by the external electric field is calculated easily and is given by,

$$\mathcal{E}_1|\mathbf{E}|^2 = -\frac{32}{3}(a+b)\left[\int_0^\Lambda \frac{k_0^5 dk_0}{(2k_0-\mathcal{E}_0)^2}\right]\left[\int_0^\Lambda \frac{k_0^4 dk_0}{(2k_0-\mathcal{E}_0)^2}\right]^{-1} < 0 \ . \tag{14}$$

Equation (14) means that if we consider up to $O(|\mathbf{E}|^2)$, the energy eigenvalue of the Cooper state becomes smaller, therefore the critical coupling α_c' for $\mathbf{E} \neq 0$ has the tendency to become smaller than α_c for $\mathbf{E} = 0$. (Fig.1)

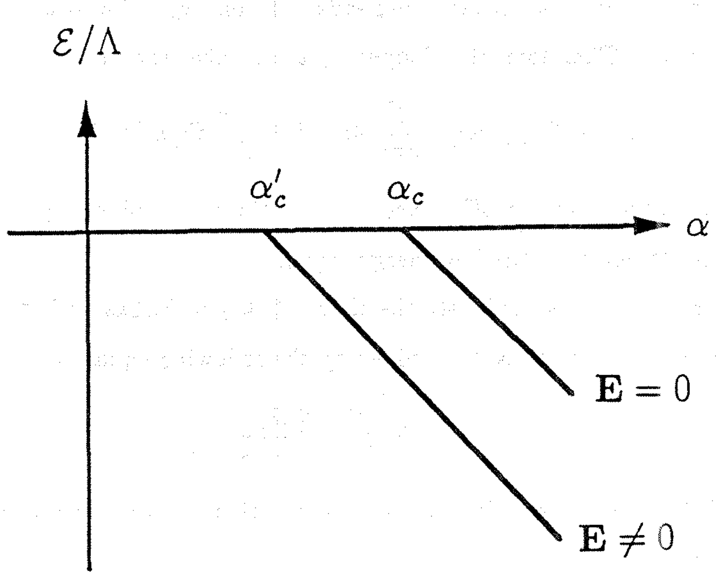

Fig.1 The relation between the energy $\mathcal{E}/\Lambda$ and the fine structure constant α in the absence of the electric field ($\mathbf{E} = 0$) and under the presence of the electric field ($\mathbf{E} = 0$).

4. Comments

In this talk, we have seen the Cooper instability of photon can occur in the massive QED. Therefore, the next task, which is of great interest, is to consider the characteristics of the stable condensed vacuum just as in the theory of the superconductivity. Since many observable phenomena in this strong coupling phase would be dependent upon the condenced nature of the vacuum, the formulation has to be established just like the Bogoliubov transformation in the superconductor theory, to discuss the characteristics of this phase.

Especially in connection with the chiral symmetry breaking, it is also very interesting whether the photon pairing phenomenon occurs in *massless* QED because this theory is very unstable in the infrared region. If this phenomenon occurred, what would the critical coupling be?

Although our results bring us a hope for explaining the anomalous GSI event, the photon pairing under the strong electromagnetic field should be studied of cource before applying our conclusions to the real experiment.

References

1) K. Yamawaki; "Rescurrecting Technicolour - Scale invariant Technicolor and a Technidilaton", in the Proceeding of the "International Workshop on the New Trends in Strong Coupling Gauge Theories", Nagoya Aug. 1988, ed. by M. Bando, T. Muta and K.Yamawaki, World Scientific Pub., Singapole, 1988. and the references therein.

2) P. Fomin, V. Gusynin, V. Miransky and Yu. Sitnenko; Riv. Nuovo.Ciment. 6, (1983), 1. and the references therein.

3) R. Fukuda; Phys. Rev. Lett. 63, (1988), 482.

4) Y. J. Ng and Y. Kikuchi; Phys. Rev. D36,(1987), 2880. D. G. Caldi and A. Chodos; Phys. Rev. D36, (1987), 2876. C. W. Wong; Phys. Rev. D37, (1988), 3206. R. D. Peccei, J. Solà and C. Watterrich; Phys. Rev. D37, (1988), 2492. L. S. Celenza, A. Pantrizis, C. M. Shakin and Hui-Wen Wang; Nucl. Phys. A489, (1988), 751.

5) T. Inagaki, M. Komachiya and R. Fukuda; Keio Univ Preprint (1989). to appear in Mod. Phys. Lett. A.

6) H. Euler; Ann. d. Phys. 26, (1936), 398. W. Heisenberg and H. Euler; Zeit. für Phys. 98, (1936), 714. J. Schwinger; Phys. Rev. 82, (1951), 664. J. Schwinger; "Particles, Sources and Fields", Addison-Wesley, Reading, MA, 1973. R. Karplus and M. Neumann; Phys. Rev. 80, (1950), 380. J. M. Jauch and F. Rohlich; "The Theory of Photons and Electrons; the relativistic quantum field theory of charged particles with spin one-half", Springer Verlag, New York, 1976.