Chiral and flavor oscillations in lepton-antineutrino spin correlations

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Abstract. We study the evolution of quantum correlations in a lepton-antineutrino pair, as produced in weak interactions (e.g. pion decay). Assuming an inital state entangled in the spins of the two particles, we show that both chiral and flavor (neutrino) oscillations affect spin correlations. Such corrections are relevant in the non-relativistic regime. In the second part we focused our attention on the weak process $n + \nu_e \to p + e$ in which the results found in the previous sections could be observed.

1. Introduction

A remarkable property of (charged) weak interactions is their parity violating character: this is mathematically accounted for by the presence of chiral projectors in the interaction term, which allow only left handed fermion field components to interact with W bosons [1, 2]. On the other hand, weak charged currents are also characterized by the fact that only fields with definite flavors enter in the vertex. As a consequence of these facts, a fermion generated in a charged weak interaction process will have definite flavor and chirality.

States with definite chirality and finite mass undergo chiral oscillations. In fact, the Dirac equation, describing the temporal evolution of a massive fermion, includes a term which couples left and right-handed components of the bispinor, which yields chirality oscillations [3, 4], characterized by the ratio between the momentum and the mass. Chiral oscillations have been studied recently [5–7] in connection with their possible relevance for relic neutrinos of the CνB background [8] and more in general in the non-relativistic regime. In particular, the effects of chiral oscillations on the spin-spin correlations of a lepton-antineutrino pair has been investigated in Ref. [9].

In this paper we extend the analysis of Ref. [9], by taking into account also flavor oscillations for the (anti)neutrino involved in the considered process. Thus, the initial entanglement encoded only in the spins is redistributed, during time evolution, due to the chiral and flavor oscillations which occur with their characteristic time and energy scales. The result is a hyperentangled state of the system at a later time. The spin correlations display a dependence on such oscillations, thus offering a possible way for detecting them.

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2. Chiral oscillations

In relativistic quantum mechanics the free evolution of a fermion is described by a wave function Ψ whose evolution is given by the Dirac equation (in natural units $\hbar = c = 1$)

$$(i\hat{\gamma}^{\mu}\partial_{\mu} - m)\Psi(x) = 0 \tag{1}$$

where the $\hat{\gamma}^{\mu}$ matrices satisfy the anticommutation relations $\{\hat{\gamma}^{\mu}, \hat{\gamma}^{\nu}\} = 2\eta^{\mu\nu}$ with $\eta^{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$. The solution $\Psi(x)$ is a 4−component spinor called Dirac bispinor which has two intrinsic degrees of freedom: spin and chirality. From a group theory point of view this is due to the fact that the Dirac bispinors belong to irreducible representations of the complete Lorentz group [11].

The chiral matrix is defined by

$$\hat{\gamma}_5 \equiv i\hat{\gamma}_0\hat{\gamma}_1\hat{\gamma}_2\gamma_3 \tag{2}$$

and its expectation value $\langle\Psi|\hat{\gamma^5}|\Psi\rangle$, the chirality of a state, is not a conserved quantity due to the presence of the mass term in the Dirac equation Eq.(1), which can be recast in the form

$$\hat{H}_D\,|\Psi\rangle = (\hat{\mathbf{p}} \cdot \hat{\boldsymbol{\alpha}} + m\hat{\beta})\,|\Psi\rangle = i\partial_t\,|\Psi\rangle \tag{3}$$

with $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2, \hat{p}_3) \equiv -i(\partial_{x1}, \partial_{x2}, \partial_{x3})$, $\hat{\boldsymbol{\alpha}} = (\hat{\alpha}_1, \hat{\alpha}_1, \hat{\alpha}_3) \equiv (\hat{\gamma}_0\hat{\gamma}_1, \hat{\gamma}_0\hat{\gamma}_2, \hat{\gamma}_0\gamma_3)$ and $\hat{\beta} = \hat{\gamma}^0$, with $\{\hat{\alpha}_i, \hat{\alpha}_j\} = 2\delta_{ij}I_4$ and $\{\hat{\alpha}_i, \hat{\beta}\} = 0$. In terms of the $\hat{\alpha}_i$, the chiral operator becomes

$$\hat{\gamma}_5 = -i\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3, \tag{4}$$

which commutes only with the first term in $\hat{H}_D$. The mass term does not commute with $\hat{\gamma}^5$:

$$[\hat{\gamma}^5, \hat{H}_D] = -2im\hat{\beta}\hat{\gamma}^5 \tag{5}$$

Thus, a state with a definite chirality whose dynamics is described by the Dirac equation will undergo chiral oscillations. We can further appreciate this fact by considering chiral representation of the Dirac matrices in which [10]

$$\hat{\alpha}_i = \begin{bmatrix} \hat{\sigma}_i & 0 \\ 0 & \hat{\sigma}_i \end{bmatrix}, \hat{\beta} = \begin{bmatrix} 0 & \hat{I}_2 \\ \hat{I}_2 & 0 \end{bmatrix}, \hat{\gamma}_5 = \begin{bmatrix} \hat{I}_2, & 0 \\ 0 & -\hat{I}_2 \end{bmatrix}. \tag{6}$$

In this representation the chiral operator is diagonal with two double degenerate eigenvalues ± 1. Any Dirac bispinor can be decomposed into left and right-handed components

$$|\Psi\rangle = \begin{bmatrix} |\xi_R\rangle \\ |\xi_L\rangle \end{bmatrix} \tag{7}$$

such that the Dirac equation can be rewritten as a the set of coupled equations

$$\hat{\mathbf{p}} \cdot \hat{\boldsymbol{\sigma}}\,|\xi_R\rangle + m\,|\xi_L\rangle = i\partial_t\,|\xi_R\rangle\,, \qquad -\hat{\mathbf{p}} \cdot \hat{\boldsymbol{\sigma}}\,|\xi_L\rangle + m\,|\xi_R\rangle = i\partial_t\,|\xi_L\rangle \tag{8}$$

Note the mass term mixes the left and chiral part which allows the chiral oscillations for a state initially with a definite chirality.

In order to describe the dynamics of a given initial bispinor, we adopt the standard procedure and divide a solution of the Dirac equation into positive and negative energy solutions given by

$$|\Psi_+\rangle = e^{-i(Et - \mathbf{p}\cdot\mathbf{x})}\,|u_s(\mathbf{p}, m)\rangle\,, \qquad |\Psi_-\rangle = e^{+i(Et - \mathbf{p}\cdot\mathbf{x})}\,|v_s(\mathbf{p}, m)\rangle \tag{9}$$

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where s represent the spin polarization of the particle, E and $\mathbf{p}$ are the energy and the momentum with $E^2 = p^2 + m^2$ with $p = |\mathbf{p}|$. Following Ref. [5], the solutions of the Dirac equation appearing in (9) are given in terms of the bispinors

$$|u_s(\mathbf{p},m)\rangle = N \begin{bmatrix} f_+ |\eta_s(\mathbf{p})\rangle \\ f_- |\eta_s(\mathbf{p})\rangle \end{bmatrix}, \qquad |v_s(\mathbf{p},m)\rangle = N \begin{bmatrix} f_+ |\eta_s(\mathbf{p})\rangle \\ -f_- |\eta_s(\mathbf{p})\rangle \end{bmatrix} \tag{10}$$

with

$$f_\pm = f_\pm(\mathbf{p},m) = 1 \pm \frac{\hat{\mathbf{p}}\cdot\hat{\boldsymbol{\sigma}}}{E+m}, \qquad N = N(\mathbf{p},m) = \sqrt{\frac{E+m}{4E}} \tag{11}$$

In the above equations $|\eta_s(\mathbf{p})\rangle$ is a two-component spinor encoding the polarization of the particle.

Following the scheme adopted by [7] we take $|\eta_s(\mathbf{p})\rangle$ as eigenstates of the helicity operator

$$\hat{h} = \frac{\hat{\mathbf{p}}\cdot\hat{\boldsymbol{\Sigma}}}{p} \tag{12}$$

where

$$\hat{\Sigma}_i = \begin{bmatrix} \hat{\sigma}_i & 0 \\ 0 & \hat{\sigma}_i \end{bmatrix} \tag{13}$$

are the generalized Pauli matrices. Since $[\hat{h}, \hat{H}_D] = 0$, helicity is conserved during the free evolution. Taking also the momentum along the z axis, $\mathbf{p} = p\hat{e}_z$, the bispinors u_s and v_s become

$$|u_\pm(\mathbf{p},m)\rangle = N \begin{bmatrix} f_\pm |\pm\rangle \\ f_\mp |\pm\rangle \end{bmatrix}, \qquad |v_\pm(\mathbf{p},m)\rangle = N \begin{bmatrix} f_\pm |\pm\rangle \\ -f_\mp |\pm\rangle \end{bmatrix} \tag{14}$$

where $|+\rangle$ ($|-\rangle$) is the state $|\uparrow\rangle$ ($|\downarrow\rangle$) with positive (negative) projection of the spin along z (eigenstates of s_z).

In [7] it is shown that starting with an initial state at $t = 0$ with both negative chirality and negative helicity

$$|\Psi(0)\rangle = [0,0,0,1]^T \tag{15}$$

using the orthonormality relations

$$\langle u_s(\mathbf{p},m)|v_{s'}(-\mathbf{p},m)\rangle = 0, \qquad \langle u_s(\mathbf{p},m)|u_{s'}(\mathbf{p},m)\rangle = \langle v_s(\mathbf{p},m)|v_{s'}(\mathbf{p},m)\rangle = \delta_{s,s'} \tag{16}$$

we can write

$$|\Psi(0)\rangle = \sum_{s=+,-} \Big(\langle\Psi(0)|u_s(\mathbf{p},m)\rangle\, |u_s(\mathbf{p},m)\rangle + \langle\Psi(0)|v_s(-\mathbf{p},m)\rangle\, |v_s(-\mathbf{p},m)\rangle \Big) \tag{17}$$

and hence the time evolution is given by

$$|\Psi(t)\rangle = e^{-i\hat{H}_D t}|\Psi(0)\rangle = N\big(f_+ e^{-iEt}|u_-(\mathbf{p},m)\rangle - f_- e^{iEt}|v_-(-\mathbf{p},m)\rangle\big). \tag{18}$$

Since helicity is a conserved quantity, the probability that the particle does not change its chirality is given by

$$P = |\langle\Psi(0)|\Psi(t)\rangle|^2 = 1 - \frac{m^2}{E^2}\sin^2(Et) \tag{19}$$

which goes to one in the limit $\frac{p}{m} \to \infty$.

3. Lepton-antineutrino entanglement and chiral oscillations

Weak interactions produce particles with a well definite chirality (left-chiral for leptons and right chiral for antileptons). If the momenta of the emitted particles are comparable with their mass, according to the time evolution of Dirac bispinors, chiral oscillations occur, as we discussed briefly in the last Section. In [9] the possibility to observe indirectly the effects of chiral oscillations has been investigated for a lepton-antineutrino pair produced by a pion decay.

The pion can decay in

$$\pi^- \to \mu^- + \bar{\nu}_\mu, \qquad \pi^- \to e^- + \bar{\nu}_e \tag{20}$$

with probability $99,9877\%$ and $0,0123\%$, respectively.

In the rest frame of the pion the two particles are emitted along opposite directions and with a well defined chirality that is right chiral for the antineutrino and left chiral for the lepton [2]. Furthermore, conservation of angular momentum imples that the total spin of the emitted particles has to vanish. Assuming that the lepton, with mass m_l, has a momentum $-p\hat{e}_z$, and the antineutrino, with mass $m_{\bar{\nu}}$, has momentum $p\hat{e}_z$, we can take the initial state in the form

$$|\Phi\rangle = \frac{|v_\uparrow(p, m_{\bar{\nu}})\rangle \otimes |u_\downarrow(-p, m_l)\rangle - |v_\downarrow(p, m_{\bar{\nu}})\rangle \otimes |u_\uparrow(-p, m_l)\rangle}{\sqrt{2}} \tag{21}$$

which is an entangled state in the spin. Since the two particles emitted in the decay have a well definite chirality, we have to project the state (21) in their respective chirality eigenstates and to normalize it, that is

$$|\Psi(0)\rangle = \frac{\hat{\Pi}_R^{\bar{\nu}} \otimes \hat{\Pi}_L^l |\Phi\rangle}{\langle\Phi|\hat{\Pi}_R^{\bar{\nu}} \otimes \hat{\Pi}_L^l|\Phi\rangle} \tag{22}$$

where the chirality projectors are given by

$$\hat{\Pi}_{R/L}^\alpha = \frac{I^\alpha \pm \hat{\gamma}_5^\alpha}{2} \tag{23}$$

with $\alpha = l, \bar{\nu}$.

After some algebraic manipulations, the state (22) can be written in the form [9]

$$|\Psi(0)\rangle = A(p, m_l, m_{\bar{\nu}})\, |\bar{\nu}_\uparrow(0)\rangle \otimes |l_\downarrow(0)\rangle - B(p, m_l, m_{\bar{\nu}})\, |\bar{\nu}_\downarrow(0)\rangle \otimes |l_\uparrow(0)\rangle \tag{24}$$

with

$$|\bar{\nu}_{\uparrow(\downarrow)}(0)\rangle = \begin{bmatrix} \uparrow (\downarrow) \\ 0 \end{bmatrix}, \qquad |l_{\uparrow(\downarrow)}(0)\rangle = \begin{bmatrix} 0 \\ \uparrow (\downarrow) \end{bmatrix} \tag{25}$$

and

$$A(p, m_l, m_{\bar{\nu}}) = N_l N_{\bar{\nu}} f_+^{\bar{\nu}} f_-^l \left[\frac{1}{2} - \frac{p^2}{2E_l E_{\bar{\nu}}}\right]^{-\frac{1}{2}}, \qquad B(p, m_l, m_{\bar{\nu}}) = N_l N_{\bar{\nu}} f_-^{\bar{\nu}} f_+^l \left[\frac{1}{2} - \frac{p^2}{2E_l E_{\bar{\nu}}}\right]^{-\frac{1}{2}} \tag{26}$$

with the short notation $f_{+(-)}^\alpha \equiv f_{+(-)}(p, m_\alpha)$ for $\alpha = l, \bar{\nu}$.

Note that in the relativistic limit, for the antineutrino $p \gg m_{\bar{\nu}}$ and we have $f_-^{\bar{\nu}} \to 0$: hence $A \gg B$ and only the first term in (24) contributes to the state. Time evolution of the superposition is obtained by writing explicitly $|\Psi(t)\rangle$ in terms of the bispinors (10), which are the plane wave solutions of the Dirac equation. This gives

$$|\Psi(t)\rangle = A(p, m_l, m_{\bar{\nu}})\, |\bar{\nu}_\uparrow(t)\rangle \otimes |l_\downarrow(t)\rangle - B(p, m_l, m_{\bar{\nu}})\, |\bar{\nu}_\downarrow(t)\rangle \otimes |l_\uparrow(t)\rangle \tag{27}$$

with

$$|\bar{\nu}_{\uparrow}(t)\rangle = N_{\bar{\nu}}[e^{-iE_{\bar{\nu}}t} f_{+}^{\bar{\nu}} |u_{\uparrow}(p, m_{\bar{\nu}})\rangle + e^{iE_{\bar{\nu}}t} f_{-}^{\bar{\nu}} |v_{\uparrow}(-p, m_{\bar{\nu}})\rangle] \quad (28)$$

$$|\bar{\nu}_{\downarrow}(t)\rangle = N_{\bar{\nu}}[e^{-iE_{\bar{\nu}}t} f_{-}^{\bar{\nu}} |u_{\downarrow}(p, m_{\bar{\nu}})\rangle + e^{iE_{\bar{\nu}}t} f_{+}^{\bar{\nu}} |v_{\downarrow}(-p, m_{\bar{\nu}})\rangle] \quad (29)$$

$$|l_{\uparrow}(t)\rangle = N_l[e^{-iE_l t} f_{+}^{l} |u_{\uparrow}(-p, m_l)\rangle - e^{iE_l t} f_{-}^{l} |v_{\uparrow}(p, m_l)\rangle] \quad (30)$$

$$|l_{\downarrow}(t)\rangle = N_l[e^{-iE_l t} f_{-}^{l} |u_{\downarrow}(-p, m_l)\rangle - e^{iE_l t} f_{+}^{l} |v_{\downarrow}(p, m_l)\rangle] \quad (31)$$

In [9] it has been shown that the quantum entanglement initially encoded only between the spins is redistributed by chiral oscillations to other partitions of the system. Such effects become relevant when the mass of the neutrino is comparable with its momentum. To compute the temporal evolution of the spin-spin entanglement we first obtain the spin density matrix by tracing out the chiralities

$$\rho_{S_{\bar{\nu}},S_l}(t) = \mathrm{Tr}_{C_{\bar{\nu}},C_l}\big[\rho(t)\big] = \mathrm{Tr}_{C_{\bar{\nu}},C_l}\Big[\, |\Psi(t)\rangle \langle\Psi(t)|\, \Big] \quad (32)$$

The negativity for the spin part is [9]

$$\mathcal{N}\Big[\rho_{S_{\bar{\nu}},S_l}(t)\Big] = ||\rho^{T}_{S_{\bar{\nu}},S_l}(t)|| - 1 = \mathcal{N}\Big[\rho_{S_{\bar{\nu}},S_l}(0)\Big]\Gamma(t) \quad (33)$$

with

$$\Gamma(t) = \left[1 - \frac{p^2}{m_{\bar{\nu}}^2}(\langle\gamma_5^{\bar{\nu}}\rangle(t) - 1)^2\right]^{\frac{1}{2}} \left[1 - \frac{p^2}{m_l^2}(\langle\gamma_5^{l}\rangle(t) + 1)^2\right]^{\frac{1}{2}} \equiv \Gamma_{\bar{\nu}}\Gamma_l \quad (34)$$

and

$$\langle\gamma_5^{l}\rangle(t) = -1 + \frac{m_l^2}{E_l^2}\Big[1 - \cos(2E_l t)\Big] \quad (35)$$

At $t = 0$, the negativity between the spins reads

$$\mathcal{N}[\rho_{S_l,S_{\bar{\nu}}}(0)] = 2|AB| = 2|A(p, m_l, m_{\bar{\nu}})B(p, m_l, m_{\bar{\nu}})| \quad (36)$$

Also it has been shown in [9] that the trace of the squared density operator is

$$\mathrm{Tr}[\rho^2_{S_{\bar{\nu}},S_l}(t)] = 1 - \frac{\mathcal{N}\big[\rho_{S_{\bar{\nu}},S_l}(0)\big]}{2}(1 - \Gamma(t)^2). \quad (37)$$

4. Flavor and chirality in a lepton-antineutrino entangled pair

The results in [9] revised in the previous section do not include flavor oscillations. Flavor is another intrinsic degree of freedom for the neutrino. Charged weak interactions produce neutrinos with a well defined flavor, which is not conserved during free evolution; therefore we expect that, as for the chirality, flavor oscillations will also redistribute the initial entanglement between spins. Let us consider only two flavor oscillations (electronic neutrino e and muonic neutrino μ) and suppose to start with an electronic neutrino. The free time evolution is given by [5]:

$$|\bar{\nu}_{\uparrow}(t)\rangle = (\cos^2\theta\, |\bar{\nu}_{m1\uparrow}(t)\rangle + \sin^2\theta\, |\bar{\nu}_{m2\uparrow}(t)\rangle) \otimes |\bar{\nu}_e\rangle + \sin\theta\cos\theta(|\bar{\nu}_{m1\uparrow}(t)\rangle - |\bar{\nu}_{m2\uparrow}(t)\rangle) \otimes |\bar{\nu}_{\mu}\rangle \quad (38)$$

$$|\bar{\nu}_{\downarrow}(t)\rangle = (\cos^2\theta\, |\bar{\nu}_{m1\downarrow}(t)\rangle + \sin^2\theta\, |\bar{\nu}_{m2\downarrow}(t)\rangle) \otimes |\bar{\nu}_e\rangle + \sin\theta\cos\theta(|\bar{\nu}_{m1\downarrow}(t)\rangle - |\bar{\nu}_{m2\downarrow}(t)\rangle) \otimes |\bar{\nu}_{\mu}\rangle \quad (39)$$

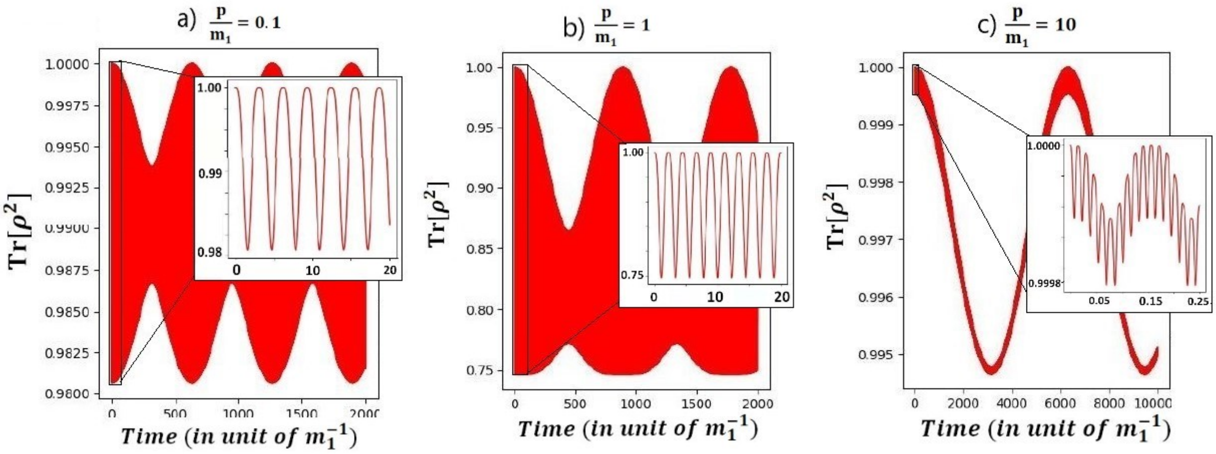


Figure 1: Tr$[\rho^2_{S_l,S_\nu}]$ as a function of time for $\frac{p}{m_{\bar\nu_1}} = 0.1; 1; 10$. We used the values $\frac{\Delta m^2}{m_{\bar\nu_1}} = 0,001$ with $\Delta m^2 = m^2_{\bar\nu_2} - m^2_{\bar\nu_1}$ and $\frac{m_l}{m_{\bar\nu_1}} = 100$. Also we take $\sin^2\theta = 0,306$ in a) for flavor mixing.

Defining the left-(right-)chiral part by $|C_\alpha^{+1(-1)}\rangle$, renaming the coefficients (11) as $f^\alpha_{+(-)} \equiv f^\alpha_{+1(-1)}$ and the bispinors (10) as $|u_\alpha\rangle \equiv |+1_\alpha\rangle$, $|v_\alpha\rangle \equiv |-1_\alpha\rangle$ for both spin up and spin down and both flavors $\alpha = \bar\nu_l, l$, Eqs. (28)-(31) become

$$|\bar\nu_\uparrow(t)\rangle = N_{\bar\nu} \sum_{d=\pm1} e^{-idE_{\bar\nu}t} f_d^{\bar\nu} |d_{\uparrow\bar\nu}(dp)\rangle\,, \qquad |\bar\nu_\downarrow(t)\rangle = N_{\bar\nu} \sum_{d=\pm1} e^{-idE_{\bar\nu}t} f_{-d}^{\bar\nu} |d_{\downarrow\bar\nu}(dp)\rangle \quad (40)$$

$$|l_\uparrow(t)\rangle = N_l \sum_{d=\pm1} e^{-idE_l t} f_d^l |d_{\uparrow l}(-dp)\rangle\,, \qquad |l_\downarrow(t)\rangle = N_l \sum_{d=\pm1} e^{-idE_l t} f_{-d}^l |d_{\downarrow l}(-dp)\rangle\,. \quad (41)$$

By taking into account flavor oscillations[1], the state (27) acquires the form

$$|\Psi(t)\rangle = \sum_{c,c'} \Big(\gamma_1(c,c')\, |C^c_{\bar\nu}\rangle \otimes |\uparrow_{\bar\nu}\rangle \otimes |\bar\nu_e\rangle \otimes |C_l^{c'}\rangle \otimes |\downarrow_l\rangle + \gamma_2(c,c')\, |C^c_{\bar\nu}\rangle \otimes |\uparrow_{\bar\nu}\rangle \otimes |\bar\nu_\mu\rangle \otimes |C_l^{c'}\rangle \otimes |\downarrow_l\rangle$$
$$- \gamma_3(c,c')\, |C^c_{\bar\nu}\rangle \otimes |\downarrow_{\bar\nu}\rangle \otimes |\bar\nu_e\rangle \otimes |C_l^{c'}\rangle \otimes |\uparrow_l\rangle - \gamma_4(c,c')\, |C^c_{\bar\nu}\rangle \otimes |\downarrow_{\bar\nu}\rangle \otimes |\bar\nu_\mu\rangle \otimes |C_l^{c'}\rangle \otimes |\uparrow_l\rangle \Big) \quad (42)$$

with $c, c' = \pm 1$. The coefficients are given by

$$\gamma_1(c,c') = A(\omega_1^{(1)}(c,c')\cos^2\theta + \omega_2^{(1)}(c,c')\sin^2\theta), \qquad \gamma_2(c,c') = A\omega^{(2)}(c,c')\sin\theta\cos\theta,$$

$$\gamma_3(c,c') = B(\omega_1^{(3)}(c,c')\cos^2\theta + \omega_2^{(3)}(c,c')\sin^2\theta), \qquad \gamma_4(c,c') = B\omega^{(4)}(c,c')\sin\theta\cos\theta \quad (43)$$

[1] Note that the flavor oscillations here considered take into account also the non-standard oscillation term also found in QFT [12, 13].

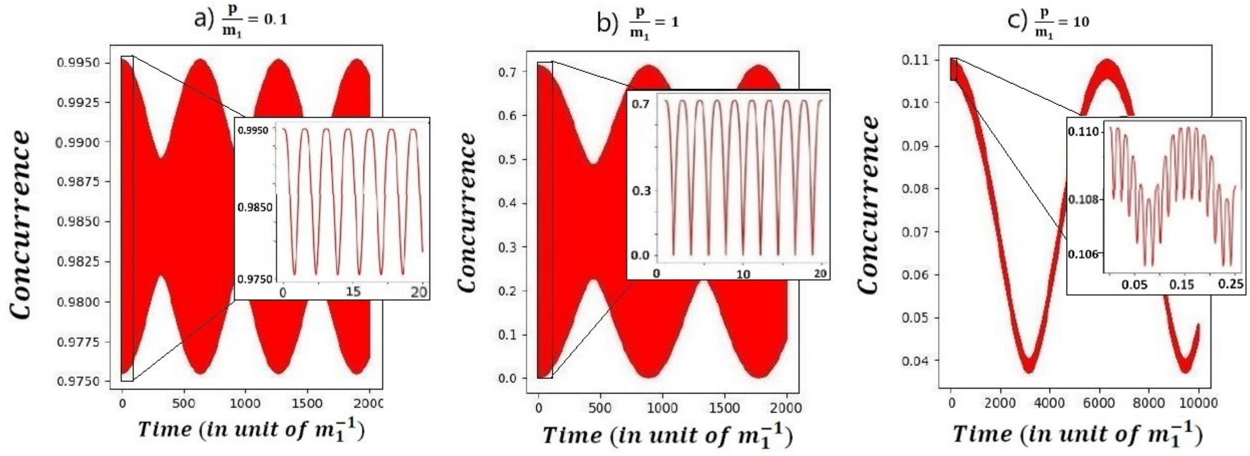


Figure 2: Concurrence $\mathcal{C}$ as a function of time for $\frac{p}{m_{\bar{\nu}_1}} = 0.1; 1; 10$. Values of the parameters are the same of Fig.1.

where A and B are given by the (26) and

$$
\begin{aligned}
\omega_1^{(1)}(c,c') &= \left(\sum_d N_{\bar{\nu}_1}^2 c^{\frac{d-1}{2}} f_d^{\bar{\nu}_1} f_{dc}^{\bar{\nu}_1} e^{-idE_{\bar{\nu}_1}t}\right)\left(\sum_{d'} d' N_l^2 c'^{\frac{d'-1}{2}} f_{-d'}^l f_{d'c'}^l e^{-id'E_l t}\right)\\
\omega_2^{(1)}(c,c') &= \left(\sum_d N_{\bar{\nu}_2}^2 c^{\frac{d-1}{2}} f_d^{\bar{\nu}_2} f_{dc}^{\bar{\nu}_2} e^{-idE_{\bar{\nu}_2}t}\right)\left(\sum_{d'} d' N_l^2 c'^{\frac{d'-1}{2}} f_{-d'}^l f_{d'c'}^l e^{-id'E_l t}\right)\\
\omega^{(2)}(c,c') &= \left[\sum_d \left(N_{\bar{\nu}_1}^2 c^{\frac{d-1}{2}} f_d^{\bar{\nu}_1} f_{dc}^{\bar{\nu}_1} e^{-idE_{\bar{\nu}_1}t} - N_{\bar{\nu}_2}^2 c^{\frac{d-1}{2}} f_d^{\bar{\nu}_2} f_{dc}^{\bar{\nu}_2} e^{-idE_{\bar{\nu}_2}t}\right)\right]\times\\
&\quad \times\left(\sum_{d'} d' N_l^2 c'^{\frac{d'-1}{2}} f_{-d'}^l f_{d'c'}^l e^{-id'E_l t}\right) \qquad (44)
\end{aligned}
$$

and

$$
\omega_1^{(3)}(c,c') = -cc'(\omega_1^1(c,c'))^*, \quad \omega_2^{(3)}(c,c') = -cc'(\omega_2^1(c,c'))^*, \quad \omega^{(4)}(c,c') = -cc'(\omega^2(c,c'))^* \quad (45)
$$

with the notation $\bar{\nu}_i = \bar{\nu}_{m_i}$ for $i = 1, 2$.

We note that the state of the lepton-antineutrino system (42) at time t is in fact a *hyperentangled state* [14], since entanglement in the flavor degrees of freedom adds up to the initial entanglement in the spins (helicity).

In the spirit of Ref. [9], we want to investigate the effects of both chiral and flavor oscillations on the spin correlations. In order to do this, we trace out flavor and chirality from $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ to obtain the reduced density operator for the spin part. After some algebra, we get

$$
\rho_{S_l,S_{\bar{\nu}}} = \begin{bmatrix} A^2 & \rho_{12} \\ \rho_{12}^* & B^2 \end{bmatrix} \qquad (46)
$$

with

$$
\begin{aligned}
\rho_{12} = AB\Big\{&\cos^4\theta(h_l h_{\bar{\nu}_1} + g_l g_{\bar{\nu}_1}) + \sin^4\theta(h_l h_{\bar{\nu}_2} + g_l g_{\bar{\nu}_2}) + \sin^2\theta\cos^2\theta[h_l(h_{\bar{\nu}_1} + h_{\bar{\nu}_2}) + g_l(g_{\bar{\nu}_1} + g_{\bar{\nu}_2})]\\
&+ i\cos^4\theta(g_l h_{\bar{\nu}_1} - g_{\bar{\nu}_1} h_l) + i\sin^4\theta(g_l h_{\bar{\nu}_2} - g_{\bar{\nu}_2} h_l) + i\sin^2\theta\cos^2\theta[g_l(h_{\bar{\nu}_1} + h_{\bar{\nu}_2}) - h_l(g_{\bar{\nu}_1} + g_{\bar{\nu}_2})]\Big\} \quad (47)
\end{aligned}
$$

and

$$h_i = 1 - \frac{2p^2}{E_i^2}\sin^2(E_i t), \quad g_i = \frac{p}{E_i}\sin(2E_i t) \tag{48}$$

for $i = l,\, \bar{\nu}_1,\, \bar{\nu}_2$.

Now it is possible to calculate the purity for the spin part and the result is

$$\begin{aligned}\mathrm{Tr}[\rho^2_{S_{\bar{\nu}},S_l}(t)] &= 1 - \frac{\mathcal{N}[\rho^2_{S_{\bar{\nu}},S_l}(0)]}{2}\Bigg(1 - \Gamma_l^2\Big\{\cos^8\theta\Gamma^2_{\bar{\nu}_1} + \sin^8\theta\Gamma^2_{\bar{\nu}_2} + 2\sin^2\theta\cos^2\theta\Big[2\cos^4\theta\Gamma^2_{\bar{\nu}_1} \\ &\quad +2\sin^4\theta\Gamma^2_{\bar{\nu}_1} + \sin^2\theta\cos^2\theta(\Gamma^2_{\bar{\nu}_1} + \Gamma^2_{\bar{\nu}_2}) + 2(h_{\bar{\nu}_1}h_{\bar{\nu}_2} + g_{\bar{\nu}_1}g_{\bar{\nu}_2})\Big]\Big\}\Bigg) \end{aligned} \tag{49}$$

with

$$\Gamma_i = h_i^2 + g_i^2$$

and $i = l, \bar{\nu}_1, \bar{\nu}_2$.

As a check, we observe that when flavor mixing is absent ($\theta = 0$), the above expression reduces to

$$\mathrm{Tr}[\rho^2_{S_{\bar{\nu}},S_l}(t)] = 1 - \frac{\mathcal{N}[\rho^2_{S_{\bar{\nu}},S_l}(0)]}{2}\Big(1 - \Gamma_l^2\Gamma^2_{\bar{\nu}_1}\Big) \tag{50}$$

which coincides with Eq.(33) as we expected.

The plot of the quantity in (49) as function of time is given in Fig.1 for different values of the ratio $\frac{p}{m_{\bar{\nu}_1}}$: we see that the purity of the reduced density operator decreases in the case of $p \simeq m_{\bar{\nu}}$ and hence the redistribution of the entanglement between the spins is maximal in this case. This resonance effect has been noticed also in [9]. In the present, more general case, we can also see that the entanglement redistribution of the spins is modulated by the flavor oscillations of the antineutrino which are slower than the chiral oscillations.

It is also interesting to look how the concurrence related to the entanglement between the spins evolves in time. It is given by [15]

$$\mathcal{C} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \tag{51}$$

where λ_i are the squared roots of the eigenvalues (in decreasing order) of the operator $\rho\tilde{\rho}$ where $\tilde{\rho} \equiv \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$ is the spin flipped density operator ρ. In our case the plots of concurrence are given in Fig.2. At $t = 0$ we have $\mathcal{C}(0) = 2|AB| = \mathcal{N}(0)$.

As expected, the concurrence decreases if the ratio $\frac{p}{m}$ increases since the coefficients A and B become more different from each other and $B \to 0$ when $\frac{p}{m} \to \infty$. Furthermore the concurrence is also modulated by both the flavor and the chiral oscillations. The effect is more relevant in the case $p = m_{\bar{\nu}_1}$ as noticed also in [9].

4.1. General case

In the previous Sections we have seen that entanglement between spins in a lepton-antineutrino pair is redistributed by both chiral and flavor oscillations during the free time evolution. Nevertheless, this scheme can be used only as a toy model because the momentum p of the two particles has been taken as a free parameter and the effects of the chiral oscillations on the entanglement between spins are relevant only in the non-relativistic limit for both particles.

Since the neutrino is much lighter than the electron ($m_{\bar{\nu}_e} \simeq 2\cdot 10^{-6} m_e$) and the non-relativistic limit is reached in the case $\frac{p}{m_{\bar{\nu}}} \lesssim 1$, this cannot happen in the case of the pion decay, because the momentum p is fixed by the relativistic conservation laws. Indeed, in the rest frame of the pion we have

$$\begin{cases} m_\pi^2 = m_l^2 + p_l^2 + m_{\bar{\nu}_l}^2 + p_{\bar{\nu}_l}^2 \\ 0 = \mathbf{p}_l + \mathbf{p}_{\bar{\nu}_l} \end{cases}$$

which gives $p_l = p_{\bar{\nu}_l} = p$ and

$$p^2 = \frac{\Delta M_\pi^2}{2} \tag{52}$$

with $\Delta M_\pi^2 = m_\pi^2 - m_l^2 - m_{\bar{\nu}_l}^2$. In the case that the emitted particle is a muon, since $m_\mu \simeq 105.6$MeV, $m_{\bar{\nu}_\mu} \simeq 1$eV and $m_\pi \simeq 139.6$MeV, we obtain $p^2 \simeq 17$MeV: thus the antineutrino is always relativistic and the entanglement between the spins goes to zero since the coefficient (36) also goes to zero because $B \to 0$.

To avoid this situation, let us consider a generalization of the above toy model with two initial particles a and b with masses m_a, m_b and momenta $\mathbf{p}_a$, $\mathbf{p}_b$ colliding. Suppose they emit two particles c and d with masses m_c, m_d and momenta $\mathbf{p}_c$, $\mathbf{p}_d$ and that we consider only the particles emitted on the same direction of that formed by $\mathbf{p}_a$ and $\mathbf{p}_b$. In this case the conservation laws give

$$\begin{cases} m_a^2 + p_a^2 + m_b^2 + p_b^2 = m_c^2 + p_c^2 + m_d^2 + p_d^2 \\ p_a - p_b = p_c + p_d \end{cases}$$

with $p_{a(b)} \equiv |\mathbf{p}_{a(b)}|$. Therefore the momenta of the emitted particles along the initial momentum direction become

$$p_{c,d} = \frac{p_a - p_b \pm \sqrt{\Delta M^2 + (p_a - p_b)^2}}{2} \tag{53}$$

where $\Delta M^2 \equiv m_a^2+m_b^2-(m_c^2+m_d^2)$. Note that the process is possible only if $m_a^2+m_b^2+(p_a-p_b)^2 \geq m_c^2 + m_d^2$.

Therefore, by changing the momentum of the initial particle p_a, we can search for a regime in which both particles are non-relativistic (i.e. $p_c \simeq m_c$ and $p_d \simeq m_d$) and the effects of chiral oscillations on the redistribution of the entanglement between the spins are significant. We assume an initial entangled state for the spin part of the form

$$|\Phi\rangle = \frac{|v_\uparrow(p_c, m_c)\rangle \otimes |u_\downarrow(p_d, m_d)\rangle - |v_\downarrow(p_c, m_c)\rangle \otimes |u_\uparrow(p_d, m_d)\rangle}{\sqrt{2}} \tag{54}$$

that is the analogous of Eq.(21). The system state at $t = 0$ is obtained by the application of chirality projectors which gives

$$|\Psi(0)\rangle = A(p_c, p_d, m_c, m_d)\, |c_\uparrow(0)\rangle \otimes |d_\downarrow(0)\rangle - B(p_c, p_d, m_c, m_d)\, |c_\downarrow(0)\rangle \otimes |d_\uparrow(0)\rangle \tag{55}$$

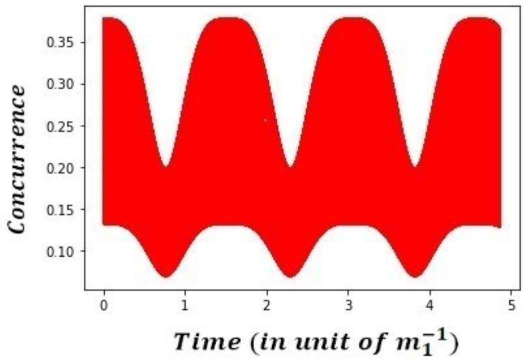
where

$$A(p_c, p_d, m_c, m_d) = \frac{\sqrt{2} f_{+c} f_{-d}}{N_c N_d (f_{+c}^2 f_{-d}^2 + f_{-c}^2 f_{+d}^2)}; \; B(p_c, p_d, m_c, m_d) = \frac{\sqrt{2} f_{-c} f_{+d}}{N_c N_d (f_{+c}^2 f_{-d}^2 + f_{-c}^2 f_{+d}^2)} \tag{56}$$

Note that in this case, if $p_c \gg m_c$ we have $f_{-c} \to 0$ and hence $B \to 0$, while in the case $p_d \gg m_d$ we have $f_{-d} \to 0$ and hence $A \to 0$. The concurrence for the spin part is still given by Eq.(51) and at $t = 0$ (where it reaches its maximum value) it is

$$\mathcal{C}[\rho_{S_b,S_c}(0)] = 2|AB| = 2|A(p_b, p_c, m_b, m_c) B(p_b, p_c, m_b, m_c)| \tag{57}$$

which vanishes in both limit cases.



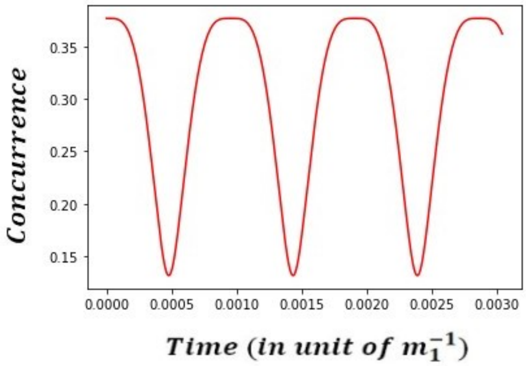


Figure 3: Concurrence shared between spins of the emitted particles as a function of time for two differents time scales for the weak process $n + \nu_e \to p + e$, supposing that the initial and final total spins of the particle is zero and that the emitted particle are entangled in the spin at $t = 0$. In this case, both particle are non-relativistic and hence the entanglement is non-negligible. The values for parameters are (in unit of m_e) $p_n = 2700$ and the real masses of the particles $m_n = 1879$, $m_p = 1876$, $m_e = 1$.

In the case of pion decay with momentum $p_a = p_\pi$ we obtain

$$p_{c,d} = \frac{p_\pi \pm \sqrt{\Delta M_\pi^2 + p_\pi^2}}{2}. \tag{58}$$

If we look for a value of the momentum p_π of the pion such that both emitted particles are non-relativistic, so that the entanglement between the spins becomes non-negligible, we see that in this case this is not allowed due to the smallness of the neutrino mass.

If we renounce to observe the effects of flavor oscillations and focus only on the chiral oscillations, we can consider another kind of decay, that is

$$n + \nu_e \to p + e \tag{59}$$

Neglecting flavor mixing, we see in Fig.3 that the concurrence is non-negligible for some values of the initial parameters and hence the effects of chiral oscillations on the entanglement between spins could in principle be observed.

5. Conclusions

In this paper, we have studied the evolution in time of spin correlations in an initially entangled state for a lepton-antineutrino system produced in a weak interaction process. In the non-relativistic regime, both chiral and flavor (neutrino) oscillations take place and affect spin correlations.

The present work represents a generalization of Ref. [9], with the addition of flavor oscillations: this makes the dynamics of oscillations much richer due to the appearance of different oscillation lengths and indeed the state of the system at time t becomes hyperentangled.

Finally, we investigated the possibility of observing the effects of chiral oscillations by looking at the entanglement between proton and electron spins produced in the process $n + \nu_e \to p + e$. We found that in this case, it is indeed possible to have both the final particles in the non-relativistic regime and thus oscillations in the spin correlations due to chiral oscillations can be relevant.

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