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Baryon Masses and Hadronic Decay Widths with Explicit Pionic Contributions

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Abstract We report results from studies of baryon ground and resonant states by taking explicit mesonic degrees of freedom into account. We are following a relativistic coupled-channels approach relying on a Poincaré-invariant mass operator in matrix form. Generally, it corresponds to a bare particle that is coupled to a number of further mesonic channels. Here we present results, where the bare particle is either a bare nucleon or a bare Delta coupled to pion–nucleon and pion–Delta channels, respectively. For the pion–baryon vertices we employ coupling constants and form factors from different models in the literature. From the mass-operator eigenvalue equation we obtain the pion-dressing effects on the nucleon mass as well as the mass and pion-decay width of the Delta. The dressed masses become smaller than the bare ones, and a finite width of the Delta is naturally generated. The results are relevant for the construction of constituent-quark models for baryons, which have so far not included explicit mesonic degrees of freedom, but have rather relied on three-quark configurations only.

A proper description of hadron resonances poses considerable problems in all current approaches to quantum chromodynamics (QCD). Along constituent-quark models resonances have hitherto usually been considered as excited bound states rather than as genuine resonances with finite decay widths. Covariant predictions for decay widths of baryons have shown shortcomings usually underestimating data from phenomenology [1–4]. A possible remedy consists in coupling to decay channels and thereby including mesonic degrees of freedom explicitly.

In this spirit we construct a relativistically invariant coupled-channels (CC) mass operator

$$\begin{pmatrix} M_{\tilde{B}} & K \\ K^{\dagger} & M_{\tilde{B}+\pi} \end{pmatrix} \begin{pmatrix} |\psi_B\rangle \\ |\psi_{B+\pi}\rangle \end{pmatrix} = m_B \begin{pmatrix} |\psi_B\rangle \\ |\psi_{B+\pi}\rangle \end{pmatrix}. \tag{1}$$

Herein $M_{\tilde{B}}$ is the mass operator of a bare particle $\tilde{B}$, which in our case is either a bare nucleon $\tilde{N}$ or a bare Delta $\tilde{\Delta}$. The second channel with the mass operator $M_{\tilde{B}+\pi}$ contains in addition an explicit interaction-free π. The transition between the channels is governed by the operator K. The mass eigenvalue m_B becomes real for the physical nucleon N (no decay channel open) and complex for the physical Delta Δ (as a genuine resonance acquiring a finite decay width).

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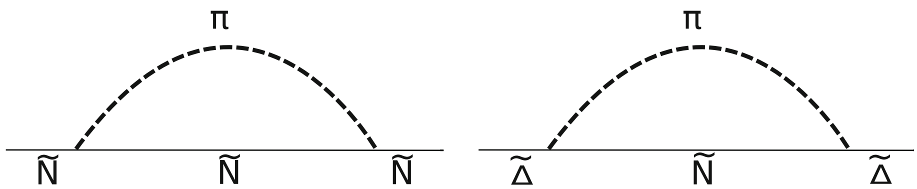

Fig. 1 π-Loop diagrams considered for the dressing of a bare $\tilde{N}$ and a bare $\tilde{\Delta}$

Table 1 Parameters of bare $\pi\tilde{N}\tilde{N}$ and $\pi\tilde{N}\tilde{\Delta}$ vertex form factors entering into Eq. (4)

	RCQM	SL	PR multipole	PR Gaussian
$\pi\tilde{N}\tilde{N}$				
$f^2_{\pi\tilde{N}\tilde{N}}/4\pi$	0.0691	0.08	0.013	0.013
λ_1	0.451	0.453	0.945	
λ_2	0.931	0.641	1.102	
λ				0.665
$\pi\tilde{N}\tilde{\Delta}$				
$f^2_{\pi\tilde{N}\tilde{\Delta}}/4\pi$	0.188	0.334	0.167	0.167
λ_1	0.594	0.4583	0.853	
λ_2	0.998	0.648	1.014	
λ				0.603

The first three columns correspond to the multipole type as on the l.h.s. of Eq. (5) and the last column to the Gaussian type as on the r.h.s. of Eq. (5). The corresponding parametrizations are taken from Refs. [5,6], respectively. All (bare) coupling constants belong to $\mathbf{k}_\pi^2 = 0$. RCQM refers to the predictions of the relativistic constituent-quark model of Ref. [7], SL to the πN meson-exchange model by Sato and Lee [8], and PR to the Nijmegen soft-core model of Polinder and Rijken [6]. All cut-off parameters are in GeV

We apply a Feshbach reduction to Eq. (1) and are left with the following eigenvalue problem

$$\left[M_{\tilde{B}} - K\left(m_B - M_{\tilde{B}+\pi} + i0\right)^{-1} K^\dagger\right] |\psi_B >= m_B\, |\psi_B > . \tag{2}$$

The optical potential $K(m_B - M_{\tilde{B}+\pi} + i0)^{-1}K^\dagger$ corresponds to the diagrams in Fig. 1, where at the vertices the π-coupling to either a bare $\tilde{N}$ or a bare $\tilde{\Delta}$ is furnished by K (resp. $K^\dagger$) and is given by the following pseudovector Lagrangian densities with the π-field $\boldsymbol{\phi}(x)$ coupling the $\tilde{N}$ Dirac fields $\psi(x)$ and the $\tilde{\Delta}$ Rarita–Schwinger fields ψ^μ, respectively, [5]

$$\mathcal{L}_{\pi\tilde{N}\tilde{N}}(x) = -\frac{f_{\pi\tilde{N}\tilde{N}}}{m_\pi}\bar{\psi}(x)\gamma^\mu\gamma_5\boldsymbol{\tau}\psi(x)\cdot\partial_\mu\boldsymbol{\phi}(x) \;\text{ and }\; \mathcal{L}_{\pi\tilde{N}\tilde{\Delta}}(x) = -\frac{f_{\pi\tilde{N}\tilde{\Delta}}}{m_\pi}\bar{\psi}(x)\mathbf{T}\psi^\mu(x)\cdot\partial_\mu\boldsymbol{\phi}(x) + h.c. \tag{3}$$

The eigenvalue equation (2) then adopts the following explicit form

$$\begin{aligned} m_{\tilde{B}} + \int \frac{d^3k_\pi}{(2\pi)^3}\frac{1}{2\omega_\pi 2\omega_{\tilde{N}} 2m_{\tilde{B}}} F_{\pi\tilde{N}\tilde{B}}(\mathbf{k}_\pi^2)\langle\tilde{B}|\mathcal{L}_{\pi\tilde{N}\tilde{B}}(0)|\tilde{N},\pi;\mathbf{k}_\pi\rangle \\ \times\left(m_B - \omega_{\tilde{N}} - \omega_\pi + i0\right)^{-1} F_{\pi\tilde{N}\tilde{B}}(\mathbf{k}_\pi^2)\langle\tilde{N},\pi;\mathbf{k}_\pi|\mathcal{L}^\dagger_{\pi\tilde{N}\tilde{B}}(0)|\tilde{B}\rangle = m_B, \end{aligned} \tag{4}$$

where $m_{\tilde{B}}$ is the bare baryon mass, ω_π and $\omega_{\tilde{N}}$ are the π- and $\tilde{N}$-energies, respectively, and the integration is over the intermediate π-momentum $\mathbf{k}_\pi$. For the extended $\pi\tilde{N}$ and $\pi\tilde{\Delta}$ vertices we employ (bare) form factors taken from prescriptions or models in the literature. They are parametrized through the formulae

$$F_{\pi\tilde{N}\tilde{B}}(\mathbf{k}_\pi^2) = \frac{1}{1 + (\frac{\mathbf{k}_\pi}{\lambda_1})^2 + (\frac{\mathbf{k}_\pi}{\lambda_2})^4} \quad\text{or}\quad F_{\pi\tilde{N}\tilde{B}}(\mathbf{k}_\pi^2) = \exp^{-\mathbf{k}_\pi^2/2\lambda^2} \tag{5}$$

in case of multipole or Gaussian types, respectively. The values of the coupling constants and cut-off parameters for the different models used here are given in Table 1. The momentum dependences of the corresponding form factors are depicted in Figs. 2 and 3.

With these ingredients we solve the eigenvalue equation (4) by calibrating the mass eigenvalue m_N to the physical N mass $m_N = 0.939$ GeV. Thereby we can predict the difference to the bare mass $m_{\tilde{N}}$. The

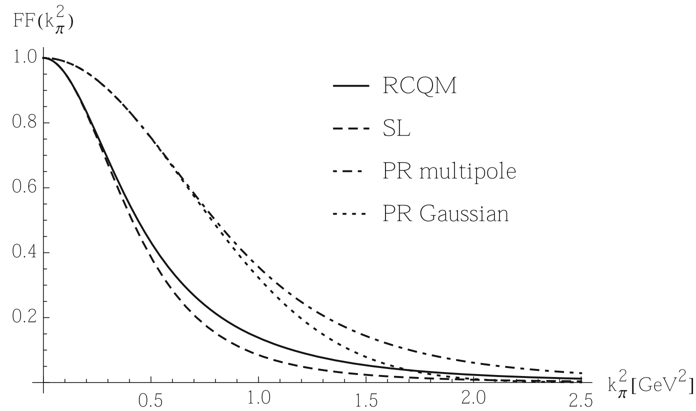

Fig. 2 Momentum dependences of the different (bare) form-factor models in the case of the $\pi\tilde{N}\tilde{N}$ system

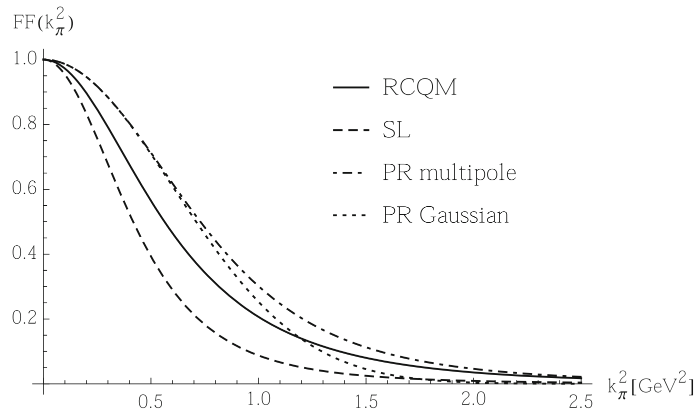

Fig. 3 Momentum dependences of the different (bare) form-factor models in case of the $\pi\tilde{N}\tilde{\Delta}$ system

Table 2 π-Loop effects in the N mass $m_N = 0.939$ GeV according to the l.h.s. diagram of Fig. 1

	RCQM	SL	PR multipole	PR Gaussian
$m_{\tilde{N}}$ (GeV)	1.067	1.031	1.051	1.025
$m_{\tilde{N}} - m_N$ (GeV)	0.128	0.092	0.112	0.086

corresponding effects of the pion-loop diagram on the l.h.s. of Fig. 1 are given in Table 2. It is seen that the dressing effects are all of the same order of magnitude for the different form-factor models employed. The biggest mass difference is obtained in case of the RCQM. The individual results from different models stem from an interplay of the magnitude of the coupling constant $f_{\pi\tilde{N}\tilde{N}}$ and the momentum dependence of the vertex form factor (Fig. 2).

For the Δ we proceed in an analogous manner calibrating the real part of the mass eigenvalue m_Δ to the physical Δ mass $m_\Delta = 1.232$ GeV. Thereby we can predict the difference to the bare mass $m_{\tilde{\Delta}}$ and the $\Delta \to N\pi$ decay width Γ. The corresponding effects of the π-loop diagram on the r.h.s. of Fig. 1 are given in Table 3. Again the results for the dressing effect on the Δ masses are of the same order of magnitude for the different form-factor models. This time the effects are biggest in case of both PR models. Still the predictions for the π-decay widths are all far too small as compared to the experimental value of about 0.12 GeV.

An improvement by about one order of magnitude for the π-decay widths is obtained, when replacing the bare $\tilde{N}$ in the π-loop diagram on the r.h.s. of Fig. 1 by the dressed N, i.e. always the physical mass $m_N = 0.939$ GeV is used instead of the respective bare masses $m_{\tilde{N}}$, while the coupling constants and form factors are kept the same (i.e. bare) as before. In this way the phase space for the hadronic decay is increased and thus about half of the value of the experimental π-decay width is reached (e.g., in the SL model); cf. the results in Table 4. This represents already a significant improvement over the even smaller values for the covariant results of π-decay widths of $\Delta(1232)$ obtained with relativistic constituent-quark models before [1].

Table 3 π-Loop effects in the Δ mass $m_\Delta = \text{Re}\,(m_\Delta) = 1.232$ GeV and in the π-decay width Γ according to the r.h.s. diagram of Fig. 1, where the bare $\tilde{N}$ masses $m_{\tilde{N}}$ in the intermediate states are the same as in Table 2

	RCQM	SL	PR multipole	PR Gaussian
$m_{\tilde{\Delta}}$ (GeV)	1.300	1.295	1.337	1.323
$m_{\tilde{\Delta}} - m_\Delta$ (GeV)	0.068	0.060	0.104	0.090
$\Gamma = 2\,\text{Im}\,(m_\Delta)$ (GeV)	0.003	0.017	0.005	0.010

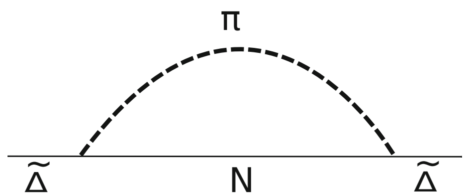

Fig. 4 π-Loop diagram considered for the dressing of a bare $\tilde{\Delta}$, where in the intermediate state a physical N with mass $m_N = 0.939$ GeV is employed

Table 4 π-Loop effects in the Δ mass $m_\Delta = \text{Re}\,(m_\Delta) = 1.232$ GeV and in the π-decay width Γ according to the diagram in Fig. 4, where in the intermediate state always $m_N = 0.939$ GeV

	RCQM	SL	PR multipole	PR Gaussian
$m_{\tilde{\Delta}}$ (GeV)	1.318	1.305	1.358	1.339
$m_{\tilde{\Delta}} - m_\Delta$ (GeV)	0.086	0.073	0.125	0.107
$\Gamma = 2\,\text{Im}\,(m_\Delta)$ (GeV)	0.042	0.069	0.039	0.04

In summary the CC approach allows in a promising way to include explicit couplings to mesonic channels in a fully relativistic framework. In this way resonant states are naturally generated with complex eigenvalues and thus with finite decay widths. So far π-loop effects involving only one π have been investigated. Here we presented the corresponding results for the N and Δ. Presumably, for a more realistic description multiple π-contributions are still necessary and a π-interaction in the second channel should be foreseen. These issues are presently under study or planned. Likewise we are now in the process of applying the same approach to N^* resonances.

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References

1. T. Melde, W. Plessas, R.F. Wagenbrunn, Covariant calculation of mesonic baryon decays. Phys. Rev. C **72**, 015207 (2005); ibid. C **74**, 069901 (2006)
2. T. Melde, W. Plessas, B. Sengl, Covariant calculation of nonstrange decays of strange baryon resonances. Phys. Rev. C **76**, 025204 (2007)
3. B. Sengl, T. Melde, W. Plessas, Covariant calculation of strange decays of baryon resonances. Phys. Rev. D **76**, 054008 (2007)
4. T. Melde, W. Plessas, B. Sengl, Quark-model identification of baryon ground and resonant states. Phys. Rev. D **77**, 114002 (2008)
5. T. Melde, L. Canton, W. Plessas, Structure of meson-baryon interaction vertices. Phys. Rev. Lett. **102**, 132002 (2009)
6. H. Polinder, T.A. Rijken, Soft-core meson-baryon interactions. I. One-hadron-exchange potentials. Phys. Rev. C **72**, 065210; Soft-core meson-baryon interactions. II. πN and K^+N scattering, ibid. 065211 (2005)
7. L.Y. Glozman, W. Plessas, K. Varga, R.F. Wagenbrunn, Unified description of light and strange baryon spectra. Phys. Rev. D **58**, 094030 (1998)
8. T. Sato, T.S.H. Lee, Meson exchange model for πN scattering and $\gamma N \to \pi N$ reaction. Phys. Rev. C **54**, 2660 (1996)