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Considerations about the Bellow

A.G. Ruggiero

To allow a minimum of transverse and longitudinal movement the superconducting magnets of the Energy Doubler are joined together by bellows. At the moment these bellows are provided of RF fingers and extensions of the vacuum chamber to shield them electromagnetically from the beam.

The RF fingers though constitute a nuisance for the installation of the magnets and enormous care must be taken to make sure they really short the item. From a practical point of view it would be better if they could be removed. The result of the analysis we show here requires then that also the vacuum chamber extensions are removed, leaving the bellow completely exposed to the beam. In our analysis we shall look at the effect of the bellow in the two different situations as shown in Figs. 1 and 2. In the case of shielded bellow, though, we shall not consider the RF fingers. It is obvious that if the RF fingers could do their job there would not be any problem. The parameters for the bellow are shown in Table I.

1. Transmission Line Model

To calculate the impedance the bellow presents to the beam we shall make use of the transmission line model. In the case of the shielded bellow (Fig. 1) the transmission line opening is between the point A and B as shown in the figure; the line is shorted at the other end. In the other case of unshielded bellow (Fig. 2) the previous transmission line obviously disappears but we can analyze the effect of the gap between A and B (see Fig. 2) now as a transmission line. Also in this case the line can be assumed shorted at the other end.

In either case the impedance is given by [1]

$$Z = -Z_o \tanh \alpha$$

where

$Z_o = \sqrt{X/Y}$ is the characteristic impedance,

$\alpha = \lambda\sqrt{XY}$ is the propagation constant, (λ = either ℓ or τ)

$Y = -i\omega C$ is the admittance per unit length,

$X = -i\omega L + P$ is the impedance per unit length,

C and L are respectively the capacitance and the inductance per unit length. In the vacuum the following relation exists

$$CL = c^{-2}$$

c being the light velocity. Finally P is the resistivity of the wall.

In the approximation

$$\frac{|P|}{\omega L} << 1 \tag{1}$$

we have

$$Z_o = \sqrt{\frac{L}{C}}$$

and

$$\tanh \alpha = \frac{-\sinh \frac{\lambda P_r}{Z_o} + i \sin 2\lambda\omega\sqrt{LC}}{\cosh \frac{\lambda P_r}{Z_o} + \cos 2\lambda\omega\sqrt{LC}}$$

where P_r is the real part of P.

Resonances occur at

$$\omega_k = \frac{\pi(1+2k)}{2\lambda\sqrt{LC}} \quad , \quad k = 0,1,2,3... \tag{2}$$

Let $r_k = \lambda P_r$ be half of the wall resistance of the line calculated at the k-th resonance frequency, then in the limit

$$r_k << Z_o \tag{3}$$

the shunt impedance at the resonance is

$$R_k = 2 \frac{Z_o^2}{r_k} \quad . \tag{4}$$

In conclusion, the total impedance associated to a bellow (either shielded or not), in the case the two approximations (1) and (3) hold, can be approximated by a series of resonances occuring at the (angular) frequencies (2), with shunt impedance (4) and figure of merit

$$Q_k = \frac{\pi(1+2k)Z_o}{\sqrt{3}\ r_k} \tag{5}$$

which, as usual, is defined as the ratio of the resonance frequency to the range of frequencies over which the total impedance is reduced by a factor two.

For our practical applications

$$P_r = \frac{1}{2\pi b} \sqrt{\frac{\omega_k \, \rho \, R_o}{2c}}$$

$$= 5\times10^{-3} \sqrt{\frac{1+2k}{\lambda_{cm}}} \quad \text{ohm/cm} \tag{6}$$

where R_o = 377 ohm.

A cut-off exists in correspondence of those wavelengths which are comparable or smaller than the transverse dimension ε (τ or w) of the line representing the bellow or a wiggle. Therefore there is a maximum number of resonances which is given by

$$k_{max} = 2\frac{\lambda}{\varepsilon}\, c\sqrt{LC} \ . \tag{7}$$

2. <u>Results for the Shielded Bellow</u>

It is well-known that in the small frequency range M unshielded bellows are associated to the following impedance[2]

$$|Z/n| = R_o \frac{\tau}{b} \frac{M\ell}{2\pi R} = 0.27 \text{ ohm} \qquad (\tau << b)$$

where n is the harmonic number. From this one can estimate the equivalent inductance per unit length associated to a bellow

$$L_B = \frac{0.9 \text{ nH}}{2.75 \text{ cm}} = 0.33 \text{ nH/cm} \ .$$

Therefore for the shielded bellow we have to take

$$L = 1/c^2C + L_B$$

and the first term is an order of magnitude smaller than the second one. The main results are shown in Table II. As one can see the conditions (1) and (3) are easily satisfied over the range $0 \leq k \leq k_{max}$. Since there are about 1000 bellows around the Energy Doubler a total shunt impedance of several 100 kΩ is anticipated. This figure is considerably larger than the one that can be tolerated from beam stability considerations.[3] Our conclusion is then that if the RF fingers do not work one might face a dangerous situation.

3. Results for the Unshielded Bellow

In this case

$$1/\sqrt{LC} = c.$$

The main results are shown in Table III.

The resonance frequencies are now considerably larger and the shunt impedances smaller. If we take again a total of 1000 bellows the largest expected value for the shunt impedance, total around the ring, is about 270 kΩ at 12 GHz. This number also might be larger than the limit can be tolerated according to beam stability considerations (~30 kΩ).

Our advice is, in conclusion, to remove both the RF fingers and the vacuum chamber wall extensions, if we do not have enough confidence on their performance.

References

1. See for example, A.G. Ruggiero, Fermilab TM-500, June 1974
2. R.J. Briggs and V.K. Neil, UCRL-14407-T, 31-8-65
3. A.G. Ruggiero, "Bunch-to-Bunch Longitudinal Instabilities", Fermilab January 1979, unpublished

Table I. Bellow's Parameters

ℓ, length	2.75 cm
b, pipe radius	3.94 cm
τ, wiggle amplitude and distance of bellow from the vacuum chamber wall	0.635 cm
w, separation between wiggles	0.131 cm
M, total number (about)	1000
R, Energy Doubler average radius	1000 m
ρ, stainless steel resistivity at 42° k	52 $\mu\Omega$x cm
number of wiggles	21

Table II. Impedance Parameters for the Shielded Bellow

λ	2.75 cm (ℓ)
ε	0.635 cm (τ)
$C = 2\pi b/1.5\tau$	29 pF/cm
L	0.37 nH/cm
Z_o	3.6 ohm
k_{max}	27
$P_r^{(*)}$	$0.0146\sqrt{1+2k}$ ohm/cm
$\omega_k/2\pi$	0.878 (1+2k) GHz
r_k	$0.04\sqrt{1+2k}$
r_k/Z_o	$0.0111\sqrt{1+2k}$
$\lvert P\rvert/\omega_k L$	$0.01/\sqrt{1+2k}$
R_k	$648/\sqrt{1+2k}$ ohm
Q_k	$163\sqrt{1+2k}$

(*)The r.h. side of (6) has been multiplied by the ratio τ/w to take into account the larger surface involved in the bellow.

Table III. Impedance Parameters for the Unshielded Bellow

λ	0.635 cm (τ)
ε	0.131 cm (w)
$C = 2\pi b/w$	210 pF/cm
L	5.3 pH/cm
Z_o	0.16 ohm
k_{max}	10
P_r	$6.3\times10^{-3}\sqrt{1+2k}$ ohm/cm
$\omega_k/2\pi$	11.8(1+2k) GHz
r_k	$4.0\times10^{-3}\sqrt{1+2k}$ ohm
r_k/Z_o	$0.025\sqrt{1+2k}$
$\|P\|/\omega_k L$	$0.0227/\sqrt{1+2k}$
R_k(*)	$269/\sqrt{1+2k}$ ohm
Q_k	$73\sqrt{1+2k}$

(*)This is Eq. (4) multiplied by the number of wiggles per bellow.

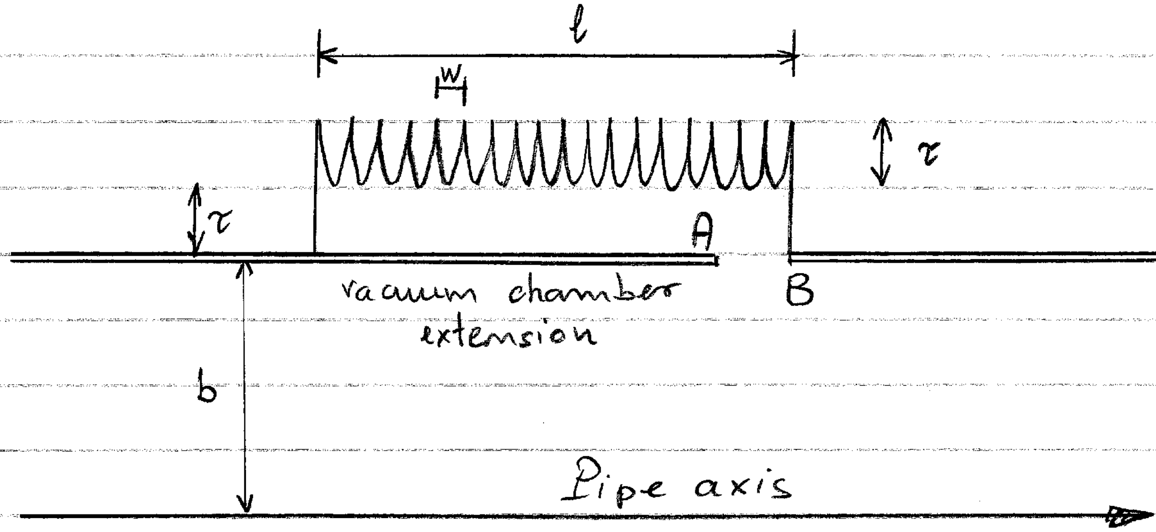

Fig. 1 Shielded Bellow

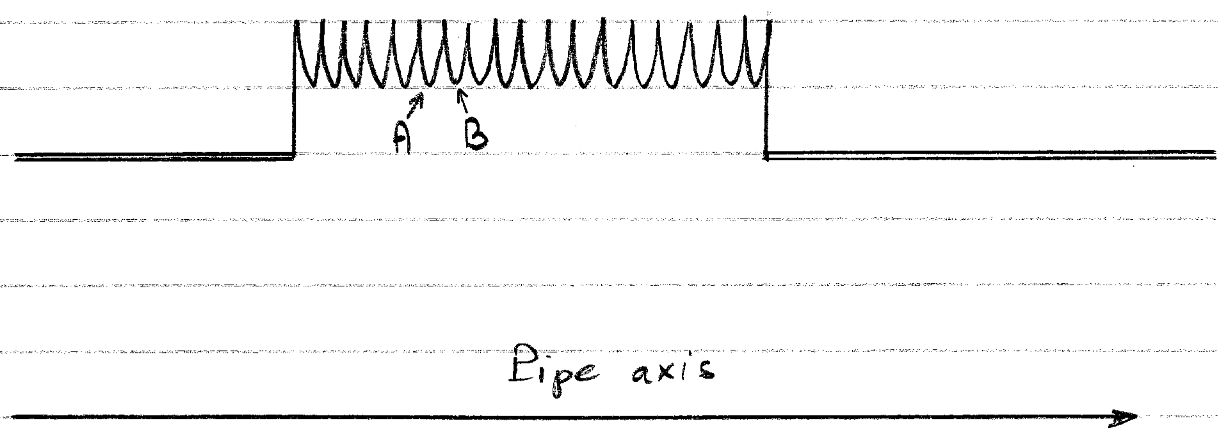

Fig. 2 Un-shielded Bellow