博士論文

論文題目　　Signatures of low-scale string models at the LHC

（LHC 実験における低スケール弦模型のシグナル）

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Signatures of low-scale string models at the LHC

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Introduction

Discovery of the Standard Model Higgs boson

The discovery of a Standard Model (SM) Higgs-like particle on 4th July in 2012 made a large impact on almost all particle physicists. The ATLAS and CMS experiments of the Large Hadron Collider (LHC) reported that mass of the new particle is about 125 GeV [1, 2]. Moreover, they also reported that spin and parity of the new particle is 0^+ [3, 4], which led to a conclusion that it is the SM Higgs boson.

The Standard Model is the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge theory with three generations of quarks and leptons, and it has been quite well confirmed by past various experiments except for the Higgs sector. The Higgs field of the SM is one doublet scalar of $\mathrm{SU}(2)_L$. It breaks the electroweak gauge symmetry spontaneously by its vacuum expectation value (VEV) and gives masses to not only the electroweak gauge bosons but also the fermions through the Yukawa couplings. The Higgs boson which is a fluctuation around the VEV has been observed through the couplings with massive particles.

Naturalness and Hierarchy problem in the Standard Model

Due to the discovery of the SM Higgs boson, the SM seems almost complete. However, most of particle physicists consider the SM as a low-energy effective theory up to a mass scale where particles of new physics beyond the SM begin to appear. The scale of the new physics should be at most the Planck scale, $M_{\mathrm{Pl}} \sim 10^{19}\,\mathrm{GeV}$, where gravitational interactions at quantum level become visible, since the field theory cannot describe the gravitational interaction in a renormalizable form.

Here, note that the mass square parameter of the Higgs boson receives finite radiative corrections written by a square of the mass scale of the new physics,

$$\delta m_{\mathrm{H}}^2 \sim M_{\text{new phys.}}^2 + \cdots .$$

Thus, the radiative corrections to the Higgs boson mass include the mass scale which has nothing to do with the Higgs boson mass. This is caused by the fact that the Higgs boson mass square parameter is an unnatural parameter which is not guaranteed to be zero by some symmetries. This problem is called *naturalness problem.*

On the other hand, when the graviton mediating the gravitational interaction is defined as a fluctuation around a flat background, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, one-loop diagrams which include the graviton $h_{\mu\nu}$ and contribute to the radiative corrections to the Higgs boson mass are shown in Fig.0.1. If there is a renormalizable theory of gravity, the Higgs boson mass is expected to receive the radiative corrections of the order of a square of the Planck mass,

$$\delta m_{\mathrm{H}}^2 \sim M_{\mathrm{Pl}}^2 + \cdots .$$

Therefore, the renormalized parameter of the Higgs boson mass square should cancel out the square of the Planck mass and results in the physical mass of the Higgs boson which is 125 GeV.

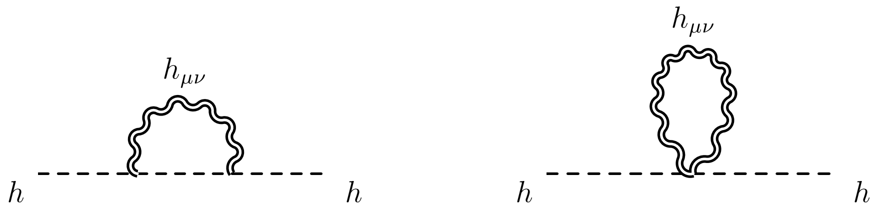

Figure 0.1: The one-loop diagrams which contribute the radiative corrections to the Higgs boson mass, including the graviton $h_{\mu\nu}$.

Very artificial fine-tuning is required since the Planck mass is much larger than the mass of the Higgs boson. This problem essentially comes from the fact that a hierarchy between the mass of the Higgs boson and the Planck mass has not been understood. The problem is called *hierarchy problem.*

TeV scale gravity and Large extra dimension

We focus on the hierarchy problem. One of solutions to the hierarchy problem is that there is a more fundamental gravitational scale of the order of TeV, rather than the Planck scale. It is expected as a solution since the fundamental scale is the same order as the mass of the Higgs boson. The solution can be realized by introducing extra space dimensions than three and compactifying the extra dimensions into a space with large volume. It is called *large extra dimension* which was proposed in ref.[5] in the light of Quantum Field Theory (QFT), and in ref.[6, 7] in the light of string theory.[1]

If the SM gauge interactions live in our four-dimensional space-time but only the gravitational interaction are spread on the extra dimensions, higher-dimensional Einstein-Hilbert action is written by

$$S_{\rm EH} = \frac{1}{16\pi G_N^{(D)}} \int d^D x \sqrt{-g}\, R \,,$$

where $G_N^{(D)}$ is the Newton constant in a D-dimensional space-time. Since the extra dimensions should be compactified into a space with finite volume, the action is reduced to the four-dimensional Einstein-Hilbert action,

$$S_{\rm EH} = \frac{1}{16\pi G_N^{(D)}} V_{D-4} \int d^4 x \sqrt{-g}\, R = \frac{1}{16\pi G_N^{(4)}} \int d^4 x \sqrt{-g}\, R \,,$$

where $G_N^{(4)}$ is the four-dimensional Newton constant and $G_N^{(4)} = 1/M_{\rm Pl}^2$. When the D-dimensional gravitational scale is denoted by M_D, the D-dimensional Newton constant $G_N^{(D)} = 1/M_D^{D-2}$, and the Planck scale is described as $M_{\rm Pl}^2 = V_{D-4}\, M_D^{D-2}$. If the volume of the compactified space V_{D-4} is very large, we can choose the D-dimensional gravitational scale M_D to be of the order

[1] There is an another problem why the compactified space has large volume. The problem can be expected to be solved in a framework of string theory.

of TeV. In this case, M_D is the more fundamental gravitational scale than the Planck scale and it is almost the same order as the mass of the Higgs boson.

Why low-scale string models?

There are field theoretical models which can solve the hierarchy problem with extra dimensions, such as the ADD [5] and the RS model [8]. However, since such models cannot describe the gauge interactions in a renormalizable form, they are also low-energy effective theories with cutoff scales. Moreover, since they are based on QFT, they of course cannot describe the gravitational interaction at quantum level. Since we require more fundamental theories which can solve the hierarchy problem, we consider models which are based on *string theory*.

The reasons why we consider string theory are:

- String theory is a strong candidate for quantum gravity. There is a closed string which includes the graviton with spin 2.
- String theory does not need a renormalization. There are not ultraviolet divergences which should be canceled out by a renormalization, as in QFT.
- String theory necessarily introduces extra dimensions. The absence of Weyl anomaly in world-sheet theory requires that the space-time dimension is $D = 10$ in superstring theory.

In models based on string theory, if the extra dimensional space has large volume, the fundamental scale which is called the string scale M_{s} or the Regge slope $\alpha' = 1/M_{\text{s}}^2$ can be chosen of the order of TeV. The models are called *low-scale string models*.

In string theory, there is a compactification mechanism which compactifies the extra dimensions into a space with large volume. Since in string theory, the volume of the compactified space is determined by an expectation value of a moduli field, a mechanism which stabilize the value of the moduli field is required. As the mechanism of stabilization of the moduli, the Type IIB Calabi-Yau flux compactification was proposed by KKLT [9]. Using the KKLT mechanism, large volume compactification is possible [10].

In order to introduce matter fermions of the SM, we should start with superstring theory. If we assume that not the Minimal Supersymmetric Standard Model (MSSM) but the SM is realized in low-scale string models, supersymmetry (SUSY) is explicitly broken at TeV scale.

String resonances and Model independence

Since the string scale M_{s} is of the order of TeV, low-scale string models have a possibility of being confirmed or excluded by the LHC. Signatures of low-scale string models at the LHC are different from typical signatures of other field theoretical models with extra dimensions, such as the ADD and the RS.

A characteristic feature in low-scale string models is the appearance of *string excited states*. String excited states are higher vibrational modes of open strings, and the open strings live in a D-brane which is a subspace where the gauge interactions are localized. Therefore, the open strings on D-branes can realize the SM gauge bosons and matters, and string excited states can have the same gauge quantum number as that of the corresponding SM particle.

At the LHC, colored string excited states can be produced dominantly in dijet events [11, 12]. Scattering amplitudes of gluons and quarks with exchanges of string excited states are calculated in ref.[13] (see ref.[14, 15] for earlier discussions). The string excited states can be observed as resonances with mass of the order of M_{s} in dijet invariant mass distributions. The resonances which are called *string resonances* have not been observed at the LHC yet, and the value of M_{s} has already been constrained in dijet events at the 7 TeV and 8 TeV LHC, to be larger than 3.61 TeV by the ATLAS [16] and 4.78 TeV by the CMS [17].

If D-branes have directions of extra dimensions, there are not only string excited states but also Kaluza-Klein (KK) modes of the SM particles. However, in dominant processes at the LHC, the KK modes cannot be produced alone, because of the momentum conservation in the direction of extra dimensions on D-branes. Since the KK modes appear by compactifying the extra dimensions, signatures of processes with the KK mode exchanges depend on the detail of model buildings, such as the geometry of compactified space and the configurations of D-branes. Therefore, signatures of the dominant processes at the LHC without the KK mode exchanges are *independent* of the detail of model buildings. Although in string theory there are various kinds of the ways of model buildings, we can test low-scale string models *model independently* at the LHC.

Distinction of low-scale string models at the LHC

If a new heavy resonance is discovered at the LHC, it is very important to specify what classes of models cause the resonance. In ref.[18], Kitazawa proposed some analyses for dijet events at the LHC which can distinguish the resonances in low-scale string models from that in the other "new physics". In ref.[19, 20], adding a new analysis, Kitazawa and the author practically performed two analyses using Monte Carlo (MC) simulations for the LHC, and we actually confirmed the possibility of identifying string resonances in dijet events at the LHC. This thesis is based on these studies [19, 20].

In order to identify a new resonance as a string resonance, we focus on the following unique properties that string excited states have:

- String excited states are degenerate with a variety of spins higher than that of the corresponding SM particle.
- Their degenerate mass is $M_n = \sqrt{n}M_{\mathrm{s}}$ for nth string excited states.

The highest spin of nth string excited states is $J_{\mathrm{max}} = j_0 + n$, where j_0 is the spin of the corresponding SM particle. For example, first string excited states of gluons are degenerate with spin $J = 0$, 1 and 2, and first string excited states of quarks are degenerate with $J = 1/2$ and $3/2$. All of them have the same mass of M_s.

Using the above properties, two analyses for dijet events can be considered:

- One is observing degeneracy of string excited states with higher spins, by an angular distribution analysis on the resonance in dijet invariant mass distributions.
- The other is observing second string excited states with the characteristic masses, by a search for a second resonance in dijet invariant mass distributions.

The former analysis is useful as a discriminator. The appearance of heavy colored states with higher spins is quite characteristic of low-scale string models.

The latter analysis is also useful. First, there must be second string excited states in low-scale string models, while there is no second state in the other "new physics", such as the axigluon models [21] and the color-octet scalar models [22]. Second, the masses of second string excited states must be $\sqrt{2}$ times of that of first string excited states in low-scale string models, while typical masses of second KK modes are 2 times of that of first KK modes, in the other "new physics" with extra dimensions, such as the five-dimensional Universal Extra Dimension (UED) models [23].[2]

The process of $qg \to qg$ almost dominates over all the other two-parton scattering processes at the LHC with several TeV collision energies, and string excited states of quarks are dominant in string resonances in dijet invariant mass distributions. In the angular distribution analysis, by comparing the angular distributions with both $J = 1/2$ and $J = 3/2$ states to that with a $J = 1/2$ state only, we try to confirm the degeneracy with higher spin. In the second resonance analysis, by calculating amplitudes with exchanges of second string excited states and generating dijet invariant mass distributions with second string resonances, we try to obtain a significance of the second string resonances. Then, we have to take into account the property that the second string excited states can decay into both the SM particles and the first string excited states.

About this thesis

This thesis consists of the following parts and sections.

Part I is a review part. Throughout this part, we refer to the textbook [24, 25]. In Sec.1, the basics of superstring theory such as the world-sheet action and quantization of the world-sheet fields are reviewed, and the type I and type II theories are introduced. In Sec. 2, after introduction of D-branes, we review the idea of intersecting D-branes which give rise to chiral fermions. Next, in order to obtain realistic models, we consider intersecting D6-branes in the type IIA orientifolds with toroidal compactifications. At the end of this section, we review a

[2]In the six-dimensional UED models, typical masses of second KK modes with KK parity +1 is $\sqrt{2}$ times of that of first KK modes with KK parity +1. It may be possible to confirm low-scale string models by a search for a third resonance, since third string excited states have $\sqrt{3}$ times of that of first string excited states.

semi-realistic model with the SM-like spectrum. In Sec.3, using the fact that the SM spectrum can be obtained in intersecting D-brane models, we introduce low-scale string models in which the extra dimensions are compactified to general spaces with large volume. When we discuss the LHC phenomenology in the next part, the low-scale string models are considered.

Part II is an original part of this thesis. In Sec.4, at first, the present constraints on the string scale from the LHC is mentioned. Next, the model independence of signatures of low-scale string models at the LHC is explained. We reduce the open-string amplitudes calculated in ref.[13] to scattering amplitudes with exchanges of string excited states, and calculate widths of string excited states, in Sec.4.3 and Appendix A. Using the scattering amplitudes and the widths, we obtain dijet invariant mass distributions with string resonances, both at parton level and using Monte Carlo simulations. In Sec.5 and Sec.6, we confirm the possibility of identifying low-scale string models at the LHC, using the two analyses mentioned above.

In the last of this thesis, we summarize this thesis and mention some future prospects. The original works in this thesis are presented in Sec.4.3 and Appendix A, Sec.4.5, Sec.5 and Sec.6. These results are based on ref.[19, 20] with Noriaki Kitazawa.

Part I

Basics of superstring theory and Construction of low-scale string models

In this part, we review construction of low-scale string models which contain the particle content of the Standard Model and whose string scale is of the order of TeV.

1 Basics of superstring theory

1.1 The world-sheet theory

The world-sheet action of the bosonic string

A relativistic (bosonic) string, in contrast to a point particle, is parametrized by not only a time-like parameter τ but also a space-like one σ. The Nambu-Goto action, the action of the relativistic string, is proportional to the area of the world-sheet which the string sweep out,

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \big[-\det(\partial_a X^\mu \partial_b X_\mu)\big]^{1/2}\,, \tag{1.1}$$

where the indices a, b run over (τ, σ).[3] The factor of $1/2\pi\alpha'$ is the string tension, where α' is the Regge slope and is related to the string scale $M_{\rm s}$ by $\alpha' = 1/M_{\rm s}^2$. The Nambu-Goto action is recast into the Polyakov action, introducing the world-sheet metric γ_{ab} as an auxiliary field,

$$S_P = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \sqrt{-\gamma}\,\gamma^{ab}\partial_a X^\mu \partial_b X_\mu\,, \tag{1.2}$$

where $\gamma = \det \gamma_{ab}$ and γ_{ab} has the Lorentzian signatures $(-, +)$. Note that the Polyakov action describes a two-dimensional field theory of D massless scalar fields X^μ.

The Polyakov action is invariant under the following gauge transformations:

- the two-dimensional general coordinate transformation (diffeomorphism),

$$\gamma'_{ab}(\tau', \sigma') = \frac{\partial \sigma^c}{\partial \sigma'^a}\frac{\partial \sigma^d}{\partial \sigma'^b}\gamma_{cd}(\tau, \sigma)\,, \tag{1.3}$$

- the Weyl transformation,

$$\gamma'_{ab}(\tau, \sigma) = \exp[2\omega(\tau, \sigma)]\gamma_{ab}\,. \tag{1.4}$$

[3]In this thesis, the space-time metric is $\eta_{\mu\nu} = {\rm diag}(-, +, +, +)$

Gauge fixing

For the quantization of the world-sheet theory, we need to fix the gauge degrees of freedom associated with the local symmetries (1.3) and (1.4). After the Minkowskian world-sheet metric γ_{ab} in eq.(1.2) is replaced with an Euclidean world-sheet metric g_{ab} with signatures $(+,+)$, we consider the path integral,

$$Z[g] = \int \frac{[dX\, dg]}{V_{\text{diff}\times\text{Weyl}}} \exp(-S_X - \lambda\chi)\,, \tag{1.5}$$

where $V_{\text{diff}\times\text{Weyl}}$ is the volume of the local symmetry group and χ is the Euler number of the world-sheet. S_X is the Euclidean one of the world-sheet action of scalar fields of eq.(1.2),

$$S_X = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g}\, g^{ab} \partial_a X^\mu \partial_b X_\mu\,. \tag{1.6}$$

where the indices a, b run over (σ^1, σ^2).[4] We perform the gauge fixing by setting the metric g_{ab} to be some fiducial metric $\hat{g}_{ab}$. Inserting $1 = \Delta_{FP}(g) \int [d\zeta] \delta(g - \hat{g}^\zeta)$, where Δ_{FP} is the Faddeev-Popov measure and ζ denotes a combination of diff and Weyl transformations, the path integral of eq.(1.5) becomes

$$Z[\hat{g}] = \int [dX\, db\, dc] \exp(-S_X - S_{\text{g}} - \lambda\chi)\,. \tag{1.7}$$

Here the ghost action S_{g} is given by

$$S_{\text{g}} = \frac{1}{2\pi} \int d^2\sigma \sqrt{\hat{g}}\, b_{ab} \hat{\nabla}^a c^b\,, \tag{1.8}$$

where b_{ab} is the traceless symmetric tensor field and c^a is the vector field, and both of them have fermionic statistics. Let us choose the unit gauge of $\hat{g}_{ab} = \delta_{ab}$ and introduce a complex coordinate $z = \sigma^1 + i\sigma^2$ and $\overline{z} = \sigma^1 - i\sigma^2$. Then, the gauge-fixed action can be written as[5]

$$S = S_X + S_{\text{g}}\,, \tag{1.9}$$

with

$$S_X = \frac{1}{4\pi} \int d^2z\, \frac{2}{\alpha'} \partial X^\mu \overline{\partial} X_\mu\,, \qquad S_{\text{g}} = \frac{1}{2\pi} \int d^2z\, \left[b\overline{\partial}c + \widetilde{b}\partial\widetilde{c} \right]\,, \tag{1.10}$$

where we introduced (b, c) and $(\widetilde{b}, \widetilde{c})$ defined by (b_{zz}, c^z) and $(b_{\overline{z}\overline{z}}, c^{\overline{z}})$ in eq.(1.8).

The world-sheet action of the superstring

Let us next introduce the world-sheet theory of the superstring. In order to introduce spacetime fermions, we extend the bosonic string action into the world-sheet supersymmetric one. Though one might think that the space-time supersymmetry (SUSY) should be introduced rather than

[4] The Euclidean coordinate is obtained by replacing τ with $-i\sigma^2$ and σ with σ^1.

[5] In the following, we use the notation $\partial \equiv \partial_z = \frac{1}{2}(\partial_1 - i\partial_2)$ and $\overline{\partial} \equiv \partial_{\overline{z}} = \frac{1}{2}(\partial_1 + i\partial_2)$.

the world-sheet SUSY, the world-sheet SUSY results in the same spectrum as the space-time SUSY does. Then, the matter action of the superstring is given by [6]

$$S_{\rm m} = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma\, \eta^{ab}\partial_a X^\mu \partial_b X_\mu - \frac{1}{4\pi}\int d\tau d\sigma\, i\overline{\psi}^\mu \gamma^a \partial_a \psi_\mu \,, \tag{1.11}$$

where ψ^μ is a two-dimensional Majorana fermion, and the two-dimensional γ matrices are

$$\gamma^\tau = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma^\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{1.12}$$

and $\overline{\psi}^\mu = \psi^{\mu T}\gamma^\tau$. The action (1.11) is invariant under the following infinitesimal SUSY transformation for the world-sheet fields,

$$\delta X^\mu = \sqrt{\alpha'}\,\bar{\varepsilon}\psi^\mu \,, \qquad \delta\psi^\mu = -\frac{1}{\sqrt{\alpha'}}\, i\gamma^a \partial_a X^\mu \varepsilon \,, \tag{1.13}$$

where ε is a transformation parameter.

Equation of motion and boundary conditions

The equations of motion for X^μ and ψ^μ can be obtained by varying the action (1.11) with respect to X^μ and ψ^μ

$$\partial^a \partial_a X^\mu = 0 \,, \qquad i\gamma^a \partial_a \psi^\mu = 0 \,. \tag{1.14}$$

Suppose that the world-sheet has boundaries and $0 \leq \sigma \leq \ell$. Then, the variation of the action induces the surface terms

$$\int_{-\infty}^{\infty} d\tau \Big[\partial_\sigma X^\mu\, \delta X_\mu\Big]_{\sigma=0}^{\sigma=\ell}, \qquad \int_{-\infty}^{\infty} d\tau \Big[\psi_+^\mu\, \delta\psi_{+\mu} - \psi_-^\mu\, \delta\psi_{-\mu}\Big]_{\sigma=0}^{\sigma=\ell}, \tag{1.15}$$

where $\psi = (\psi_+, \psi_-)$ in eq.(1.11). In order for the surface terms to vanish, we need to impose some boundary conditions on the world-sheet fields. If we respect the Poincaré invariance, one of the following two boundary conditions should be imposed:

- the periodic boundary condition,

$$X^\mu(\tau, \sigma = \ell) = X^\mu(\tau, \sigma = 0) \,, \tag{1.16}$$

$$\psi_+^\mu(\tau, \sigma = \ell) = e^{-2\pi i\tilde{\nu}}\psi_+^\mu(\tau, \sigma = 0) \,, \qquad \psi_-^\mu(\tau, \sigma = \ell) = e^{2\pi i\nu}\psi_-^\mu(\tau, \sigma = 0) \,, \tag{1.17}$$

- the Neumann boundary condition,

$$\partial_\sigma X^\mu(\tau, \sigma = \ell) = \partial_\sigma X^\mu(\tau, \sigma = 0) = 0 \tag{1.18}$$

$$\psi_-^\mu(\tau, \sigma = \ell) = e^{2\pi i\nu}\psi_+^\mu(\tau, \sigma = \ell) \,, \qquad \psi_-^\mu(\tau, \sigma = 0) = \psi_+^\mu(\tau, \sigma = 0) \,, \tag{1.19}$$

[6] Here we set the world-sheet metric to be the flat one to fix the gauge degrees of freedom: $\eta_{ab} = {\rm diag}(-, +)$.

where ν and $\widetilde{\nu}$ take the values 0 and $\frac{1}{2}$. The periodic boundary condition gives a closed string and the Neumann boundary condition gives an open string. The boundary condition with $\nu = 0$ is the Ramond (R) sector, while that with $\nu = \frac{1}{2}$ is the Neveu-Schwarz (NS) sector. The world-sheet fermions ψ_+ and ψ_- have degrees of freedom of the sign at boundaries, and they are independent for the closed string but not for the open string. For the open string, we can also impose another boundary condition, which breaks the Poincaré invariance:

- the Dirichlet boundary condition,

$$X^\mu(\tau, \sigma = \ell) = X^\mu(\tau, \sigma = 0) = \text{const.}\,, \tag{1.20}$$

$$\psi_-^\mu(\tau, \sigma = \ell) = -e^{2\pi i \nu}\psi_+(\tau, \sigma = \ell)\,, \qquad \psi_-^\mu(\tau, \sigma = 0) = -\psi_+(\tau, \sigma = 0)\,. \tag{1.21}$$

Superconformal symmetry

Let us next discuss the full action of the superstring. As in the case of the bosonic string, we consider the world-sheet with the Euclidean metric and introduce a complex coordinate $z = \sigma^1 + i\sigma^2$ and $\overline{z} = \sigma^1 - i\sigma^2$. Then, the gauge-fixed action in the unit gauge of $\hat{g}_{ab} = \delta_{ab}$ is given by

$$S = S_\text{m} + S_\text{g}\,, \tag{1.22}$$

where

$$S_\text{m} = \frac{1}{4\pi}\int d^2z \left[\frac{2}{\alpha'}\partial X^\mu \overline{\partial} X_\mu + \psi^\mu \overline{\partial}\psi_\mu + \widetilde{\psi}^\mu \partial \widetilde{\psi}_\mu\right], \tag{1.23}$$

$$S_\text{g} = \frac{1}{2\pi}\int d^2z \left[b\overline{\partial}c + \widetilde{b}\partial\widetilde{c} + \beta\overline{\partial}\gamma + \widetilde{\beta}\partial\widetilde{\gamma}\right], \tag{1.24}$$

where $\psi = (i^{-1/2}\widetilde{\psi}, i^{1/2}\psi)$ in eq.(1.11). The bosonic fields β, γ and $\widetilde{\beta}$, $\widetilde{\gamma}$ are superpartners of b, c and $\widetilde{b}$, $\widetilde{c}$, respectively. The equations of motion in the complex coordinate are

$$\partial\overline{\partial}X^\mu = 0\,, \qquad \overline{\partial}\psi = \partial\widetilde{\psi} = 0\,, \tag{1.25}$$

$$\overline{\partial}b = \overline{\partial}c = \partial\widetilde{b} = \partial\widetilde{c} = 0\,, \qquad \overline{\partial}\beta = \overline{\partial}\gamma = \partial\widetilde{\beta} = \partial\widetilde{\gamma} = 0\,. \tag{1.26}$$

Therefore, when the equations of motion are satisfied, ∂X^μ is a holomorphic function of z and $\overline{\partial}X^\mu$ is an antiholomorphic function of $\overline{z}$. Similarly, ψ^μ, b, c are holomorphic and $\widetilde{\psi}^\mu$, $\widetilde{b}$, $\widetilde{c}$ are antiholomorphic.

Consider the diff transformation by a holomorphic function of z and the Weyl transformation by a specific function,

$$z' = f(z)\,, \qquad \omega = \ln|\partial_z f(z)|\,. \tag{1.27}$$

Then the combination of eq.(1.27), which is a conformal transformation, keeps the unit metric invariant. In other words, the gauge-fixed world-sheet action (1.22) has a conformal symmetry. In the quantum theory, however, the conformal symmetry is broken (the Weyl anomaly), unless the space-time dimension $D = 10$. The conformal symmetry combines with an invariance under the global SUSY transformation (1.13) into a local superconformal symmetry. Therefore, the world-sheet theory of the superstring is a two-dimensional superconformal field theory.

The current of conformal symmetry is the energy-momentum tensor,

$$T_{ab} = -\frac{1}{\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \tfrac{1}{2}\eta_{ab}\partial^c X^\mu \partial_c X_\mu\right) - \frac{1}{2}\left(i\overline{\psi}^\mu \gamma_a \partial_b \psi_\mu - \tfrac{1}{2}\eta_{ab}\, i\overline{\psi}^\mu \gamma^c \partial_c \psi_\mu\right), \tag{1.28}$$

for the Minkowskian action (1.11). The energy-momentum tensor is a traceless symmetric tensor and satisfies the conservation law $\partial^a T_{ab} = 0$. These properties give holomorphic current denoted by T_B and antiholomorphic one denoted by $\widetilde{T}_B$ in the complex coordinate,

$$T_B(z) = T_B^{\mathrm{m}}(z) + T_B^{\mathrm{g}}(z)\,, \qquad \widetilde{T}_B(z) = \widetilde{T}_B^{\mathrm{m}}(z) + \widetilde{T}_B^{\mathrm{g}}(z)\,, \tag{1.29}$$

$$T_B^{\mathrm{m}}(z) = -\frac{1}{\alpha'}\partial X^\mu \partial X_\mu - \frac{1}{2}\psi^\mu \partial \psi_\mu\,, \qquad \widetilde{T}_B^{\mathrm{m}}(\overline{z}) = -\frac{1}{\alpha'}\overline{\partial} X^\mu \overline{\partial} X_\mu - \frac{1}{2}\widetilde{\psi}^\mu \overline{\partial}\widetilde{\psi}_\mu\,, \tag{1.30}$$

$$T_B^{\mathrm{g}}(z) = -\lambda\partial(bc) + (\partial b)c - \lambda'\partial(\beta\gamma) + (\partial\beta)\gamma\,, \quad \widetilde{T}_B^{\mathrm{g}}(\overline{z}) = -\lambda\overline{\partial}(\widetilde{b}\widetilde{c}) + (\overline{\partial}\widetilde{b})\widetilde{c} - \lambda'\overline{\partial}(\widetilde{\beta}\widetilde{\gamma}) + (\overline{\partial}\widetilde{\beta})\widetilde{\gamma}\,, \tag{1.31}$$

where $\lambda = 2$ and $\lambda' = \frac{3}{2}$.

The SUSY current is obtained from the global SUSY transformation (1.13) in the Minkowskian action of eq.(1.11),

$$J^a = \frac{i}{\sqrt{2\alpha'}}\gamma^b \gamma^a \partial_b X^\mu \psi_\mu\,, \tag{1.32}$$

The SUSY current is also conserved and gives holomorphic current denoted by T_F and anti-holomorphic one denoted by $\widetilde{T}_F$ in the complex coordinate,[7]

$$T_F(z) = T_F^{\mathrm{m}}(z) + T_F^{\mathrm{g}}(z)\,, \qquad \widetilde{T}_F(z) = \widetilde{T}_F^{\mathrm{m}}(z) + \widetilde{T}_F^{\mathrm{g}}(z)\,, \tag{1.33}$$

$$T_F^{\mathrm{m}}(z) = i\sqrt{\frac{2}{\alpha'}}\,\partial X^\mu \psi_\mu\,, \qquad \widetilde{T}_F^{\mathrm{m}}(\overline{z}) = i\sqrt{\frac{2}{\alpha'}}\,\overline{\partial} X^\mu \widetilde{\psi}_\mu\,, \tag{1.34}$$

$$T_F^{\mathrm{g}}(z) = \lambda'\partial(\beta c) - \frac{1}{2}(\partial\beta)c - 2b\gamma\,, \qquad \widetilde{T}_F^{\mathrm{g}}(\overline{z}) = \lambda'\overline{\partial}(\widetilde{\beta}\widetilde{c}) - \frac{1}{2}(\overline{\partial}\widetilde{\beta})\widetilde{c} - 2\widetilde{b}\widetilde{\gamma}\,. \tag{1.35}$$

The world-sheet fields are covariant under the conformal transformation (1.27). A field is called a tensor when it is transformed under the conformal transformation as

$$\mathcal{O}'(z', \overline{z}') = \frac{1}{(\partial_z z')^h (\partial_{\overline{z}} \overline{z}')^{\widetilde{h}}}\mathcal{O}(z, \overline{z})\,, \tag{1.36}$$

where $(h, \widetilde{h})$ is a conformal weight of the tensor field. The world-sheet fields are tensor fields, and their conformal wights are

$$\begin{aligned}
\partial X^\mu &: (h,\widetilde{h}) = (1,0)\,, & \psi^\mu &: (h,\widetilde{h}) = (\tfrac{1}{2},0)\,,\\
b &: (h,\widetilde{h}) = (2,0)\,, & c &: (h,\widetilde{h}) = (-1,0)\,,\\
\beta &: (h,\widetilde{h}) = (\tfrac{3}{2},0)\,, & \gamma &: (h,\widetilde{h}) = (-\tfrac{1}{2},0)\,,
\end{aligned} \tag{1.37}$$

[7] Here, the Minkowski current and the Euclidean current are related to each other as $J = (i^{-1/2}\widetilde{T}_F, i^{1/2}T_F)$, just as in the case of ψ.

for holomorphic fields as well as for antiholomorphic fields except for a replacement of $h \leftrightarrow \widetilde{h}$. The weights of bc ghosts and $\beta\gamma$ ghosts correspond to the weights of the energy-momentum tensor (1.29) and the SUSY current (1.33), respectively. The energy-momentum tensor itself is not a tensor since it is transformed under the conformal transformation as

$$T'_B(z') = \frac{1}{(\partial_z z')^2} T_B(z) - \frac{c}{12} \frac{2\partial_z^3 z' \partial_z z' - 3(\partial_z^2 z')^2}{2(\partial_z z')^4}, \tag{1.38}$$

where c is the central charge. Since the last term (1.38) is an origin of the Weyl anomaly, the central charge should vanish. The central charge in superstring theory is calculated as

$$\begin{aligned} c = c^{\mathrm{m}} + c^{\mathrm{g}} &= D + \tfrac{D}{2} + \left[-3(2\lambda - 1)^2 + 1\right] + \left[3(2\lambda' - 1)^2 - 1\right] \\ &= \tfrac{3D}{2} - 15 \,. \end{aligned} \tag{1.39}$$

If the space-time dimension $D = 10$, the central charge is zero.

1.2 Mode expansions and physical states

The world-sheet theory in the unit gauge (1.22) is a free theory, and therefore, the world-sheet fields can be easily quantized by imposing canonical commutation relation on the fields, following the procedure of ordinary canonical quantization.

Let us choose two complex coordinates, $w = \sigma^1 + i\sigma^2$ and

$$z = e^{-iw} = \exp(-i\sigma^1 + \sigma^2) \,. \tag{1.40}$$

For a closed string which is periodic, setting the space coordinate $\sigma^1 \sim \sigma^1 + 2\pi$ and the time one $-\infty < \sigma^2 < \infty$, the w-coordinate forms an infinite cylinder. On the other hand, in the z-coordinate, time runs radically and the origin in the z-plane corresponds to the infinite past. For an open string with a boundary, setting the space coordinate $0 \leq \sigma^1 \leq \pi$, the w- coordinate forms an infinite strip. The z-coordinate forms the upper complex plane and the real axis in the z-plane corresponds to the boundary.[8] These coordinates are related by the conformal transformation.

Closed string

In the case of the closed string, since the world-sheet scalar satisfies the periodic boundary condition (1.16) which corresponds to $X^\mu(w + 2\pi, \overline{w} + 2\pi) = X^\mu(w, \overline{w})$ in the w-coordinate, it can be expanded in terms of modes by just a Fourier transformation,

$$\partial_w X^\mu(w) = -\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \alpha_m^\mu e^{imw} \,, \qquad \partial_{\overline{w}} X^\mu(\overline{w}) = \sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \widetilde{\alpha}_m^\mu e^{-im\overline{w}} \,, \tag{1.41}$$

for the holomorphic (left-moving) and antiholomorphic (right-moving) fields. Since the world-sheet scalar ∂X^μ has a conformal weight (1.37), the Fourier expansion (1.41) is transformed to

[8] Note that for an open string, z is defined as $z = -e^{-iw}$.

the z coordinate as follows:

$$\partial X^\mu(z) = -i\sqrt{\frac{\alpha'}{2}}\sum_{m=-\infty}^{\infty}\frac{\alpha_m^\mu}{z^{m+1}}\,, \qquad \overline{\partial}X^\mu(\overline{z}) = -i\sqrt{\frac{\alpha'}{2}}\sum_{m=-\infty}^{\infty}\frac{\widetilde{\alpha}_m^\mu}{\overline{z}^{m+1}}\,. \tag{1.42}$$

Integrating them over z and $\overline{z}$, a mode expansion for X^μ is obtained as [9]

$$X^\mu(z,\overline{z}) = x^\mu - i\sqrt{\frac{\alpha'}{2}}\,\alpha_0^\mu \ln|z|^2 + i\sqrt{\frac{\alpha'}{2}}\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty}\frac{1}{m}\left(\frac{\alpha_m^\mu}{z^m}+\frac{\widetilde{\alpha}_m^\mu}{\overline{z}^m}\right). \tag{1.43}$$

The zero mode α_0^μ in eq.(1.43) corresponds to center-of-mass momentum of the string. Since the current of space-time translation is $j_a^\mu = \frac{i}{\alpha'}\partial_a X^\mu$, the space-time momentum is given by

$$p^\mu = \frac{1}{2\pi}\int_0^{2\pi} d\sigma^1\, j_2^\mu = \frac{i}{2\pi}\oint\left(dz\, j_z^\mu - d\overline{z}\, j_{\overline{z}}^\mu\right) = \sqrt{\frac{2}{\alpha'}}\,\alpha_0^\mu = \sqrt{\frac{2}{\alpha'}}\,\widetilde{\alpha}_0^\mu\,. \tag{1.44}$$

By imposing the canonical commutation relation on the fields, commutation relations of mode operators are obtained as

$$\left[\alpha_m^\mu, \alpha_{-n}^\nu\right] = \left[\widetilde{\alpha}_m^\mu, \widetilde{\alpha}_{-n}^\nu\right] = m\,\eta^{\mu\nu}\delta_{m,n}\,, \tag{1.45}$$

$$\left[x^\mu, p^\nu\right] = i\eta^{\mu\nu}\,. \tag{1.46}$$

The world-sheet fermions also satisfy the periodic boundary conditions (1.17) which correspond to $\psi^\mu(w+2\pi) = e^{2\pi i\nu}\psi(w)$ and $\widetilde{\psi}^\mu(\overline{w}+2\pi) = e^{-2\pi i\widetilde{\nu}}\widetilde{\psi}^\mu(\overline{w})$ in the w-coordinate, and they can be expanded by a Fourier transformation,

$$\psi^\mu(w) = i^{-1/2}\sum_{m=-\infty}^{\infty}\psi_{m+\nu}^\mu e^{i(m+\nu)w}\,, \qquad \widetilde{\psi}^\mu(\overline{w}) = i^{1/2}\sum_{m=-\infty}^{\infty}\widetilde{\psi}_{m+\widetilde{\nu}}^\mu e^{-i(m+\widetilde{\nu})\overline{w}}\,. \tag{1.47}$$

Using a collective notation $r = m+\nu$, the expansions (1.47) can be expressed in the z coordinate as

$$\psi^\mu(z) = \sum_{r\in\mathbf{Z}+\nu}\frac{\psi_r^\mu}{z^{r+\frac{1}{2}}}\,, \qquad \widetilde{\psi}^\mu(\overline{z}) = \sum_{r\in\mathbf{Z}+\widetilde{\nu}}\frac{\widetilde{\psi}_r^\mu}{\overline{z}^{r+\frac{1}{2}}}\,. \tag{1.48}$$

Then, anti-commutation relations are given by

$$\left\{\psi_r^\mu, \psi_{-s}^\nu\right\} = \left\{\widetilde{\psi}_r^\mu, \widetilde{\psi}_{-s}^\nu\right\} = \eta^{\mu\nu}\delta_{r,s}\,. \tag{1.49}$$

The ghost fields satisfy the same boundary condition as the energy-momentum tensor (1.29) for the bc ghosts and as the SUSY current (1.33) for the $\beta\gamma$ ghosts. Therefore, they satisfy the same boundary condition as ∂X^μ for the bc and ψ^μ for the $\beta\gamma$,

$$b(z) = \sum_{m=-\infty}^{\infty}\frac{b_m}{z^{m+2}}\,, \qquad c(z) = \sum_{m=-\infty}^{\infty}\frac{c_m}{z^{m-1}}\,, \tag{1.50}$$

$$\beta(z) = \sum_{r\in\mathbf{Z}+\nu}\frac{\beta_r}{z^{r+\frac{3}{2}}}\,, \qquad \gamma(z) = \sum_{r\in\mathbf{Z}+\nu}\frac{\gamma_r}{z^{r-\frac{1}{2}}}\,, \tag{1.51}$$

[9] Single-valuedness and hermiticity of X^μ implies that $\alpha_0^\mu = \widetilde{\alpha}_0^\mu$, $(\alpha_m^\mu)^\dagger = \alpha_{-m}^\mu$ and $(\widetilde{\alpha}_m^\mu)^\dagger = \widetilde{\alpha}_{-m}^\mu$.

as well as the antiholomorphic ghost fields. Similarly, anti-commutation and commutation relations are obtained as

$$\{b_m, c_{-n}\} = \{\widetilde{b}_m, \widetilde{c}_{-n}\} = \delta_{m,n}\,, \tag{1.52}$$

$$[\gamma_r, \beta_{-s}] = [\widetilde{\gamma}_r, \widetilde{\beta}_{-s}] = \delta_{r,s}\,. \tag{1.53}$$

Open string

In the case of the open string, the world-sheet scalar satisfies the Neumann boundary condition (1.18) which corresponds to $\partial X^\mu(\mathrm{Im}\, z = 0) = \overline{\partial} X^\mu(\mathrm{Im}\, \overline{z} = 0)$ in the z-coordinate. Then, the mode operators of the left-moving and right-moving are equivalent, $\alpha^\mu_m = \widetilde{\alpha}^\mu_m$,

$$\partial X^\mu(z) = -i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^\mu_m}{z^{m+1}}\,, \qquad \overline{\partial} X^\mu(\overline{z}) = -i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^\mu_m}{\overline{z}^{m+1}}\,, \tag{1.54}$$

$$X^\mu(z,\overline{z}) = x^\mu - i\sqrt{\frac{\alpha'}{2}}\,\alpha^\mu_0 \ln|z|^2 + i\sqrt{\frac{\alpha'}{2}} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{\alpha^\mu_m}{m}\left(\frac{1}{z^m} + \frac{1}{\overline{z}^m}\right). \tag{1.55}$$

Note that space-time momentum of an open string is half of that of a closed string, $p^\mu = \frac{1}{\sqrt{2\alpha'}}\alpha^\mu_0$.

The world-sheet fermions also satisfy the Neumann boundary conditions (1.19) which correspond to $\psi^\mu(z) = e^{2\pi i\nu}\widetilde{\psi}^\mu(\overline{z})$ for $\sigma^1 = \pi$ and $\psi^\mu(z) = \widetilde{\psi}^\mu(\overline{z})$ for $\sigma^1 = 0$, in the z-coordinate. The mode operators of the left-moving and right-moving are equivalent, $\psi^\mu_r = \widetilde{\psi}^\mu_r$,

$$\psi^\mu(z) = \sum_{r\in\mathbf{Z}+\nu} \frac{\psi^\mu_r}{z^{r+\frac{1}{2}}}\,, \qquad \widetilde{\psi}^\mu(\overline{z}) = \sum_{r\in\mathbf{Z}+\nu} \frac{\psi^\mu_r}{\overline{z}^{r+\frac{1}{2}}}\,. \tag{1.56}$$

States

We now focus on the open string or the holomorphic part of the closed string. For the world-sheet scalar, as it is found from the commutation relation of mode operators in eq.(1.45), one can consider α^μ_m for $m < 0$ as a creation operator and α^μ_m for $m > 0$ as an annihilation operator which satisfies

$$\alpha^\mu_m|0;k\rangle = 0\,, \qquad \text{for } m > 0\,, \tag{1.57}$$

where $|0;k\rangle$ is a ground state with space-time momentum k^μ. Then,

$$p^\mu|0;k\rangle = \tfrac{1}{\sqrt{2\alpha'}}\alpha^\mu_0|0;k\rangle = k^\mu|0;k\rangle\,. \tag{1.58}$$

For the bc ghosts, let us define the bc ghost vacuum $|0\rangle_{bc}$ as a state annihilated by mode operators b_m and c_m for $m > 0$:

$$b_m|0\rangle_{bc} = c_m|0\rangle_{bc} = 0\,, \qquad \text{for } m > 0\,. \tag{1.59}$$

The zero modes b_0 and c_0 form two independent vacua which satisfy

$$b_0|{\downarrow}\rangle = 0\,, \quad b_0|{\uparrow}\rangle = |{\downarrow}\rangle\,, \qquad c_0|{\downarrow}\rangle = |{\uparrow}\rangle\,, \quad c_0|{\uparrow}\rangle = 0\,. \tag{1.60}$$

We consider b_0 as an annihilation operator and c_0 as a creation operator, and define the bc ghost vacuum as $|0\rangle_{bc} \equiv |\downarrow\rangle$.

Consider the world-sheet fermion. For the NS sector with $\nu = \frac{1}{2}$, as it is found from the mode expansion (1.56), mode operators ψ^μ_r are labeled by $r = \frac{1}{2}, \frac{3}{2}, \ldots$, and they have no zero modes. One can define the NS sector vacuum $|0\rangle_{\rm NS}$ as a state annihilated by ψ^μ_r for $r > 0$:

$$\psi^\mu_r |0\rangle_{\rm NS} = 0\,, \qquad \text{for } r = \tfrac{1}{2}, \tfrac{3}{2}, \ldots\,. \tag{1.61}$$

On the other hand, for the Ramond sector with $\nu = 0$, when we define the Ramond sector vacuum $|0\rangle_{\rm R}$ as a state annihilated by ψ^μ_r for $r > 0$, the zero modes ψ^μ_0 form degenerate vacua,

$$|0\rangle_{\rm R}\,, \quad \psi^\mu_0|0\rangle_{\rm R}\,, \quad \psi^\mu_0\psi^\nu_0|0\rangle_{\rm R}\,, \quad \psi^\mu_0\psi^\nu_0\psi^\rho_0|0\rangle_{\rm R}\,, \quad \ldots\,. \tag{1.62}$$

Recall the commutation relation (1.49), the ψ^μ_0 satisfy the Dirac gamma matrix algebra when $\Gamma^\mu \cong \sqrt{2}\psi^\mu_0$,

$$\{\psi^\mu_0, \psi^\nu_0\} = \eta^{\mu\nu} \quad \Leftrightarrow \quad \{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{1.63}$$

Therefore, the degenerate vacua (1.62) form a spinor representation of $\mathrm{SO}(1, D-1)$. Due to $D = 10$, Γ^μ are grouped into five pairs of raising and lowering operators,

$$\Gamma^{a\pm} = \begin{cases} \frac{1}{2}\big(\pm\Gamma^0 + \Gamma^1\big) & a = 0\,, \\ \frac{1}{2}\big(\Gamma^{2a} \pm i\Gamma^{2a+1}\big) & a = 1, \ldots, 4\,, \end{cases} \tag{1.64}$$

which satisfy $\{\Gamma^{a\pm}, \Gamma^{b\mp}\} = \delta^{ab}$ and $\{\Gamma^{a\pm}, \Gamma^{b\pm}\} = 0$. Then, we can take a basis of eigenstates of the Lorentz generator $S_a = \Gamma^{a+}\Gamma^{a-} - \frac{1}{2}$ with the eigenvalues $s_a = \pm\frac{1}{2}$,[10]

$$|\mathbf{s}\rangle = |s_0, s_1, s_2, s_3, s_4\rangle = \big(\Gamma^{0+}\big)^{s_0+\frac{1}{2}} \cdots \big(\Gamma^{4+}\big)^{s_4+\frac{1}{2}} |-,-,-,-,-\rangle\,. \tag{1.65}$$

Thus, the ground state in the Ramond sector, $|\mathbf{s}\rangle_{\rm R}$, corresponds to an eigenstate of the space-time spin $\pm\frac{1}{2}$ in eq.(1.65). Therefore, we can expect to obtain space-time fermions.

For the $\beta\gamma$ ghosts, the $\beta\gamma$ NS vacuum is defined in the same manner as the NS vacuum for the world-sheet fermion (1.61),

$$\beta_r|0\rangle^{\beta\gamma}_{\rm NS} = \gamma_r|0\rangle^{\beta\gamma}_{\rm NS} = 0\,, \qquad \text{for } r = \tfrac{1}{2}, \tfrac{3}{2}, \ldots\,. \tag{1.66}$$

The $\beta\gamma$ Ramond vacuum is given just as the bc ghost vacuum (1.59) by

$$\beta_r|0\rangle^{\beta\gamma}_{\rm R} = 0\,, \quad \text{for } r \geq 0\,, \qquad \gamma_r|0\rangle^{\beta\gamma}_{\rm R} = 0\,, \quad \text{for } r > 0\,, \tag{1.67}$$

where the zero mode β_0 is defined as an annihilation operator, and γ_0 as a creation operator.

[10] Here $|-,-,-,-,-\rangle$ in eq.(1.65) is a state with all spin eigenvalues $-\frac{1}{2}$.

The Virasoro generators

The energy-momentum tensor which is the current of conformal symmetry (1.29) and the SUSY current (1.33) are also expanded by a Laurant expansion as well as the world-sheet fields,

$$T_B(z) = \sum_{m=-\infty}^{\infty} \frac{L_m}{z^{m+2}}\,, \qquad \widetilde{T}_B(\overline{z}) = \sum_{m=-\infty}^{\infty} \frac{\widetilde{L}_m}{\overline{z}^{m+2}}\,, \tag{1.68}$$

$$T_F(z) = \sum_{r\in\mathbf{Z}+\nu} \frac{G_r}{z^{r+\frac{3}{2}}}\,, \qquad \widetilde{T}_F(\overline{z}) = \sum_{r\in\mathbf{Z}+\widetilde{\nu}} \frac{\widetilde{G}_r}{\overline{z}^{r+\frac{3}{2}}}\,. \tag{1.69}$$

The expansions are obtained by the correspondence to the boundary conditions of ∂X^μ and ψ^μ, for both closed and open string. Then, $L_m = \widetilde{L}_m$ and $G_r = \widetilde{G}_r$ for an open string.

The expansion coefficients in (1.68) and (1.69) correspond to conserved quantities of each current. Therefore, L_m and $\widetilde{L}_m$ are the generators of conformal transformation (called the Virasoro generators), while G_r and $\widetilde{G}_r$ are the SUSY generator.

Since the energy-momentum tensor and the SUSY current are written by products of the world-sheet fields in eqs.(1.29) and (1.33), the Virasoro generators and the SUSY generators are expanded in terms of mode operators of the world-sheet fields. For the open string,

$$L_m = L_m^{\mathrm{m}} + L_m^{\mathrm{g}}\,, \tag{1.70}$$

where

$$L_m^{\mathrm{m}} = \begin{cases} \dfrac{1}{2}\displaystyle\sum_{n=-\infty}^{\infty} {}^\circ_\circ \alpha^\mu_{m-n}\alpha_{n\mu} {}^\circ_\circ + \dfrac{1}{4}\sum_{r\in\mathbf{Z}+\nu} (2r-m) {}^\circ_\circ \psi^\mu_{m-r}\psi_{r\mu} {}^\circ_\circ & m \neq 0\,, \\ \alpha' p^2 + \displaystyle\sum_{n=1}^{\infty} {}^\circ_\circ \alpha^\mu_{-n}\alpha_{n\mu} {}^\circ_\circ + \sum_{r\in\mathbf{N}-\nu} r {}^\circ_\circ \psi^\mu_{-r}\psi_{r\mu} {}^\circ_\circ + a^{\mathrm{m}} & m = 0\,, \end{cases} \tag{1.71}$$

$$L_m^{\mathrm{g}} = \begin{cases} \displaystyle\sum_{n=-\infty}^{\infty} (m+n) {}^\circ_\circ b_{m-n} c_n {}^\circ_\circ + \dfrac{1}{2}\sum_{r\in\mathbf{Z}+\nu} (m+2r) {}^\circ_\circ \beta_{m-r}\gamma_r {}^\circ_\circ & m \neq 0\,, \\ \displaystyle\sum_{n=1}^{\infty} n {}^\circ_\circ (b_{-n}c_n + c_{-c}b_n) {}^\circ_\circ + \sum_{r\in\mathbf{N}-\nu} r {}^\circ_\circ (\beta_{-r}\gamma_r - \gamma_{-r}\beta_r) {}^\circ_\circ + a^{\mathrm{g}} & m = 0\,, \end{cases} \tag{1.72}$$

and

$$G_r = G_r^{\mathrm{m}} + G_r^{\mathrm{g}} = \sum_{n=-\infty}^{\infty} \alpha^\mu_n \psi_{r-n\mu} - \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}(2r+n)\beta_{r-n}c_n + 2b_n\gamma_{r-n}\right], \tag{1.73}$$

where ${}^\circ_\circ\ {}^\circ_\circ$ denotes the creation-annihilation normal ordering, and a^{m} and a^{g} are normal ordering constants. The normal ordering constant appears only in the zero mode of the Virasoro generator, L_0. A sum of the normal ordering constants, $a = a^{\mathrm{m}} + a^{\mathrm{g}}$, corresponds to zero-point energy, and it is calculated using a result obtained by an analytical continuation[11] as follows,

$$a = a^X + a^\psi + a^{bc} + a^{\beta\gamma} = \begin{cases} -\dfrac{D}{24} + \dfrac{D}{24} + \dfrac{1}{12} - \dfrac{1}{12} = 0 & \text{for R sector}\,, \\ -\dfrac{D}{24} - \dfrac{D}{48} + \dfrac{1}{12} + \dfrac{1}{24} = -\dfrac{1}{2} & \text{for NS sector}\,. \end{cases} \tag{1.74}$$

[11] The result is $\sum_{n=1}^{\infty}(n-\theta) = \frac{1}{24} - \frac{1}{8}(2\theta-1)^2$.

Physical states

Finally, we discuss physical states based on the old covariant quantization approach, where we simply ignore the ghosts and impose the conditions that the Virasoro and SUSY lowering operators in matter sector annihilate physical states:

$$\begin{aligned} L^{\rm m}_m|{\rm phys}\rangle &= \widetilde{L}^{\rm m}_m|{\rm phys}\rangle = 0\,, \qquad \text{for } m>0\,,\\ G^{\rm m}_r|{\rm phys}\rangle &= \widetilde{G}^{\rm m}_r|{\rm phys}\rangle = 0\,, \qquad \text{for } r\geq 0\,. \end{aligned} \tag{1.75}$$

These conditions implies that matrix elements between the physical states do not depend on the gauge, since classically the energy-momentum tensor in the matter sector varies the world-sheet metric.

Furthermore, we impose the condition that the Virasoro zero operator in the matter and the ghost sector annihilates physical states:

$$\big(L^{\rm m}_0 + L^{\rm g}_0\big)|{\rm phys}\rangle = \big(\widetilde{L}^{\rm m}_0 + \widetilde{L}^{\rm g}_0\big)|{\rm phys}\rangle = 0\,. \tag{1.76}$$

The condition implies that the world-sheet Hamiltonian and translation operator annihilate physical states, since

$$H = \frac{1}{2\pi}\int_0^{2\pi} d\sigma^1\, T_{22} = L_0 + \widetilde{L}_0\,, \qquad P = \frac{1}{2\pi}\int_0^{2\pi} d\sigma^1\, T_{12} = L_0 - \widetilde{L}_0\,. \tag{1.77}$$

The condition (1.76) determines the space-time mass of the state.

For the open string, let us impose the physical state conditions (1.75) and (1.76) on the following lowest three states:

- The NS sector ground state, $|0;k\rangle_{\rm NS}$,

$$(L^{\rm m}_0 + L^{\rm g}_0)|0;k\rangle_{\rm NS} = (\alpha' k^2 - \tfrac{1}{2})|0;k\rangle_{\rm NS} = 0\,. \tag{1.78}$$

  Since $m^2 = -k^2$, the state has tachyonic mass, $m^2 = -\frac{1}{2\alpha'}$.

- The NS sector first excited state, $|e;k\rangle_{\rm NS} = e_\mu \psi^\mu_{-1/2}|0;k\rangle_{\rm NS}$,

$$\begin{aligned} (L^{\rm m}_0 + L^{\rm g}_0)|e;k\rangle_{\rm NS} &= \alpha' k^2 |e;k\rangle_{\rm NS} = 0\,,\\ G^{\rm m}_{1/2}|e;k\rangle_{\rm NS} &= \sqrt{2\alpha'}\, e_\mu k^\mu |0;k\rangle_{\rm NS} = 0\,, \end{aligned} \tag{1.79}$$

  where e_μ is the polarization vector. The state is massless, $m^2 = 0$. Since $e\cdot k = 0$, it has no unphysical time-like and longitudinal polarization. Therefore, it forms a vector representation of SO(8).

  The state is physically equivalent to a state including itself and

$$G^{\rm m}_{-1/2}|0;k\rangle_{\rm NS} = \sqrt{2\alpha'}\, k_\mu \psi^\mu_{-1/2}|0;k\rangle_{\rm NS}\,, \tag{1.80}$$

  which implies that there is the space-time gauge symmetry, $e_\mu \cong e_\mu + \lambda k_\mu$.

- The Ramond sector ground state, $|u;k\rangle_{\rm R} = |\mathbf{s};k\rangle_{\rm R}\, u_{\mathbf{s}}$,

$$\begin{aligned}(L_0^{\rm m} + L_0^{\rm g})|u;k\rangle_{\rm R} &= \alpha' k^2 |u;k\rangle_{\rm R} = 0\,,\\ G_0^{\rm m}|u;k\rangle_{\rm R} &= \sqrt{\alpha'}\, k_\mu |\mathbf{s}';k\rangle_{\rm R}\, \Gamma^\mu_{\mathbf{s}'\mathbf{s}} u_{\mathbf{s}} = 0\,,\end{aligned} \tag{1.81}$$

where $u_{\mathbf{s}}$ is the polarization spinor. The state is massless, and it satisfies the Dirac equation, $k\cdot\Gamma_{\rm s's}\,u_{\rm s} = 0$. This means that a spinor representation of SO(1,9) in eq.(1.65) is reduced to two spinor representations of SO(8), with chirality $\pm$.

1.3 Type I and type II superstring theory

The GSO projection

In spite of imposing the physical state conditions, unwanted states remain. For example, the NS sector ground state $|0;k\rangle_{\rm NS}$ is a tachyon, and the NS sector second excited state $\psi^\mu_{-1/2}\psi^\nu_{-1/2}|0;k\rangle_{\rm NS}$ is a space-time tensor but has fermionic statistics.[12] These unphysical states are projected out by the GSO projection.

The GSO parity operator is defined as $e^{\pi i F}$, where F is a sum of the world-sheet fermion number, a sum of spin eigenvalues of the space-time fermion, and the $\beta\gamma$ ghost number,[13]

$$F \equiv \sum_{a=0}^{4} S_a + N_{\beta\gamma}\,, \qquad S_a = \begin{cases} +\dfrac{1}{2}\displaystyle\sum_{r\in\mathbf{Z}+\nu}\left[\psi^0_r, \psi^1_{-r}\right] & a=0\,,\\ -\dfrac{i}{2}\displaystyle\sum_{r\in\mathbf{Z}+\nu}\left[\psi^{2a}_r, \psi^{2a+1}_{-r}\right] & a=1,\ldots,4\,.\end{cases} \tag{1.82}$$

S_a is the Lorentz generator, and it has the property of counting the number of the world-sheet fermion operator, $\left[S_a, (\psi^{2a}_r \pm i\psi^{2a+1}_r)\right] = \pm(\psi^{2a}_r \pm i\psi^{2a+1}_r)$. For the space-time fermion, $\sum_{a=0}^4 S_a$ gives a sum of the spin eigenvalues. Then, $\exp\left(\pi i\sum_{a=0}^4 s_a\right)$ gives (i times) chirality of the space-time fermion which takes the value $+1$ when the spin eigenvalues s_a include an even number of $-\frac{1}{2}$ and -1 for an odd number of $-\frac{1}{2}$.

We now consider a case of the open string. The NS sector ground state $|0;k\rangle_{\rm NS}$ has the GSO parity $-$, since the state has the $\beta\gamma$ ghost number -1,

$$e^{\pi i F}|0;k\rangle_{\rm NS} = -|0;k\rangle_{\rm NS}\,. \tag{1.83}$$

The NS sector first excited state and second excited state have the GSO parity $+$ and $-$, respectively,

$$\begin{aligned} e^{\pi i F}\psi^\mu_{-1/2}|0;k\rangle_{\rm NS} &= +\psi^\mu_{-1/2}|0;k\rangle_{\rm NS}\,,\\ e^{\pi i F}\psi^\mu_{-1/2}\psi^\nu_{-1/2}|0;k\rangle_{\rm NS} &= -\psi^\mu_{-1/2}\psi^\nu_{-1/2}|0;k\rangle_{\rm NS}\,.\end{aligned} \tag{1.84}$$

If we restrict to only states with the GSO parity $+$ for the NS sector, unwanted states such as the tachyon and the space-time tensor with fermionic statistics are projected out.

[12] $|0;k\rangle_{\rm NS}$ has the bc ghost number $+1$ in the z coordinate. It has fermionic statistics.

[13] The reason why F includes the $\beta\gamma$ ghost number is that the $\beta\gamma$ ghosts are associated with the SUSY current which is the world-sheet spinor.

The Ramond sector ground state $|\mathbf{s}\rangle_{\rm R}$ has the GSO parity corresponding to the chirality of the state, since it has the $\beta\gamma$ ghost number $-\frac{1}{2}$,

$$e^{\pi i F}|\mathbf{s}\rangle_{\rm R} = |\mathbf{s}'\rangle_{\rm R}\,\Gamma_{\mathbf{s}'\mathbf{s}}\,, \tag{1.85}$$

where Γ is the chirality matrix. The GSO projection for the Ramond sector which restricts to states with the GSO parity + leaves only fermions with chirality +. As a result, the Ramond sector ground state $|\mathbf{s}\rangle_{\rm R}$ becomes a superpartner of the NS sector first excited state $\psi^{\mu}_{-1/2}|0;k\rangle_{\rm NS}$. The former is a gaugino and the latter is a gauge boson. This means $\mathcal{N}=1$ SUSY in a ten-dimensional space-time.

Thus, there are two possible choices of the GSO projections which leave only physical states and keep supersymmetry, NS+ and R+, or NS+ and R−, in the case of the open string.

Type IIA and IIB theory

We now consider the closed string. The on-shell condition (1.76) requires that the left-moving and right-moving of the closed string satisfy the following relation,

$$m^2 = \frac{4}{\alpha'}\big(N_\alpha + N_\psi - \nu\big) = \frac{4}{\alpha'}\big(\widetilde{N}_\alpha + \widetilde{N}_\psi - \widetilde{\nu}\big)\,, \tag{1.86}$$

where $N_\alpha + N_\psi - \nu$ is the left-moving level and $\widetilde{N}_\alpha + \widetilde{N}_\psi - \widetilde{\nu}$ is the right-moving level of the closed string state. Here, N_α and N_ψ are excitation levels of the state due to the respective mode operators, α^{μ}_{n} and ψ^{μ}_{r},

$$\begin{aligned} N_\alpha &= \sum_{n=1}^{\infty} {:}\alpha^{\mu}_{-n}\alpha_{n\mu}{:} = 0,1,2,\dots\,, \\ N_\psi &= \sum_{r\in\mathbf{N}-\nu} r\,{:}\psi^{\mu}_{-r}\psi_{r\mu}{:} = \begin{cases} 0,1,2,\dots & \text{for R sector}\,, \\ 0,\frac{1}{2},1,\frac{3}{2},2,\dots & \text{for NS sector}\,. \end{cases} \end{aligned} \tag{1.87}$$

Therefore, only the NS sector with the GSO parity − takes the values of $(N_\psi - \nu)$ half-integers, and the NS− sector cannot pair with the other three sectors, NS+, R+ and R−.

Possible tachyonic and massless states of the closed string are as follows,

- (NS−,NS−) sector,

$$|0,\widetilde{0};k\rangle_{\text{NS-NS}}\,, \tag{1.88}$$

- (NS+,NS+) sector,

$$\psi^{\mu}_{-1/2}\widetilde{\psi}^{\nu}_{-1/2}|0,\widetilde{0};k\rangle_{\text{NS-NS}}\,, \tag{1.89}$$

- (R,R) sector,

$$|\mathbf{s},\widetilde{\mathbf{s}};k\rangle_{\text{R-R}}\,, \tag{1.90}$$

- (NS+,R) and (R,NS+) sector,

$$\psi^{\mu}_{-1/2}|0,\widetilde{\mathbf{s}};k\rangle_{\text{NS-R}}\,, \qquad \widetilde{\psi}^{\mu}_{-1/2}|\mathbf{s},\widetilde{0};k\rangle_{\text{R-NS}}\,. \tag{1.91}$$

Only the (NS−,NS−) sector has tachyonic mass, $m^2 = -\frac{2}{\alpha'}$, and all the remaining sectors are massless.

Since the NS+ sector forms a SO(8) vector denoted by $\mathbf{8}_v$, and the R± sectors form two SO(8) spinors with chirality ± denoted by $\mathbf{8}$ and $\mathbf{8}'$, products of them are decomposed into representations under the Lorentz transformation. In the (NS+, NS+) sector, $\mathbf{8}_v \times \mathbf{8}_v = [0] + [2] + (2)$, where [0] is a scalar, [2] is an antisymmetric two-tensor, and (2) is a traceless symmetric two-tensor. These representations correspond to the dilaton, the antisymmetric B-field, and the graviton. In the (NS+, R+) sector, $\mathbf{8}_v \times \mathbf{8} = \mathbf{8}' + \mathbf{56}$, where $\mathbf{56}$ is a vector-spinor. The vector-spinor corresponds to the gravitino.

Now, let us search for the GSO projections which project out the tachyonic state, are consistent with modular invariance[14], close the sector and conserve the GSO parity as follows

$$\mathrm{NS} \times \mathrm{NS} = \mathrm{NS}\,, \qquad \mathrm{R} \times \mathrm{R} = \mathrm{NS}\,, \qquad \mathrm{NS} \times \mathrm{R} = \mathrm{R}\,, \tag{1.92}$$

$$+ \times + \ = +\,, \qquad - \times - = +\,, \qquad + \times - = -\,. \tag{1.93}$$

There are two possible choices of the GSO projections, called type IIA and IIB,

$$\mathrm{IIA} \ : \ (\mathrm{NS}+, \mathrm{NS}+) \ (\mathrm{R}+, \mathrm{R}-) \ (\mathrm{NS}+, \mathrm{R}-) \ (\mathrm{R}+, \mathrm{NS}+)\,, \tag{1.94}$$

$$\mathrm{IIB} \ : \ (\mathrm{NS}+, \mathrm{NS}+) \ (\mathrm{R}+, \mathrm{R}+) \ (\mathrm{NS}+, \mathrm{R}+) \ (\mathrm{R}+, \mathrm{NS}+)\,, \tag{1.95}$$

in addition to those replaced by opposite chirality in the Ramond sector of eqs.(1.94) and (1.95).

Both of the type IIA and IIB have two massless gravitinos in the NS-R and R-NS sectors, which means $\mathcal{N} = 2$ SUSY. In the type IIA, the gravitinos have opposite chiralities and in the type IIB, they have the same chirality. A difference between the IIA and IIB is also in the R-R sector, and the representations are decomposed as

$$\mathbf{8} \times \mathbf{8} = [0] + [2] + [4]_+ \qquad \text{for } (\mathrm{R}+, \mathrm{R}+) \text{ sector}\,, \tag{1.96}$$

$$\mathbf{8} \times \mathbf{8}' = [1] + [3] \qquad \text{for } (\mathrm{R}+, \mathrm{R}-) \text{ sector}\,, \tag{1.97}$$

where $[n]$ is an anti-symmetric n-tensor. Then, the massless spectra in the IIA and IIB are

$$\mathrm{IIA} \ : \ [0] + [1] + [2] + [3] + (2) + \mathbf{8} + \mathbf{8}' + \mathbf{56} + \mathbf{56}'\,, \tag{1.98}$$

$$\mathrm{IIB} \ : \ [0] + [0] + [2] + [2] + [4]_+ + (2) + \mathbf{8}' + \mathbf{8}' + \mathbf{56} + \mathbf{56}\,, \tag{1.99}$$

which form supergravity multiplets of ten-dimensional $\mathcal{N} = (1, 1)$ and $\mathcal{N} = (2, 0)$ SUSY, for the IIA and IIB, respectively. The type IIA and IIB are superstring theories which are consistent in a flat ten-dimensional space-time, including the closed string only.

[14]The modular invariance is required to make one-loop amplitudes consistent. As a condition for the modular invariance, we require at least one left-moving R sector and at least one right-moving R sector.

Type I theory

We have obtained the type IIA and IIB as the closed string theories in a ten-dimensional space-time. How is the open string introduced? Since one can expect an interaction between two open strings generates one closed string, there is always the closed string. Therefore, we have to introduce the open string in a closed string theory such as the type IIA and IIB. However, we confirmed that the GSO projection for the open string resulted in $\mathcal{N}=1$ SUSY in a ten-dimensional space-time. Therefore, in order to introduce the open string, we should break $\mathcal{N}=2$ into $\mathcal{N}=1$.

Let us consider imposing the world-sheet parity projection on the type IIB. The world-sheet parity transformation for the closed string is

$$\Omega \ : \ (\tau, \sigma) \to (\tau, 2\pi - \sigma)\,, \tag{1.100}$$

which reverses the left-moving and right-moving. It is found that the type IIB has symmetry under Ω. The mode operators of the world-sheet fermion and the NS-NS and NS-R sector ground states are transformed under Ω as follows,

$$\Omega\, \psi^\mu_r\, \Omega^{-1} = e^{-2\pi i r} \widetilde{\psi}^\mu_r\,, \qquad \Omega\, \widetilde{\psi}^\mu_r\, \Omega^{-1} = e^{2\pi i r} \psi^\mu_r\,, \tag{1.101}$$

$$\Omega\,|0,\widetilde{0};k\rangle_{\text{NS-NS}} = -\,|0,\widetilde{0};k\rangle_{\text{NS-NS}}\,, \qquad \Omega\,|0,\widetilde{\mathbf{s}};k\rangle_{\text{NS-R}} = -\,|\mathbf{s},\widetilde{0};k\rangle_{\text{R-NS}}\,. \tag{1.102}$$

Consequently, the antisymmetric two-tensor in the NS-NS sector and an antisymmetric combination of the vector spinors in the NS-R and R-NS sectors have the world-sheet parity odd,

$$\Omega\big(\psi^\mu_{-1/2}\widetilde{\psi}^\nu_{-1/2} - \psi^\nu_{-1/2}\widetilde{\psi}^\mu_{-1/2}\big)|0,\widetilde{0};k\rangle_{\text{NS-NS}} = -\big(\psi^\mu_{-1/2}\widetilde{\psi}^\nu_{-1/2} - \psi^\nu_{-1/2}\widetilde{\psi}^\mu_{-1/2}\big)|0,\widetilde{0};k\rangle_{\text{NS-NS}}\,, \tag{1.103}$$

$$\Omega\big(\psi^\mu_{-1/2}|0,\widetilde{\mathbf{s}};k\rangle_{\text{NS-R}} - \widetilde{\psi}^\mu_{-1/2}|\mathbf{s},\widetilde{0};k\rangle_{\text{R-NS}}\big) = -\big(\psi^\mu_{-1/2}|0,\widetilde{\mathbf{s}};k\rangle_{\text{NS-R}} - \widetilde{\psi}^\mu_{-1/2}|\mathbf{s},\widetilde{0};k\rangle_{\text{R-NS}}\big)\,, \tag{1.104}$$

and the antisymmetric two-tensor in the R-R sector has the world-sheet parity even,

$$\Omega\big(|\mathbf{s},\widetilde{\mathbf{s}}';k\rangle_{\text{R-R}} - |\mathbf{s}',\widetilde{\mathbf{s}};k\rangle_{\text{R-R}}\big) = +\big(|\mathbf{s},\widetilde{\mathbf{s}}';k\rangle_{\text{R-R}} - |\mathbf{s}',\widetilde{\mathbf{s}};k\rangle_{\text{R-R}}\big)\,. \tag{1.105}$$

If we restrict to only states with the world-sheet parity even, some of the massless states in the type IIB are projected out. Thus, we obtain a theory with the world-sheet parity projection, which is called the type I. A closed string part of the massless spectrum in the type I is

$$\text{I}_{\text{closed}} \ : \ [0] + [2] + (2) + \mathbf{8}' + \mathbf{56}\,. \tag{1.106}$$

The spectrum contains only one gravitino, which means that there is ten-dimensional $\mathcal{N}=1$ SUSY in the type I theory.

Before considering the world-sheet parity projection for the open string, let us add new degrees of freedom to each end of the open string. For example, the NS first excited state is then

$$e_\mu \psi^\mu_{-1/2}|0;k;i,\overline{\jmath}\rangle_{\text{NS}}\,, \tag{1.107}$$

where i and $\bar{j}$ denotes states of $\sigma = 0$ and $\sigma = \pi$ endpoints, running from 1 to N. Taking N^2 Hermitian matrices which are the representation matrices of $\mathrm{U}(N)$ as a complete set,

$$e_\mu \psi^\mu_{-1/2}|0;k;a\rangle_{\mathrm{NS}} = \sum_{i,\bar{j}} e_\mu \psi^\mu_{-1/2}|0;k;i,\bar{j}\rangle_{\mathrm{NS}} \, (T^a)_i{}^{\bar{j}} \,, \tag{1.108}$$

and one can expect that the massless vector bosons are associated with a $\mathrm{U}(N)$ gauge symmetry. T^a is called the Chan-Paton factor.

Now consider the world-sheet parity transformation for the open string,

$$\Omega \;\;:\;\; (\tau, \sigma) \to (\tau, \pi - \sigma) \,, \tag{1.109}$$

which reverses the endpoints of the open string. The NS sector first excited state are transformed under Ω as follows,

$$\Omega \, e_\mu \psi^\mu_{-1/2}|0;k;i,j\rangle_{\mathrm{NS}} = -e_\mu \psi^\mu_{-1/2}|0;k;j,i\rangle_{\mathrm{NS}} \,. \tag{1.110}$$

If we restrict states with the world-sheet parity even, the Chan-Paton factors should be real antisymmetric matrices. Since they are the representation matrices of $\mathrm{SO}(N)$, one can expect that the massless vector bosons are associated with a $\mathrm{SO}(N)$ gauge symmetry.

Thus, we obtain the type I theory including both the closed string and open string, by the world-sheet parity projection of the type IIB. The massless spectrum is

$$\mathrm{I} \;\;:\;\; [0] + [2] + (2) + \mathbf{8}' + \mathbf{56} + (\mathbf{8}_v + \mathbf{8})_{\mathrm{SO}(N)} \,. \tag{1.111}$$

Tadpole cancellation

In string theory including the open string, there is the open-closed string duality which means that one-loop amplitudes of the open string are understood as tree-level amplitudes of the closed string. Each string amplitude corresponds to a specific world-sheet diagram with a certain topology. An one-loop diagram of the open string is an annulus while a tree-level diagram of the closed string is a cylinder, and they have the same topology with two boundaries. The open-closed duality also means that one-loop vacuum amplitudes of the open string always contain contributions of tree-level vacuum-to-vacuum amplitudes of the closed string, and that there are infrared divergences by a tree-level propagation of the massless closed string.

In the type I theory with the world-sheet parity projection, there are more diagrams which contribute the vacuum-to-vacuum amplitudes. Another one-loop diagram of the open string is the Möbius strip with a boundary and a crosscap. Another tree-level digram of the closed string is the Klein bottle with two crosscaps. The vacuum-to-vacuum amplitudes which are contributed by these diagrams are canceled out, if a gauge symmetry of the type I theory is SO(32).

This is called the tadpole cancellation. In the next section, we introduce "D-branes" in order to include the open string in the type IIA and IIB theories. In general configurations of D-branes, there are cases in which the tadpoles are not canceled. The tadpole which is understood as an exchange of the NS-NS closed string (the dilaton, the antisymmetric B-field, and the graviton) is called the NS-NS tadpole, while the tadpole as an exchange of the Ramond-Ramond closed string is called the R-R tadpole. In particular, the R-R tadpole must be canceled so that gauge symmetries on D-branes are anomaly free. On the other hand, the NS-NS tadpole does not necessarily have to be canceled. However, in non-SUSY system, a practical problem remains, which there are infrared divergences by an exchange of the massless NS-NS closed string.

2 Intersecting D-brane models

2.1 Introduction of D-branes

Dirichlet boundary condition

In the previous section, we have imposed the Neumann boundary condition on the open string. However, as it is shown in eq.(1.20), the Dirichlet boundary condition can be also imposed. Since the Dirichlet boundary condition fix the position of endpoints of the open string, it breaks the Poincaré invariance in a ten-dimensional space-time.

Let us consider the open string with the Neumann boundary condition in a four-dimensional space-time $\mu = 0, \ldots, 3$, and the Dirichlet boundary condition in a six-dimensional space $i = 4, \ldots, 9$. Since eq.(1.20) can be recast into $\partial_\tau X^i(\tau, \sigma = \ell) = \partial_\tau X^i(\tau, \sigma = 0) = 0$, the Dirichlet boundary conditions on the world-sheet fields are given in the z-coordinate by

$$\partial X^i(\operatorname{Im} z = 0) = -\overline{\partial} X^i(\operatorname{Im} \overline{z} = 0)\,, \tag{2.1}$$

$$\psi^i(z) = -e^{2\pi i \nu} \widetilde{\psi}^i(\overline{z}) \quad \text{for } \sigma^1 = \pi\,, \qquad \psi^i(z) = -\widetilde{\psi}^i(\overline{z}) \quad \text{for } \sigma^1 = 0\,. \tag{2.2}$$

Then, the world-sheet fields in the Dirichlet direction are expanded as follows:

$$\partial X^i(z) = -i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^i_m}{z^{m+1}}\,, \qquad \overline{\partial} X^i(\overline{z}) = +i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^i_m}{\overline{z}^{m+1}}\,, \tag{2.3}$$

$$\psi^i(z) = \sum_{r\in \mathbf{Z}+\nu} \frac{\psi^i_r}{z^{r+\frac{1}{2}}}\,, \qquad \widetilde{\psi}^i(\overline{z}) = -\sum_{r\in \mathbf{Z}+\nu} \frac{\psi^i_r}{\overline{z}^{r+\frac{1}{2}}}\,. \tag{2.4}$$

Note that the sign of the antiholomorphic field is opposite to the Neumann case (1.42). There is no center-of-mass momentum in the Dirichlet direction as it is found from eq.(1.44), and a mode expansion for X^i is

$$X^i(z,\overline{z}) = y^i + i\sqrt{\frac{\alpha'}{2}} \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} \frac{\alpha^i_m}{m}\left(\frac{1}{z^m} - \frac{1}{\overline{z}^m}\right)\,, \tag{2.5}$$

where y^i is a constant vector. Commutation and anti-commutation relations of the mode operators are the same as the Neumann case,

$$\left[\alpha^i_m, \alpha^j_{-n}\right] = m\,\delta^{ij}\delta_{m,n}\,, \qquad \left\{\psi^i_r, \psi^j_{-s}\right\} = \delta^{ij}\delta_{r,s}\,. \tag{2.6}$$

The energy-momentum tensor and the SUSY current can be separated into the Neumann and the Dirichlet direction,

$$T_B^{\rm m}(z) = -\frac{1}{\alpha'}\big(\partial X^\mu \partial X_\mu + \partial X^i \partial X^i\big) - \frac{1}{2}\big(\psi^\mu \partial \psi_\mu + \psi^i \partial \psi^i\big)\,, \tag{2.7}$$

$$T_F^{\rm m}(z) = i\sqrt{\frac{2}{\alpha'}}\big(\partial X^\mu \psi_\mu + \partial X^i \psi^i\big)\,. \tag{2.8}$$

They are expanded so that they satisfy the Dirichlet boundary conditions (2.1) and (2.2), and result in the same as the Neumann case (1.68) and (1.69). The on-shell condition (1.76) determine mass in the four-dimensional space-time. For the NS sector first excited state, the state in the Neumann direction $e_\mu \psi^\mu |0;k\rangle_{\rm NS}$ acts as a gauge boson with $e \cdot k = 0$ and $e_\mu \cong e_\mu + \lambda k_\mu$, while the state in the Dirichlet direction $\psi^i|0;k\rangle_{\rm NS}$ acts as just a scalar:

$$e_\mu \psi^\mu_{-1/2}|0;k\rangle_{\rm NS} \qquad \text{for } \mu = 0,\ldots,3\,, \tag{2.9}$$

$$\psi^i_{-1/2}|0;k\rangle_{\rm NS} \qquad \text{for } i = 4,\ldots,9\,. \tag{2.10}$$

All the physical states are localized in the space-time consist of dimensions with the Neumann boundary condition.

D-branes

Thus, if we impose the Neumann boundary condition on the open string in a $(p+1)$-dimensional space-time and the Dirichlet boundary conditions in a $(9-p)$-dimensional space, we can define the open string which is localized in a p-dimensional space. Since one can expect that an interaction between two open strings generates a closed string, the closed string can be emitted from and absorbed into the p-dimensional space. Therefore, the p-dimensional space acts as a object which scatters the closed string and excites the open string by the absorption of the closed string. This p-dimensional object is called Dp-brane.

The D-brane has a coupling to the NS-NS closed string, and the low-energy effective action which describes this coupling is known as the Dirac-Born-Infeld action,

$$S_{{\rm D}p} = -T_p \int d^{p+1}x\, {\rm Tr}\big\{e^{-\Phi}\big[-\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})\big]^{1/2}\big\}\,, \tag{2.11}$$

where Φ, $G_{\mu\nu}$ and $B_{\mu\nu}$ are components of the NS-NS fields parallel to the Dp-brane, and $F_{\mu\nu}$ is a gauge field living on the Dp-brane. T_p, a tension of the Dp-brane, is given by

$$T_p = \frac{\sqrt{\pi}}{\kappa_{10}}(4\pi^2\alpha')^{\frac{3-p}{2}}\,, \tag{2.12}$$

where κ_{10} is the parameter in the low-energy effective action of the type II (discussed later).

The D-brane also has a coupling to the R-R closed string. At low energies this coupling is described by

$$T_p \int_{p+1} C_{p+1} = T_p \int_{p+1} \tfrac{1}{(p+1)!}\, C_{\mu_1\cdots\mu_{p+1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{p+1}}\,, \tag{2.13}$$

where T_p is the Dp-brane tension (2.12) and $C_{\mu_1 \cdots \mu_{p+1}}$ is the $(p+1)$-dimensional antisymmetric $(p+1)$-tensor which is the R-R field originating from the R-R massless states. As it is found from eqs.(1.96) and (1.97), the R-R massless states in the type IIA are the ten-dimensional antisymmetric tensors of rank 1 and 3, while in the type IIB, they are that of rank 0, 2 and 4. Note that the dimension p of the Dp-brane to which the R-R field can couple is different between the type IIA and IIB. There can be only the D-branes of dimension even in the type IIA and that of dimension odd in the type IIB.

Let us introduce a D-brane in the type IIA or IIB theory. Then, $\mathcal{N} = 2$ SUSY in the IIA or IIB is broken in half on the D-brane. This is the same that $\mathcal{N} = 2$ SUSY in the type IIB must be broken into $\mathcal{N} = 1$ in the type I. The ten-dimensional $\mathcal{N} = 2$ SUSY has 32 supersymmetries. For example, if we introduce a D3-brane, 32 supersymmetries are broken into 16 supersymmetries, and four-dimensional $\mathcal{N} = 4$ SUSY is realized on the D3-brane.

Next, let us introduce a stack of N D-branes. The open string on the stack has N^2 degrees of freedom that each end of the open string attaches on N D-branes. This fact justifies the Chan-Paton factor of eq.(1.108) which is originally added by hand in the type I theory. We can understand that the type I is constructed from the type IIB by the world-sheet parity projection and introduction of a stack of 32 D9-branes. Thus, introducing a stack of n D-branes results in a gauge symmetry on the stack of U(N) without the world-sheet parity projection, and SO(N) or USp(N) with the world-sheet parity projection. The massless vector bosons of the open string on the stack correspond to gauge bosons associated with the gauge symmetry.

One can also consider two parallel D-branes separated by some distance y. Then, there is a stretching open string whose ends attach on the respective D-branes. Because of the string tension, the ground states obtain the mass proportional to the distance:

$$m^2 = \frac{y^2}{(2\pi\alpha')^2} \,. \tag{2.14}$$

2.2 Intersecting D-branes

Neumann-Dirichlet boundary condition

Let us consider a system including two D-branes of different dimensions, a Dp-brane and a Dp'-brane. There are three kinds of open strings, p-p, p'-p' and p-p', with ends on the respective D-branes. The spectra of the p-p and p'-p' strings are the same as obtained in the previous section. However, the p-p' string is new.

When $p < p'$, the p-p' string has the $(p+1)$ Neumann-Neumann boundary conditions, the $(9-p')$ Dirichlet-Dirichlet boundary conditions, and $(p'-p)$ Neumann-Dirichlet boundary conditions.[15] Here note that the Neumann-Dirichlet boundary condition imposes the Neumann condition on one end of the open string, $\sigma^1 = 0$, and imposes the Dirichlet condition on the other, $\sigma^1 = \pi$. With two D-branes of different dimensions, there is always the open string with such a boundary condition.

[15] The p'-p string has $(p'-p)$ Dirichlet-Neumann boundary conditions.

The Neumann-Dirichlet (ND) boundary conditions on the world-sheet fields are given by

$$\partial X^I(z) = -\overline{\partial} X^I(\overline{z}) \qquad \text{for } \sigma^1 = \pi\,, \qquad \partial X^I(z) = \overline{\partial} X^I(\overline{z}) \qquad \text{for } \sigma^1 = 0\,, \tag{2.15}$$

$$\psi^I(z) = -e^{2\pi i\nu}\widetilde{\psi}^I(\overline{z}) \qquad \text{for } \sigma^1 = \pi\,, \qquad \psi^I(z) = \widetilde{\psi}^I(\overline{z}) \qquad \text{for } \sigma^1 = 0\,, \tag{2.16}$$

for $I = 1, \ldots, p' - p$. Then, the world-sheet fields in the ND direction are expanded differently from the NN and DD cases of eqs.(1.42) and (2.3), by $\frac{1}{2}$ in terms of modes,[16]

$$\partial X^I(z) = -i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^I_{m+\frac{1}{2}}}{z^{m+\frac{3}{2}}}\,, \qquad \overline{\partial} X^I(\overline{z}) = -i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^I_{m+\frac{1}{2}}}{\overline{z}^{m+\frac{3}{2}}}\,, \tag{2.17}$$

$$\psi^I(z) = \sum_{r\in\mathbf{Z}+\nu} \frac{\psi^I_{r+\frac{1}{2}}}{z^{r+1}}\,, \qquad \widetilde{\psi}^I(\overline{z}) = \sum_{r\in\mathbf{Z}+\nu} \frac{\psi^I_{r+\frac{1}{2}}}{\overline{z}^{r+1}}\,. \tag{2.18}$$

The mode operator $\alpha^I_{m+\frac{1}{2}}$ does not have zero mode, and there is no momentum in the ND direction as well as the Dirichlet direction. Therefore, a mode expansion for X^I is

$$X^I(z,\overline{z}) = y^I + i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^I_{m+\frac{1}{2}}}{m+\frac{1}{2}} \left(\frac{1}{z^{m+\frac{1}{2}}} + \frac{1}{\overline{z}^{m+\frac{1}{2}}}\right). \tag{2.19}$$

Commutation and anti-commutation relations can be obtained in a similar way as before:

$$\left[\alpha^I_{m+\frac{1}{2}}, \alpha^J_{-(n+\frac{1}{2})}\right] = \left(m+\tfrac{1}{2}\right)\delta^{IJ}\delta_{m,n}\,, \qquad \left\{\psi^I_{r+\frac{1}{2}}, \psi^J_{-(s+\frac{1}{2})}\right\} = \delta^{IJ}\delta_{r,s}\,. \tag{2.20}$$

However, the mode operator $\psi^I_{r+\frac{1}{2}}$ has a zero mode in the NS sector, while it does not in the R sector. This situation is opposite to the NN and DD cases. Therefore, the zero mode ψ^I_0 forms degenerate vacua as eq.(1.65) in the NS sector,

$$|\{s_a\}; k\rangle_{\rm NS}\,, \tag{2.21}$$

where $s_a = \pm\frac{1}{2}$ and the number of a is $\frac{p'-p}{2}$.

The energy-momentum tensor similarly can be separated as,

$$T^{\rm m}_B(z) = -\frac{1}{\alpha'}\left(\partial X^\mu \partial X_\mu + \partial X^i \partial X^i + \partial X^I \partial X^I\right) - \frac{1}{2}\left(\psi^\mu \partial\psi_\mu + \psi^i\partial\psi^i + \psi^I\partial\psi^I\right), \tag{2.22}$$

and the matter sector Virasoro zero operator is

$$L^{\rm m}_0 = L^X_0 + L^\psi_0\,, \tag{2.23}$$

where

$$\begin{aligned} L^X_0 &= \alpha' p^2 + \sum_{n=1}^{\infty} {}^\circ_\circ \alpha^\mu_{-n}\alpha_{n\mu} {}^\circ_\circ + \sum_{n=1}^{\infty} {}^\circ_\circ \alpha^i_{-n}\alpha^i_n {}^\circ_\circ + \sum_{n=1}^{\infty} {}^\circ_\circ \alpha^I_{-n+\frac{1}{2}}\alpha^I_{n-\frac{1}{2}} {}^\circ_\circ + a^X\,, \\ L^\psi_0 &= \sum_{r\in\mathbf{N}-\nu} r\, {}^\circ_\circ \psi^\mu_{-r}\psi_{r\mu} {}^\circ_\circ + \sum_{r\in\mathbf{N}-\nu} r\, {}^\circ_\circ \psi^i_{-r}\psi^i_r {}^\circ_\circ + \sum_{r\in\mathbf{N}+\nu} \left(r-\tfrac{1}{2}\right) {}^\circ_\circ \psi^I_{-r+\frac{1}{2}}\psi^I_{r-\frac{1}{2}} {}^\circ_\circ + a^\psi\,. \end{aligned} \tag{2.24}$$

[16]The antiholomorphic fields with the Dirichlet-Neumann (DN) boundary condition have opposite sign to that with the ND boundary condition.

The normal ordering constants, a^X and a^ψ, are sums of contributions from the NN and DD directions and the ND or DN direction. Since the modes differ by $\frac{1}{2}$, the normal ordering constants from the ND direction also differ from that from the NN and DD direction (1.74). Including contributions of the ghosts,

$$
\begin{aligned}
a &= a^X + a^\psi + a^{bc} + a^{\beta\gamma} \\
&= \begin{cases} -\dfrac{d}{24} + \dfrac{p'-p}{48} + \dfrac{d}{24} - \dfrac{p'-p}{48} + \dfrac{1}{12} - \dfrac{1}{12} = 0 & \text{for R sector}\,, \\ -\dfrac{d}{24} + \dfrac{p'-p}{48} - \dfrac{d}{48} + \dfrac{p'-p}{24} + \dfrac{1}{12} + \dfrac{1}{24} = -\dfrac{1}{2} + \dfrac{p'-p}{8} & \text{for NS sector}\,, \end{cases}
\end{aligned}
\tag{2.25}
$$

where d is the number of dimensions with the NN and DD boundary conditions, $d = D-(p'-p)$. Consequently, if $p'-p$ is four, the zero-point energies in the R and NS sector are equal. This means that there is supersymmetry in the system including two D-branes of different dimensions by four.

D5-D9 system

As a concrete example, let us consider the case of $p = 5$ and $p' = 9$. The system is constructed by introducing a D5-brane and a D9-brane to the type IIB theory.

The introduction of a D-brane breaks ten-dimensional $\mathcal{N} = 2$ SUSY in half. On the D9-brane, there is ten-dimensional $\mathcal{N} = 1$ SUSY and on the D5-brane, there is six-dimensional $\mathcal{N} = 2$ SUSY. The existence of 5-9 and 9-5 strings breaks supersymmetry further in half. The system has eight unbroken supersymmetries, which means that there is six-dimensional $\mathcal{N} = 1$ SUSY. The massless states of 5-5 and 9-9 strings separate into a vector and hypermultiplet of six-dimensional $\mathcal{N} = 1$ SUSY. The massless states of 5-9 and 9-5 strings form a hypermultiplet of six-dimensional $\mathcal{N} = 1$ SUSY, and they carry both charges under gauge symmetries on the D5- and D9-branes.

The bosonic part of the action which is determined by supersymmetry is written as,[17]

$$
S_{\text{D5-D9}} = -\frac{1}{4g_{\text{D9}}^2}\int d^{10}x\, F_{MN}F^{MN} - \frac{1}{4g_{\text{D5}}^2}\int d^6x\, F'_{MN}F'^{MN} - \int d^6x \left[D_\mu\chi^\dagger D^\mu\chi + \cdots\right], \tag{2.26}
$$

where $M, N = 0, \ldots, 9$ and $\mu = 0, \ldots, 6$. The covariant derivative is $D_\mu = \partial_\mu + iA_\mu - iA'_\mu$ where A_μ and A'_μ are gauge fields on the D9- and D5-branes. The field χ is a doublet consisting of the hypermultiplet scalars of 5-9 string, and χ carries charge -1 and $+1$ under gauge symmetries on the D9- and D5-branes. For 5-5 string, ten-dimensional vector fields separate as $A'_M = (A'_\mu, A'_i)$ where $i = 6, \ldots, 9$. The gauge couplings $g_{\text{D}p}$ will be given later.

Since the NS sector zero-point energy (2.25) vanishes, the massless states of 5-9 string are the NS and R sector ground states,

$$
|s_3, s_4; k\rangle_{\text{NS}}\,, \qquad |\mathbf{s}; k\rangle_{\text{R}} = |s_1, s_2; k\rangle_{\text{R}}\,. \tag{2.27}
$$

[17] The last dots in eq.(2.26) mean the potential term required by supersymmetry.

These spin eigenstates are generated by $\Gamma^{a+} = \frac{1}{\sqrt{2}}(\psi_0^{2a} + i\psi_0^{2a+1})$ in eq.(1.64). For the ND or DN direction, the $(6,7,8,9)$-directions, there are the zero modes in not the R sector but the NS sector. Since the GSO parity of these states are as follows,[18]

$$\begin{aligned} e^{\pi i F}|s_3, s_4; k\rangle_{\rm NS} &= -e^{\pi i (s_3+s_4)}|s_3, s_4; k\rangle_{\rm NS}\,, \\ e^{\pi i F}|s_1, s_2; k\rangle_{\rm R} &= e^{\pi i (s_1+s_2)}|s_1, s_2; k\rangle_{\rm R}\,, \end{aligned} \tag{2.28}$$

the GSO projection requires $s_3 = s_4$ and $s_1 = -s_2$. The states (2.27) with $s_3 = s_4$ and $s_1 = -s_2$ are related by SUSY transformation.

D-branes at angles

Now, consider D-branes intersecting each other at general angles. Let us specify two D6-branes, in which both initially are extended in the $(1,2,3)$- and $(4,6,8)$-directions and one of two is rotated by an angle ϕ_2 in the $(4,5)$-plane, ϕ_3 in the $(6,7)$-plane and ϕ_4 in the $(8,9)$-plane. We call this rotation ρ. Then, the two D6-branes are on top of the four-dimensional space-time.

How many supersymmetries are broken by introducing two D-branes depends on the D-brane configuration. In this case, when an eigenvalue of twice the rotation ρ^2 is 1, supersymmetry is left unbroken. In the $\mathbf{s}$-basis, the eigenvalue of ρ^2 is $\exp\big(2i \sum_{a=2}^{4} s_a\phi_a\big)$. Therefore, for example,

- For general ϕ_a, there are no unbroken supersymmetries.
- For $\phi_2 + \phi_3 + \phi_4 = 0 \mod 2\pi$, there are four unbroken supersymmetries, which means $\mathcal{N} = 1$ SUSY in the four-dimensional space-time.
- For $\phi_2 + \phi_3 = \phi_4 = 0 \mod 2\pi$, there are eight unbroken supersymmetries, which means $\mathcal{N} = 2$ SUSY in the four-dimensional space-time.

Let us join the $(4,\ldots,9)$-coordinates into complex pairs,

$$Z^a = X^{2a} + iX^{2a+1} \qquad \text{for } a = 2,3,4\,. \tag{2.29}$$

Then, the rotation ρ on Z^a is an U(3) matrix, $\mathrm{diag}\big[e^{i\phi_2}, e^{i\phi_3}, e^{i\phi_4}\big]$. When $\phi_2 + \phi_3 + \phi_4 = 0 \mod 2\pi$, the determinant of ρ is 1 and the rotation is a SU(3) matrix. Consequently, a general U(3) rotation breaks all the supersymmetry, a SU(3) rotation breaks three-fourths, and a SU(2) rotation breaks half. These ideas were originally pointed out in ref.[26].

We consider an open string in which one end $\sigma^1 = 0$ attaches on the unrotated D6-brane and the other $\sigma^1 = \pi$ attaches on the rotated D6′-brane. Such an open string follows the boundary conditions,

$$\begin{cases} \partial_1 \,\mathrm{Re}\, Z^a = \partial_1 X^{2a} = 0 \\ \partial_2 \,\mathrm{Im}\, Z^a = \partial_2 X^{2a+1} = 0 \end{cases} \qquad \text{for } \sigma^1 = 0\,, \tag{2.30}$$

[18] The spin s_0 can be fixed by $\frac{1}{2}$ using the Dirac equation, and s_0 can cancel the $\beta\gamma$ ghost number $-\frac{1}{2}$ of the Ramond vacuum.

$$\begin{cases} \partial_1 \operatorname{Re}\left[e^{-i\phi_a} Z^a\right] = \partial_1\left(+\cos\phi_a X^{2a} + \sin\phi_a X^{2a+1}\right) = 0 \\ \partial_2 \operatorname{Im}\left[e^{-i\phi_a} Z^a\right] = \partial_2\left(-\sin\phi_a X^{2a} + \cos\phi_a X^{2a+1}\right) = 0 \end{cases} \quad \text{for } \sigma^1 = \pi\,. \tag{2.31}$$

Namely, the open string in the $\cos\phi_a X^{2a}$ direction follows the NN boundary condition, that in the $\cos\phi_a X^{2a+1}$ does the DD boundary condition, in the $-\sin\phi_a X^{2a}$ the ND boundary condition, and in the $\sin\phi_a X^{2a+1}$ the DN boundary condition.

Here, we use the doubling trick. First, we separate the mode expansion $X(w,\overline{w})$ in the NN direction of eq.(1.55) into a holomorphic part $\mathcal{X}(w)$ and an antiholomorphic part $\widetilde{\mathcal{X}}(\overline{w})$,

$$X(w,\overline{w}) = \mathcal{X}(w) + \widetilde{\mathcal{X}}(\overline{w})\,, \tag{2.32}$$

where

$$\mathcal{X}(w) = \frac{x}{2} + \sqrt{\frac{\alpha'}{2}}\left[\alpha_0(-w) + i\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty} \frac{\alpha_m}{m}\, e^{imw}\right], \quad \widetilde{\mathcal{X}}(\overline{w}) = \frac{x}{2} + \sqrt{\frac{\alpha'}{2}}\left[\alpha_0\overline{w} + i\sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty} \frac{\alpha_m}{m}\, e^{-im\overline{w}}\right]. \tag{2.33}$$

$\mathcal{X}(w)$ and $\mathcal{X}(\overline{w})$ are defined only in the region $0 \leq w, \overline{w} \leq \pi$, so let us define $\mathcal{X}(w)$ in the region $\pi \leq w \leq 2\pi$ as follows,

$$\mathcal{X}(w) \equiv \widetilde{\mathcal{X}}(\overline{w}') \qquad \text{for } \pi \leq w \leq 2\pi\,, \tag{2.34}$$

where $w' = 2\pi - \overline{w}$. Then, $X(w,\overline{w})$ of eq.(2.32) is defined in the region $0 \leq w \leq 2\pi$.

Similarly, the mode expansions $X(w,\overline{w})$ in the DD, ND and DN directions of eqs.(2.5) and (2.19) are separated as eq.(2.32), and the holomorphic part is given by

$$\mathcal{X}(w) = i\sqrt{\frac{\alpha'}{2}} \sum_{\substack{m=-\infty\\ m\neq 0}}^{\infty} \frac{\alpha_m}{m}\, e^{imw} \quad \text{for DD}\,, \quad \mathcal{X}(w) = i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha_{m+\frac{1}{2}}}{m+\frac{1}{2}}\, e^{i(m+\frac{1}{2})w} \quad \text{for ND, DN}\,. \tag{2.35}$$

It is found that they satisfy $\mathcal{X}(w+2\pi) = \mathcal{X}(w)$ for the NN and DD directions and $\mathcal{X}(w+2\pi) = -\mathcal{X}(w)$ for the ND and DN directions. Then, the complex coordinates $Z^a(w,\overline{w})$ are also separated into $\mathcal{Z}^a(w)$ and $\widetilde{\mathcal{Z}}^a(\overline{w})$, and $\mathcal{Z}^a(w)$ satisfies the following boundary condition,

$$\begin{aligned} \mathcal{Z}^a(w) &= e^{-i\phi_a}\left[e^{i\phi_a}\mathcal{Z}^a(w)\right] \\ &= e^{-i\phi_a}\left[\cos\phi_a \mathcal{X}^{2a}(w) + i\sin\phi_a \mathcal{X}^{2a}(w) + i\left(\cos\phi_a \mathcal{X}^{2a+1}(w) + i\sin\phi_a \mathcal{X}^{2a+1}(w)\right)\right] \\ &= e^{-2i\phi_a}\mathcal{Z}^a(w+2\pi)\,. \end{aligned} \tag{2.36}$$

This implies the mode expansion

$$\mathcal{Z}^a(w) = i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\alpha^a_{m+\theta_a}}{m+\theta_a}\, e^{i(m+\theta_a)w}\,, \tag{2.37}$$

where $\theta_a = \phi_a/\pi$. The complex conjugate of Z^a is denoted by $\overline{Z}^a = X^{2a} - iX^{2a+1}$. A holomorphic part of $\overline{Z}^a$, $\overline{\mathcal{Z}}^a$, satisfies the boundary condition $\overline{\mathcal{Z}}^a(w) = e^{2i\phi_a}\overline{\mathcal{Z}}^a(w+2\pi)$, and the mode expansion is

$$\overline{\mathcal{Z}}^a(w) = i\sqrt{\frac{\alpha'}{2}} \sum_{m=-\infty}^{\infty} \frac{\overline{\alpha}^a_{m-\theta_a}}{m-\theta_a}\, e^{i(m-\theta_a)w}\,. \tag{2.38}$$

Thus, for the open string with ends on the respective D-branes intersecting at angles, we expand the complexified space-time coordinates. The mode expansions (2.37) and (2.38) are different from the NN and DD cases by θ_a in terms of modes, and when $\theta_a = \frac{1}{2}$, they are the same as the mode expansions in the ND direction of eq.(2.19).

The world-sheet fermions in the $(4,\ldots,9)$-directions also combine into complex pairs,

$$\Psi^a = \psi^{2a} + i\psi^{2a+1} \qquad \text{for } a = 2,3,4\,, \tag{2.39}$$

and the complex conjugate is denoted by $\overline{\Psi}^a = \psi^{2a} - i\psi^{2a+1}$. The mode expansions of the world-sheet fields are in the z-coordinate,

$$\partial\mathcal{Z}^a(z) = -i\sqrt{\frac{\alpha'}{2}}\sum_{m=-\infty}^{\infty}\frac{\alpha^a_{m+\theta_a}}{z^{m+\theta_a+1}}\,, \qquad \partial\overline{\mathcal{Z}}^a(z) = -i\sqrt{\frac{\alpha'}{2}}\sum_{m=-\infty}^{\infty}\frac{\overline{\alpha}^a_{m-\theta_a}}{z^{m-\theta_a+1}}\,, \tag{2.40}$$

$$\Psi^a(z) = \sum_{r\in\mathbf{Z}+\nu}\frac{\psi^a_{r+\theta_a}}{z^{r+\theta_a+\frac{1}{2}}}\,, \qquad \overline{\Psi}^a(z) = \sum_{r\in\mathbf{Z}-\nu}\frac{\overline{\psi}^a_{r-\theta_a}}{z^{r-\theta_a+\frac{1}{2}}}\,, \tag{2.41}$$

where the mode operators $\alpha^a_{m+\theta_a}$, $\overline{\alpha}^a_{m-\theta_a}$, $\psi^a_{r+\theta_a}$ and $\overline{\psi}^a_{r-\theta_a}$ satisfy the following commutation and anti-commutation relations,

$$\big[\alpha^a_{m+\theta_a}, \overline{\alpha}^b_{-n-\theta_b}\big] = 2(m+\theta_a)\,\delta^{ab}\delta_{m,n}\,, \qquad \big\{\psi^a_{r+\theta_a}, \overline{\psi}^b_{-s-\theta_b}\big\} = 2\,\delta^{ab}\delta_{r,s}\,. \tag{2.42}$$

One can consider $\alpha^a_{m+\theta_a}$, $\overline{\alpha}^a_{m-\theta_a}$, $\psi^a_{r+\theta_a}$ and $\overline{\psi}^a_{r-\theta_a}$ for $m, r > 0$ as annihilation operators, and that for $m, r < 0$ as creation operators.

The energy-momentum tensor is again separated,

$$T^{\rm m}_B(z) = -\frac{1}{\alpha'}\big(\partial X^\mu\partial X_\mu + \partial\mathcal{Z}^a\partial\overline{\mathcal{Z}}^a\big) - \frac{1}{2}\big(\psi^\mu\partial\psi_\mu + \tfrac{1}{2}(\Psi^a\partial\overline{\Psi}^a + \overline{\Psi}^a\partial\Psi^a)\big)\,, \tag{2.43}$$

and the Virasoro zero operators are

$$\begin{aligned} L^X_0 &= \alpha' p^2 + \sum_{n=1}^{\infty} {}^\circ_\circ\alpha^\mu_{-n}\alpha_{n\mu}{}^\circ_\circ + \frac{1}{2}\sum_{n=-\infty}^{\infty} {}^\circ_\circ\alpha^a_{-n+\theta_a}\overline{\alpha}^a_{n-\theta_a}{}^\circ_\circ + a^X\,, \\ L^\psi_0 &= \sum_{r\in\mathbf{N}-\nu} r\,{}^\circ_\circ\psi^\mu_{-r}\psi_{r\mu}{}^\circ_\circ + \frac{1}{2}\sum_{r\in\mathbf{Z}+\nu}(r-\theta_a)\,{}^\circ_\circ\psi^a_{-r+\theta_a}\overline{\psi}^a_{r-\theta_a}{}^\circ_\circ + a^\psi\,. \end{aligned} \tag{2.44}$$

Then, the normal ordering constants are

$$\begin{aligned} a &= a^X + a^\psi + a^{bc} + a^{\beta\gamma} \\ &= \begin{cases} \displaystyle\sum_{a=2}^{4}\frac{\theta_a(1-\theta_a)}{2} - \frac{d}{24} - \sum_{a=2}^{4}\frac{\theta_a(1-\theta_a)}{2} + \frac{d}{24} + \frac{1}{12} - \frac{1}{12} = 0 & \text{for R sector}\,, \\ \displaystyle\sum_{a=2}^{4}\frac{\theta_a(1-\theta_a)}{2} - \frac{d}{24} + \sum_{a=2}^{4}\frac{\theta_a^2}{2} - \frac{d}{48} + \frac{1}{12} + \frac{1}{24} = -\frac{1}{2} + \sum_{a=2}^{4}\frac{\theta_a}{2} & \text{for NS sector}\,, \end{cases} \end{aligned} \tag{2.45}$$

where $d = 3$. Thus, the zero-point energy in the NS sector depends on rotation angles. This is coincident with the above analysis that there are unbroken supersymmetries in cases of $\theta_2 + \theta_3 + \theta_4 = 0 \mod 2$, $\theta_2 + \theta_3 = \theta_4 = 0 \mod 2$ and so on.

Let us form lower states of the 6-6′ string. The NS sector ground state has the mass

$$|0;k\rangle_{\rm NS} \quad : \quad m^2 = \frac{1}{2\alpha'}\big(-1 + \theta_2 + \theta_3 + \theta_4\big)\,, \tag{2.46}$$

however this state is projected out by the GSO projection. In the NS sector first excited states, there are four kinds of states with lowest mass,

$$\begin{aligned} \psi^{\mu}_{-\frac{1}{2}}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{1}{2\alpha'}\big(+\theta_2 + \theta_3 + \theta_4\big)\,, \\ \psi^{2}_{-\frac{1}{2}+\theta_2}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{1}{2\alpha'}\big(-\theta_2 + \theta_3 + \theta_4\big)\,, \\ \psi^{3}_{-\frac{1}{2}+\theta_3}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{1}{2\alpha'}\big(+\theta_2 - \theta_3 + \theta_4\big)\,, \\ \psi^{4}_{-\frac{1}{2}+\theta_4}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{1}{2\alpha'}\big(+\theta_2 + \theta_3 - \theta_4\big)\,. \end{aligned} \tag{2.47}$$

The NS sector second excited states are projected out by the GSO projection, and the following NS sector third excited state has lower mass,

$$\psi^{2}_{-\frac{1}{2}+\theta_2}\psi^{3}_{-\frac{1}{2}+\theta_3}\psi^{4}_{-\frac{1}{2}+\theta_4}|0;k\rangle_{\rm NS} \quad : \quad m^2 = \frac{1}{\alpha'}\big(1 - \tfrac{1}{2}(\theta_2 + \theta_3 + \theta_4)\big)\,, \tag{2.48}$$

The R sector ground state is massless due to the vanishing R sector zero-point energy,

$$|s_1;k\rangle_{\rm R} \quad : \quad m^2 = 0\,. \tag{2.49}$$

Since the mode operators $\psi^a_{r+\theta_a}$ have no zero modes, they do not form degenerate ground states which have spins in the $(4,5)$, $(6,7)$ and $(8,9)$-planes. The GSO projection restricts $|s_1;k\rangle_{\rm R}$ to a state with $s_1 = +\frac{1}{2}$, and since s_1 is a space-time spin, the state is chiral.

Note that the state $\psi^{\mu}_{-\frac{1}{2}}|0;k\rangle_{\rm NS}$ does not act as a gauge boson, even if there are unbroken supersymmetries, since it comes from the open string stretched between the different D6-branes. In case of $\theta_2 + \theta_3 + \theta_4 = 0$, the state combine the massless state in the R sector, $|s_1;k\rangle_{\rm R}$, into a chiral multiplet of the four-dimensional $\mathcal{N} = 1$ SUSY. The chiral multiplet is in the bifundamental representation under gauge symmetries on the D6- and D6′-branes, as discussed above eq.(2.26).

Thus, we obtain bifundamental chiral fermions from the open string with ends on the respective D-branes intersecting at angles. Then, we can expect that the chiral matters in the Standard Model are obtained using the intersecting D-branes.

2.3 Type IIA orientifolds

Now, let us consider construction of a realistic model using D-branes. We have discussed that the open string on a stack of N D-branes gives massless gauge bosons (and scalars) associated with $\mathrm{U}(N)$ gauge symmetry, and the open string on intersecting D-branes gives massless chiral fermions with both charges under gauge symmetries on the D-branes. Therefore, we can expect that the gauge groups and chiral matters in the Standard Model are obtained by specific D-brane configurations.

However, the D-brane configurations are not always consistent. They must satisfy the Ramond-Ramond tadpole cancellation, which is required to make gauge symmetries on the D-branes anomaly free. For example, in the type I theory which is projected by the world-sheet parity, the R-R tadpoles are canceled when the gauge symmetry is SO(32). Thus, the world-sheet parity projection can be used for the R-R tadpole cancellation in general situations. We want to consider the type IIA with intersecting D6-branes as discussed in the previous section. The type IIA does not have the world-sheet parity symmetry, but it does have a symmetry under a combination of the world-sheet parity and odd times space-time reflections. A theory which is projected by such an action is generally called orientifold. Referring to the earlier constructions [27], let us consider the type IIA orientifolds with intersecting D6-branes.

Type IIA orientifolds

First, we consider compactification of extra six-dimensional space. As a simple choice, we consider a six-dimensional torus which is factorizable into $T^2 \times T^2 \times T^2$. Let us introduce D6-branes on top of the four-dimensional space-time and wrapped on three-cycles in the six-dimensional torus. In this case, the three-cycles are products of one-cycles in each of three T^2s. Then, we consider stacks of N_a D6$_a$-branes which are wrapped on the cycle with the wrapping numbers (n_a^i, m_a^i) in the i-th T^2.

The type IIA is changed by the world-sheet parity transformation Ω into the type IIA$'$,

$$\mathrm{IIA}' \;:\; (\mathrm{NS}+,\mathrm{NS}+)\;(\mathrm{R}-,\mathrm{R}+)\;(\mathrm{NS}+,\mathrm{R}+)\;(\mathrm{R}-,\mathrm{NS}+)\,. \tag{2.50}$$

On the other hand, a space-time reflection on a single axis,

$$X^\mu \to -X^\mu\,, \qquad \psi^\mu \to -\psi^\mu\,, \qquad \widetilde{\psi}^\mu \to -\widetilde{\psi}^\mu\,, \tag{2.51}$$

reverses the chirality in the R sector. Therefore, the type IIA has a symmetry under a combination of Ω and the odd times space-time reflections.

We mod out the type IIA by the orientifold action ΩR, where Ω is the world-sheet parity transformation (1.109), and R acts as

$$R \quad : \quad (Z^1, Z^2, Z^3) \to (\overline{Z}^1, \overline{Z}^2, \overline{Z}^3)\,, \tag{2.52}$$

where Z^i's are complex coordinates in the $T^2 \times T^2 \times T^2$. Here, Z^i's are corresponding to eq.(2.29). Then, there are orientifold fixed planes in the $(4, 6, 8)$-directions, which are called O6-planes. Furthermore, the symmetry under the orientifold action requires a image or mirror of a D6-brane about the O6-planes. The orientifold images of the D6$_a$-branes, D6$_{a'}$-branes, are wrapped on the cycle with the wrapping numbers $(n_a^i, -m_a^i)$ in the i-th T^2, if the T^2s are rectangular.

The R-R tadpole cancellation

Next, we consider the R-R tadpole cancellation conditions on the D6-brane configurations in this type IIA orientifold. Here, we define the homology class of the three-cycles wrapped by the $\mathrm{D6}_a$-branes,

$$\Pi_a = \prod_{i=1}^{3} \left(n_a^i [a_i] + m_a^i [b_i] \right) , \tag{2.53}$$

where $[a_i]$ and $[b_i]$ denote the $(1,0)$ and $(0,1)$ homology one-cycles in the i-th T^2, for rectangular tori. Similarly, the homology class of the cycles wrapped by the $\mathrm{D6}_{a'}$-branes is defined as

$$\Pi_{a'} = \prod_{i=1}^{3} \left(n_a^i [a_i] - m_a^i [b_i] \right) . \tag{2.54}$$

We also define the homology class of the cycles wrapped by the O6-planes,

$$\Pi_{\mathrm{O6}} = [a_1] \times [a_2] \times [a_3] . \tag{2.55}$$

The R-R tadpole is proportional to a charge of the D6-brane under the R-R field which is an antisymmetric seven-tensor originating from the R-R closed string. Therefore, the R-R tadpole cancellation is a simple Gauss law, namely the cancellation of the total R-R charge. The R-R charge carried by all the D6-branes and the orientifold images is

$$\sum_a N_a \, \Pi_a + \sum_a N_a \, \Pi_{a'} , \tag{2.56}$$

while the R-R charge carried by the O6-planes is,

$$-4 \times 8 \times \Pi_{\mathrm{O6}} , \tag{2.57}$$

where -4 is a charge of an O6-plane and 8 is the number of O6-planes. Then, the requirement that the total R-R charge is canceled gives the following constraints on the wrapping numbers and the multiplicity of the D6-branes,

$$\begin{aligned} &\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 , \qquad &&\sum_a N_a n_a^1 m_a^2 m_a^3 = 0 , \\ &\sum_a N_a m_a^1 n_a^2 m_a^3 = 0 , \qquad &&\sum_a N_a m_a^1 m_a^2 n_a^3 = 0 . \end{aligned} \tag{2.58}$$

Thus, the constraints (2.58) define a consistent D-brane configuration in the type IIA orientifold compactified on a torus.

Spectrum

Let us discuss the spectrum of the open string in this theory. We denote the open string in which one end attaches on a stack of $\mathrm{D6}_a$-branes and the other attaches on a stack of $\mathrm{D6}_b$-branes, by the *ab* sector. These stacks of the D6-branes intersect at general angles, and the open string stretched between the different D6-branes is localized on the intersection plane (in this case, the four-dimensional space-time).

- The aa sector :
  There are a vector multiplet and three chiral multiplets in the $\mathrm{U}(N_a)$ adjoint representation. When the D6$_a$-branes are on top of the O6-planes, the vector multiplet and the chiral multiplets are in the $\mathrm{SO}(N_a)$ or $\mathrm{USp}(N_a)$ adjoint representation.

  The scalar components of the chiral multiplets come from the Dirichlet components (2.10) in the ten-dimensional vector.

- The $ab + ba$ sector :
  There are I_{ab} chiral fermions in the bifundamental representation $(N_a, \overline{N}_b)$, where I_{ab} is the intersection number of the wrapped cycles,

$$I_{ab} = \Pi_a \cdot \Pi_b = \prod_{i=1}^{3} \left(n_a^i m_b^i - m_a^i n_b^i \right) . \tag{2.59}$$

  The sign of I_{ab} denotes a chirality of the chiral fermion.

- The $ab' + b'a$ sector :
  There are $I_{ab'}$ chiral fermions in the bifundamental representation (N_a, N_b), where

$$I_{ab'} = \Pi_a \cdot \Pi_{b'} = -\prod_{i=1}^{3} \left(n_a^i m_b^i + m_a^i n_b^i \right) . \tag{2.60}$$

- The $aa' + a'a$ sector :
  There are chiral fermions in the two-index symmetric and antisymmetric representations of $\mathrm{U}(N_a)$. The two indices correspond to ends of the open string. The number of the symmetric and antisymmetric representations are

$$n_{\square\!\square} = -\frac{1}{2}\left(I_{aa'} - \frac{4}{2^k} I_{a,\mathrm{O6}} \right) , \qquad n_{\begin{smallmatrix}\square\\[-2pt]\square\end{smallmatrix}} = -\frac{1}{2}\left(I_{aa'} + \frac{4}{2^k} I_{a,\mathrm{O6}} \right) , \tag{2.61}$$

  where k is the number of tilted tori and

$$I_{aa'} = \Pi_a \cdot \Pi_{a'} = -8 \prod_{i=1}^{3} n_a^i m_a^i \, , \qquad I_{a,\mathrm{O6}} = \Pi_a \cdot \Pi_{\mathrm{O6}} = -\prod_{i=1}^{3} m_a^i \, . \tag{2.62}$$

There are undesirable chiral fermions in the aa and $aa' + a'a$ sectors. The adjoint chiral fermions in the aa sector can obtain mass due to their hermiticity, and we can exclude them from the massless spectrum. The antisymmetric fermions in the $aa' + a'a$ sector can be the representation under the Standard Model gauge group, in particular, $\mathrm{SU}(3)_C$, but the symmetric fermions are always exotics. In order to exclude them from the massless spectrum, we can require that the cycles wrapped by D6-branes satisfy $\prod_{i=1}^{3} m^i = 0$. Then, as it is found from eq.(2.62), there are no fermions in both the symmetric and antisymmetric representations. This condition means that for any D6-branes, at least one of the three cycles in the T^2s should be parallel to the O6-planes.

Thus, we can obtain the massless spectrum in which there are no chiral exotics. The generation number of the Standard Model can be obtained as the intersection number of the wrapped cycles in case of the toroidal compactification.

Note that, in order to obtain three generations, at least one of the three T^2s should be tilted. Consider that only the third T^2 is tilted. The homology class of the cycle wrapped by the O6-planes, (2.55), becomes

$$\widetilde{\Pi}_{\text{O6}} = [a_1] \times [a_2] \times (2[a_3] - [b_3]) \,, \tag{2.63}$$

and then the R-R charge (2.57) carried by the O6-planes is divided by two. When the cycle wrapped by the D6$_a$-branes has the wrapping numbers (n_a^i, m_a^i) in the tilted T^2, the cycle wrapped by the images has the wrapping numbers $(n_a^i, -n_a^i - m_a^i)$, so the homology class of the cycle wrapped by the D6$_{a'}$-branes becomes

$$\widetilde{\Pi}_{a'} = \left(n_a^1[a_1] - m_a^1[b_1]\right) \times \left(n_a^2[a_2] - m_a^2[b_2]\right) \times \left(n_a^3[a_3] - (n_a^3 + m_a^3)[b_3]\right) . \tag{2.64}$$

Then, the constraints (2.58) required by the R-R tadpole cancellation are modified as

$$\begin{aligned} &\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 \,, \qquad &&\sum_a N_a n_a^1 m_a^2 \widetilde{m}_a^3 = 0 \,, \\ &\sum_a N_a m_a^1 n_a^2 \widetilde{m}_a^3 = 0 \,, \qquad &&\sum_a N_a m_a^1 m_a^2 n_a^3 = 0 \,, \end{aligned} \tag{2.65}$$

where $\widetilde{m} = m + \frac{n}{2}$. The modification is simply to replace $m \to \widetilde{m}$. The half-integer wrapping number realizes three generations.

SUSY conditions

As discussed in the previous section, the system in which one stack of D6-branes are related to the other stack by a SU(3) rotation has four-dimensional $\mathcal{N} = 1$ SUSY. Denoting by θ^i the intersection angles between the D6-branes in the i-th T^2, configurations preserving $\mathcal{N} = 1$ SUSY must satisfy

$$\theta^1 + \theta^2 + \theta^3 = 0 \mod 2\pi \,. \tag{2.66}$$

If one stack of the D6-branes is parallel to the O6-planes in a i-th T^2, the intersection angle depends on the ratio of the radii in the i-th T^2 and the wrapping numbers of the other stack, for rectangular tori,

$$\theta^i = \arctan\left(\frac{m^i}{n^i} \frac{R_{2i}}{R_{2i-1}}\right) , \tag{2.67}$$

where R_{2i-1} and R_{2i} are the radii in the vertical and horizontal directions in the i-th T^2.

For general θ^i, $\mathcal{N} = 1$ SUSY is broken. Then, the R sector ground state $|s_1; k\rangle_{\text{R}}$ (2.49) remains massless. However, the NS sector lower states (2.47) and (2.48) have masses which depend on the intersection angles. In order for the system to be stable, they must not have tachyonic masses. Setting $\theta^i \geq 0$, the absence of tachyons require θ^i to be within a regular tetrahedron in the $(\theta^1, \theta^2, \theta^3)$-space.

Intersection	Matter	$(\mathrm{SU}(3)_C, \mathrm{SU}(2)_L)$	(Q_a, Q_b, Q_c, Q_d)	Y
(a, b)	Q_L	$(\mathbf{3}, \mathbf{2})$	$(1, -1, 0, 0)$	$\frac{1}{6}$
(a, b')	q_L	$2 \times (\mathbf{3}, \mathbf{2})$	$(1, 1, 0, 0)$	$\frac{1}{6}$
(a, c)	$(u_R)^c$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})$	$(-1, 0, 1, 0)$	$-\frac{2}{3}$
(a, c')	$(d_R)^c$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})$	$(-1, 0, -1, 0)$	$\frac{1}{3}$
(b, d')	ℓ_L	$3 \times (\mathbf{1}, \mathbf{2})$	$(0, -1, 0, -1)$	$-\frac{1}{2}$
(c, d)	$(e_R)^c$	$3 \times (\mathbf{1}, \mathbf{1})$	$(0, 0, -1, 1)$	1
(c, d')	$(\nu_R)^c$	$3 \times (\mathbf{1}, \mathbf{1})$	$(0, 0, 1, 1)$	0

Table 2.1: The chiral fermion spectrum of the semi-realistic four stack model.

2.4 Getting the Standard Model spectrum

Now, we specify a D-brane configuration which gives the massless spectrum of the Standard Model, considering the type IIA orientifolds with intersecting D6-branes compactified on a torus. There are so many constructions with the SM spectrum, but in most cases, there are extra chiral fermions (the right-handed neutrinos, and so on) and Higgs fields. However, it is true that one can obtain the particle content quite close to that of the SM, using intersecting D-branes with simple toroidal compactification. Here, we present a semi-realistic but simple proto-type model [28] which contains the SM-like spectrum and features of D-brane models.

Non-supersymmetric semi-realistic model

In ref.[28], the authors considered a non-SUSY model. Using a bottom-up approach, they introduced four stacks of D6-branes with $N_a = 3$, $N_b = 2$, $N_c = 1$ and $N_d = 1$. The stacks yield the gauge group $\mathrm{U}(3)_a \times \mathrm{U}(2)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d$, in the sense of the SM gauge group,

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_a \times \mathrm{U}(1)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d \,. \tag{2.68}$$

A charge of an additional $\mathrm{U}(1)_a$ factor is denoted by Q_a. The desired chiral fermions and representations are listed in Table 2.1. In the Table 2.1, Q_L is one left-handed quark antidoublet, and q_L are two left-handed quark doublets. This is required to cancel $\mathrm{U}(2)_b$ anomalies, and then ℓ_L are left-handed lepton antidoublets. ν_R are also required to cancel $\mathrm{U}(1)_d$ anomaly. Thus, such models with only bifundamental matter necessarily contain the right-handed neutrinos.

Considering mixed anomalies of the four U(1) factors with the non-Abelian ones $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$, it is found that $(Q_a + 3Q_d)$ and Q_c are free of triangle anomalies. Then, the hypercharge is given by the linear combination

$$Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d \,. \tag{2.69}$$

The other orthogonal combinations are anomalous, however, gauge bosons of the anomalous U(1)s receive masses via the Green-Schwarz mechanism [29], which can be realized by an exchange of the R-R field. All the massive U(1) gauge symmetries survive as global symmetries,

N	$(n^1, \widetilde{m}^1)$	$(n^2, \widetilde{m}^2)$	$(n^3, \widetilde{m}^3)$
$N_a = 3$	$(1/\beta^1, 0)$	$(n_a^2, \epsilon\beta^2)$	$(1/\rho, 1/2)$
$N_b = 2$	$(n_b^1, -\epsilon\beta^1)$	$(1/\beta^2, 0)$	$(1, 3\rho/2)$
$N_c = 1$	$(n_c^1, 3\epsilon\rho\beta^1)$	$(1/\beta^2, 0)$	$(0, 1)$
$N_d = 1$	$(1/\beta^1)$	$(n_d^2, -\epsilon\beta^2/\rho)$	$(1, 3\rho/2)$

Table 2.2: The D6-brane wrapping numbers of the semi-realistic non-SUSY model. The parameters are defined as $\beta^{1,2} = 1, \frac{1}{2}$, a parameter $\rho = 1, \frac{1}{3}$, a phase $\epsilon = \pm 1$ and four integers $n_a^2, n_b^1, n_c^1, n_d^2 \in \mathbf{Z}$.

and Q_a and Q_d can be identified with the baryon number and the lepton number. They stabilize the proton and prevent Majorana neutrino masses.

If the wrapping numbers of the four stacks of D6-branes are chosen as shown in Table 2.2, the intersection numbers between the four stacks are

$$\begin{aligned} I_{ab} &= 1\,, & I_{ab'} &= 2\,, & I_{ac} &= -3\,, & I_{ac'} &= -3\,, \\ I_{bd} &= 0\,, & I_{bd'} &= -3\,, & I_{cd} &= -3\,, & I_{cd'} &= 3\,, \end{aligned} \tag{2.70}$$

and all the other vanish. They give rise to the number of generation of the desired chiral fermions listed in Table 2.1. In the configurations of Table 2.2, all but the first of the R-R tadpole cancellation conditions (2.58) are automatically satisfied, and the first cancellation condition read[19]

$$\frac{3n_a^2}{\rho\beta^1} + \frac{2n_b^1}{\beta^2} + \frac{n_d^2}{\beta^1} = 16\,. \tag{2.71}$$

In order for the $U(1)_Y$ to remain massless, further conditions are imposed as

$$n_c^1 = \frac{\beta^2}{2\beta^1}\left(n_a^2 + 3\rho n_d^2\right) \tag{2.72}$$

on the parameters in Table 2.2.

The possible Higgs configurations in this model are listed in Table 2.3. The Higgs fields appear in pairs analogous to the Higgs sector of the Minimal Supersymmetric Standard Model (MSSM). The $U(2)_b$ branes are parallel to the $U(1)_c$ brane in the second T^2, and there are no massless chiral fermions at the intersection. However, if there is a small distance z between these stacks of the branes in the second T^2, the open strings can stretch between them and lead light scalars with the masses,

$$\begin{aligned} \psi^1_{-\frac{1}{2}+\theta^1}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{z^2}{(2\pi\alpha')^2} + \frac{1}{2\alpha'}\left(-\theta^1 + \theta^3\right)\,, \\ \psi^3_{-\frac{1}{2}+\theta^3}|0;k\rangle_{\rm NS} \quad &: \quad m^2 = \frac{z^2}{(2\pi\alpha')^2} + \frac{1}{2\alpha'}\left(+\theta^1 - \theta^3\right)\,. \end{aligned} \tag{2.73}$$

[19] One can always relax the constraint (2.71) by adding extra hidden D6-branes, for example, N' D6-branes with $m'^i = 0$ (parallel to the O6-planes). Then the constraint is replaced by

$$\frac{3n_a^2}{\rho\beta^1} + \frac{2n_b^1}{\beta^2} + \frac{n_d^2}{\beta^1} + N'n'^1n'^2n'^3 = 16\,.$$

Intersection	Matter	$(\mathrm{SU}(3)_C, \mathrm{SU}(2)_L)$	(Q_a, Q_b, Q_c, Q_d)	Y
(b,c)	h_1	$n_h \times (\mathbf{1},\mathbf{2})$	$(0,1,-1,0)$	$\frac{1}{2}$
(b,c)	h_2	$n_h \times (\mathbf{1},\mathbf{2})$	$(0,-1,1,0)$	$-\frac{1}{2}$
(b,c')	H_1	$n_H \times (\mathbf{1},\mathbf{2})$	$(0,-1,-1,0)$	$\frac{1}{2}$
(b,c')	H_2	$n_H \times (\mathbf{1},\mathbf{2})$	$(0,1,1,0)$	$-\frac{1}{2}$

Table 2.3: The possible Higgs field configurations of the semi-realistic model.

Then, the Higgs fields have the quadratic potential similar to that of the MSSM,

$$\begin{aligned} V_{\text{quad.}} = {}& m_h^2\big(|h_1|^2+|h_2|^2\big) + m_H^2\big(|H_1|^2+|H_2|^2\big) \\ & + m_b^2 h_1 h_2 + \text{h.c.} + m_B^2 H_1 H_2 + \text{h.c.}\,, \end{aligned} \tag{2.74}$$

where

$$\begin{aligned} m_h^2 &= \frac{{z_{bc}}^2}{(2\pi\alpha')^2}\,, \qquad & m_H^2 &= \frac{{z_{bc'}}^2}{(2\pi\alpha')^2}\,, \\ m_b^2 &= \frac{1}{2\alpha'}|\theta_{bc}^1-\theta_{bc}^3|\,, \qquad & m_B^2 &= \frac{1}{2\alpha'}|\theta_{bc'}^1-\theta_{bc'}^3|\,. \end{aligned} \tag{2.75}$$

If the distances z_{bc} and $z_{bc'}$ are small compared to the string scale $M_{\mathrm{s}} = 1/\sqrt{\alpha'}$, there appear flat directions along $\langle h_1\rangle = \langle h_2\rangle$ and $\langle H_1\rangle = \langle H_2\rangle$ in the scalar potential, which may give rise to the electroweak symmetry breaking below the string scale. However, note that this requires the string scale to be not far above the weak scale.

The numbers of the Higgs fields, n_h and n_H, are given by the numbers of times the $\mathrm{U}(2)_b$ branes intersect with the $\mathrm{U}(1)_c$ branes in the first and third T^2s, depending on the parameters in Table 2.2,

$$n_h = \beta^1|n_c^1 + 3\rho n_b^1|\,, \qquad n_H = \beta^1|n_c^1 - 3\rho n_b^2|\,. \tag{2.76}$$

The Yukawa couplings which are allowed by the symmetries have the general form,

$$y_U^i(u_R^i)^\dagger Q_L h_1 + y_D^i(d_R^i)^\dagger Q_L H_2 + y_u^{ij}(u_R^i)^\dagger q_L^j H_1 + y_d^{ij}(d_R^i)^\dagger q_L^j h_2 + y_e^{ij}(e_R^i)^\dagger \ell_L^j H_2 + y_\nu^{ij}(\nu_R^i)^\dagger \ell_L^j h_1 + \text{h.c.}\,. \tag{2.77}$$

If $n_h = 0$ and $n_H = 1$, only two up-type quarks and one down-type quark can get masses. One would identify them with the top, charm and bottom quarks. Thus, at this level, the strange, down and up quarks remain massless, as well as the neutrinos.[20] If $n_h = 1$ and $n_H = 1$, the observed hierarchy of fermion masses is a consequence of the different values of the Higgs vacuum expectation values and hierarchical values of the Yukawa couplings. The Yukawa couplings are calculated from open string three-point amplitudes for the left- and right-handed fermion and the Higgs states appearing at the D-brane intersections, and they are proportional

[20]Since the $\mathrm{U}(1)_b$ symmetry has mixed $\mathrm{SU}(3)_C$ anomalies, it can be regarded as the Peccei-Quinn symmetry. If the symmetry is broken by strong interaction effects, one expects that effective Yukawa couplings $u_R^\dagger Q_L H_1$ and $d_R^\dagger q_L H_2$, and dimension six operators $\frac{1}{M_{\mathrm{s}}^2}(\nu_R^\dagger \ell_L)(u_R^\dagger Q_L)^*$ from an exchange of massive string excited states are allowed. Then, the neutrino masses are of the order of $\Lambda_{\mathrm{QCD}}^3/M_{\mathrm{s}}^2$.

to $\prod_i \exp(-A_i/2\pi\alpha')$ where A_i is the area of the triangle formed by the three intersecting D-branes in the i-th T^2 [30, 31]. One also has to check whether the flavor-changing neutral current (FCNC) from a Higgs exchange are sufficiently suppressed (see ref.[32, 33]).

Thus, we have discussed that there are D-brane configurations which give the massless spectrum of the SM except for the Higgs sector, when we consider the type IIA orientifolds with intersecting D6-branes compactified on a torus. For the MSSM, there is also a proto-type D-brane configuration [34, 35] which keeps $\mathcal{N} = 1$ SUSY and gives the MSSM spectrum. For other phenomenological issues, see ref.[36] for review. Note that most non-SUSY constructions assume a low string scale $M_{\rm s}$ which is of the order of TeV, in order to avoid large radiative corrections to the Higgs boson mass, $\delta m_H^2 \sim M_{\rm s}^2$.[21] However, in case of such a low string scale, extra space should be compactified to not a simple torus but more general spaces. In the next section, we consider D-brane models with the low string scale, which are called low-scale string models.

3 Low-scale string models

Now, we consider low-scale string models in which the string scale $M_{\rm s} = 1/\sqrt{\alpha'}$ is of the order of TeV. Such a low string scale is possible due to the existence of large extra dimensions, and these models are expected to solve the hierarchy problem of the Standard Model. Since the string scale $M_{\rm s}$ is of the order of TeV, low-scale string models have a possibility of being confirmed or excluded by the LHC. We will assume the low-scale string models which are explained in this section, in discussions of the LHC phenomenology in the next part.

3.1 Large extra dimensions in string compactifications

Gravitational and gauge coupling constants

Here, we discuss the gravitational and gauge coupling constants in the context of string compactifications. While the gauge interactions are localized on the D-brane world volume, the gravitational interactions are spread into the transverse space. This gives qualitatively different quantities for their couplings.

The low-energy effective action of the type II theory which includes only the NS-NS closed string is

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,(-G)^{1/2}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - \frac{1}{2}H_{MNL}H^{MNL}\right), \tag{3.1}$$

where H_{MNL} is the field strength of the antisymmetric B-field, $H_{MNL} = \partial_M B_{NL} + \partial_N B_{LM} + \partial_L B_{MN}$. Let us make field redefinitions of the forms

$$\widetilde{G}_{MN} = \exp\big(\tfrac{-\widetilde{\Phi}}{2}\big)G_{MN}\,, \qquad \widetilde{\Phi} = \Phi - \Phi_0\,, \tag{3.2}$$

[21] Note that in one-loop calculations in the non-SUSY system, there is a problem that infrared divergence from an exchange of the NS-NS closed string is not canceled.

where Φ_0 is a background value of the dilaton field and $\widetilde{G}_{MN}$ is called the Einstein metric. In the Einstein frame, the space-time action (3.1) has the standard Hilbert form,

$$S = \frac{1}{2\kappa^2} \int d^{10}x\, (-\widetilde{G})^{1/2} \left(\widetilde{R} - \frac{1}{2}\partial_M \widetilde{\Phi} \widetilde{\partial}^M \widetilde{\Phi} - \frac{1}{2} e^{-\widetilde{\Phi}} H_{MNL} \widetilde{H}^{MNL} \right). \tag{3.3}$$

The constant $\kappa = \kappa_{10} e^{\Phi_0}$ is the gravitational coupling. Hereafter, we denote Φ_0 simply by Φ. The extra six-dimensional space in the ten-dimensional space-time should be compactified to an internal space with the finite volume V_6. Then, the gravitational coupling κ is reduced to the four-dimensional Newton constant G_N, and since $G_N = 1/M_{\rm Pl}^2$,

$$\frac{V_6}{2\kappa^2} = \frac{1}{16\pi G_N} = \frac{M_{\rm Pl}^2}{16\pi}\,. \tag{3.4}$$

On the other hand, from the ratio of the string tension $1/2\pi\alpha'$ and the D1-brane tension $T_1 = \frac{\sqrt{\pi}}{\kappa_{10}} 4\pi^2\alpha'$ in eq.(2.12), we obtain the relation between κ and α',

$$\kappa^2 = \tfrac{1}{2}(2\pi)^7 e^{2\Phi} \alpha'^4\,. \tag{3.5}$$

Then, the Planck scale $M_{\rm Pl}$ (3.4) can be written by the string scale $M_{\rm s}$ and the volume of the internal space V_6,

$$M_{\rm Pl}^2 = 8\, e^{-2\Phi} M_{\rm s}^8 \frac{V_6}{(2\pi)^6}\,. \tag{3.6}$$

Therefore, if the string coupling $g_{\rm s} \equiv e^{\Phi/2}$ is small and the internal volume V_6 is as large as $M_{\rm s}^6 V_6 \sim 10^{32}$, we can take the string scale $M_{\rm s}$ of the order of TeV.

On the other hand, the low-energy effective action which describes a coupling between a Dp-brane and the NS-NS closed string is shown in eq.(2.11),

$$S_{{\rm D}p} = -T_p \int d^{p+1}x\, {\rm Tr}\left\{ e^{-\Phi} \left[-\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}) \right]^{1/2} \right\}, \tag{3.7}$$

where T_p is the Dp-brane tension (2.12). Expanding the action in terms of $F_{\mu\nu}$, it is just the Yang-Mills action in the $(p+1)$-dimensional space-time,

$$S_{{\rm D}p} = -\frac{1}{4g_{{\rm D}p}^2} \int d^{p+1}x\, (-G)^{1/2}\, {\rm Tr}\left\{ {F_{\mu\nu}}^2 \right\} + \cdots, \tag{3.8}$$

where $g_{{\rm D}p}$ is the gauge coupling constant on the Dp-brane, as it appears also in eq.(2.26),

$$\frac{1}{g_{{\rm D}p}^2} = T_p\, e^{-\Phi} (2\pi\alpha')^2 = e^{-\Phi} \frac{\alpha'^{\frac{3-p}{2}}}{(2\pi)^{p-2}}\,. \tag{3.9}$$

When the extra $(p-3)$-dimensional space on the Dp-brane is compactified to a space with the volume V_{p-3}, the gauge coupling constant $g_{{\rm D}p}$ is reduced to a four-dimensional gauge coupling constant g,

$$\frac{1}{g^2} = \frac{V_{p-3}}{g_{{\rm D}p}^2} = e^{-\Phi} M_{\rm s}^{p-3} \frac{V_{p-3}}{(2\pi)^{p-2}}\,. \tag{3.10}$$

Therefore, if the string scale $M_{\rm s}$ is of the order of TeV, in order to keep the four-dimensional gauge coupling small, the volume on the Dp-brane V_{p-3} must be as small as $M_{\rm s}^{p-3}V_{p-3}\sim 1$.

However, the intersecting D6-branes with the simple toroidal compactification that we have discussed in the previous section cannot realize such a low string scale, because there is no direction in the internal space which is transverse to all the D6-branes. In the case of toroidal compactification, V_{p-3} is the volume of the $(p-3)$-cycles wrapped by the Dp-brane. In particular, for $p=6$,

$$V_3=(2\pi)^3\prod_{i=1}^{3}\sqrt{(n^iR_{2i-1})^2+(m^iR_{2i})^2}\,, \tag{3.11}$$

where (n^i,m^i) are the wrapping numbers of the one-cycle wrapped by the D6-branes, and R_{2i-1} and R_{2i} are the radii in the i-th T^2. On the other hand, the total internal volume V_6 is

$$V_6=(2\pi)^6\prod_{i=1}^{6}R_i\,. \tag{3.12}$$

Then, there is no radius that we can choose large in the internal space. In order to keep the four-dimensional gauge coupling small, the total volume V_6 must be also as small as $M_{\rm s}^6V_6\sim 1$, and the string scale $M_{\rm s}$ is restricted to be a few orders of magnitude of the Planck scale.

Thus, consistent string compactifications with large extra dimensions can be possible not for a simple torus but for more general Calabi-Yau spaces which can have large transverse dimensions to all the D-branes.

"Local models"

Let us consider large extra dimensions in the context of string compactifications. As explained above, in order to combine D-branes with the SM particle content with the scenario of large extra dimensions, one has to consider specific Calabi-Yau compactifications (on the Calabi-Yau large volume compactification, see ref.[10]). The three or four stacks of the intersecting D-branes which give rise to the SM spectrum such as Table 2.2 are local modules embedded into a global large volume Calabi-Yau manifold, as shown in Fig.3.1 [13].[22]

The hidden sector in Fig.3.1 is required to satisfy the R-R tadpole cancellation and stability conditions for the absence of tachyons. It is also responsible for spontaneously SUSY breaking in the system which has four-dimensional $\mathcal{N}=1$ SUSY for calculations of string amplitudes [13]. The supersymmetry breakdown is transferred to the SM branes by gravitational interactions or by gauge interactions via some vector-like messenger fields. The hidden sector is also responsible for moduli stabilization and cosmic inflation in the early universe (see ref.[37], for review).

We will use the results of open string four-point amplitudes involving the SM gauge and matter fields. In particular, the amplitudes involving four gauge bosons or two gauge bosons

[22] Note that the stacks of the D-branes cannot be wrapped on untwisted cycles of a compact torus or of toroidal orientifolds. One has to consider twisted and blowing-up cycles of an orbifold or more general Calabi-Yau spaces with blowing-up cycles.

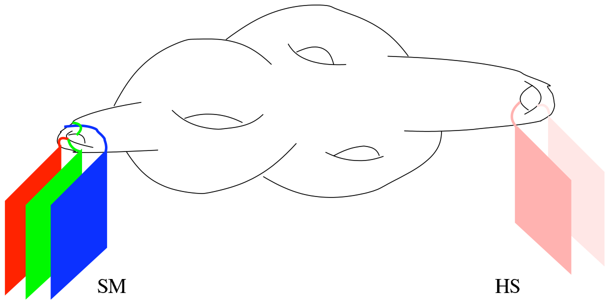

Figure 3.1: Some stacks of D-branes giving rise to the SM spectrum are embedded into a large Calabi-Yau space [13].

and two matter fermions do not depend on the geometry of the Calabi-Yau spaces. However, in order to calculate the string amplitudes, we have to assume that the SM branes are wrapped on flat or toroidal like cycles.

Thus, for discussions of the LHC phenomenology in the next part, we will assume such "local models", in which the intersecting D-branes giving rise to the SM spectrum are wrapped on local and flat cycles, and the cycles are embedded into a global large volume Calabi-Yau manifold giving rise to the low string scale.

3.2 Spectrum of low-scale string models

Let us summarize the spectrum of low-scale string models. This is useful for later discussions on phenomenology at the LHC.

The Standard Model gauge bosons and chiral fermions

The SM branes in Fig.3.1 are constructed of three or four stacks of intersecting D-branes such as Table 2.2 which yield the SM gauge group with additional U(1) factors and give rise to the SM chiral fermions at their intersections.

The SM gauge boson is realized by the open string with both ends on a stack of N D-branes. It is the NS sector massless state,

$$T^a e_\mu \psi^\mu_{-1/2}|0;k\rangle_{\rm NS}\,, \tag{3.13}$$

where T^a is the Chan-Paton factor in the adjoint representation of $\mathrm{U}(N)$ with $a = 1,\ldots,N^2$. The polarization vector e_μ satisfies the physical state condition $e\cdot k = 0$ and the equivalence condition $e_\mu \cong e_\mu + \lambda k_\mu$.

The SM chiral fermion is realized by the open string stretched between stacks of N_a D_a-branes and N_b D_b-branes. It is the R sector massless state,

$$u_{\mathbf{s}}|\mathbf{s};k;\alpha,\beta\rangle_{\rm R}\,, \tag{3.14}$$

where α and β are the Chan-Paton indices in the fundamental representations of $\mathrm{U}(N_a)$ and $\mathrm{U}(N_b)$ with $\alpha = 1,\ldots,N_a$ and $\beta = 1,\ldots,N_b$, respectively. The polarization spinor $u_{\mathbf{s}}$ satisfies the physical state condition $k \cdot \Gamma_{\mathbf{s}'\mathbf{s}}\, u_{\mathbf{s}} = 0$ and the state which is in the spinor representation of SO(1,3) is reduced to that in the spinor representation of SO(2). The GSO projection, $\Gamma_{\mathbf{s}'\mathbf{s}} u_{\mathbf{s}} = +u_{\mathbf{s}}$, restricts to the state with $s_1 = +\frac{1}{2}$.

String excited states

We can obtain massive states by exciting the massless states such as (3.13) and (3.14) with the mode operators, and such massive states are called string excited states. In the case of the open string, the mass of the string excited state is given by

$$m^2 = \frac{1}{\alpha'}\big(N_\alpha + N_\psi - \nu\big)\,, \tag{3.15}$$

where $N \equiv N_\alpha + N_\psi - \nu$ is the level of the state, and N_α and N_ψ are excitation levels due to the mode operators α^μ_n and ψ^μ_r, as shown in eq.(1.87).

Let us focus on the case of $N = 1$. The first string excited state of the SM gauge boson,

$$T^a\big[(e_1)_\mu \psi^\mu_{-3/2} + (e_2)_{\mu\nu}\alpha^\mu_{-1}\psi^\nu_{-1/2} + (e_3)_{\mu\nu\rho}\psi^\mu_{-1/2}\psi^\nu_{-1/2}\psi^\rho_{-1/2}\big]|0;k\rangle_{\rm NS}\,, \tag{3.16}$$

has the mass of $m^2 = 1/\alpha'$. The polarization three-tensor $(e_3)_{\mu\nu\rho}$ is antisymmetric. The physical state conditions derived from eq.(1.75) on the polarization tensors are given by

$$\begin{aligned} \sqrt{2\alpha'}(e_1)\cdot k + (e_2)_{\mu\nu}\eta^{\mu\nu} &= 0\,,\\ (e_1)_\nu + \sqrt{2\alpha'}k^\mu (e_2)_{\mu\nu} &= 0\,,\\ (e_2)_{\mu\nu} - (e_2)_{\nu\mu} + 6\sqrt{2\alpha'}k^\rho (e_3)_{\mu\nu\rho} &= 0\,. \end{aligned} \tag{3.17}$$

Counting the physical degrees of freedom in the four-dimensional space-time, we can understand that the degrees of freedom correspond to one spin $J = 2$, two $J = 1$ and two $J = 0$ modes [38].

On the other hand, the first string excited state of the SM chiral fermion,

$$\big[(u_1)_{\mu\mathbf{s}}\alpha^\mu_{-1} + (u_2)_{\mu\mathbf{s}}\psi^\mu_{-1}\big]|\mathbf{s};k;\alpha,\beta\rangle_{\rm R}\,, \tag{3.18}$$

also has the mass of $m^2 = 1/\alpha'$. The physical state conditions on the polarization vector-spinors are given by

$$\begin{aligned} k\cdot\Gamma_{\mathbf{s}'\mathbf{s}}(u_1)_{\mu\mathbf{s}} = k\cdot\Gamma_{\mathbf{s}'\mathbf{s}}(u_2)_{\mu\mathbf{s}} &= 0\,,\\ \Gamma_{\mathbf{s}'\mathbf{s}}\cdot(u_1)_{\mathbf{s}} + 2\sqrt{\alpha'}k\cdot(u_2)_{\mathbf{s}} &= 0\,,\\ 4\sqrt{\alpha'}k\cdot(u_1)_{\mathbf{s}} + \Gamma_{\mathbf{s}'\mathbf{s}}\cdot(u_2)_{\mathbf{s}} &= 0\,. \end{aligned} \tag{3.19}$$

The GSO projection impose $\Gamma_{\mathbf{s}'\mathbf{s}}(u_1)_{\mu\mathbf{s}} = +(u_1)_{\mu\mathbf{s}}$ and $\Gamma_{\mathbf{s}'\mathbf{s}}(u_2)_{\mu\mathbf{s}} = -(u_2)_{\mu\mathbf{s}}$ on the state. The remaining physical degrees of freedom correspond to one spin $J = 3/2$ and one $J = 1/2$ modes.

Kaluza-Klein and winding modes

In addition to the string excited states, there are other massive states in low-scale string models. The massive states appear when the extra six-dimensional space is compactified to an internal space with finite volume. They are called Kaluza-Klein and winding modes.

Although one has to consider more general Calabi-Yau compactifications in low-scale string models, here, for simplicity, let us consider compactifications on a six-dimensional torus which is factorizable into $T^1 \times \cdots \times T^1$. Then, the six-dimensional coordinates have the following periodicity,

$$X^i \cong X^i + 2\pi R_i \qquad \text{for } i = 4, \ldots, 9\,, \tag{3.20}$$

where R_i is the radius in the i-th T^1.

Let us consider the open string realizing the SM gauge bosons. We specify that the open string is on a stack of D6-branes and has the Neumann boundary condition in the four-dimensional space-time with $\mu = 0, 1, 2, 3$ and the three-dimensional space with $i_{\rm N} = 4, 5, 6$ and the Dirichlet boundary condition in the three-dimensional space with $i_{\rm D} = 7, 8, 9$. Note that in the case of the non-compact ten-dimensional space-time, there are the open string center-of-mass momenta only in the Neumann directions not in the Dirichlet directions. However, with the compact periodic dimensions (3.20), there are quantized momenta not only in the Neumann directions but also the Dirichlet directions.

First, the operator $\exp(2\pi i R_{i_{\rm N}} p^{i_{\rm N}})$ which translates the open string around the periodic Neumann directions $i_{\rm N} = 4, 5, 6$ must leave states invariant. The center-of-mass momenta in the Neumann directions are quantized,

$$k^{i_{\rm N}} = \frac{n^{i_{\rm N}}}{R_{i_{\rm N}}} \qquad \text{for } n^{i_{\rm N}} \in \mathbf{Z}\,. \tag{3.21}$$

The modes with the nonzero momenta (3.21) are Kaluza-Klein (KK) modes. On the other hand, the open string may wind around the periodic Dirichlet directions $i_{\rm D} = 7, 8, 9$ and then it satisfies the extended Dirichlet boundary condition,

$$X^{i_{\rm D}}(\sigma = \pi) = X^{i_{\rm D}}(\sigma = 0) + 2\pi R_{i_{\rm D}} w^{i_{\rm D}} \qquad \text{for } w^{i_{\rm D}} \in \mathbf{Z}\,, \tag{3.22}$$

where $w^{i_{\rm D}}$ is the winding number. Since the coordinates in the Dirichlet directions are expanded as eq.(2.3), the change of the Dirichlet coordinates in going around the open string is

$$2\pi R_{i_{\rm D}} w^{i_{\rm D}} = \int_0^\pi d\sigma^1\, \partial_1 X^{i_{\rm D}} = \int \left(dz\, \partial X^{i_{\rm D}} + d\bar{z}\, \bar{\partial} X^{i_{\rm D}} \right) = 2\pi \sqrt{\frac{\alpha'}{2}}\, \alpha_0^{i_{\rm D}}\,, \tag{3.23}$$

and the momenta in the Dirichlet directions are also quantized,

$$k^{i_{\rm D}} = \frac{w^{i_{\rm D}} R_{i_{\rm D}}}{\alpha'}\,. \tag{3.24}$$

The modes with the nonzero momenta (3.24) are winding modes. Then, the mass of the Kaluza-Klein and winding mode of the open string is given by

$$m^2 = \frac{(n^{i_{\rm N}})^2}{R_{i_{\rm N}}^2} + \frac{(w^{i_{\rm D}})^2\, R_{i_{\rm D}}^2}{\alpha'^2}\,. \tag{3.25}$$

In low-scale string models, the radius in the compact Neumann direction R_{i_N} corresponds to ${V_{p-3}}^{\frac{1}{p-3}}$ which is determined by the size of the four-dimensional gauge coupling constants in eq.(3.10), and the radius in the Dirichlet direction R_{i_D} corresponds to ${V_6}^{1/6}$ which is determined by the Planck scale in eq.(3.6). If the string coupling g_s is small and the string scale M_s is of the order of TeV, ${V_6}^{1/6} \sim (10\,\mathrm{MeV})^{-1}$ and the winding mass is $M_\mathrm{s}^2 {V_6}^{1/6} \sim 10^8\,\mathrm{GeV}$. Ignoring the winding modes due to its heaviness, in order to keep the gauge coupling constants small, ${V_{p-3}}^{\frac{1}{p-3}} \sim (1\mathrm{TeV})^{-1}$ and the KK mode mass of the open string is of almost the same order as the string excited state mass (3.15).

However, we will use the results of open string four-point amplitudes which do not depend on the geometry and size of the internal space, namely, which do not include poles of the KK and winding modes of the SM gauge and matter fields. Eventually, we can ignore both exchanges of the KK and winding modes.

Closed string states

While the gravitational coupling κ of eq.(3.5) is proportional to the string coupling g_s^2, the gauge coupling constant g of eq.(3.10) is proportional to g_s. Therefore, the open string four-point amplitudes with poles of closed string states are suppressed by g_s^2 in contrast to that with poles of open string states. This is also understood from the open-closed string duality that tree-level amplitudes of the closed string are understood as one-loop amplitudes of the open string. Thus, due to such one-loop suppressions, we can also ignore the closed string states including the closed string excited states and the KK and winding modes of the closed string.

In low-scale string models, one has to consider the large Calabi-Yau spaces shown in Fig.3.1, and the total volume V_6 must be so large as $M_\mathrm{s}^6 V_6 \sim 10^{32}$. Therefore, we can expect that the mass of the Kaluza-Klein graviton is so small as $V_6^{-1/6} \sim 10\,\mathrm{MeV}$, assuming that all the six dimensions in the Calabi-Yau space are similar in size. If the six dimensions are extremely asymmetric in size [39], for example, so that $M_\mathrm{s}^4 V_4 \sim 1$ and $M_\mathrm{s}^2 V_2 \sim 10^{32}$, we can expect that the lightest KK graviton mass is so much small as $V_2^{-1/2} \sim 10^{-4}\,\mathrm{eV}$. It is important to investigate experimental bounds on the KK graviton with a very small mass.

The model-independent signature is energy loss due to the KK graviton emission, which can be observed as missing energy. The emission cross sections are sizable due to enormous phase space of the KK gravitons. Even if each KK graviton couples with the four-dimensional gravitational strength, $1/M_\mathrm{Pl}^2$, a sum over all the modes converts this small coupling to the ten-dimensional gravitational strength, $1/M_\mathrm{s}^8$. Searches for missing energy in monojet events at the LHC set limits of $M_\mathrm{s} \gtrsim 4.54\,\mathrm{TeV}$ for the asymmetric case and $M_\mathrm{s} \gtrsim 2.51\,\mathrm{TeV}$ for the symmetric case [40]. The SN1987A observations which agree with standard calculations of supernovae energy loss also set limits of $M_\mathrm{s} \gtrsim 10\,\mathrm{TeV}$ for the asymmetric case and $M_\mathrm{s} \gtrsim 10\,\mathrm{GeV}$ for the symmetric case (see ref.[39] and therein).

Signatures coming from the virtual KK graviton exchange are model-dependent, since they are assumed that there are no exotic decay processes. Searches for signatures in dilepton events at the LHC set limits of $M_{\rm s} \gtrsim 3.8\,{\rm TeV}$ for the asymmetric case and $M_{\rm s} \gtrsim 2.7\,{\rm TeV}$ for the symmetric case [41]. In the case that the KK gravitons have branching fractions into photons and gluons, the absence of γ-ray signals in the EGRET satellite and considerations of neutron-star cooling give limits of $M_{\rm s} \gtrsim 40\,{\rm TeV}$ and $M_{\rm s} \gtrsim 700\,{\rm TeV}$ for the asymmetric case, and $M_{\rm s} \gtrsim 40\,{\rm GeV}$ and $M_{\rm s} \gtrsim 200\,{\rm GeV}$ for the symmetric case, respectively (see ref.[39] and therein).

Anomalous U(1) gauge bosons

The three or four stacks of intersecting D-branes yield the SM gauge group with additional U(1) factors, and some linear combinations of the U(1)s are anomalous. The anomalous U(1) gauge bosons receive masses via the Green-Schwarz mechanism which can be realized by an exchange of the R-R field originating from the R-R closed string.

The coupling of the Dp-brane with the R-R field is described by eq.(2.13). Under T-dualities it transforms to Chern-Simons terms of the form

$$T_p \int_{p+1} {\rm Tr}\big[\exp(B_2 + 2\pi\alpha' F_2) \wedge \sum_q C_q\big]\,, \tag{3.26}$$

where B_2 is the NS-NS antisymmetric B-field parallel to the Dp-brane, F_2 is the gauge field on the Dp-brane and C_q is the R-R antisymmetric q-tensor. Consider a D6_a-brane wrapped on a three-cycle π_a. Expanding (3.26) in terms of F_2, there is a Chern-Simons term of the form

$$2\pi\alpha'\, T_6 \int_{\mathbf{R}^{1,3}\times\pi_a} C_5 \wedge {\rm Tr}(F_2^a)\,. \tag{3.27}$$

The five-tensor C_5 on the D6_a-brane wrapped on the three-cycle π_a is reduced to four two-tensors B_2^I with $I = 0, 1, 2, 3$ in the four-dimensional space-time. The Chern-Simons term (3.27) results in the following coupling

$$c_a^I \int_{\mathbf{R}^{1,3}} B_2^I \wedge {\rm Tr}(F_2^a)\,, \tag{3.28}$$

where c_a^I is a constant. For example, in the case of compactification on a factorizable six-dimensional torus $T^6 = T^2 \times T^2 \times T^2$, considering a stack of N_a D6_a-branes wrapped on a cycle with wrapping numbers (n_a^i, m_a^i) in the i-th T^2, the constants c_a^I are

$$c_a^I = \begin{cases} N_a m_a^1 m_a^2 m_a^3 & \text{for } I = 0\,, \\ N_a n_a^J n_a^K m_a^I & \text{for } I = 1,2,3\,, \quad I \neq J \neq K \neq I\,. \end{cases} \tag{3.29}$$

Note that the $B \wedge F$ couplings (3.28) are not nonzero only for an U(1) gauge field F_2.

In order to understand the mechanism giving a mass to the U(1) gauge boson, let us consider the following Lagrangian of an Abelian gauge field A_μ and an antisymmetric two-tensor $B_{\mu\nu}$,

$$\mathcal{L} = -\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{c}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}\,, \tag{3.30}$$

gauge bosons	level	spin	mass	chiral fermions	level	spin	mass
SM gauge bosons	$N=0$	$J=1$	0	SM chiral fermions	$N=0$	$J=1/2$	0
string excited states	$N=1$	$J=0$	M_s	string excited states	$N=1$	$J=1/2$	M_s
		$J=1$	M_s			$J=3/2$	M_s
		$J=2$	M_s				
	$N=2$	$J=0$	$\sqrt{2}M_\mathrm{s}$		$N=2$	$J=1/2$	$\sqrt{2}M_\mathrm{s}$
		$J=1$	$\sqrt{2}M_\mathrm{s}$			$J=3/2$	$\sqrt{2}M_\mathrm{s}$
		$J=2$	$\sqrt{2}M_\mathrm{s}$			$J=5/2$	$\sqrt{2}M_\mathrm{s}$
		$J=3$	$\sqrt{2}M_\mathrm{s}$				
Kaluza-Klein modes	$n=1$	$J=1$	$V_{p-3}^{-\frac{1}{p-3}}$	Kaluza-Klein modes	$n=1$	$J=1/2$	$V_{q-3}^{-\frac{1}{q-3}}$
winding modes	$w=1$	$J=1$	$M_\mathrm{s}^2 V_6^{1/6}$	winding modes	$w=0$	$J=1/2$	$M_\mathrm{s}^2 V_6^{1/6}$
anomalous U(1)s	$N=0$	$J=1$	$\sim gM_\mathrm{s}$	gravitons	level	spin	mass
string excited states	$N=1$	$J=0$	$\sim M_\mathrm{s}$	massless gravitons	$N=0$	$J=2$	0
		$J=1$	$\sim M_\mathrm{s}$	Kaluza-Klein modes	$n=1$	$J=2$	$V_6^{-1/6}$
		$J=2$	$\sim M_\mathrm{s}$	winding modes	$w=1$	$J=2$	$M_\mathrm{s}^2 V_6^{1/6}$

Table 3.1: The spectrum of lower states in low-scale string models.

where $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We can rewrite (3.30) in terms of the arbitrary field $H_{\mu\nu\rho}$ so that the constraint $H = dB$ is imposed by introducing a Lagrange multiplier field η,

$$\mathcal{L}_0 = -\frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{c}{6} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} A_\sigma - \frac{c}{6} \eta \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma} \,. \tag{3.31}$$

Integrating by parts the last term in eq.(3.31), we can find a quadratic action for H and immediately solve for H,

$$H^{\mu\nu\rho} = -c\, \epsilon^{\mu\nu\rho\sigma} (A_\sigma + \partial_\sigma \eta) \,. \tag{3.32}$$

Inserting (3.32) into (3.31), we can find

$$\mathcal{L}_A = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - \frac{c^2}{2} (A_\sigma + \partial_\sigma \eta)^2 \,, \tag{3.33}$$

which is a mass term of the Abelian gauge field A_μ "eating" the scalar η and acquiring a mass $m^2 = g^2 c^2$.

Thus, the $B \wedge F$ couplings (3.28) give masses to some linear combinations of additional U(1) gauge bosons. The masses of the U(1) gauge bosons are explicitly given by

$$m_{ab}^2 = g_a g_b M_\mathrm{s}^2 \sum_I c_a^I c_b^I \,, \tag{3.34}$$

where the sum runs over the R-R fields with nonzero $B \wedge F$ couplings and g_a is the gauge coupling constant of $\mathrm{U}(1)_a$. Therefore, the anomalous U(1) gauge bosons have masses of the order of $M_\mathrm{s} \sim 1\,\mathrm{TeV}$.

After electroweak symmetry breaking, the anomalous U(1) gauge bosons form mass eigenstates of neutral gauge bosons together with the third component of $\mathrm{SU}(2)_L$. The mass of Z boson receives corrections induced by the additional U(1) gauge bosons. The electroweak precision measurements give constraints on the ρ parameter, $\rho = 1.0004^{+0.0003}_{-0.0004}$ [42], and this can be used to impose bounds on the string scale M_s. For example, in case of $n_H = 1$ and $n_h = 0$ in eq.(2.76), $M_\mathrm{s} \gtrsim 10\,\mathrm{TeV}$, while in case of $n_H = 0$ and $n_h = 1$, $M_\mathrm{s} \gtrsim 5\,\mathrm{TeV}$ [43]. However, note that these bounds highly depend on the detail of the Higgs sector.

We will use the results of open string four-point amplitudes which do not include poles of the anomalous U(1) gauge bosons as well as the KK modes. We can ignore the anomalous U(1) exchanges. However, note that the amplitudes do include poles of string excited states of the anomalous U(1) gauge bosons.

The spectrum of lower states in low-scale string models which have been discussed above is summarized in Table 3.1. We will eventually consider only string excited states in the next part, however, it is important to understand the spectrum of the models that we will assume in discussions on phenomenology at the LHC.

Part II

Signatures of low-scale string models at the LHC

In this part, we explore a possibility of observing signatures of low-scale string models in dijet events at the LHC. This part is original of this thesis based on ref.[19, 20].

4 String resonances at the LHC

4.1 Dijet events at the LHC

We focus on dijet events. String excited states which are characteristic modes in low-scale string models have the color quantum number and can be produced in scattering processes of partons. Since string excited states do not have a characteristic parity, such as the R-parity in the MSSM (for review, in ref.[44]) and the KK parity in the UED models [23], they can be produced alone in two-parton scattering processes. In these processes with exchanges of the states with masses of the order of TeV, the two partons in the final state are very energetic and hadronized into hadrons resulting in two jets.

Dijet events consist of two jets with higher transverse momentum, p_{T}, which is the size of momenta in the plane vertical to the beam pipe. In the QCD processes in the high-p_{T} regime, t-channel processes with small scattering angles from the beam pipe are dominant. Since at the LHC which is a proton-proton collider, the parton scattering processes cannot be in the center-of-mass frame, the Lorentz invariant observables are important. One of the Lorentz invariant observables in dijet events is the dijet invariant mass,

$$M_{jj} = \sqrt{(E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2}, \tag{4.1}$$

where E_i is the energy and $\mathbf{p}_i$ is the momentum of each jet. The dijet invariant mass distributions at the LHC are shown in Fig.4.1. The results are fitted well with the QCD prediction.

If string excited states are produced in the two-parton scattering processes, they can be observed as "resonances" in the dijet invariant mass distributions, since scattering amplitudes of the processes have s-channel poles. The resonances create a deviation from the QCD prediction or an excess of events localized in the dijet invariant mass. They are most clear signature for the discovery of colored new particles.

Constraints at the LHC

The string scale M_{s}, the mass of the first string excited state, is constrained by the ATLAS and CMS experiments at the LHC. The value of the string scale smaller than 3.61 TeV and 4.78 TeV has been excluded by the ATLAS [16] and by the CMS [17], respectively. This is obtained

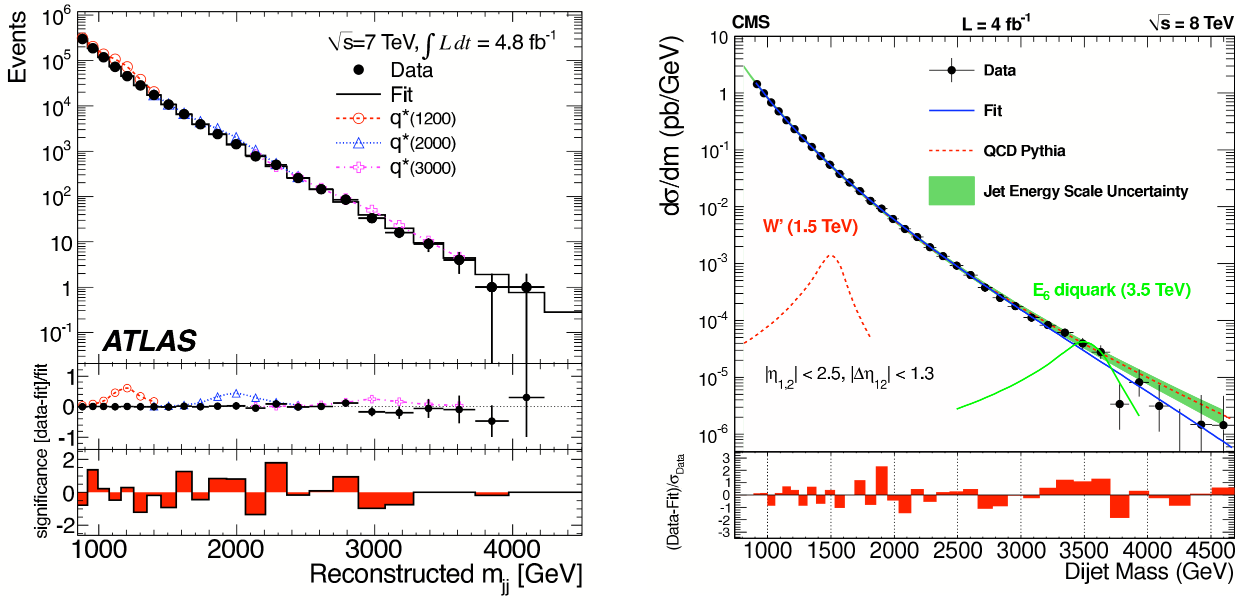

Figure 4.1: The dijet invariant mass distributions fitted with a function fitting to the QCD prediction at the ATLAS (the left figure) [16] and the CMS (the right figure) [17], with $4.8\,\text{fb}^{-1}$ for $\sqrt{s} = 7\,\text{TeV}$ and with $4\,\text{fb}^{-1}$ for $\sqrt{s} = 8\,\text{TeV}$, respectively. The right figure shows together with the QCD prediction. The middle panel in the left figure shows the data minus the fit divided by the fit, and the bottom panels in the left and right figures show the bin-by-bin significance between the data and the fit.

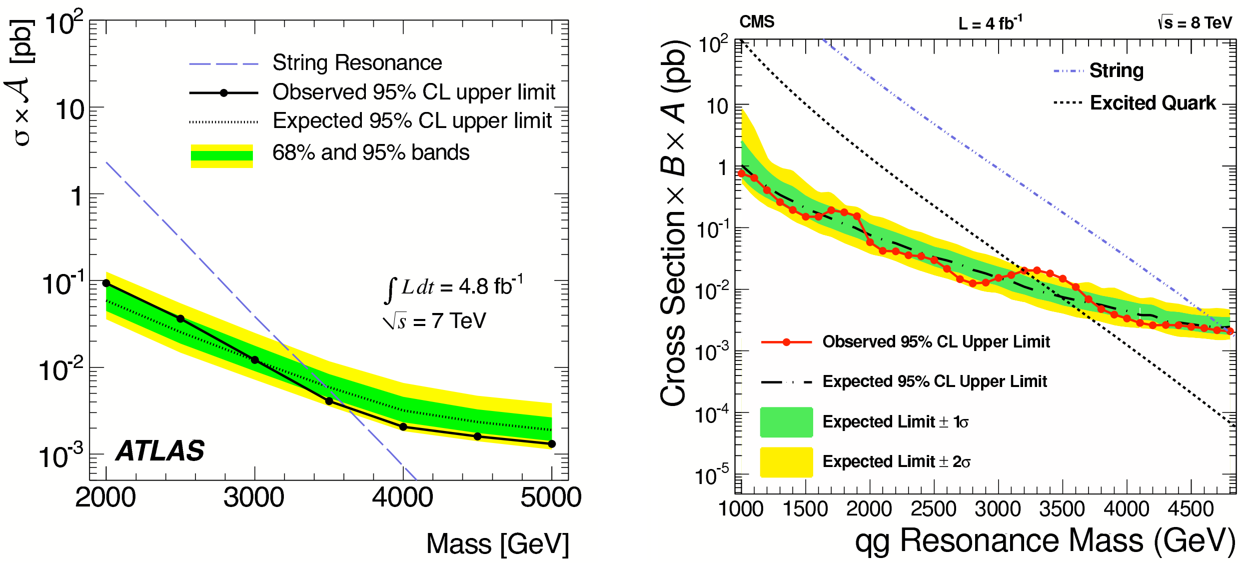

Figure 4.2: The 95% Confidence Level (CL) upper limit on the cross section times acceptance for string resonances at the ATLAS (the right figure) [16] with $4.8\,\text{fb}^{-1}$ for $\sqrt{s} = 7\,\text{TeV}$, and the cross section times branching fraction times acceptance for resonances decaying to qg final state at the CMS (the right figure) [17] with $4\,\text{fb}^{-1}$ for $\sqrt{s} = 8\,\text{TeV}$. The acceptance ranges from 45% to 48% for the ATLAS, and A ≈ 0.6 for the CMS.

from Fig.4.2. The black filled circles or the red filled circles in Fig.4.2 are the observed 95% Confidence Level (CL) upper limit on the cross section for string resonances. The value of the string scale where the cross section is larger than the observed upper limit has been excluded at 95% CL.

Photon+jet events

Besides dijet events, other interesting events at the LHC have also the possibility of observing string resonances, such as γ+jet events [45, 46] and $t\bar{t}$ events [38]. In particular, γ+jet events include a process which is absent in the SM, and they are available for confirming string models.

The process of $gg \to g\gamma$ in γ+jet events does not occurs at tree-level in the SM but does in string models using D-branes. Since the open string on a stack of N D-branes realize U(N) gauge bosons, in scattering processes of a SU(N) and an U(1) gauge bosons on the same stack, string excited states of the SU(N) gauge bosons can be exchanged. The $\mathrm{U}(1)_Y$ hypercharge gauge boson of the SM is a linear combination of some U(1) gauge bosons on some stacks of D-branes, and the string excited states of the SU(N) gauge bosons can decay into a photon. Therefore, string excited states of the $\mathrm{SU}(3)_{\mathrm{color}}$ gauge bosons can be exchanged at tree-level in $gg \to g\gamma$, and string resonances can be also observed in the γ+jet invariant mass distributions.

The γ+jet invariant mass distributions at the ATLAS are fitted well with the SM prediction [47], and the recorded highest invariant mass is 2.57 TeV. Although an additional parameter which is the U(1) gauge boson fraction of a photon is required, γ+jet events are interesting and available for confirming D-brane models.

4.2 Model independence at the LHC

We concentrate on dijet events. Let us consider two-parton scattering processes which result in dijet events and where string excited states can be exchanged. The two-parton scattering processes are $gg \to gg$, $gg \to q\bar{q}$, $q\bar{q} \to gg$, $qg \to qg$, $\bar{q}g \to \bar{q}g$, $qq \to qq$, $qq' \to qq'$ and $q\bar{q} \to q\bar{q}$. Since quarks and gluons are massless modes of open strings on stacks of D-branes, scattering amplitudes of these processes are calculated from open string amplitudes with external four massless states of the open strings. The open string amplitudes are reduced to the QCD amplitudes in a low-energy limit of $\sqrt{s} \ll M_{\mathrm{s}}$, where $\sqrt{s}$ is the center-of-mass energy in the parton CM frame.

When $\sqrt{s} \approx M_{\mathrm{s}}$, the open string amplitudes have all the s, t and u-channel poles of all the infinitely excited open string states. Including all the poles of all the infinitely excited states is just a string effect, and the string amplitudes cannot be reduced to field theoretical amplitudes without some approximation. We should expand them by any one of the s, t and u-channel poles in order to add widths of string excited states in the poles. Since we want to observe a resonance in a mass distribution, we expand the open string amplitudes with a sum over the s-channel poles. Then the amplitudes have already been correct only near the s-channel poles.

We consider only the two-parton scattering processes with s-channels, $gg \to gg$, $gg \to q\overline{q}$, $q\overline{q} \to gg$, $qg \to qg$, $\overline{q}g \to \overline{q}g$ and $q\overline{q} \to q\overline{q}$.

Kaluza-Klein and winding modes

In the spectrum of low-scale string models, in addition to string excited states of the open strings, there are other states which can be exchanged in the two-parton scattering processes at tree-level. They are Kaluza-Klein modes and characteristic modes of strings, winding modes.

Generally, D-branes realizing the SM gauge group have extra space dimensions than three. When the extra dimensions on the SM branes are compactified, Kaluza-Klein (KK) and winding modes of open strings on the SM branes appear in our four-dimensional space-time. They have the same gauge quantum numbers as that of the SM particles. The KK modes come from that momenta along the compact dimensions parallel to the SM brane are quantized, and the winding modes come from that the open strings can wind around the compact dimensions transverse to the SM branes. If the compact dimensions are flat, and radii of the parallel and transverse dimensions are $R_{\rm N}$ and $R_{\rm D}$, respectively, the KK and winding modes have masses,

$$M_n^{\rm KK} = \frac{n}{R_{\rm N}} , \qquad M_w^{\rm wind.} = w M_{\rm s}^2 R_{\rm D} , \tag{4.2}$$

for the KK modes with nth level, and for the winding modes with the winding number w. Both of them appear from momenta along the compact dimensions.

Thus, the masses of the KK and winding modes depend on the size of the compact dimensions and the geometry of the compact space. Therefore, processes where the KK and winding modes can be exchanged also depend on such the detail of model-buildings. Since there are a lot of possible ways of model-buildings in string models, it does not seem worthwhile to discuss each individual model.

The KK and winding modes have the possibility of being exchanged in the two-parton scattering processes at tree-level. However, if momenta along the extra dimensions are conserved, they are produced in pairs but not alone, like the KK parity conservation in the UED models. In fact, among the two-parton scattering processes with s-channels, in the processes of $gg \to gg$, $gg \to q\overline{q}$, $q\overline{q} \to gg$, $qg \to qg$ and $\overline{q}g \to \overline{q}g$, only the string excited states can be exchanged (Fig.4.3), on the other hand, in the process of $q\overline{q} \to q\overline{q}$, both the string excited states and the KK and winding modes can be exchanged (Fig.4.4). The former processes are *model-independent*, and the latter is *model-dependent*.

In the rest of this subsection, let us discuss the momentum conservation in each process.

4.2.1 $gg \to gg$, $gg \to q\overline{q}$ and $q\overline{q} \to gg$

In the processes of $gg \to gg$, $gg \to q\overline{q}$ and $q\overline{q} \to gg$, there may be exchanges of the SM gluons, the string excited gluons and the KK gluons which are realized by an open string on the $\mathrm{U}(3)_{\rm color}$ branes. In particular, $gg \to gg$ is a scattering process including only states realized by the open string. Since the $\mathrm{U}(3)_{\rm color}$ brane is one stack of three D-branes, momenta in the process are

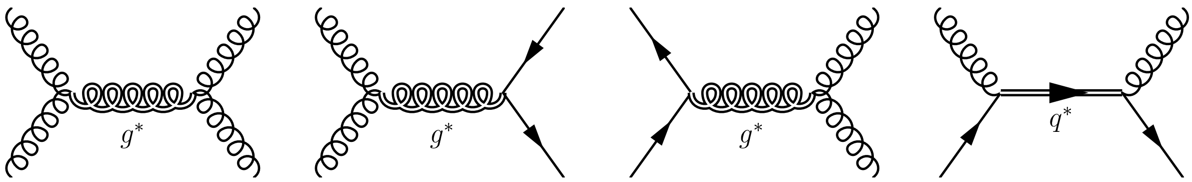

Figure 4.3: The model independent two-parton scattering processes with exchanges of the string excited states of gluons and quarks, g^* and q^*, respectively.

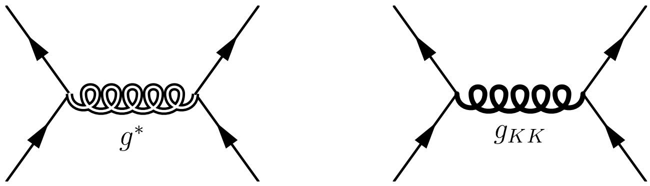

Figure 4.4: The model-dependent processes with an exchange of the KK gluon, $g_{\rm KK}$.

conserved. The single KK gluon cannot be produced in $gg \to gg$ and the pair of the KK gluons can be produced as shown in Fig.4.5. However, the process such as Fig.4.5 is at one-loop level beyond a scope of this thesis. The momentum conservation is similarly true in $gg \to q\overline{q}$ and $q\overline{q} \to gg$, because these processes have the same interaction vertices $gg \to g^*$ as $gg \to gg$ has, where g^* denotes the first string excited gluon. If momenta are conserved at the vertex, they are also conserved through the processes.

4.2.2 $qg \to qg$ and $\overline{q}g \to \overline{q}g$

In the processes of $qg \to qg$ and $\overline{q}g \to \overline{q}g$, there may be exchanges of the SM quarks, the string excited quarks and the KK quarks which are realized by an open string whose ends attach on two different stacks of D-branes intersecting each other. For example, the left-handed quark doublets Q_L are realized as massless modes of the open string with ends on both the $\rm U(3)_{color}$ and the $\rm U(2)_{left}$ branes. An intersection plane between the two different stacks of D-branes contains our three-dimensional space, and the open string is localized on the intersection plane. If the intersection plane does not have extra dimensions, there are not KK modes of the open string including the KK quarks. Even if the intersection plane has extra dimensions, momenta on the intersection plane are conserved. Therefore, the KK quarks cannot be exchanged in $qg \to qg$ and $\overline{q}g \to \overline{q}g$.

4.2.3 $q\overline{q} \to q\overline{q}$

In the process of $q\overline{q} \to q\overline{q}$, there may be exchanges of the SM gluons, the string excited gluons and the KK gluons which are realized by the open string on the $\rm U(3)_{color}$ branes. On the other hand, since the process is a scattering process between a quark and an antiquark realized by the open strings with ends on the two different stacks of D-branes, momenta in the process are not conserved. The process of $q\overline{q} \to q\overline{q}$ is a pair-annihilation and pair-production, and each quark

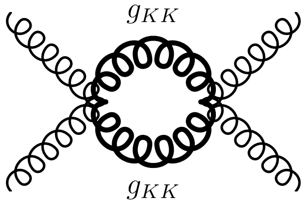

Figure 4.5: The $gg \to gg$ process with an exchange of a pair of the KK gluons, $g_{\rm KK}$.

current is conserved but an internal charge through the process is not conserved. In $q\overline{q} \to q\overline{q}$, the KK gluons and the other KK modes can be exchanged, and the process is model-dependent.

As mentioned above, it does not seem worthwhile to discuss the model-dependent process in detail. Fortunately, considering dijet events at the LHC which is a proton-proton collider, we may ignore antiquarks in the initial state because antiquark fractions of a proton are suppressed by the parton distribution functions (PDFs). In fact, although $q\overline{q}$ initial states contribute larger than gg initial states since $q\overline{q}$ contains an initial quark, qg initial states have largest contributions of all the initial states. Therefore, we can consider only $qg \to qg$ in later analyses, and eventually we can say leading signatures of low-scale string models at the LHC are independent of the detail of model-buildings.

Here, we will simply ignore the model-dependent process, $q\overline{q} \to q\overline{q}$, and concentrate on only the model-independent processes, $gg \to gg$, $gg \to q\overline{q}$ ($q\overline{q} \to gg$) and $qg \to qg$ ($\overline{q}g \to \overline{q}g$).[23]

4.3 Reduction of string amplitudes and widths of string excited states

As mentioned in the previous subsection, in low-scale string models, scattering amplitudes of the two-parton scattering processes are calculated from open string amplitudes with external four massless states of the open strings. The tree-level amplitude with external four gluons has been calculated in ref.[48, 49]. The amplitudes with external two gluons and two quarks at tree-level have been calculated in ref.[13], where it is assumed that the SM chiral fermions are realized by the open string stretching between D-branes intersecting each other. [24]

The open string amplitudes should be reduced to field theoretical amplitudes in order to add widths of string excited states in their poles. In this subsection, the reduction of the open string amplitudes and calculation of the widths of string excited sates are reviewed briefly, and see Appendix A in detail.

[23]Although contributions of $qq \to qq$ are very large due to initial quarks, the process does not contribute dijet resonances because it has no s-channel. Contributions of t-channel poles of KK modes in the process are analyzed in ref.[12].

[24]A brief review of calculation of the four-point open string amplitudes is in Appendix C.

The open-string amplitudes are reduced to the QCD amplitudes in the low-energy limit where the scattering energy $\sqrt{s}$ is much less than the string scale $M_{\rm s}$, $\sqrt{s} \ll M_{\rm s}$. When $\sqrt{s} \approx M_{\rm s}$, string effects appear in the open string amplitudes through "string form factor" functions such as

$$V(s,t,u) = \frac{\Gamma(1-s/M_{\rm s}^2)\Gamma(1-u/M_{\rm s}^2)}{\Gamma(1+t/M_{\rm s}^2)}\,, \tag{4.3}$$

where s, t and u are the Mandelstam variables of scattering particles, and $\Gamma(x)$ is the ordinary Gamma function. The form factor functions have all the s, t and u-channel poles of all the infinitely excited open string states, which is just the string effect. We expand them by a sum over the s-channel poles in order to reduce to field theoretical amplitudes,

$$V(s,t,u) \simeq \sum_{n=1}^{\infty} \frac{1}{(n-1)!}\frac{1}{(M_{\rm s}^2)^{n-1}}\frac{1}{s-nM_{\rm s}^2}\prod_{J=0}^{n-1}(u+JM_{\rm s}^2)\,, \tag{4.4}$$

which is an approximation near each nth pole, $s \simeq nM_{\rm s}^2$. These poles correspond to string excited states which have masses of $M_n = \sqrt{n}M_{\rm s}$. This expansion implies that the open string four-point amplitudes are reduced to two-body scattering amplitudes of SM particles, in which string excited states are exchanged in s-channel.

The string form factor functions can be expanded in terms of $1/M_{\rm s}^2$. The expansion corresponds to the low-energy limit of $M_{\rm s} \to \infty$. In the expansion, the string form factor functions become unity,

$$V(s,t,u) = 1 - \frac{\pi^2}{6}\,su\,\frac{1}{M_{\rm s}^4} + \mathcal{O}\left(\frac{1}{M_{\rm s}^6}\right). \tag{4.5}$$

The unity in eq.(4.5) describes an exchange of a massless state, and the open string amplitudes are reduced to the QCD amplitudes. On the other hand, the term of the order of $1/M_{\rm s}^4$ in eq.(4.5) describes a string contact interaction as a result of summing up all string excited states. Note that there is no term of the order of $1/M_{\rm s}^2$ in eq.(4.5).

In the open string amplitudes expanded by s-channel poles, there is a new angular dependence which is described by a factor $\prod_{J=0}^{n-1}(u+JM_{\rm s}^2)$ in eq.(4.4). The new angular dependence is multiplied by an original angular dependence in the SM amplitudes and decomposed into a sum over the Wigner d-functions. The Wigner d-function, $d^J_{\lambda_1-\lambda_2,\,\lambda_3-\lambda_4}(\theta)$, represents an angular dependence of a process through a state with spin J, in which initial helicities along z-axis are λ_1, λ_2, and final helicities along z'-axis are λ_3, λ_4. The angle θ is that between the z-axis and z'-axis. Then, the two-body scattering amplitude with exchanges of nth string excited states are

$$\mathcal{M}_{n\text{th}}(\lambda_1,\,\lambda_2 \to \lambda_3,\,\lambda_4) \simeq \frac{1}{s-M_n^2}\sum_{J=J_{\rm min}}^{J_{\rm max}} d^{\,J}_{\,\lambda_1-\lambda_2,\,\lambda_3-\lambda_4}(\theta)\,F_n{}^{J\,*}_{\lambda_3,\lambda_4}\,F_n{}^{J}_{\lambda_1,\lambda_2}\,, \tag{4.6}$$

where $F_n{}^{J}_{\lambda_i,\lambda_j}$ is a matrix element of a decay of the nth string excited state with spin J into states with helicities λ_i, λ_j. This decomposition implies that nth string excited states are degenerate in mass of M_n with various spin J which ranges from $J_{\rm min}$ to $J_{\rm max}$. The value of the highest spin is $J_{\rm max} = j_0 + n$, where j_0 is original spin of the corresponding SM particle.

String excited states are unstable and decay into not only the SM particles but also other light string excited states or KK modes. The s-channel poles corresponding to nth string excited states in eq.(4.4) should be softened to the Breit-Wigner form,

$$\frac{1}{s-M_n^2} \to \frac{1}{s-M_n^2+iM_n\,\Gamma_n^J}\,, \tag{4.7}$$

where Γ_n^J is a total decay width of the nth string excited state with spin J. The width is calculated as

$$\Gamma_n^J = \frac{1}{16\pi M_n}\,\frac{1}{2J+1}\sum_{\lambda_i,\lambda_j}\left|F_n{}^J_{\lambda_i,\lambda_j}\right|^2, \tag{4.8}$$

by extracting $F_n{}^J_{\lambda_i,\lambda_j}$ from the two-body scattering amplitudes with exchanges of the nth string excited states in eq.(4.6).

Loop-level amplitudes may lead to an imaginary part in a pole written by a width of an exchanged state. However, for the purpose of this thesis, we consider only tree-level amplitudes, and we add the width to the imaginary part in the s-channel pole by hand.

4.3.1 First string excited state exchanges

The model-independent two-parton scattering processes with exchanges of the first string excited states are $gg \to gg$, $gg \to q\overline{q}$, $q\overline{q} \to gg$, $qg \to qg$ and $\overline{q}g \to \overline{q}g$ in Fig.4.3. The scattering amplitudes of the processes are calculated in ref.[11, 12], and the calculations are reviewed in Appendix A. The squared amplitudes averaged over spins and colors in the initial state and summed in the final state are

$$\begin{aligned}\left|\mathcal{M}_{1\mathrm{st}}(gg\to gg)\right|^2 = \frac{8g^4}{M_\mathrm{s}^4}\,\frac{1}{N^2}\Bigg\{&\frac{(N^2-4)^2}{4(N^2-1)}\left[\frac{M_\mathrm{s}^8}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{g^*}^{J=0})^2}+\frac{\hat{u}^4+\hat{t}^4}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{g^*}^{J=2})^2}\right]\\&+\left[\frac{M_\mathrm{s}^8}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{C^*}^{J=0})^2}+\frac{\hat{u}^4+\hat{t}^4}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{C^*}^{J=2})^2}\right]\Bigg\},\end{aligned} \tag{4.9}$$

$$\left|\mathcal{M}_{1\mathrm{st}}(gg\to q\overline{q})\right|^2 = \frac{2g^4}{M_\mathrm{s}^4}\,\frac{N_f}{N(N^2-1)}\left[\frac{N^2-4}{2}\,\frac{\hat{t}\hat{u}^3+\hat{u}\hat{t}^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{g^*}^{J=2})^2}+\frac{\hat{t}\hat{u}^3+\hat{u}\hat{t}^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{C^*}^{J=2})^2}\right]. \tag{4.10}$$

$$\left|\mathcal{M}_{1\mathrm{st}}(q\overline{q}\to gg)\right|^2 = \frac{2g^4}{M_\mathrm{s}^4}\quad\frac{N^2-1}{N^3}\quad\left[\frac{N^2-4}{2}\,\frac{\hat{t}\hat{u}^3+\hat{u}\hat{t}^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{g^*}^{J=2})^2}+\frac{\hat{t}\hat{u}^3+\hat{u}\hat{t}^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{C^*}^{J=2})^2}\right], \tag{4.11}$$

$$\begin{aligned}\left|\mathcal{M}_{1\mathrm{st}}(qg\to qg)\right|^2 &= \left|\mathcal{M}_{1\mathrm{st}}(\overline{q}g\to \overline{q}g)\right|^2\\&=\frac{2g^4}{M_\mathrm{s}^2}\,\frac{N^2-1}{4N^2}\left[\frac{M_\mathrm{s}^4(-\hat{u})}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{q^*}^{J=1/2})^2}+\frac{(-\hat{u})^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{q^*}^{J=3/2})^2}\right],\end{aligned} \tag{4.12}$$

$$\begin{aligned}\left|\mathcal{M}_{1\mathrm{st}}(qg\to gq)\right|^2 &= \left|\mathcal{M}_{1\mathrm{st}}(\overline{q}g\to g\overline{q})\right|^2\\&=\frac{2g^4}{M_\mathrm{s}^2}\,\frac{N^2-1}{4N^2}\left[\frac{M_\mathrm{s}^4(-\hat{t})}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{q^*}^{J=1/2})^2}+\frac{(-\hat{t})^3}{(\hat{s}-M_\mathrm{s}^2)^2+(M_\mathrm{s}\Gamma_{q^*}^{J=3/2})^2}\right],\end{aligned} \tag{4.13}$$

where $N=3$, $N_f=6$ and g is the gauge coupling constant of strong interaction. Here, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the Mandelstam variables of the scattering partons. $\Gamma^J_{g^*,C^*,q^*}$ are total decay widths of the first string excited states of gluons, the $\mathrm{U}(1)_{\text{color}}$ gauge bosons and quarks with spin J, respectively,

$$\Gamma^{J=0}_{g^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{N}{4}, \qquad \Gamma^{J=2}_{g^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{1}{5}\left(\frac{N}{2}+\frac{N_f}{8}\right), \tag{4.14}$$

$$\Gamma^{J=0}_{C^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{N}{2}, \qquad \Gamma^{J=2}_{C^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{1}{5}\left(N+\frac{N_f}{8}\right), \tag{4.15}$$

$$\Gamma^{J=1/2}_{q^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{1}{2}\frac{N}{4}, \qquad \Gamma^{J=3/2}_{q^*} = \frac{g^2 M_\mathrm{s}}{4\pi}\frac{1}{4}\frac{N}{4}, \tag{4.16}$$

which are calculated in ref.[50].

The first string excited states of gluons and the $\mathrm{U}(1)_{\text{color}}$ gauge boson have spins not only $J=0$ and $J=2$ but also $J=1$, which is verified by counting physical degrees of freedom of them as in ref.[38]. However, as it is found from eqs.(4.10) and (4.11), the string excited gluons with $J=1$ cannot be accessible even in the process of $q\overline{q}\to gg$.[25]

The string excited states of the $\mathrm{U}(1)_{\text{color}}$ gauge boson can be also exchanged in scattering processes of gluons and quarks, since the open string on $\mathrm{U}(3)_{\text{color}}$ branes realizes not only gluons but also the $\mathrm{U}(1)_{\text{color}}$ gauge boson. The anomalous U(1) gauge bosons including the $\mathrm{U}(1)_{\text{color}}$ gauge boson obtain masses of the order of M_s by the Green-Schwartz mechanism if the theory is consistent or anomaly free.

4.3.2 Second string excited state exchanges and interference effects

The amplitudes of the model-independent two-parton scattering processes with exchanges of the second string excited states are calculated in ref.[38, 19]. The authors of ref.[38] calculate them for the processes of $gg\to gg$, $gg\to q\overline{q}$, and we calculated for the processes of $qg\to qg$ and $\overline{q}g\to\overline{q}g$ in ref.[19]. The squared amplitudes of the processes are

$$\begin{aligned}
&\left|\mathcal{M}_{\text{2nd}}(gg\to gg)\right|^2_{\text{non int.}} \\
&= \frac{g^4}{8M_\mathrm{s}^8}\frac{N^2}{N^2-1}\left\{\frac{16M_\mathrm{s}^8(-\hat{u}+\hat{t})^2}{(\hat{s}-2M_\mathrm{s}^2)^2+(\sqrt{2}M_\mathrm{s}\Gamma^{J=1}_{g^{**}})^2}\right. \\
&\qquad\left. +\left[\frac{4}{9}\frac{4M_\mathrm{s}^4(\hat{u}^4+\hat{t}^4)}{(\hat{s}-2M_\mathrm{s}^2)^2+(\sqrt{2}M_\mathrm{s}\Gamma^{J=2}_{g^{**}})^2}+\frac{1}{9}\frac{\hat{u}^4(-\hat{u}+5\hat{t})^2+\hat{t}^4(-\hat{t}+5\hat{u})^2}{(\hat{s}-2M_\mathrm{s}^2)^2+(\sqrt{2}M_\mathrm{s}\Gamma^{J=3}_{g^{**}})^2}\right]\right\},
\end{aligned} \tag{4.17}$$

$$\begin{aligned}
&\left|\mathcal{M}_{\text{2nd}}(gg\to q\overline{q})\right|^2_{\text{non int.}} \\
&= \frac{g^4}{8M_\mathrm{s}^8}\frac{N_f\cdot N}{2(N^2-1)}\left[\frac{1}{9}\frac{4M_\mathrm{s}^4\left(\hat{t}\hat{u}^3+\hat{u}\hat{t}^3\right)}{(\hat{s}-2M_\mathrm{s}^2)^2+(\sqrt{2}M_\mathrm{s}\Gamma^{J=2}_{g^{**}})^2}+\frac{4}{9}\frac{\hat{t}\hat{u}^3(-\hat{u}+2\hat{t})^2+\hat{u}\hat{t}^3(-\hat{t}+2\hat{u})^2}{(\hat{s}-2M_\mathrm{s}^2)^2+(\sqrt{2}M_\mathrm{s}\Gamma^{J=3}_{g^{**}})^2}\right],
\end{aligned} \tag{4.18}$$

[25] The string excited gluons with $J=1$ can be identified with a intermediate state in scattering processes of $\tilde{g}\tilde{g}\to\tilde{g}\tilde{g}$ and $\tilde{g}\tilde{g}\to q\overline{q}$, where $\tilde{g}$ denotes a gluino, a superpartner of gluon. Widths of the string excited gluons with $J=1$ are calculated in ref.[50] and Appendix A.

$$\left|\mathcal{M}_{\rm 2nd}(q\overline{q}\to gg)\right|^2_{\text{non int.}}$$
$$=\frac{g^4}{8M_s^8}\quad\frac{N^2-1}{2N}\quad\left[\frac{1}{9}\frac{4M_s^4\big(\hat{t}\hat{u}^3+\hat{u}\hat{t}^3\big)}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=2}_{g^{**}})^2}+\frac{4}{9}\frac{\hat{t}\hat{u}^3(-\hat{u}+2\hat{t})^2+\hat{u}\hat{t}^3(-\hat{t}+2\hat{u})^2}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=3}_{g^{**}})^2}\right], \tag{4.19}$$

$$\left|\mathcal{M}_{\rm 2nd}(qg\to qg)\right|^2_{\text{non int.}}=\left|\mathcal{M}_{\rm 2nd}(\overline{q}g\to \overline{q}g)\right|^2_{\text{non int.}}$$
$$=\frac{g^4}{4M_s^6}\frac{N^2-1}{4N^2}\left\{\left[\frac{1}{9}\frac{16M_s^8(-\hat{u})}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=1/2}_{q^{**}})^2}+\frac{4}{9}\frac{4M_s^4(-\hat{u})(-\hat{u}+2\hat{t})^2}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=3/2}_{q^{**}})^2}\right]\right.$$
$$\left.+\left[\frac{9}{25}\frac{4M_s^4(-\hat{u})^3}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=3/2}_{q^{**}})^2}+\frac{4}{25}\frac{(-\hat{u})^3(-\hat{u}+4\hat{t})^2}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=5/2}_{q^{**}})^2}\right]\right\}, \tag{4.20}$$

$$\left|\mathcal{M}_{\rm 2nd}(qg\to gq)\right|^2_{\text{non int.}}=\left|\mathcal{M}_{\rm 2nd}(\overline{q}g\to g\overline{q})\right|^2_{\text{non int.}}$$
$$=\frac{g^4}{4M_s^6}\frac{N^2-1}{4N^2}\left\{\left[\frac{1}{9}\frac{16M_s^8(-\hat{t})}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=1/2}_{q^{**}})^2}+\frac{4}{9}\frac{4M_s^4(-\hat{t})(-\hat{t}+2\hat{u})^2}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=3/2}_{q^{**}})^2}\right]\right.$$
$$\left.+\left[\frac{9}{25}\frac{4M_s^4(-\hat{t})^3}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=3/2}_{q^{**}})^2}+\frac{4}{25}\frac{(-\hat{t})^3(-\hat{t}+4\hat{u})^2}{(\hat{s}-2M_s^2)^2+(\sqrt{2}M_s\Gamma^{J=5/2}_{q^{**}})^2}\right]\right\}. \tag{4.21}$$

Note that the second string excited states of the $\mathrm{U}(1)_{\rm color}$ gauge boson do not appear as intermediate states in $gg\to gg$, $gg\to q\overline{q}$ and $q\overline{q}\to gg$. It is a general fact that string excited states of an Abelian gauge boson with even level do not appear as poles in open string amplitudes.

$\Gamma^J_{g^{**},q^{**}}$ are decay widths of the second string excited gluons and quarks with spin J, respectively,

$$\Gamma^{J=1}_{g^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{3}\left(\frac{N}{4}+\frac{N_f}{360}\right),\quad \Gamma^{J=2}_{g^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{5}\left(\frac{N}{3}+\frac{N_f}{48}\right),\quad \Gamma^{J=3}_{g^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{7}\left(\frac{N}{6}+\frac{N_f}{15}\right), \tag{4.22}$$
$$\Gamma^{J=1/2}_{q^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{2}\frac{N}{12},\qquad \Gamma^{J=3/2}_{q^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{4}\frac{19N}{60},\qquad \Gamma^{J=5/2}_{q^{**}}=\frac{g^2\sqrt{2}M_s}{4\pi}\frac{1}{6}\frac{N}{10}. \tag{4.23}$$

Note that in calculations of the decay widths (4.22) and (4.23), it is assumed that the second string excited states decay into only the SM particles. The decay widths are calculated as eq.(4.8) by extracting matrix elements from the two-body scattering amplitudes (4.6). However, since the second string excited states are heavier than and have coupling with the first string excited states, we should consider decay widths of the string excited states into the first string excited states. Since there are some technical complication to calculate open string amplitudes with external first string excited states, we do not calculate the decay widths of 2nd → 1st+SM. We will discuss this issue in Sec.6.

Interference effect between second string excited states

As it is understood from eq.(4.6), string excited states are degenerate in mass with various spin J. Therefore, we should consider interference effects between degenerate string excited states with different spins.

The necessity to consider the interference effects appear in the scattering amplitudes with exchanges of the second string excited states for the first time. For one example, in the processes of $g^\pm g^\mp \to g^\pm g^\mp$ with exchanges of intermediate states with $J_z = \pm 2$, both the second string excited gluons with $J = 2$ and $J = 3$ are exchanged. For the other example, in the processes of $q^\pm g^\pm \to q^\pm g^\pm$ with exchanges of states with $J_z = \pm 1/2$, both the second string excited quarks with $J = 1/2$ and $J = 3/2$ are exchanged. Similarly, in the processes of $q^\pm g^\mp \to q^\pm g^\mp$ with $J_z = \pm 3/2$, both the second string excited quarks with $J = 3/2$ and $J = 5/2$ are exchanged.

The interference effects in the above processes between the second string excited states are

$$\begin{aligned}&\left|\mathcal{M}_{\rm 2nd}(gg \to gg)\right|^2_{\rm int.}\\ &= \frac{g^4}{4M_{\rm s}^8}\frac{N^2}{N^2-1}\left[\frac{2}{9}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=2}_{g^{**}}\Gamma^{J=3}_{g^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}\frac{2M_{\rm s}^2\left(\hat{u}^4(-\hat{u}+5\hat{t})+\hat{t}^4(-\hat{t}+5\hat{u})\right)}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=3}_{g^{**}})^2}\right],\end{aligned} \tag{4.24}$$

$$\begin{aligned}&\left|\mathcal{M}_{\rm 2nd}(gg \to q\overline{q})\right|^2_{\rm int.}\\ &= \frac{g^4}{4M_{\rm s}^8}\frac{N_f\cdot N}{2(N^2-1)}\left[\frac{2}{9}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=2}_{g^{**}}\Gamma^{J=3}_{g^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}\frac{2M_{\rm s}^2\left(\hat{t}\hat{u}^3(-\hat{u}+2\hat{t})+\hat{u}\hat{t}^3(-\hat{t}+2\hat{u})\right)}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right],\end{aligned} \tag{4.25}$$

$$\begin{aligned}&\left|\mathcal{M}_{\rm 2nd}(q\overline{q} \to gg)\right|^2_{\rm int.}\\ &= \frac{g^4}{4M_{\rm s}^8}\frac{N^2-1}{2N}\left[\frac{2}{9}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=2}_{g^{**}}\Gamma^{J=3}_{g^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}\frac{2M_{\rm s}^2\left(\hat{t}\hat{u}^3(-\hat{u}+2\hat{t})+\hat{u}\hat{t}^3(-\hat{t}+2\hat{u})\right)}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right],\end{aligned} \tag{4.26}$$

$$\begin{aligned}&\left|\mathcal{M}_{\rm 2nd}(qg \to qg)\right|^2_{\rm int.} = \left|\mathcal{M}_{\rm 2nd}(\overline{q}g \to \overline{q}g)\right|^2_{\rm int.}\\ &= \frac{g^4}{2M_{\rm s}^6}\frac{N^2-1}{4N^2}\left\{\left[\frac{2}{9}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=1/2}_{q^{**}}\Gamma^{J=3/2}_{q^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=1/2}_{q^{**}})^2}\frac{8M_{\rm s}^6\,(-\hat{u})(-\hat{u}+2\hat{t})}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}})^2}\right]\right.\\ &\qquad\left.+\left[\frac{6}{25}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}}\Gamma^{J=5/2}_{q^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}})^2}\frac{2M_{\rm s}^2\,(-\hat{u})^3(-\hat{u}+4\hat{t})}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=5/2}_{q^{**}})^2}\right]\right\},\end{aligned} \tag{4.27}$$

$$\begin{aligned}&\left|\mathcal{M}_{\rm 2nd}(qg \to gq)\right|^2_{\rm int.} = \left|\mathcal{M}_{\rm 2nd}(\overline{q}g \to g\overline{q})\right|^2_{\rm int.}\\ &= \frac{g^4}{2M_{\rm s}^6}\frac{N^2-1}{4N^2}\left\{\left[\frac{2}{9}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=1/2}_{q^{**}}\Gamma^{J=3/2}_{q^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=1/2}_{q^{**}})^2}\frac{8M_{\rm s}^6\,(-\hat{t})(-\hat{t}+2\hat{u})}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}})^2}\right]\right.\\ &\qquad\left.+\left[\frac{6}{25}\frac{(\hat{s}-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}}\Gamma^{J=5/2}_{q^{**}}}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}})^2}\frac{2M_{\rm s}^2\,(-\hat{t})^3(-\hat{t}+4\hat{u})}{(\hat{s}-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=5/2}_{q^{**}})^2}\right]\right\}.\end{aligned} \tag{4.28}$$

If the scattering amplitudes with exchanges of the first and second string excited states are correct even in the region of $M_{\rm s}^2 < s < 2M_{\rm s}^2$, we can also consider interference effects between the first and second string excited states. However, the string form factor functions which are expanded by s-channel poles in eq.(4.4) are good approximations only near the s-channel poles, $s \simeq M_s^2$ and $s \simeq 2M_s^2$. The interference effects between the first and second string excited states are calculated in ref.[19] and considered in later analyses. The inclusion of the interference effects may make the analyses imprecise. However, in the analyses, we focus on only the neighborhood of the s-channel poles, and the interference effects are negligible.

4.4 Dijet invariant mass distributions with string resonances

The squared amplitudes of the two-parton scattering processes with exchanges of the first and second string excited states have been shown in the previous subsection. We can obtain dijet invariant mass distributions with string resonances at parton level, assuming the LHC.

Formulation for dijet invariant mass distributions

A formula for the dijet invariant mass distribution is derived in the following way. A cross section of dijet events caused by two-parton scattering processes is described by

$$\begin{aligned}&\sigma\big(\mathrm{p}_1(P_1), \mathrm{p}_2(P_2) \to j_1(p_1), j_2(p_2), X\big)\\ &\qquad = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1, Q) f_j(x_2, Q)\, \sigma\big(i(p_i), j(p_j) \to k(p_k), \ell(p_\ell)\big)\,,\end{aligned} \tag{4.29}$$

where p_1, p_2 denote the incoming protons, j_1, j_2 denote observed jets and X denote the QCD remnants. The functions of $f_i(x_1, Q)$ and $f_j(x_2, Q)$ are the PDFs of a proton for the initial parton i and j, with momentum transfer Q and momentum fractions

$$x_1 = \frac{p_i}{P_1}\,, \qquad x_2 = \frac{p_j}{P_2}\,, \tag{4.30}$$

respectively, where p_i and p_j are each momentum of the initial parton i and j. The cross section (4.29) is described in terms of a differential cross section of a specific two-parton scattering process $ij \to k\ell$,

$$\sigma(\mathrm{p}_1\mathrm{p}_2 \to j_1 j_2) = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2\, f_i(x_1, Q) f_j(x_2, Q) \int d\hat{t}\, \frac{d\sigma(ij \to k\ell)}{d\hat{t}}\,, \tag{4.31}$$

where the differential cross section can be recast by a squared amplitude $|\mathcal{M}(ij \to k\ell)|^2$ into

$$\frac{d\sigma(ij \to k\ell)}{d\hat{t}} = \frac{\big|\mathcal{M}(ij \to k\ell)\big|^2}{16\pi \hat{s}^2}\,. \tag{4.32}$$

We want to transform the integrating variables in eq.(4.31) into observables in dijet events. The observables are the dijet invariant mass M, the transverse momentum p_{T} and rapidity y or pseudo-rapidity η of the observed jet which are defined as

$$\tanh y \equiv \frac{|\mathbf{p}|}{E}\,, \qquad \tanh \eta \equiv \frac{|\mathbf{p}|}{\mathrm{p}_z}\,, \tag{4.33}$$

and so on.

In the parton center-of-mass frame, rapidities of the final parton k and ℓ are opposite in sign: $y \equiv y_k^* = -y_\ell^*$, since these partons are produced back-to-back in this frame. We define a boost velocity from the parton center-of-mass frame to the proton center-of-mass frame as $\beta \equiv \tanh Y$. Then pseudo-rapidities of the observed jets, y_1 and y_2, can be written by

$$y_1 = y + Y\,, \qquad y_2 = -y + Y\,. \tag{4.34}$$

As the observables in dijet events, we use the following quantities independent each other,

$$y = \frac{1}{2}(y_1 - y_2)\,, \qquad Y = \frac{1}{2}(y_1 + y_2)\,, \tag{4.35}$$

which correspond to the rapidity and the boost in the parton CM frame, respectively. Then the Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ of the scattering partons and the momentum fractions x_1 and x_2 can be described in terms of y, Y and the dijet invariant mass M,

$$\hat{s} = M^2\,, \qquad \hat{t} = -\frac{M^2}{2}\frac{e^{-y}}{\cosh y}\,, \qquad \hat{u} = -\frac{M^2}{2}\frac{e^{y}}{\cosh y}\,, \tag{4.36}$$

$$x_1 = \sqrt{\frac{M^2}{s}}\,e^{Y}\,, \qquad x_2 = \sqrt{\frac{M^2}{s}}\,e^{-Y}\,, \tag{4.37}$$

where $\sqrt{s}$ is the proton center-of-mass energy.

Then, we obtain the dijet invariant mass distribution by transforming the integrating variables in eq.(4.31) into the observables in dijet events,

$$\frac{d\sigma(\mathrm{pp} \to jj)}{dM} = \sum_{i,j} M \int_{-Y_{\max}}^{Y_{\max}} dY\, x_1 f_i(x_1, M)\, x_2 f_j(x_2, M) \int_{-y_{\max}}^{y_{\max}} dy\, \frac{1}{\cosh^2 y} \frac{\left|\mathcal{M}(ij \to k\ell)\right|^2}{16\pi \hat{s}^2}\,. \tag{4.38}$$

Kinematical cuts for dijet invariant mass distributions

We impose cuts on kinematical observables event by event in order to suppress background events and relatively enhance signal events. According to ref.[51], we impose kinematical cuts for the dijet invariant mass distributions as follows,

$$p_{\mathrm{T},i} > p_{\mathrm{T,cut}}\,, \qquad |y_i| < y_{i,\mathrm{cut}}\,, \qquad |y| < y_{\mathrm{cut}}\,. \tag{4.39}$$

where $p_{\mathrm{T},i}$ is the transverse momentum and y_i is the pseudo-rapidity of the observed jet. Dijet events consist of two jets with high-p_T, since momenta along the beam pipe are not conserved but the transverse momenta vertical to the beam pipe are conserved at a hadron collider. The pseudo-rapidity cut in eq.(4.39) is also required, since jets near the beam pipe cannot be observed. The last cut in eq.(4.39) on the rapidity in the parton CM frame y is imposed to suppress the QCD background events. The dominant QCD processes in the high-p_T regime are t-channel processes with small scattering angles from the beam pipe. Small y means a large scattering angle because of the following relation with a scattering angle in the parton CM frame θ_*,

$$y = \frac{1}{2}\ln\left[\frac{1+\cos\theta_*}{1-\cos\theta_*}\right]. \tag{4.40}$$

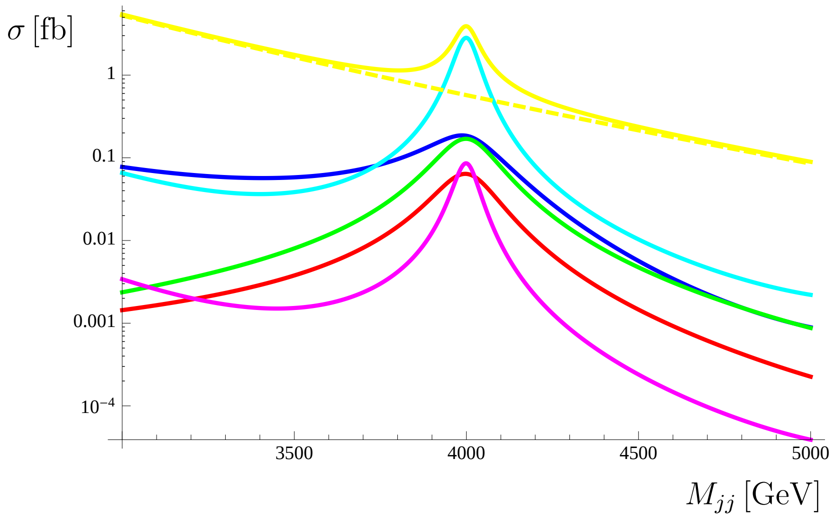

Figure 4.6: The dijet invariant mass distribution $d\sigma/dM$ [fb/GeV] of the first string resonances for $M_s = 4\,\text{TeV}$ in the processes of $gg \to gg$ (the blue line), $gg \to q\overline{q}$ (the red line), $q\overline{q} \to gg$ (the green line), $qg \to qg$ (the cyan line) and $\overline{q}g \to \overline{q}g$ (the magenta line), of the first string resonances in the all processes and the SM background (the yellow solid line), and of the SM background only (the yellow dashed line), at the 14 TeV LHC.

Therefore, the rapidity cut on y is useful to suppress the background events.

Taken into account the kinematical cuts (4.39), the integrating region of y and Y in eq.(4.38) are

$$\begin{aligned} y_{\max} &= \min(y_{\text{cut}}\,,\ y_{i,\max} - Y) \quad \text{for } \ Y > 0 \\ &= \min(y_{\text{cut}}\,,\ y_{i,\max} + Y) \quad \text{for } \ Y < 0\,, \end{aligned} \tag{4.41}$$

$$Y_{\max} = \min\left(y_{i,\max}\,,\ \ln\sqrt{\frac{s}{M^2}} \right), \tag{4.42}$$

where

$$y_{i,\max} = \min\left(y_{i,\text{cut}}\,,\ \frac{1}{2}\ln\left[1+\sqrt{1-\frac{p_{\text{T,cut}}^2}{(M/2)^2}}\right] \Big/ \left[1-\sqrt{1-\frac{p_{\text{T,cut}}^2}{(M/2)^2}}\right] \right). \tag{4.43}$$

Dominance of $qg \to qg$ process

The dijet invariant mass distributions of the first and second string resonances for $M_s = 4\,\text{TeV}$ are shown in Fig.4.6 and Fig.4.7. We take into account the kinematical cuts (4.39) with $p_{\text{T,cut}} = 350\,\text{GeV}$, $y_{i,\text{cut}} = 2.3$ and $y_{\text{cut}} = 0.85$. We use `mstw2008` [52] as the PDFs, and we use the following function as the dijet invariant mass distribution of the SM background [51],

$$f(x) = p_0\,\frac{(1-x)^{p_1}}{x^{p_2+p_3\ln x}}\,, \tag{4.44}$$

which are fitted with the QCD prediction and where $x \equiv \frac{M_{jj}}{\sqrt{s}}$ and $p_{0,1,2,3}$ are parameters.

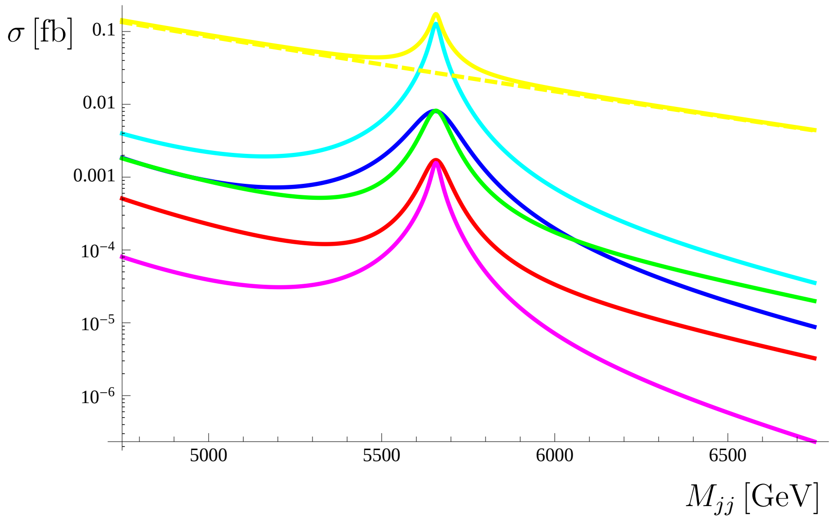

Figure 4.7: The dijet invariant mass distribution $d\sigma/dM$ [fb/GeV] of the second string resonances for $M_s = 4\,\text{TeV}$ in the processes of $gg \to gg$ (the blue line), $gg \to q\overline{q}$ (the red line), $q\overline{q} \to gg$ (the green line), $qg \to qg$ (the cyan line) and $\overline{q}g \to \overline{q}g$ (the magenta line), of the second string resonances in the all processes and the SM background (the yellow solid line), and of the SM background only (the yellow dashed line), at the 14 TeV LHC.

As it is clearly understood from Fig.4.6 and Fig.4.7, the process of $qg \to qg$ are dominant in both the first and second string resonances. The dominance of $qg \to qg$ comes from the dominance of quarks in the PDF of a proton. The process of $qg \to qg$ is remarkably dominant in case of $M_s = 4\,\text{TeV}$, while the process of $gg \to gg$ also have major contributions to the string resonances in case of $M_s = 2\,\text{TeV}$ [18].

Thus, since the process of $qg \to qg$ is dominant, we can consider only the process in later analyses. The fact that there are almost only the string excited quarks in the string resonances is useful for later analyses.

4.5 The flow of Monte Carlo simulation

In studies of this thesis, we focus on the two-parton scattering processes. The two partons in the final state are hadronized into two bundles of hadrons which result in two jets. Therefore, we should perform computer simulations for hadronization and detector simulation to reproduce results at the LHC. The setup of our Monte Carlo (MC) simulation is as follows.

First, we use `CalcHEP` [53, 54] to generate event samples at parton-level, using `MRST2007 Modified LO` [55] as the PDFs. We generate event samples for the SM background of all the two-parton scattering processes at tree-level, as well as event samples for string signal of the model-independent two-parton scattering processes in Fig.4.3. Since there was no event generator including higher spin fields in particular $J = 3/2$, we generate the event samples by adding the scattering amplitudes calculated in Sec.4.3 to `CalcHEP` programs by hand.[26] See the web page [57] about event generations for string signal in detail.

[26] We may use technique in ref.[56] to generate event samples including spin-3/2 particles.

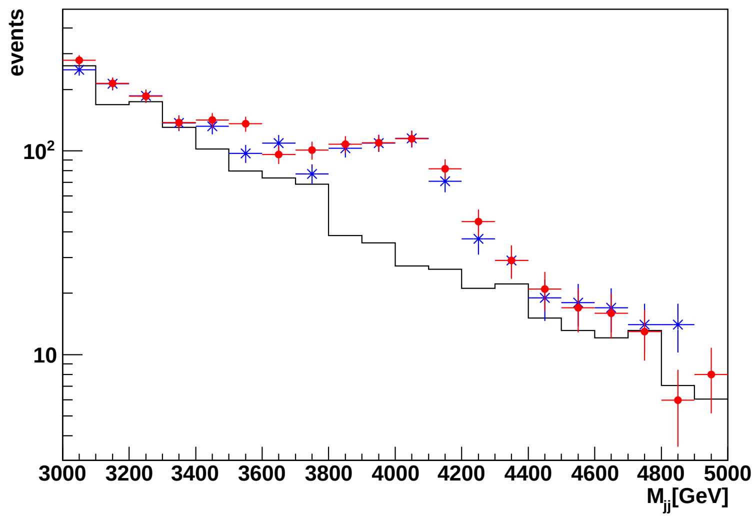

Figure 4.8: The dijet invariant mass distribution of the first string resonances for $M_s = 4\,\text{TeV}$ in the process of $qg \to qg$ only (the blue points with error bars) and the all processes (the red points with error bars) including the SM background (the histogram), using the MC simulations with $1.4\,\text{fb}^{-1}$ at the 14 TeV LHC.

Next, we use `Pythia8` [58] for hadronization of quarks and gluons in the final state, and we use `Delphes1.9` [59] for detector simulation mainly to identify jets consisting of hadrons, using its default detector card for the ATLAS. Finally, we use `ROOT` [60] to construct dijet events by choosing two jets, j_1 and j_2, with highest and second highest transverse momenta, event by event. We also use `ROOT` to suppress background events and relatively enhance signal events by imposing kinematical cuts in dijet events, as eq.(4.39).

The flow of our MC simulation is as follows:

$$\begin{aligned} \texttt{CalcHEP} \quad & \text{for event generation} \\ & \downarrow \\ \texttt{Pythia8} \quad & \text{for hadronization} \\ & \downarrow \\ \texttt{Delphes1.9} \quad & \text{for detector simulation} \\ & \downarrow \\ \texttt{ROOT} \quad & \text{for analysis}\,. \end{aligned} \tag{4.45}$$

The dijet invariant mass distribution of the first string resonances for $M_s = 4\,\text{TeV}$ using the MC simulation with $1.4\,\text{fb}^{-1}$ at the 14 TeV LHC is shown in Fig.4.8. The kinematical cuts are (4.39) with $p_{\text{T,cut}} = 330\,\text{GeV}$, $y_{i,\text{cut}} = 2.5$ and $y_{\text{cut}} = 0.65$. As it is also found from Fig.4.8, the process of $qg \to qg$ dominates over all the processes.

In Fig.4.8, there are some bins which the number of events in $qg \to qg$ only exceeds the number of events in all the processes. It is considered as the reason that each event sample for $qg \to qg$ only and for all the processes corresponds to an independent experiment, and the statistics are poor due to the integrated luminosity of $1.4\,\mathrm{fb}^{-1}$.

5 Angular analysis

If a new heavy resonance is discovered at the LHC, an angular distribution analysis is important to confirm that the resonance comes from low-scale string models. In the dominant process at the LHC, $qg \to qg$, two degenerate first string excited quarks with $J = 1/2$ and $J = 3/2$ are exchanged. If we can experimentally distinguish angular distributions with both $J = 1/2$ and $J = 3/2$ states from that with a $J = 1/2$ state only, this can be a signature of low-scale string models.

5.1 χ distributions on string resonances

We analyze χ distributions as angular distributions of dijet events. The quantity χ is defined as

$$\chi \equiv \exp\left(y_1 - y_2\right) = \frac{1+\cos\theta_*}{1-\cos\theta_*}\,, \tag{5.1}$$

where y_1 and y_2 are the pseudo rapidities of the observed jets, and θ_* is the scattering angle in the parton CM frame.

The χ distributions are used to search for non-resonant signals of new physics, such as the quantum black hole and contact interactions at the ATLAS [16] and quark compositeness at the CMS [61]. The χ distribution predicted by the QCD is relatively flat, while that by many new physics has a peak at low values of χ, since the new physics signals are more isotropic than the QCD predictions which dominantly have small scattering angles. Fig.5.1 shows the χ distributions for the dijet invariant mass bins at the ATLAS [16].

Formulation for χ distributions on string resonances

A formula for the χ distribution is derived in the following way. The cross section of dijet events caused by two-parton scattering processes is shown in eqs.(4.31) and (4.32). In case of the dijet invariant mass distributions, the integrating variables in eq.(4.31) are transformed into the observables in dijet events, $(x_1, x_2, \hat{t}) \to (y, Y, M)$. In case of the χ distributions, since χ is defined as $\chi \equiv e^{2y}$, the transformation is $(x_1, x_2, \hat{t}) \to (\chi, Y, M)$. The Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ of the scattering quark and gluon can be described in terms of χ and M,

$$\hat{s} = M^2\,, \qquad \hat{t} = -\frac{M^2}{1+\chi}\,, \qquad \hat{u} = -\frac{M^2\chi}{1+\chi}\,. \tag{5.2}$$

Then, we obtain the χ distribution of dijet events,

$$\frac{d\sigma(\mathrm{pp}\to jj)}{d\chi} = \sum_{i,j}\int_{M^2_{\mathrm{low}}}^{M^2_{\mathrm{high}}} dM^2 \int_{-Y_{\max}}^{Y_{\max}} dY\, x_1 f_i(x_1, M)\, x_2 f_j(x_2, M)\, \frac{1}{(1+\chi)^2}\frac{\left|\mathcal{M}(ij\to k\ell)\right|^2}{16\pi\hat{s}^2}\,, \tag{5.3}$$

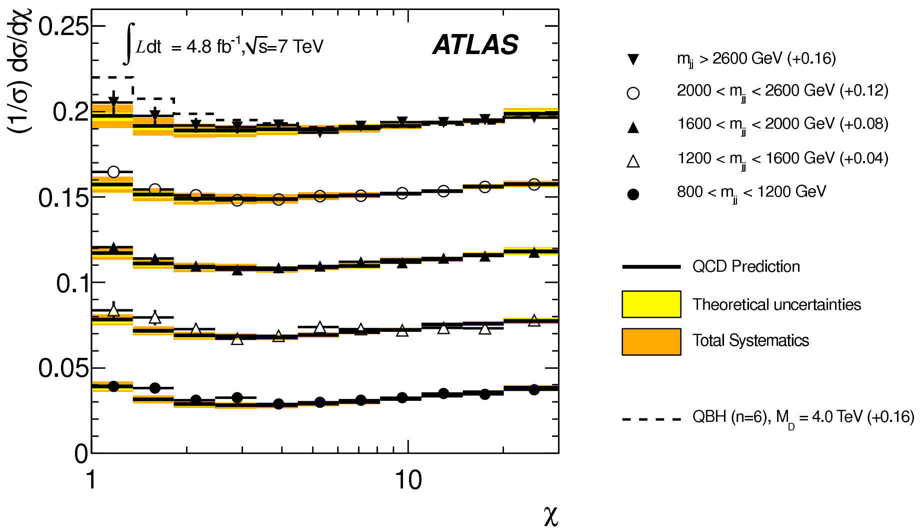

Figure 5.1: The χ distributions of the QCD prediction with theoretical and total systematic uncertainties and of the data with statistical uncertainties at the ATLAS with $4.8\,\mathrm{fb}^{-1}$ for $\sqrt{s} = 7\,\mathrm{TeV}$, for each bin of the dijet invariant mass [16]. The distributions are offset by the amount shown in parentheses.

where M_{low} and M_{high} are lowest and highest values of the dijet invariant mass bins.

We focus on the χ distribution on the first string resonance. In the first string resonance, the first string excited quarks with $J = 1/2$ and $J = 3/2$ dominate over all the other string excited states. Substituting the squared amplitudes of $qg \to qg$ of eqs.(4.12) and (4.13) into (5.3), we obtain a prediction to the χ-distribution on the first string resonance,

$$\frac{d\sigma_{\mathrm{1st}}(qg \to qg)}{d\chi} = \frac{1}{(1+\chi)^2}\left(C^{J=1/2} + C^{J=3/2}\frac{1+\chi^3}{(1+\chi)^3}\right), \tag{5.4}$$

where $C^{J=1/2}$ and $C^{J=3/2}$ are constants. The term proportional to $C^{J=1/2}$ in the parentheses represents a χ dependence due to an exchange of the first string excited quark with $J = 1/2$, and the term proportional to $C^{J=3/2}$ represents that due to an exchange of the first string excited quark with $J = 3/2$. The overall factor $1/(1+\chi)^2$ in eq.(5.4) is a kinematical factor.[27]

We consider the χ distribution on a new resonance coming from other "new physics". In the new resonance, there may be new $J = 1/2$ and $J = 1$ states. Then, the χ distribution of dijet events on the new resonance is

$$\frac{d\sigma(\mathrm{pp} \to jj)}{d\chi} = \frac{1}{(1+\chi)^2}\left(C^{J=1/2} + C^{J=1}_{J_z=0}\frac{(1-\chi)^2}{(1+\chi)^2} + C^{J=1}_{J_z=\pm1}\frac{1+\chi^2}{(1+\chi^2)}\right), \tag{5.5}$$

where $C^{J=1/2}$, $C^{J=1}_{J_z=0}$ and $C^{J=1}_{J_z=\pm1}$ are also constants. The term proportional to $C^{J=1/2}$ represents a χ dependence due to an exchange of the new $J = 1/2$ state in $qg \to qg$, and the terms proportional to $C^{J=1}_{J_z=0}$ and $C^{J=1}_{J_z=\pm1}$ represent that due to exchanges of the new $J = 1$ state in $gg \to gg$ and $q\bar{q} \to q\bar{q}$, respectively. If we want to distinguish which a new resonance comes

[27] Note that the χ dependence due to a $J = 1/2$ state exchange vanishes, and leaves only the overall kinematical factor $1/(1+\chi)^2$. The χ dependence is the same as that due to a $J = 0$ state exchange.

from low-scale string or the other “new physics”, we should consider the χ dependence due to a $J = 1$ state exchange. However, in this analysis for simplicity, we do not consider it and assume that the “new physics” include only a $J = 1/2$ state.

Kinematical cuts for χ distributions

As well as for the dijet invariant mass distributions, according to ref.[51], we impose kinematical cuts for the χ distributions as follows,

$$p_{\mathrm{T},j_1} > p_{\mathrm{T,cut}}\,, \qquad |y| < y_{\mathrm{cut}}\,, \qquad |Y| < Y_{\mathrm{cut}}\,. \tag{5.6}$$

The transverse momentum cut is imposed on the leading jet j_1 with highest p_{T}. The rapidity cut on y corresponds to constrain the small region of χ which we focus on. The last cut in eq.(5.6) on the boost Y is imposed to make the QCD prediction of the χ distribution flat. The boost cut corresponds to selection of more central events and it does not affect events with the dijet invariant mass of the order of TeV. We set the dijet invariant mass bin $M = [M_{\mathrm{low}}, M_{\mathrm{high}}]$ around the first string resonance.

In this analysis, we use a definition of $\chi \equiv e^{2|y|}$, and we can obtain twice the number of events in the χ distribution.

5.2 χ distribution analysis

We generate event samples for both the SM background only and the SM background plus string signal for $M_{\mathrm{s}} = 4\,\mathrm{TeV}$, $4.5\,\mathrm{TeV}$ and $5\,\mathrm{TeV}$, at the LHC with $\sqrt{s} = 14\,\mathrm{TeV}$. The event samples for the string signal are generated using the scattering amplitudes with the first string excited state exchanges of all the model-independent processes in Fig.4.3.

After choosing two jets with highest and second highest p_{T} to construct dijet events, the kinematical cuts (5.6) are imposed on both the SM background and the SM plus string event samples, using the following parameters

$$\begin{aligned} p_{\mathrm{T,cut}} &= 350\,\mathrm{GeV} \quad \text{for } M_{\mathrm{s}} = 4\,\mathrm{TeV}\,, \\ p_{\mathrm{T,cut}} &= 400\,\mathrm{GeV} \quad \text{for } M_{\mathrm{s}} = 4.5\,\mathrm{TeV}\,, \\ p_{\mathrm{T,cut}} &= 450\,\mathrm{GeV} \quad \text{for } M_{\mathrm{s}} = 5\,\mathrm{TeV}\,, \\ y_{\mathrm{cut}} &= 1.15\,, \qquad Y_{\mathrm{cut}} = 1.0\,. \end{aligned} \tag{5.7}$$

The dijet invariant mass bin is set by $M_{jj} = [M_{\mathrm{s}} - 250\,\mathrm{GeV},\ M_{\mathrm{s}} + 250\,\mathrm{GeV}]$ around the first string resonance.[28] In order to obtain event samples for the string signal only, we subtract the SM background from the SM plus string event samples.

Thus, we obtain a χ distribution which shows a behavior different from the QCD prediction. If the behavior comes from the first string resonance, the χ distribution shows the χ dependences

[28] The dijet invariant mass bin is set by us, however, we consider the bin to be valid because the Gaussian fits to the dijet invariant mass distributions give $M = 3995 \pm 6.2\,\mathrm{GeV}$ and $\sigma = 171.9 \pm 5.0\,\mathrm{GeV}$ for $M_{\mathrm{s}} = 4\,\mathrm{TeV}$ with $20\,\mathrm{fb}^{-1}$, $M = 4471 \pm 7.9\,\mathrm{GeV}$ and $\sigma = 206 \pm 9.0\,\mathrm{GeV}$ for $M_{\mathrm{s}} = 4.5\,\mathrm{TeV}$ with $30\,\mathrm{fb}^{-1}$, and $M = 4997 \pm 11.8\,\mathrm{GeV}$ and $\sigma = 222.4 \pm 13.7\,\mathrm{GeV}$ for $M_{\mathrm{s}} = 5\,\mathrm{TeV}$ with $50\,\mathrm{fb}^{-1}$, and $[M - \sigma, M + \sigma]$ is almost fitted with the bin.

M_s (luminosity)	$J = 1/2$ and $3/2$	$J = 1/2$ only	$J = 3/2$ only
4 TeV (18.7 fb^{-1})	0.7471	0.04058	0.
4.5 TeV (30 fb^{-1})	0.6966	0.004562	5.386×10^{-9}
5 TeV (50 fb^{-1})	0.4737	0.0466	3.198×10^{-5}

Table 5.1: p-values of the fits

(5.4) due to exchanges of the first string excited quarks with not only $J = 1/2$ but also $J = 3/2$. The other "new physics" may also cause the χ dependence due to exchanges of new $J = 1/2$ and $J = 1$ states. In this analysis, for simplicity, it is assumed that the other "new physics" includes only new $J = 1/2$ states.

Fitting χ distribution with three hypotheses

Using event samples for the string signal only, we fit the χ distribution with the function of eq.(5.4), in three cases of $C^{J=1/2} \neq 0$ and $C^{J=3/2} \neq 0$, $C^{J=1/2} \neq 0$ and $C^{J=3/2} = 0$, and $C^{J=1/2} = 0$ and $C^{J=3/2} \neq 0$. These fittings are performed by `ROOT`. These three cases correspond to the hypotheses with both $J = 1/2$ and $J = 3/2$ states, a $J = 1/2$ state only, and a $J = 3/2$ state only, respectively. The first hypothesis corresponds to low-scale string models with the first string excited quarks with $J = 1/2$ and $J = 3/2$, while the second one corresponds to the other "new physics" with new quark-like states.

Figs. 5.2, 5.3 and 5.4 show the χ-distributions of the string signal for $M_\mathrm{s} = 4$ TeV, 4.5 TeV and 5 TeV with 18.7 fb^{-1}, 30 fb^{-1} and 50 fb^{-1}, respectively, at the 14 TeV LHC. Each figure includes three figures corresponding to the fits associated with the above three hypotheses.

The χ-distributions clearly do not match the fits with $J = 3/2$ only in the bottom figures of Figs. 5.2, 5.3 and 5.4, therefore, the hypothesis with $J = 3/2$ only can be excluded. On the other hand, the other two hypotheses with both $J = 1/2$ and $J = 3/2$ in the top figures and with $J = 1/2$ only in the middle figures give rather good fits. It is difficult to judge which hypothesis are fitted better with the χ distributions, without statistical analyses. We calculate p-values of these fits using `ROOT`, and give them in Table 5.1.

The p-value is a probability that if a hypothesis is excluded in spite of that it is correct, the exclusion is an experimental error [62]. If the p-value is smaller than 5%, the hypothesis can be excluded at 95% Confidence Level. Evidently, the p-values of the hypotheses with $J = 1/2$ only and with $J = 3/2$ only are very small and smaller than 5% in Table 5.1. On the other hand, the p-value of the hypothesis with both $J = 1/2$ and $J = 3/2$ is much larger than 5%, and the hypothesis cannot be excluded, for $M_\mathrm{s} = 4$ TeV with 18.7fb^{-1}, for $M_\mathrm{s} = 4.5$ TeV with 30 fb^{-1} and for $M_\mathrm{s} = 5$ TeV with 50 fb^{-1}. Indeed, if we look at the middle figures of Figs.5.2, 5.3 and 5.4 closer, it is found that the fit with $J = 1/2$ only is systematically inconsistent, since curves of the fit falls more quickly than the MC data for large χ.

Note that in this analysis, we consider only statistical uncertainties but not systematic uncertainties. We may need to use normalized χ distributions $1/\sigma\, d\sigma/d\chi$, to reduce systematic

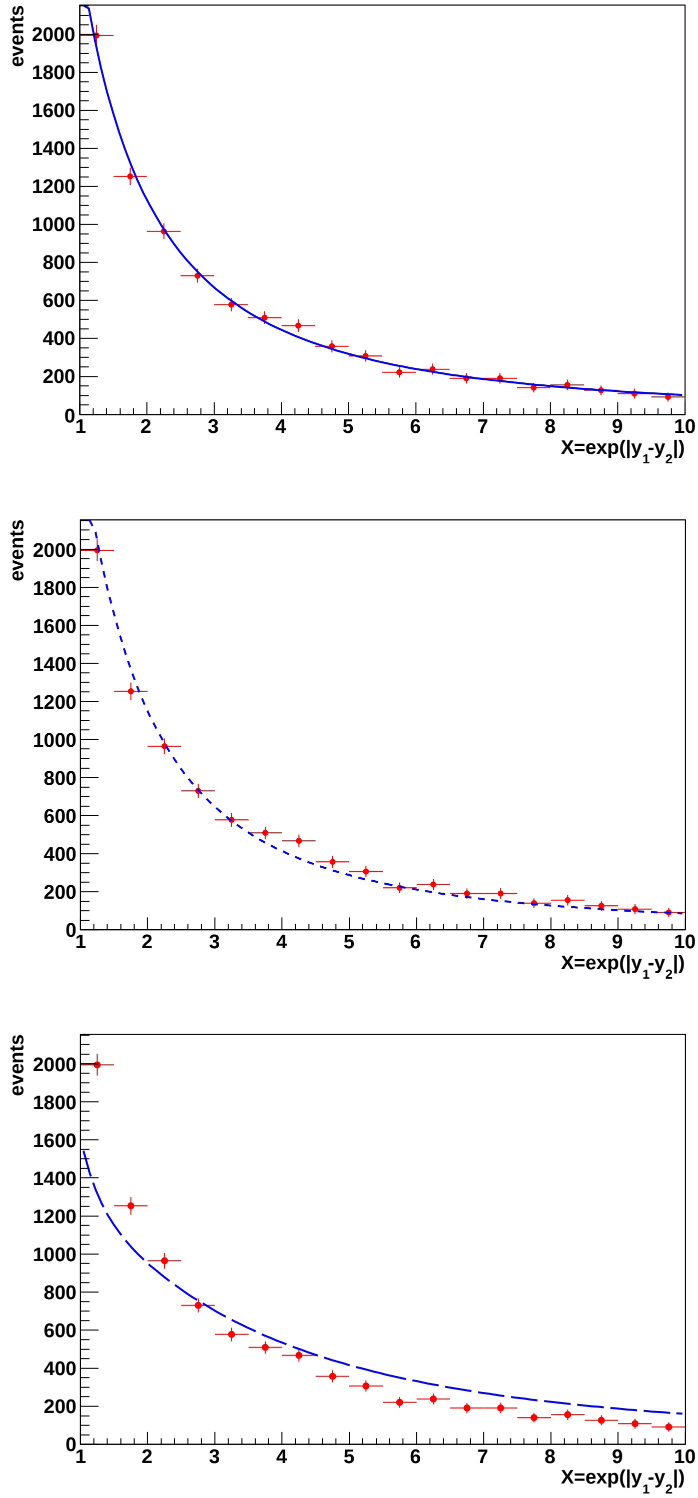

Figure 5.2: The χ-distributions of the MC data for $M_s = 4\,\text{TeV}$ with $18.7\,\text{fb}^{-1}$ at the $14\,\text{TeV}$ LHC (the points with error bars), and the fits associated with three hypotheses with both $J = 1/2$ and $J = 3/2$ (the solid line in the top figure), $J = 1/2$ only (the dashed line in the middle figure) and $J = 3/2$ only (the long dashed line in the bottom figure).

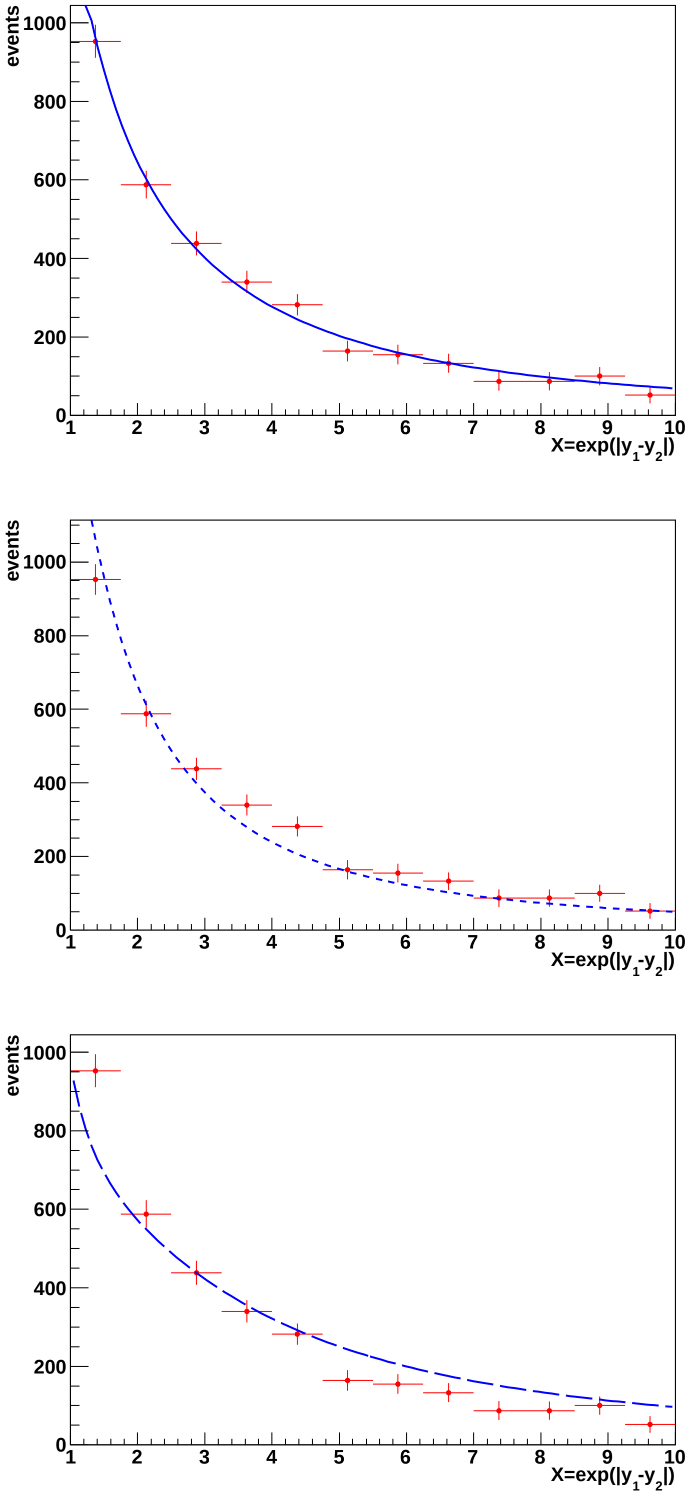

Figure 5.3: The χ-distributions of the MC data for $M_{\rm s} = 4.5\,\text{TeV}$ with $30\,\text{fb}^{-1}$ at the $14\,\text{TeV}$ LHC (the points with error bars), and the fits associated with three hypotheses with both $J = 1/2$ and $J = 3/2$ (the solid line in the top figure), $J = 1/2$ only (the dashed line in the middle figure) and $J = 3/2$ only (the long dashed line in the bottom figure).

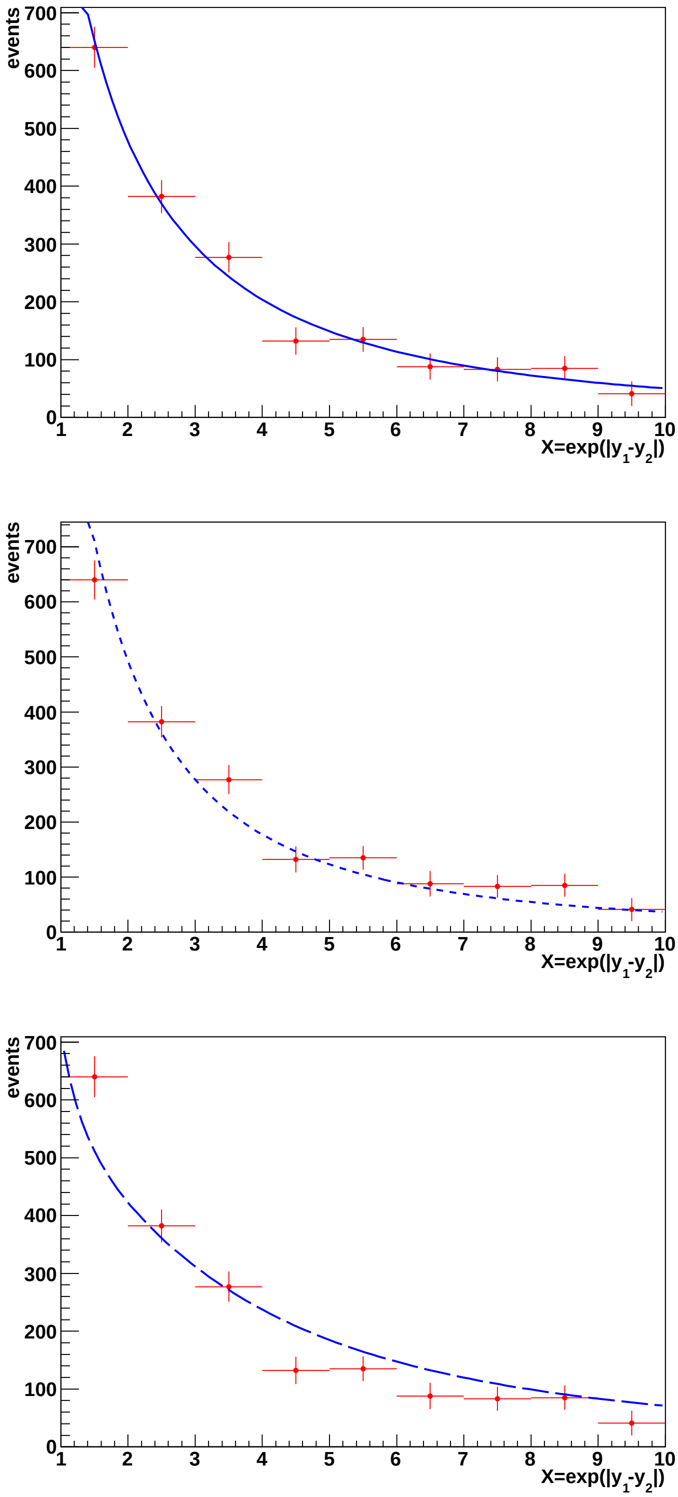

Figure 5.4: The χ-distributions of the MC data for $M_s = 5\,\text{TeV}$ with $50\,\text{fb}^{-1}$ at the 14 TeV LHC (the points with error bars), and the fits associated with three hypotheses with both $J = 1/2$ and $J = 3/2$ (the solid line in the top figure), $J = 1/2$ only (the dashed line in the middle figure) and $J = 3/2$ only (the long dashed line in the bottom figure).

uncertainties such as the jet energy scale (JES), the PDFs and the integrated luminosity, as in the analysis at the ATLAS [16]. Subtracting with the QCD background, we need to consider theoretical and systematic uncertainties in the χ distribution predicted by the QCD, $\sim 10\%$ for higher dijet invariant mass bins. However, even in this easy analysis, the wrong hypotheses can be excluded at 95% CL with $\mathcal{O}(10)\,\mathrm{fb}^{-1}$. Therefore, we can expect that the LHC has the ability to statistically identify characteristic angular dependences on the string resonance.

Thus, spin degeneracy of string excited states can be experimentally confirmed with the integrated luminosity of $\mathcal{O}(10)\,\mathrm{fb}^{-1}$, by the angular distribution analysis on the first string resonance. This can be a signature of low-scale string models.

6 Second resonance analysis

In order to identify a new resonance as the string resonance, search for a second resonance in dijet invariant mass distributions is important. The existence of the second string excited states and their characteristic masses are distinct properties of low-scale string models.

First, there must be second string excited states in low-scale string models, while there is no second state in the other "new physics", such as the axigluon models [21] and the color-octet scalar models [22]. Second, the masses of the second string excited states are $\sqrt{2}$ times of that of the first string excited states, while typical masses of second KK modes are 2 times of that of first KK modes, in the other models with extra dimensions, such as the five-dimensional Universal Extra Dimension (UED) models.[29]

In this analysis, we investigate a discovery potential of the second string resonance in dijet invariant mass distributions.

6.1 Second string resonances in dijet invariant mass distributions

We generate event samples for the SM background only and the SM background plus string signal for $M_{\mathrm{s}} = 4\,\mathrm{TeV}$, $4.5\,\mathrm{TeV}$, $4.75\,\mathrm{TeV}$ and $5\,\mathrm{TeV}$, at the LHC with $\sqrt{s} = 14\,\mathrm{TeV}$. The event samples for the string signal are generated using the scattering amplitudes with the first and second string excited state exchanges of the dominant process at the LHC, $qg \to qg$.

After constructing dijet events, the kinematical cuts for the dijet invariant mass distributions (4.39) are imposed on both the SM background and the SM plus string event samples, using

[29] In the six-dimensional UED models, typical masses of second KK modes with KK parity $+1$ is $\sqrt{2}$ times of that of first KK modes with KK parity $+1$. It may be possible to confirm low-scale string models by a search for the third resonance, since third string excited states have $\sqrt{3}$ times of that of first string excited states.

the following parameters

$$
\begin{aligned}
p_{\mathrm{T,cut}} &= 350\,\mathrm{GeV} \quad \text{for } \ M_{\mathrm{s}} = 4\,\mathrm{TeV}\,, \\
p_{\mathrm{T,cut}} &= 400\,\mathrm{GeV} \quad \text{for } \ M_{\mathrm{s}} = 4.5\,\mathrm{TeV}\,, \\
p_{\mathrm{T,cut}} &= 430\,\mathrm{GeV} \quad \text{for } \ M_{\mathrm{s}} = 4.75\,\mathrm{TeV}\,, \\
p_{\mathrm{T,cut}} &= 450\,\mathrm{GeV} \quad \text{for } \ M_{\mathrm{s}} = 5\,\mathrm{TeV}\,, \\
y_{i,\mathrm{cut}} &= 2.3\,, \qquad y_{\mathrm{cut}} = 0.85\,.
\end{aligned}
\tag{6.1}
$$

As well as the kinematical cuts for the χ distributions (5.6), the transverse momentum cut is imposed on the leading jet j_1 with highest p_{T}.

The dijet invariant mass distributions with the first and second string resonances are shown in Fig. 6.1, for $M_{\mathrm{s}} = 4\,\mathrm{TeV}$, 4.5 TeV and 5 TeV with $50\,\mathrm{fb}^{-1}$, $100\,\mathrm{fb}^{-1}$ and $300\,\mathrm{fb}^{-1}$, respectively, at the 14 TeV LHC. We can see first resonances at $M_{jj} = M_{\mathrm{s}}$ and second resonances at $M_{jj} = \sqrt{2} \times M_{\mathrm{s}} \simeq 5.66\,\mathrm{TeV}$, 6.36 TeV and 7.07 TeV for $M_{\mathrm{s}} = 4\,\mathrm{TeV}$, 4.5 TeV and 5 TeV, respectively. In particular, for $M_{\mathrm{s}} = 5\,\mathrm{TeV}$, the second resonance seems a just excess rather than a resonance, and the statistics are clearly poor.

In order to investigate the discovery potential of the second string resonance, we calculate a signal significance which represents a deviation from the SM prediction. We focus on the dijet invariant mass window of $M_{jj} = [\sqrt{2}M_{\mathrm{s}} - 250\,\mathrm{GeV},\ \sqrt{2}M_{\mathrm{s}} + 250\,\mathrm{GeV}]$ around the second string resonance. If the signal significance is larger than $5\,\sigma$, we can propose that the second string resonance can be discovered.

Calculation of significance

The significance Z is calculated in the following way. We define a statistic χ^2 as follows

$$
\chi^2 = \sum_{i=1}^{n_{\mathrm{bin}}} \frac{\left(N_{\mathrm{SM+str},i} - N_{\mathrm{SM},i}\right)^2}{\sigma^2_{\mathrm{SM+str},i} + \sigma^2_{\mathrm{SM},i}}\,, \tag{6.2}
$$

where n_{bin} is the number of bins. $N_{\mathrm{SM+str},i}$ and $N_{\mathrm{SM},i}$ are the numbers of events of ith bin in the dijet invariant mass distributions, for the SM plus string signal and for the SM background only event samples, respectively. $\sigma^2_{\mathrm{SM+str},i}$ and $\sigma^2_{\mathrm{SM},i}$ are dispersions of $N_{\mathrm{SM+str},i}$ and $N_{\mathrm{SM+str},i}$, including only statistical uncertainties.

We consider that the SM plus string event samples correspond to "experimental results" and the SM only event samples correspond to "theoretical predictions" generated by MC simulations. Therefore, the numerator of eq.(6.2) represents a deviation from the SM hypothesis, while the denominator represents a sum of the dispersions of "experiment" and "theory". Since the number of events N follows the Poisson distribution, the standard deviation is $\sigma = \sqrt{N}$ (if only statistical errors are considered). Therefore, $\sigma^2_{\mathrm{SM+str}} = N_{\mathrm{SM+str}}$ and $\sigma^2_{\mathrm{SM}} = N_{\mathrm{SM}}$, and (6.2) is recast into

$$
\chi^2 = \sum_{i=1}^{n_{\mathrm{bin}}} \frac{\left(N_{\mathrm{SM+str},i} - N_{\mathrm{SM},i}\right)^2}{N_{\mathrm{SM+str},i} + N_{\mathrm{SM},i}}\,. \tag{6.3}
$$

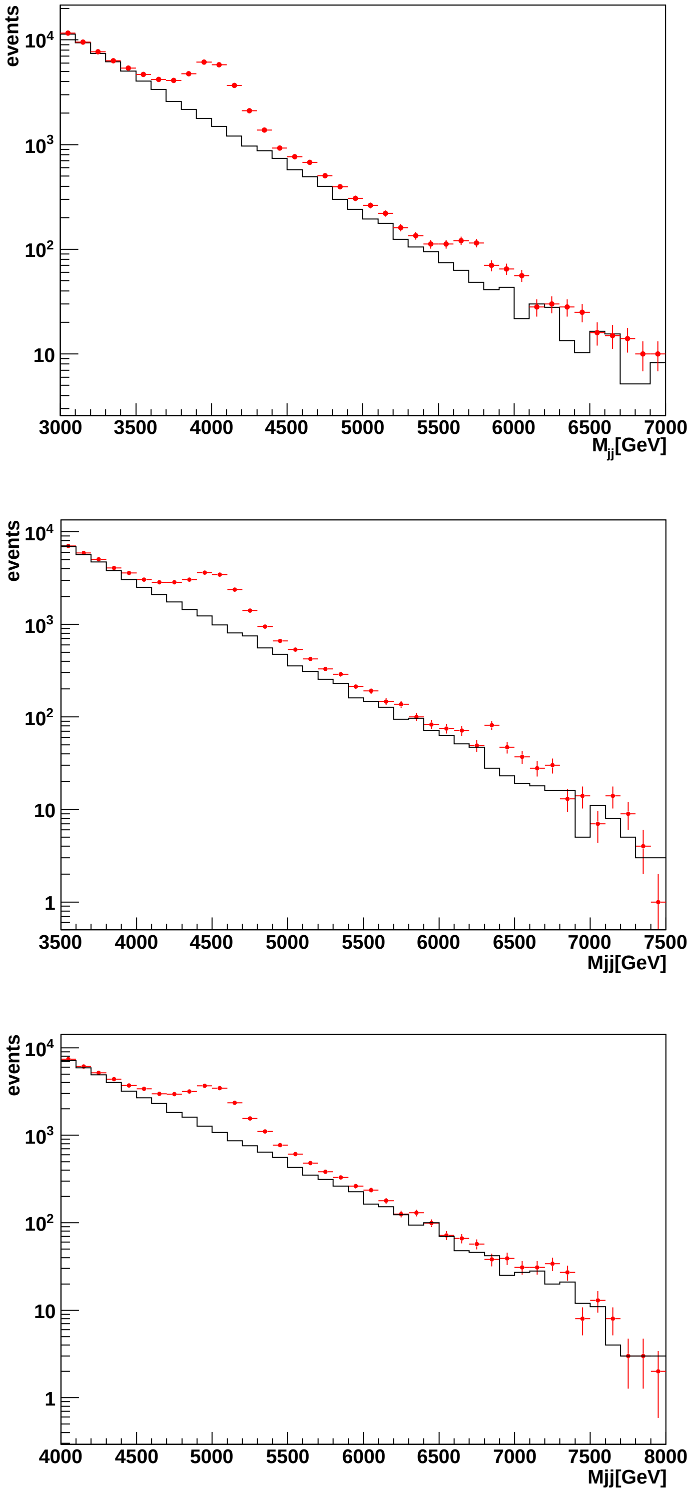

Figure 6.1: The dijet invariant mass distributions with the first and second string resonances of the MC data (the points with error bars) including the SM background (the histogram), for $M_s = 4\,\text{TeV}$ with $50\,\text{fb}^{-1}$ (the top figure), $M_s = 4.5\,\text{TeV}$ with $100\,\text{fb}^{-1}$ (the middle figure) and $M_s = 5\,\text{TeV}$ with $300\,\text{fb}^{-1}$ (the bottom figure) at the 14 TeV LHC.

If the SM hypothesis is correct and the statistics are rich, the statistic χ^2 follows the χ^2 distribution. Then, the p-value is calculated as follows,

$$p = \int_{\chi^2}^{\infty} f_{\chi^2}(z; n_{\rm bin})\, dz\,, \tag{6.4}$$

where $f_{\chi^2}(z;n)$ is the probability distribution function of the χ^2 distribution with n degrees of freedom. The significance is calculated using the p-value and the cumulative distribution function Φ of the standard Gaussian distribution with the average 0 and the standard deviation 1,

$$Z = \Phi^{-1}(1-p)\,. \tag{6.5}$$

Significance of second string resonances

We calculate the significance of the second string resonance in the above way. Fig. 6.2 shows the dijet invariant mass distributions in which the second string resonances are observed at $5\,\sigma$ level. The integrated luminosities of $50\,\text{fb}^{-1}$, $150\,\text{fb}^{-1}$, $300\,\text{fb}^{-1}$ and $600\,\text{fb}^{-1}$ are required for $M_{\rm s} = 4\,\text{TeV}$, $4.5\,\text{TeV}$, $4.75\,\text{TeV}$ and $5\,\text{TeV}$, respectively. Here, we assume that the second string excited states decay into the SM particles but not into the first string excited states, namely the second string excited states have narrow widths (4.23). We consider the decay of 2nd $\to$ 1st + SM in the next subsection.

We consider only statistics uncertainties but not systematic uncertainties. In systematic uncertainties, the normalization of the QCD background is important because the systematic uncertainties include the JES, the PDFs, the integrated luminosity and so on which are associated with the normalization. The normalization of the dijet invariant mass distributions is determined so that it is fitted with the data in a control region. However, the dijet invariant mass distributions between the first and second string resonances are raised due to string effects, and there is a possibility that the normalization is determined so that it is excessively large. Then, we will compare the dijet invariant mass distributions with the string resonances and that of the background only which is increased by some percent.

When the SM background is increased by 20 %, Fig. 6.3 shows the dijet invariant mass distributions in which the second string resonances are observed at $5\,\sigma$ level and for $M_{\rm s} = 4\,\text{TeV}$ and $4.5\,\text{TeV}$, and at $4.8\,\sigma$ level for $M_{\rm s} = 5\,\text{TeV}$. The integrated luminosities of $70\,\text{fb}^{-1}$, $350\,\text{fb}^{-1}$ and $1500\,\text{fb}^{-1}$ are required, respectively. When the SM background is increased by 50 %, the integrated luminosity of $100\,\text{fb}^{-1}$ is required to observe the second string resonance at $2.4\,\sigma$ level for $M_{\rm s} = 4\,\text{TeV}$. In this case, in order to observe the second string resonance at $5\,\sigma$ level, we need much more integrated luminosities, however, the analysis about how much luminosities are required is not performed in this thesis.

6.2 Modified decay widths of second string excited states

Since we consider only the dominant process of $qg \to qg$, exchanged states are the string excited quarks. The decay width of the second string excited quark of eq.(4.23) includes only a decay

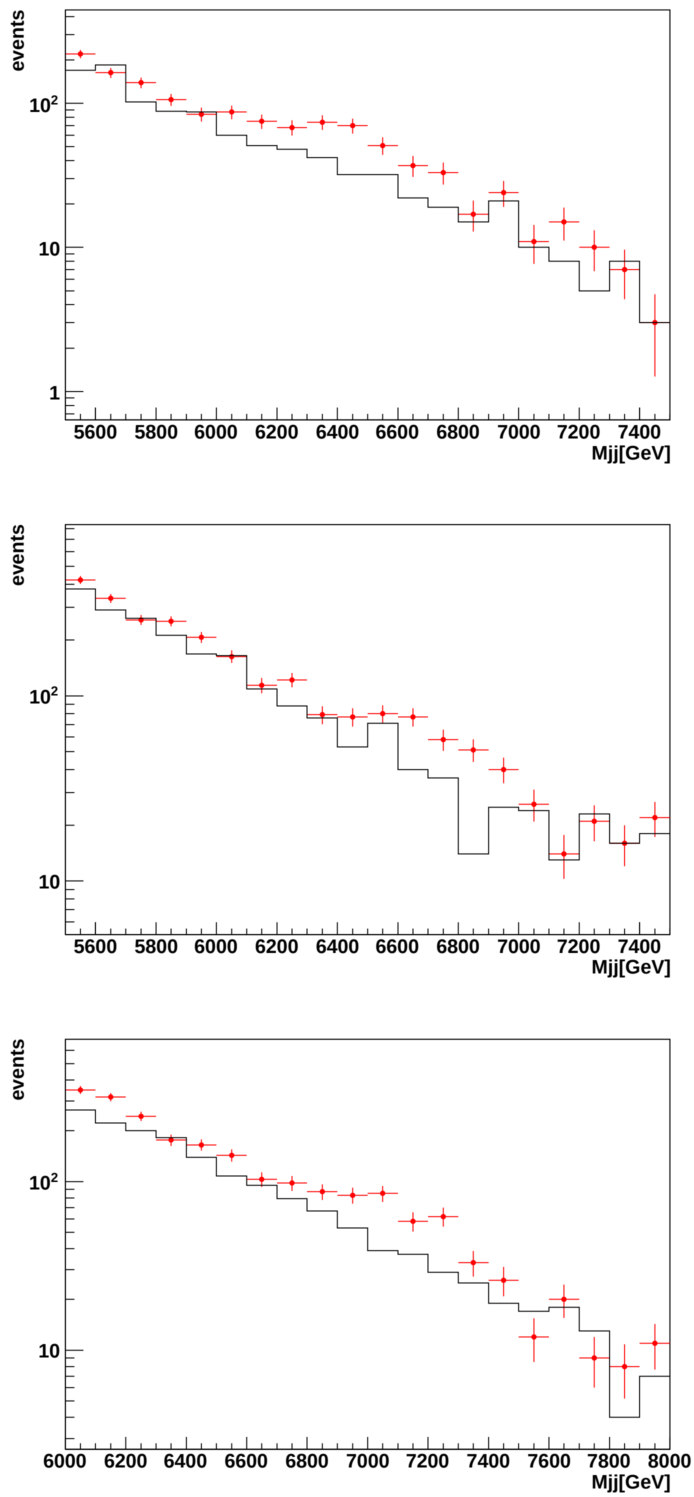

Figure 6.2: The dijet invariant mass distributions with the second string resonances with the narrow widths of the MC data (the points with error bars) including the SM background (the histogram), in which the resonances are observed at $5\,\sigma$ level for $M_{\rm s} = 4.5\,\text{TeV}$ with $150\,\text{fb}^{-1}$ (the top figure), $M_{\rm s} = 4.75\,\text{TeV}$ with $300\,\text{fb}^{-1}$ (the middle figure) and $M_{\rm s} = 5\,\text{TeV}$ with $600\,\text{fb}^{-1}$ (the bottom figure).

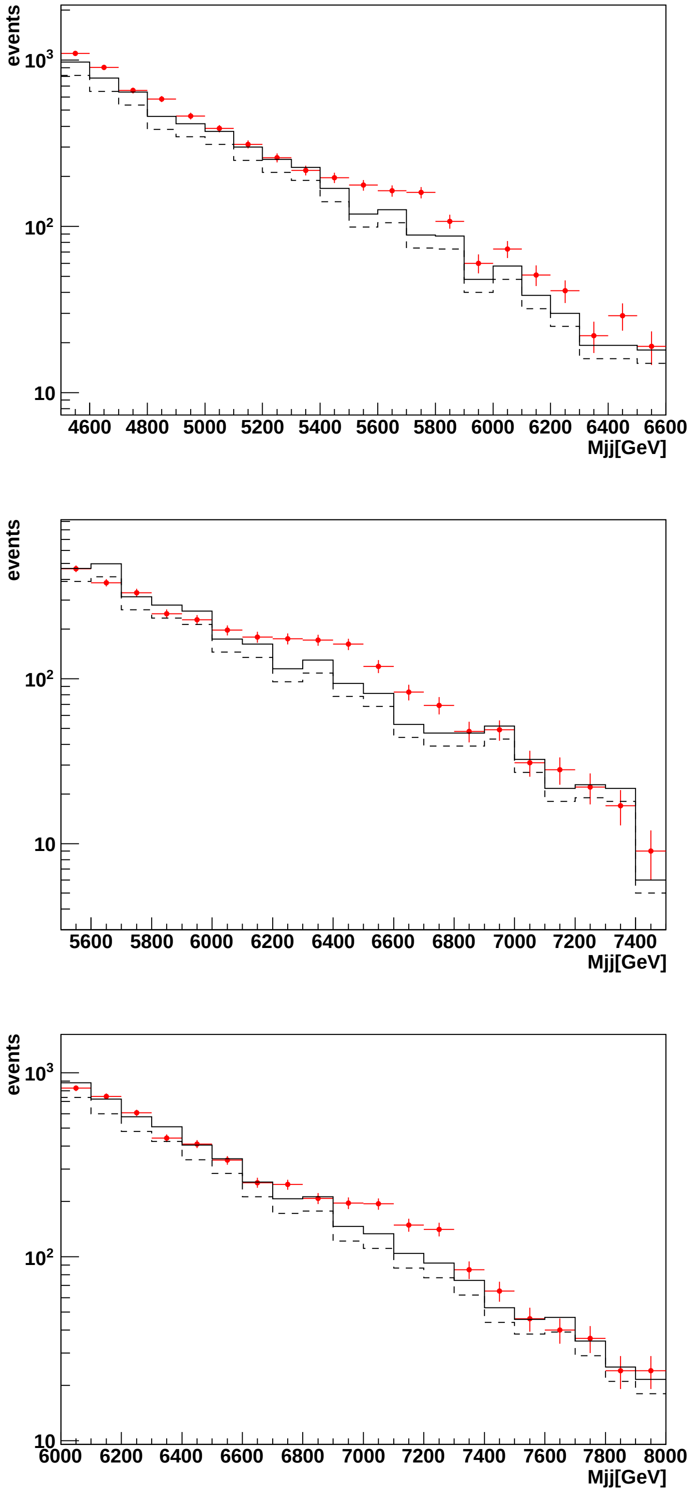

Figure 6.3: The dijet invariant mass distributions with the second string resonances with the narrow widths of the MC data (the points with error bars) including the SM background (the dashed histogram) and that increased by 20 % (the solid histogram), in which the resonances are observed at $5\,\sigma$ level for $M_s = 4\,\text{TeV}$ with $70\,\text{fb}^{-1}$ (the top figure) and $M_s = 4.5\,\text{TeV}$ with $350\,\text{fb}^{-1}$ (the middle figure), and at $4.8\,\sigma$ level for $M_s = 5\,\text{TeV}$ with $1500\,\text{fb}^{-1}$ (the bottom figure).

process of $q^{**} \to qg$. However, we should consider other decay channels into the first string excited states, where the second string excited quark decays into a quark and the first string excited gluon, $q^{**} \to qg^*$, and into a gluon and the first string excited quark, $q^{**} \to q^*g$.

Simple estimate of decay width of 2nd → 1st + SM

In order to avoid technical complication in calculations of decay widths of 2nd → 1st + SM, we make a simple estimate that the decay widths of 2nd → 1st + SM are that of 2nd → SM + SM multiplied by a factor, following in ref. [38]. In other words, we describe total decay widths of the second string excited quarks as

$$\begin{aligned}\Gamma_{q^{**},\,\mathrm{tot.}} &= \Gamma_{q^{**}\to qg} + \Gamma_{q^{**}\to qg^*} + \Gamma_{q^{**}\to q^*g} \\ &\equiv \Gamma_{q^{**}\to qg} + A_{q^{**}} \times \Gamma_{q^{**}\to qg}\,,\end{aligned} \tag{6.6}$$

where $A_{q^{**}}$ is a constant factor.

The factor $A_{q^{**}}$ in eq.(6.6) is estimated in the following way. First, we count the number of the first string excited states which are decay products of the second string excited quarks. The first string excited quarks have two states with $J = 1/2$ and $J = 3/2$, while the first string excited gluons have three states with $J = 0$, $J = 1$ and $J = 2$. The number of physical degrees of freedom of the first string excited states are counted in ref.[38]. Therefore, the number of the decay products is five.

Second, we consider phase space suppression. The phase space of a two-body decay in the center-of-mass frame is

$$\begin{aligned}d\Phi(M \to p_1, p_2) &= \prod_{i=1}^{2} \frac{d^3\mathbf{p}_i}{(2\pi)^2 2p_i^0}(2\pi)^4\delta^4(p_1 + p_2 - M) \\ &= \frac{d\Omega}{(4\pi)^2}\frac{\sqrt{(M^2 - (m_1+m_2)^2)(M^2-(m_1-m_2)^2)}}{2M^2}\,,\end{aligned} \tag{6.7}$$

where a parent particle mass is M, and each four-momentum of daughter particles is $p_i = (m_i, \mathbf{p}_i)$. If $m \equiv m_1 \neq 0$ and $m_2 = 0$, a measure of the phase space is proportional to $(M^2 - m^2)/2M^2$. If $m_1 = m_2 = 0$, that is proportional to $1/2$. A ratio of these phase spaces is $(M^2 - m^2)/M^2$. Therefore, the phase space suppression between decay channels of 2nd → 1st + SM and 2nd → SM + SM is proportional to the ratio. In the case of $M = \sqrt{2}M_\mathrm{s}$ and $m = M_\mathrm{s}$, the ratio is

$$\frac{(\sqrt{2}M_\mathrm{s})^2 - M_\mathrm{s}^2}{(\sqrt{2}M_\mathrm{s})^2} = \frac{1}{2}\,. \tag{6.8}$$

In total, the factor $A_{q^{**}}$ is estimated as $5/2$.

Significance of second string resonances with broader widths

We generate event samples for the string signal with the modified widths of the second string excited quarks, for $M_\mathrm{s} = 4.5\,\mathrm{TeV}$ and $4.75\,\mathrm{TeV}$ at the LHC with $\sqrt{s} = 14\,\mathrm{TeV}$. The modified

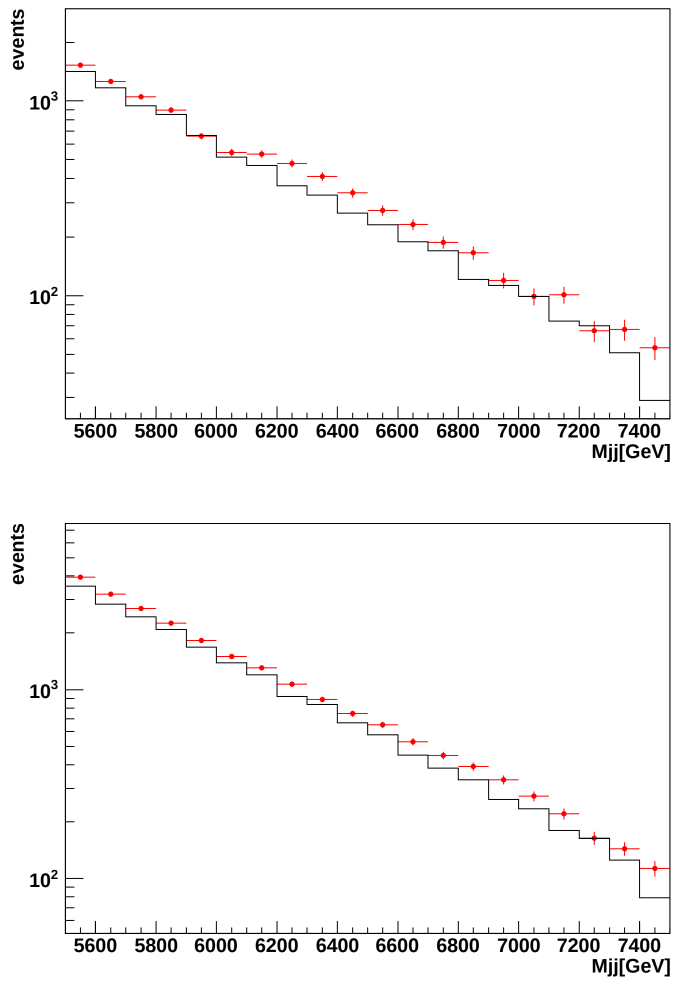

Figure 6.4: The dijet invariant mass distributions with the second string resonances with the broader widths of the MC data (the points with error bars) including the SM background (the histogram), in which the resonances are observed at $5\,\sigma$ and $4.1\,\sigma$ level, for $M_\text{s} = 4.5\,\text{TeV}$ with $1200\,\text{fb}^{-1}$ (the top figure) and $M_\text{s} = 4.75\,\text{TeV}$ with $3000\,\text{fb}^{-1}$ (the bottom one), respectively.

widths are 7/2 times of the widths of the second string excited quarks of eq.(4.23). Then, we calculate the significance of the second string resonance with the broader width. Fig.6.4 shows the dijet invariant mass distributions in which the second string resonances are observed at $5\,\sigma$ and $4.1\,\sigma$ level, for $M_\text{s} = 4.5\,\text{TeV}$ and $4.75\,\text{TeV}$, respectively. In order for the significance Z to exceed 5, the integrated luminosities of $1200\,\text{fb}^{-1}$ for $M_\text{s} = 4.5\,\text{TeV}$ and larger than $3000\,\text{fb}^{-1}$ for $M_\text{s} = 4.75\,\text{TeV}$ are required.

In Fig.6.4, the resonances almost do not form peaks. The height of the resonance becomes much lower as the width and mass are larger. In addition, the low-mass region of the resonance becomes broader due to the effects of final state radiations and parton distribution functions. On the other hand, the high-mass region of it also becomes broader due to initial state radiations, though this effect is not included in our MC simulations [63].

The existence of the second string excited states and their characteristic masses are distinct properties of low-scale string models. If the second string excited states do not decay into the first string excited states, the second string resonance in dijet events can be discovered at $5\,\sigma$ level with the integrated luminosity of $\mathcal{O}(100)\,\mathrm{fb}^{-1}$ for $M_{\rm s} \leq 5\,\mathrm{TeV}$. However, since they have the decay channels of 2nd $\to$ 1st + SM, the integrated luminosity of larger than $\mathcal{O}(1000)\,\mathrm{fb}^{-1}$ may be required to discover the second string resonance at $5\,\sigma$ level. This means that we may need the High-Luminosity LHC which is a possible future plan of the extension of the LHC.

Summary and Prospects

Summary of this thesis

In this thesis, we have discussed signatures of low-scale string models at the LHC. Low-scale string models are phenomenological models based on string theory, where the string scale $M_{\rm s}$ which is the fundamental scale in string theory is of the order of TeV. Such a low string scale is possible due to the existence of large extra dimensions, and these models are expected as solutions to the hierarchy problem and have a possibility of being confirmed or excluded by the LHC. In the following paragraphs, let us summarize what we have discussed in this thesis.

Intersecting D-branes and low-scale string models

In Part I, we reviewed intersecting D-branes which could give the SM spectrum, and we explained low-scale string models which has been considered in the studies of this thesis.

In Sec.1, we reviewed the basics of superstring theory. In Sec. 2, we reviewed the idea of intersecting D-branes which could give rise to chiral fermions. In order to obtain realistic models, we considered intersecting D6-branes in the type IIA orientifolds with toroidal compactifications. We presented the semi-realistic model with the SM-like spectrum, as the specific example. In Sec.3, we introduced low-scale string models in which the extra dimensions are compactified to general spaces with large volume, and which has been considered in the studies of this thesis.

String resonances and Distinction of low-scale string models at the LHC

In Part II, we explained the features of signatures of low-scale string models, and we performed two analyses to identify string resonances in dijet events at the LHC.

In Sec.4, we explained the features of signatures of low-scale string models. String excited states which are characteristic modes in low-scale string models can be observed as resonances in dijet invariant mass distributions at the LHC, and signatures of the dominant processes at the LHC are independent of the detail of model buildings. We reduced the open string amplitudes to the scattering amplitudes with exchanges of string excited states and calculated the widths of string excited states. We obtained the dijet invariant mass distributions with string resonances, both at parton level and using the MC simulations.

In Sec.5 and Sec.6, we performed the following two analyses using the MC simulations for the 14 TeV LHC, and we confirmed the possibility of distinguishing low-scale string models from the other "new physics".

The angular distribution analysis of dijet events is important, because the degeneracy of string excited states with higher spins is a distinct property of low-scale string models. We have shown that hypotheses for the angular distributions without higher spin states could be excluded at 95% confidence level with the integrated luminosities of $18.7\,\mathrm{fb}^{-1}$, $30\,\mathrm{fb}^{-1}$ and $50\,\mathrm{fb}^{-1}$ for $M_{\rm s} = 4\,\mathrm{TeV}$, $4.5\,\mathrm{TeV}$ and $5\,\mathrm{TeV}$, respectively.

The dijet invariant mass distribution analysis for the second string resonance is also important, because the existence of second string excited states with $\sqrt{2}$ times masses of that of the first string excited states is also a distinct property of low-scale string models. We have shown that the second string resonance in dijet invariant mass distributions could be discovered at $5\,\sigma$ level with $150\,\mathrm{fb}^{-1}$, $300\,\mathrm{fb}^{-1}$ and $600\,\mathrm{fb}^{-1}$ for $M_\mathrm{s} = 4.5\,\mathrm{TeV}$, $4.75\,\mathrm{TeV}$ and $5\,\mathrm{TeV}$, respectively. In these results, it is assumed that the second string excited states do not decay into the first string excited states.

In case of broader decay widths of the second string excited states including decay channels into the first string excited states, we found that integrated luminosities of $1200\,\mathrm{fb}^{-1}$ and more than $3000\,\mathrm{fb}^{-1}$ would be required to observe the second string resonance at $5\,\sigma$ level, for $M_\mathrm{s} = 4.5\,\mathrm{TeV}$ and $4.75\,\mathrm{TeV}$, respectively. This means that we might need the High-Luminosity LHC which is a possible future plan of the extension of the LHC.

Future prospects

Finally, we mention some future prospects after this thesis.

Calculation of decay widths of 2nd $\to$ 1st + SM

In Sec.6, we parametrized the decay widths of the second string excited states into the first string excited states, in order to avoid technical complication in calculation of the widths. In ref.[64], however, string amplitudes with external one massive state of open strings are calculated. They are tree-level amplitudes with external three gluons and one first string excited gluon, and that with external two gluons, one quark and one first string excited quark. If we can expand them by s-channel poles, we can calculate the decay widths of 2nd $\to$ 1st + SM by extracting a matrix element in decomposition of the amplitudes into the Wigner d-functions.

If the decay widths of 2nd $\to$ 1st + SM can be calculated, and if they are smaller than the parametrized widths in Sec.6, there may be a possibility of discovering the second string resonances at $5\,\sigma$ level without the High-Luminosity LHC.

Direct production of first string excited states

The amplitudes which are calculated in ref.[64] are also that of the two-parton scattering processes with a direct production of the first string excited states. If the string excited states are produced directly at the LHC, what signatures can be observed is interesting. Since the first string excited states decay into the SM particles immediately, the final states result in three-jet events. String resonances may be observed not only in dijet invariant mass distributions but also in three-jet invariant mass distributions. In ref.[65], string amplitudes with external five massless states of open strings are calculated. They are two-parton scattering processes with three partons in the final state and contribute the three-jet events. We have to investigate different interactions in the three-parton final state processes from in the direct production processes, since there may be some contact interactions in the amplitudes.

Multi-jet event analyses are performed by the CMS to search for the microscopic black holes [66]. Three-jet events give contributions at the next-to-leading order with respect to the gauge coupling constant, compared with dijet events. However, we expect that the cross sections of string excited state production are large enough to observe string resonances in three-jet invariant mass distributions.

Radiative electroweak symmetry breaking in low-scale string models

At present, the string scale has been excluded to 3.61 TeV by the ATLAS and 4.78 TeV by the CMS. The LHC can constrain to be about 7 TeV on the string scale, for the 14 TeV LHC with $100\,\mathrm{fb}^{-1}$ [11]. However, if we consider low-scale string models as a solution to the hierarchy problem in the SM, the string scale must not be larger than $\mathcal{O}(\mathrm{TeV})$.

On the other hand, if the Higgs field obtains a negative mass square at one-loop level in non-SUSY system, and the radiative electroweak symmetry breaking can occur in low-scale string models. In ref.[67], it is shown that the potential has a minimum and the Higgs field obtain a vacuum expectation value (VEV), when the Higgs field is in the adjoint representation. In ref.[68], a possibility that the Higgs field in the fundamental representation obtain a negative mass square is investigated, however, the Higgs one-loop mass square is calculated by order estimate, since there is a problem in non-SUSY system that infrared divergence from an exchange of the NS-NS closed string is not cancelled.[30]

In ref.[72], it is shown that even if the Higgs mass square is positive, the potential around the origin is lifted up by the Fayet-Iliopoulos term, and the Higgs field obtain a VEV. Then, the Higgs mass square is shown as,

$$m_H^2 \simeq \frac{1}{\omega^2}\frac{g^2}{16\pi^2}M_\mathrm{s}^2\,,$$

where g is a gauge coupling constant and $1/\omega^2$ is a factor of the order of unity. If $m_H^2/g^2 \simeq v^2$, the string scale M_s should be about $3 \times \omega$ TeV.

Thus, considering the radiative electroweak symmetry breaking in low-scale string models, we can set an upper limit on the string scale. Then, we can conclude low-scale string models with the constraint on the string scale by the LHC. However, note that the radiative electroweak symmetry breaking in low-scale string models is open to further discussions.

[30]Using a prescription of the NS-NS tadpole resummation [70, 71], one-loop corrections can be calculated even in non-SUSY system.

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A Calculation of scattering amplitudes with string excited state exchanges

A.1 Four gluon amplitudes

String amplitudes are calculated based on SuperConformal Field Theory (SCFT) in the two-dimensional world-sheet. The disk-level open string amplitude with external four gluons is a Maximally Helicity Violating (MHV) amplitude [31] shown in ref.[48, 49] as

$$\begin{aligned}\mathcal{M}(g_1^-, g_2^-, g_3^+, g_4^+) = 4g^2\langle 12\rangle^4 \Bigg[&\frac{V_t}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_4}T^{a_3}\big) \\ &+ \frac{V_u}{\langle 13\rangle\langle 34\rangle\langle 42\rangle\langle 21\rangle} \mathrm{Tr}\big(T^{a_2}T^{a_1}T^{a_3}T^{a_4} + T^{a_1}T^{a_2}T^{a_4}T^{a_3}\big) \\ &+ \frac{V_s}{\langle 14\rangle\langle 42\rangle\langle 23\rangle\langle 31\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_2}T^{a_4} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}\big)\Bigg],\end{aligned} \tag{A.1}$$

where g is the gauge coupling constant of strong interaction, and T^{a_i} are generators of $\mathrm{SU}(3)_\mathrm{C}$ gauge group. Here, g_1^-, g_2^-, g_3^+ and g_4^+ refer to incoming gluons with helicity $-$ or $+$.

In eq.(A.1), V_s, V_t and V_u are string "form factor" functions of the Mandelstem variables, s, t and u, with $s + t + u = 0$, which are defined as

$$V_t \equiv V(s,t,u) \quad , \quad V_u \equiv V(t,u,s) \quad , \quad V_s \equiv V(u,s,t) \tag{A.2}$$

where

$$V(s,t,u) = \frac{\Gamma(1 - s/M_\mathrm{s}^2)\,\Gamma(1 - u/M_\mathrm{s}^2)}{\Gamma(1 + t/M_\mathrm{s}^2)} = \frac{1}{M_\mathrm{s}^2}\frac{su}{t}\,\frac{\Gamma(-s/M_\mathrm{s}^2)\,\Gamma(-u/M_\mathrm{s}^2)}{\Gamma(t/M_\mathrm{s}^2)}\,. \tag{A.3}$$

In addition, $\langle ij\rangle$ $(i,j = 1,2,3,4)$ are standard spinor products associated with momenta k_i and k_j,

$$\langle ij\rangle \equiv \overline{u}(\mathbf{k}_i, -)\, u(\mathbf{k}_j, +)\,, \tag{A.4}$$

where $u(\mathbf{k}, \lambda)$ is the Dirac spinor wave function of a massless fermion, as an eigenstate of helicity λ. The spinor products have important properties

$$|\langle ij\rangle| = \sqrt{-2k_i \cdot k_j} \quad , \quad \langle ij\rangle = -\langle ji\rangle\,. \tag{A.5}$$

See Appendix B in detail.

Amplitudes with four gluons similar to eq.(A.1) are given by using crossing symmetry,

$$\begin{aligned}\mathcal{M}(g_1^-, g_2^+, g_3^+, g_4^-) = 4g^2\langle 14\rangle^4 \Bigg[&\frac{V_t}{\langle 14\rangle\langle 43\rangle\langle 32\rangle\langle 21\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_4}T^{a_3}T^{a_2} + T^{a_4}T^{a_1}T^{a_2}T^{a_3}\big) \\ &+ \frac{V_s}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle} \mathrm{Tr}\big(T^{a_4}T^{a_1}T^{a_3}T^{a_2} + T^{a_1}T^{a_4}T^{a_2}T^{a_3}\big) \\ &+ \frac{V_u}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_4}T^{a_2} + T^{a_3}T^{a_1}T^{a_2}T^{a_4}\big)\Bigg],\end{aligned} \tag{A.6}$$

[31] The MHV amplitudes are amplitudes with n external gauge bosons, where $(n-2)$ gauge bosons have a particular helicity and the other two have the opposite helicity. The tree-level amplitudes in which all gauge bosons have the same helicity or all but one have the same helicity vanish (helicities refer to incoming particles).

$$\mathcal{M}(g_1^-, g_2^+, g_3^-, g_4^+) = 4g^2\langle 13\rangle^4 \Bigg[\frac{V_s}{\langle 13\rangle\langle 32\rangle\langle 24\rangle\langle 41\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_2}T^{a_4} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}\big) + \frac{V_u}{\langle 12\rangle\langle 24\rangle\langle 43\rangle\langle 31\rangle} \mathrm{Tr}\big(T^{a_3}T^{a_1}T^{a_2}T^{a_4} + T^{a_1}T^{a_3}T^{a_4}T^{a_2}\big) + \frac{V_t}{\langle 14\rangle\langle 43\rangle\langle 32\rangle\langle 21\rangle} \mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_4}T^{a_3}\big)\Bigg]. \tag{A.7}$$

(A.7) and (A.6) are obtained by replacements of $2 \leftrightarrow 4$ and $2 \leftrightarrow 3$ in eq.(A.1), respectively.

***s*-channel pole expansion in form factor functions** The string "form factor" functions in eq.(A.2) can be expanded in terms of s-channel poles. That is because the Gamma function for $\varepsilon \ll 1$ has a property that it has a pole of $1/\varepsilon$,

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} + \mathcal{O}(\varepsilon^0). \tag{A.8}$$

The Gamma function also has the following properties for an integer n and $z \in \mathbb{R}$,

$$\begin{aligned} \Gamma(\varepsilon - n) &= \frac{(-1)^n}{n!}\frac{1}{\varepsilon} + \mathcal{O}(\varepsilon^0), \\ \Gamma(\varepsilon + z - n) &= \frac{(-1)^n}{\prod_{m=1}^n (m-z)} \frac{\Gamma(z+\varepsilon)}{1+\mathcal{O}(\varepsilon)} \\ &= \frac{(-1)^n}{\prod_{m=1}^n (m-z)} \Gamma(z) + \mathcal{O}(\varepsilon). \end{aligned} \tag{A.9}$$

The factor written by the Gamma functions in the string form factor function of eq.(A.3), $\big[\Gamma(-s/M_\mathrm{s}^2)\Gamma(-u/M_\mathrm{s}^2)\big]/\Gamma(t/M_\mathrm{s}^2)$, has s-channel poles of $1/(s-nM_\mathrm{s}^2)$ for an integer n. Suppose that $\varepsilon \equiv (nM_\mathrm{s}^2 - s)/M_\mathrm{s}^2$ and $z \equiv -u/M_\mathrm{s}^2$,

$$\frac{\Gamma(-s/M_\mathrm{s}^2)\,\Gamma(-u/M_\mathrm{s}^2)}{\Gamma\big(-(s+u)/M_\mathrm{s}^2\big)} = \frac{\Gamma(\varepsilon - n)\,\Gamma(z)}{\Gamma(\varepsilon + z - n)} = \frac{1}{n!}\frac{1}{\varepsilon}\prod_{m=1}^n (m-z) + \mathcal{O}(\varepsilon^0). \tag{A.10}$$

Since the s-channel poles for an integer n in eq.(A.10) should be summed over all integers, the string form factor functions are

$$\begin{aligned} V_t &= \frac{1}{M_\mathrm{s}^2}\frac{su}{t}\left[-\sum_{n=1}^{\infty}\frac{1}{n!}\frac{1}{(M_\mathrm{s}^2)^{n-1}}\frac{1}{s-nM_\mathrm{s}^2}\prod_{J=1}^{n}\big(u+JM_\mathrm{s}^2\big) + \mathcal{O}\big((s-nM_\mathrm{s}^2)^0\big)\right], \\ V_u &= \frac{1}{M_\mathrm{s}^2}\frac{st}{u}\left[-\sum_{n=1}^{\infty}\frac{1}{n!}\frac{1}{(M_\mathrm{s}^2)^{n-1}}\frac{1}{s-nM_\mathrm{s}^2}\prod_{J=1}^{n}\big(t+JM_\mathrm{s}^2\big) + \mathcal{O}\big((s-nM_\mathrm{s}^2)^0\big)\right], \\ V_s &= \frac{1}{M_\mathrm{s}^2}\frac{tu}{s}\left[\mathcal{O}\big((s-nM_\mathrm{s}^2)^0\big)\right], \end{aligned} \tag{A.11}$$

and hence V_s has no s-channel poles. The terms of $\mathcal{O}\big((s-nM_\mathrm{s}^2)^0\big)$ in eq.(A.11) can be neglected, since we consider only near each nth s-channel pole, $s \simeq nM_\mathrm{s}^2$. The each s-channel pole corresponds to the nth string excited state with mass of $M_n = \sqrt{n}M_\mathrm{s}$.

low-energy limit of form factor functions The string form factor functions can be also expanded in terms of $1/M_s^2$, by using a formula of the Gamma functions for $\varepsilon \ll 1$,

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + \frac{1}{2}\left(\gamma^2 + \frac{\pi^2}{6}\right)\varepsilon + \mathcal{O}(\varepsilon^2)\,, \tag{A.12}$$

where γ is the Euler constant. In the expansion by $1/M_s^2$ corresponding to low-energy limit of $M_s \to \infty$, the string form factor functions become unity,

$$V_t = 1 - \frac{\pi^2}{6}\, su\, \frac{1}{M_s^4} + \mathcal{O}\big(1/M_s^6\big)\,. \tag{A.13}$$

In eq.(A.13), unity describes an exchange of a massless gluon, so QCD amplitudes with external four gluons are obtained by replacements of $V_{s,t,u} \to 1$ in eqs.(A.1), (A.6) and (A.7). On the other hand, the term at the order of $1/M_s^4$ in eq.(A.13) describes a string contact interaction as a result of summing up all string excited states. Note that there is no term at the order of $1/M_s^2$ in eq.(A.13). Nevertheless, according to ref.[13], a disk-level amplitude with external four fermions yields a contact term at the order of $1/M_s^2$.

two gluon scattering amplitudes Now, we regard the four gluon amplitudes of eqs.(A.1), (A.6) and (A.7) as two gluon scattering amplitudes of processes of $g_1, g_2 \to g_3, g_4$ (g_1, g_2 refer to incoming gluons, while g_3, g_4 refer to outgoing gluons). The amplitudes of these processes are obtained by replacements of $k_{3,4} = -k_{3,4}$ in the four gluon amplitudes of eqs.(A.1), (A.6) and (A.7). Since expressions of the spinor products $\langle ij\rangle$ in the processes are given in Appendix B,

$$\begin{aligned}\mathcal{M}(g_1^-, g_2^- \to g_3^-, g_4^-) = -4g^2 s^2 \bigg[&\frac{V_t}{su}\,\mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_4}T^{a_3}\big) \\ &+ \frac{V_u}{st}\,\mathrm{Tr}\big(T^{a_2}T^{a_1}T^{a_3}T^{a_4} + T^{a_1}T^{a_2}T^{a_4}T^{a_3}\big) \\ &+ \frac{V_s}{tu}\,\mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_2}T^{a_4} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}\big)\bigg]\,,\end{aligned} \tag{A.14}$$

$$\begin{aligned}\mathcal{M}(g_1^-, g_2^+ \to g_3^-, g_4^+) = -4g^2 u^2 \bigg[&\frac{V_t}{su}\,\mathrm{Tr}\big(T^{a_1}T^{a_4}T^{a_3}T^{a_2} + T^{a_4}T^{a_1}T^{a_2}T^{a_3}\big) \\ &+ \frac{V_s}{tu}\,\mathrm{Tr}\big(T^{a_4}T^{a_1}T^{a_3}T^{a_2} + T^{a_1}T^{a_4}T^{a_2}T^{a_3}\big) \\ &+ \frac{V_u}{st}\,\mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_4}T^{a_2} + T^{a_3}T^{a_1}T^{a_2}T^{a_4}\big)\bigg]\,,\end{aligned} \tag{A.15}$$

$$\begin{aligned}\mathcal{M}(g_1^-, g_2^+ \to g_3^+, g_4^-) = -4g^2 t^2 \bigg[&\frac{V_s}{tu}\,\mathrm{Tr}\big(T^{a_1}T^{a_3}T^{a_2}T^{a_4} + T^{a_3}T^{a_1}T^{a_4}T^{a_2}\big) \\ &+ \frac{V_u}{st}\,\mathrm{Tr}\big(T^{a_3}T^{a_1}T^{a_2}T^{a_4} + T^{a_1}T^{a_3}T^{a_4}T^{a_2}\big) \\ &+ \frac{V_t}{su}\,\mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_2}T^{a_1}T^{a_4}T^{a_3}\big)\bigg]\,.\end{aligned} \tag{A.16}$$

A.1.1 First string excited gluon exchanges

Let us consider the two gluon scattering process, which has an exchange of a first string excited gluon in s-channel. We focus on only the first s-channel pole, in case of $n = 1$ in eq.(A.11). Then the string form factor functions are

$$V_t = -\frac{1}{M_{\rm s}^2}\frac{su}{t}\frac{1}{s-M_{\rm s}^2}\left(u+M_{\rm s}^2\right) \quad , \quad V_u = -\frac{1}{M_{\rm s}^2}\frac{st}{u}\frac{1}{s-M_{\rm s}^2}\left(t+M_{\rm s}^2\right) , \tag{A.17}$$

and in the limit of $s \simeq M_{\rm s}^2$,

$$V_t \simeq \frac{u}{s-M_{\rm s}^2} \quad , \quad V_u \simeq \frac{t}{s-M_{\rm s}^2} . \tag{A.18}$$

In this limit, the two gluon scattering amplitudes with the first string excited gluon exchanges are obtained from eqs.(A.14), (A.15) and (A.16),

$$\mathcal{M}_{\rm 1st}(g_1^-, g_2^- \to g_3^-, g_4^-) \simeq -4\,g^2 M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\,{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big) , \tag{A.19}$$

$$\mathcal{M}_{\rm 1st}(g_1^-, g_2^+ \to g_3^-, g_4^+) \simeq -4\,g^2 M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left(-\frac{u}{s}\right)^2{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big) , \tag{A.20}$$

$$\mathcal{M}_{\rm 1st}(g_1^-, g_2^+ \to g_3^+, g_4^-) \simeq -4\,g^2 M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left(-\frac{t}{s}\right)^2{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big) . \tag{A.21}$$

The absence of angular dependence in eq.(A.19) imply that the exchanged first string excited gluon has spin $J = 0$. On the other hand, the presence of angular dependence in eqs.(A.20) and (A.21) imply that the exchanged first string excited gluons have spin $J = 2$. This is found from that the angular dependence can be factorized into the Wigner d-function,

$$\begin{aligned} d^{\,J=0}_{\,0,0}(\theta) &= 1 , \\ d^{\,J=2}_{\,\pm2,\pm2}(\theta) &= \left(-\frac{u}{s}\right)^2 = \left(\frac{1+\cos\theta}{2}\right)^2 , \\ d^{\,J=2}_{\,\pm2,\mp2}(\theta) &= \left(-\frac{t}{s}\right)^2 = \left(\frac{1-\cos\theta}{2}\right)^2 . \end{aligned} \tag{A.22}$$

The Wigner d-function $d^{\,J}_{J_z, J_{z'}}(\theta)$ represents angular dependence of a process through a state with spin J, in which an initial state has total spin along z-axis, J_z, and a final state has total spin along z'-axis, $J_{z'}$. The angle θ is that between the z- and z'-axes.

U(N) gauge bosons and group factors We define a completely symmetric symbol $d^{\,a_i a_j a_k}$ as a symmetrized trace of generators T^{a_i},

$$d^{\,a_1 a_2 a_3} \equiv {\rm STr}\big(T^{a_1}T^{a_2}T^{a_3}\big) = \frac{1}{2}\,{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\,T^{a_3}\big) , \tag{A.23}$$

so that

$$\{T^{a_1},T^{a_2}\} = 4\sum_A d^{\,a_1 a_2 A}\,T^A . \tag{A.24}$$

Note that T^{a_i} are $\mathrm{SU}(3)_\mathrm{C}$ generators in the fundamental representation with $\mathrm{Tr}(T^{a_1}T^{a_2}) = \frac{1}{2}\,\delta^{a_1a_2}$. Then, the group factor of eqs.(A.19), (A.20) and (A.21) is

$$\mathrm{Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big) = 8\sum_A d^{\,a_1a_2A}\,d^{\,a_3a_4A}\,. \tag{A.25}$$

Since $\mathrm{U}(N)$ gauge group rather than $\mathrm{SU}(N)$ is realized in D-brane models, gluons as $\mathrm{SU}(3)_\mathrm{C}$ gauge bosons come from $\mathrm{U}(3)_\mathrm{C}$ gauge bosons. The $\mathrm{U}(3)_\mathrm{C}$ gauge bosons are $G^A = (C^0, g^a)$ where C^0 is an $\mathrm{U}(1)_\mathrm{C}$ gauge boson and g^a are gluons with $a = 1,\cdots,8$. The gauge index A in eq.(A.25) sums over string excited "gluons" exchanged in the process. The string excited "gluons" are G^* which are excited $\mathrm{U}(3)_\mathrm{C}$ gauge bosons, including not only excited gluons g^* but also an excited $\mathrm{U}(1)_\mathrm{C}$ gauge boson C^*.

The $\mathrm{U}(N)$ generator is $T^A = (T^0, T^a)$, where $T^0 = \frac{1}{\sqrt{2N}}\,\mathbb{I}_N$ is an $\mathrm{U}(1)$ generator and T^a is the $\mathrm{SU}(N)$ generator with $a = 1,\cdots,N^2-1$. The completely symmetric symbols $d^{\,A_iA_jA_k}$ defined by the $\mathrm{U}(N)$ generators are

$$d^{\,000} = \frac{1}{\sqrt{8N}} \quad,\quad d^{\,a_100} = 0 \quad,\quad d^{\,a_1a_20} = \frac{1}{\sqrt{8N}}\,\delta^{a_1a_2}\,, \tag{A.26}$$

and $d^{\,a_1a_2a_3}$ $(a_i = 1,\cdots,N^2-1)$.

instability of string excited states The string excited states are unstable and decay into SM particles or other light states associated with string. The s-channel pole of the nth string excited states in eq.(A.11) should be softened as follows,

$$\frac{1}{s-M_n^2} \to \frac{1}{s-M_n^2+i\,M_n\,\Gamma_n^J}\,, \tag{A.27}$$

where M_n is the mass of the nth string excited states, $M_n = \sqrt{n}M_\mathrm{s}$, and Γ_n^J is a total decay width of the nth string excited states with spin J.

Loop-level amplitudes necessarily lead to an imaginary part in a pole, in terms of a width of an exchanged state. For the purpose of this thesis, however, we calculate only tree-level string amplitudes, and have to add the imaginary part in the s-channel pole by hand.

calculation of widths of exchanged states Consider a decay of a massive particle into two massless particles. The decay S-matrix is given by

$$S = i(2\pi)^4\,\delta^4(P-p_3-p_4)\,\langle\,\mathbf{p}_3,\lambda_3,a_3\,;\mathbf{p}_4,\lambda_4,a_4\,|\,\mathcal{L}^J\,|\,\mathbf{P},\Lambda,A\,\rangle\,, \tag{A.28}$$

where $\mathcal{L}^J$ is an interaction Lagrangian of spin J. P and $p_{3,4}$ are four-momenta, and, A and $a_{3,4}$ are gauge indices of the parent particle and the two daughter particles, respectively. Λ is spin z-component of the parent, and $\lambda_{3,4}$ are helicities of the daughters. In a rest frame of the parent particle with mass M, $\mathbf{P} = \mathbf{0}$ and $\mathbf{p}_3 = -\mathbf{p}_4 \equiv \mathbf{p}$ with $|\mathbf{p}| = \frac{M}{2}$. The decay width is

$$\begin{aligned}\Gamma^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4} &= \int d\Pi_3\,d\Pi_4\,\frac{1}{2M}\,(2\pi)^4\,\delta^4(P-p_3-p_4)\,\big|\langle\,\mathbf{p}_3,\lambda_3,a_3\,;\mathbf{p}_4,\lambda_4,a_4\,|\,\mathcal{L}^J\,|\,\mathbf{P},\Lambda,A\,\rangle\big|^2\\ &= \frac{1}{16\pi M}\,\frac{1}{4\pi}\int d\Omega\,\big|\langle\,\mathbf{p},\lambda_3,a_3\,;-\mathbf{p},\lambda_4,a_4\,|\,\mathcal{L}\,|\,\mathbf{0},\Lambda,A\,\rangle\big|^2\,.\end{aligned} \tag{A.29}$$

The spin z-component eigenstate with zero momentum, $|\,\mathbf{0},\Lambda\,\rangle$, is expanded in terms of a spin z'-component eigenstate, $|\,\mathbf{0},\Lambda'\,\rangle$, in which the z'-axis has an angle θ with the z-axis. The expansion coefficient is the Wigner d-function,

$$\begin{aligned}|\,\mathbf{0},\Lambda\,\rangle &= \sum_{\Lambda'} |\,\mathbf{0},\Lambda'\,\rangle\langle\,\mathbf{0},\Lambda'\,|\,\mathbf{0},\Lambda\,\rangle \\ &= \sum_{\Lambda'} d^{\,J}_{\,\Lambda,\,\Lambda'}(\theta)\,|\,\mathbf{0},\Lambda'\,\rangle\,.\end{aligned} \tag{A.30}$$

The width becomes

$$\begin{aligned}\Gamma^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4} &= \frac{1}{16\pi M}\,\frac{1}{4\pi}\int d\Omega\,\left|d^{\,J}_{\,\Lambda,\,\lambda_3-\lambda_4}(\theta)\right|^2\left|\langle\,\mathbf{p},\lambda_3,a_3\,;-\mathbf{p},\lambda_4,a_4\,|\,\mathcal{L}^J\,|\,\mathbf{0},\lambda_3-\lambda_4,A\,\rangle\right|^2 \\ &= \frac{1}{16\pi M}\,\frac{1}{2J+1}\,\left|F^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4}\right|^2,\end{aligned} \tag{A.31}$$

since $\frac{1}{4\pi}\int d\Omega\left|d^{\,J}_{\,\Lambda,\,\Lambda'}(\theta)\right|^2 = \frac{1}{2J+1}$. Here, $F^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4}$ is a matrix element of a decay for spin z'-component $\lambda_3-\lambda_4$ into helicities λ_3 and λ_4 along the z'-axis.

The matrix element $F^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4}$ can be extracted from a two-body scattering amplitude among massless particles with a massive state exchange in s-channel. In the center-of-mass frame, $\mathbf{p}_1=-\mathbf{p}_2\equiv\mathbf{p}$ and $\mathbf{p}_3=-\mathbf{p}_4\equiv\mathbf{p}'$ with an angle θ between $\mathbf{p}$ and $\mathbf{p}'$. The amplitude is

$$\begin{aligned}&\langle\,\mathbf{p}_3,\lambda_3,a_3\,;\mathbf{p}_4,\lambda_4,a_4\,|\,\mathcal{M}\,|\,\mathbf{p}_1,\lambda_1,a_1\,;\mathbf{p}_2,\lambda_2,a_2\,\rangle \\ &=\langle\,\mathbf{p}',\lambda_3,a_3\,;-\mathbf{p}',\lambda_4,a_4\,;\theta\,|\,\mathcal{M}\,|\,\mathbf{p},\lambda_1,a_1\,;-\mathbf{p},\lambda_2,a_2\,;0\,\rangle \\ &=-\sum_{J,A}\frac{1}{s-M^2}\,\langle\,\mathbf{p}',\lambda_3,a_3\,;-\mathbf{p}',\lambda_4,a_4\,;\theta\,|\,\mathcal{L}^J\sum_{\Lambda,\Theta}|\,\mathbf{0},\Lambda,A\,;\Theta\,\rangle \\ &\qquad\qquad\qquad\qquad\times\langle\,\mathbf{0},\Lambda,A\,;\Theta\,|\,\mathcal{L}^J\,|\,\mathbf{p},\lambda_1,a_1\,;-\mathbf{p},\lambda_2,a_2\,;0\,\rangle \\ &=-\sum_{J,A}\frac{1}{s-M^2}\,\langle\,\mathbf{p}',\lambda_3,a_3\,;-\mathbf{p}',\lambda_4,a_4\,;\theta\,|\,\mathcal{L}^J\,|\,\mathbf{0},\lambda_1-\lambda_2,A\,;0\,\rangle\,F^{JA}_{\lambda_1,\lambda_2;\,a_1,a_2} \\ &=-\sum_{J,A}\frac{1}{s-M^2}\,\langle\,\mathbf{p}',\lambda_3,a_3\,;-\mathbf{p}',\lambda_4,a_4\,;\theta\,|\,\mathcal{L}^J\sum_{\Lambda',\Theta'}d^{\,J}_{\,\lambda_1-\lambda_2,\,\Lambda'}(\Theta')|\,\mathbf{0},\Lambda',A\,;\Theta'\,\rangle\,F^{JA}_{\lambda_1,\lambda_2;\,a_1,a_2} \\ &=-\sum_{J,A}\frac{1}{s-M^2}\,d^{\,J}_{\,\lambda_1-\lambda_2,\,\lambda_3-\lambda_4}(\theta)\,\left(F^{\,JA}_{\,\lambda_3,\lambda_4;\,a_3,a_4}\right)^\dagger F^{\,JA}_{\,\lambda_1,\lambda_2;\,a_1,a_2}\,.\end{aligned} \tag{A.32}$$

See ref.[50] for a way of calculating decay widths in detail.

widths of first string excited gluons In order to calculate widths of the first string excited gluons, G^*, comparing the amplitudes with the first excited gluon exchanges of eqs.(A.19), (A.20) and (A.21) to the general amplitude with a massive state exchange of eq.(A.32), we can extract matrix elements of G^*, $F_{G^*}{}^{\,JA}_{\,\lambda_i,\lambda_j;\,a_i,a_j}$, as

$$F_{G^*}{}^{\,J=0\,A}_{\,\pm1,\pm1;\,a_3,a_4} = F_{G^*}{}^{\,J=2\,A}_{\,\pm1,\mp1;\,a_3,a_4} = 4\sqrt{2}\,gM_{\rm s}\,d^{\,a_3a_4A}\,. \tag{A.33}$$

Once the matrix element $F_{G^*}{}^{JA}_{\lambda_i,\lambda_j;\,a_i,a_j}$ is obtained, a decay width of $G^* \to GG$ can be calculated from eq.(A.31) by summing over helicities and gauge indices in the final state,

$$\Gamma^{\,JA}_{\,G^*\to GG} = \frac{1}{2}\,\frac{1}{16\pi M_{\rm s}}\,\frac{1}{2J+1}\sum_{\lambda_3,\lambda_4}\sum_{A_3,A_4}\left|F_{G^*}{}^{JA}_{\lambda_3,\lambda_4;\,A_3,A_4}\right|^2, \tag{A.34}$$

where a factor of $\frac{1}{2}$ is introduced to take into account identical particles in the final state. For $J=0$ and $J=2$,

$$\begin{aligned}\Gamma^{\,J=0\,A}_{\,G^*\to GG} &= \frac{1}{2}\,\frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times 0+1}\sum_{A_3,A_4}\frac{1}{2}\left\{\left|F_{G^*}{}^{J=0\,A}_{+1,+1;\,A_3,A_4}\right|^2+\left|F_{G^*}{}^{J=0\,A}_{-1,-1;\,A_3,A_4}\right|^2\right\}\\ &= \frac{g^2 M_{\rm s}}{\pi}\sum_{A_3,A_4} d^{\,A_3A_4A}\,d^{\,A_3A_4A}\,,\end{aligned} \tag{A.35}$$

$$\begin{aligned}\Gamma^{\,J=2\,A}_{\,G^*\to GG} &= \frac{1}{2}\,\frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times 2+1}\sum_{A_3,A_4}\left\{\left|F_{G^*}{}^{J=2\,A}_{+1,-1;\,A_3,A_4}\right|^2+\left|F_{G^*}{}^{J=2\,A}_{-1,+1;\,A_3A_4}\right|^2\right\}\\ &= \frac{2g^2 M_{\rm s}}{\pi}\,\frac{1}{5}\sum_{A_3,A_4} d^{\,A_3A_4A}\,d^{\,A_3A_4A}\,.\end{aligned} \tag{A.36}$$

Note that a factor of $\frac{1}{2}$ for $J=0$ reflects the fact that the resonance produced through an initial helicity configuration of $(-,-)$ subsequently does not decay into a final helicity configuration of $(+,+)$, as it is found from the amplitude of the process, $g^-g^- \to g^-g^-$, of eq.(A.19).

A group factor in eqs.(A.35) and (A.36) is different for each decay process, calculated by averaging over gauge indices for the initial state,

- $g^* \to gg$

$$\frac{1}{N^2-1}\sum_{a}^{N^2-1}\sum_{a_3,a_4}^{N^2-1} d^{\,a_3a_4a}\,d^{\,a_3a_4a} = \frac{N^2-4}{16N}\,, \tag{A.37}$$

- $g^* \to gC$ [32]

$$2\times\frac{1}{N^2-1}\sum_{a}^{N^2-1}\sum_{a_3}^{N^2-1} d^{\,a_30a}\,d^{\,a_30a} = \frac{1}{4N}\,, \tag{A.38}$$

- $C^* \to gg$

$$\sum_{a_3,a_4}^{N^2-1} d^{\,a_3a_40}\,d^{\,a_3a_40} = \frac{N^2-1}{8N}\,, \tag{A.39}$$

- $C^* \to CC$

$$d^{\,000}\,d^{\,000} = \frac{1}{8N}\,, \tag{A.40}$$

[32] A factor of 2 is introduced due to not identical particles in the final state.

where $N = 3$.

As a result, widths of the first string excited gluons, g^* and C^*, with $J = 0$ and $J = 2$ for the processes of $G^* \to GG$ are

$$\Gamma^{J=0}_{g^* \to GG} = \frac{g^2 M_\mathrm{s}}{4\pi} \frac{N}{4} \quad , \quad \Gamma^{J=2}_{g^* \to GG} = \frac{g^2 M_\mathrm{s}}{4\pi} \frac{1}{5} \frac{N}{2} \, , \tag{A.41}$$

$$\Gamma^{J=0}_{C^* \to GG} = \frac{g^2 M_\mathrm{s}}{4\pi} \frac{N}{2} \quad , \quad \Gamma^{J=2}_{C^* \to GG} = \frac{g^2 M_\mathrm{s}}{4\pi} \frac{1}{5} N \, . \tag{A.42}$$

Total widths of g^* and C^* should be included processes of $G^* \to q\bar{q}$ in case of $J = 2$, and the processes are considered later.

squared amplitudes with first string excited gluon exchanges Including above results, the two gluon scattering amplitudes with the first excited gluon exchanges of eqs.(A.19), (A.20) and (A.21) are rewritten by softening the s-channel pole into the Breit-Wigner form of eq.(A.27),

$$\begin{aligned} &\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^- \to g_3^-, g_4^-) \\ &\to -32\, g^2 M_\mathrm{s}^2 \left[\frac{1}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=0}_{g^*}} \sum_a d^{\,a_1 a_2 a}\, d^{\,a_3 a_4 a} + \frac{1}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=0}_{C^*}}\, d^{\,a_1 a_2 0}\, d^{\,a_3 a_4 0} \right] , \end{aligned} \tag{A.43}$$

$$\begin{aligned} &\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to g_3^-, g_4^+) \\ &\to -32\, g^2 M_\mathrm{s}^2 \frac{1}{M_\mathrm{s}^4} \left[\frac{u^2}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=2}_{g^*}} \sum_a d^{\,a_1 a_2 a}\, d^{\,a_3 a_4 a} + \frac{u^2}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=2}_{C^*}}\, d^{\,a_1 a_2 0}\, d^{\,a_3 a_4 0} \right] , \end{aligned} \tag{A.44}$$

$$\begin{aligned} &\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to g_3^+, g_4^-) \\ &\to -32\, g^2 M_\mathrm{s}^2 \frac{1}{M_\mathrm{s}^4} \left[\frac{t^2}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=2}_{g^*}} \sum_a d^{\,a_1 a_2 a} d^{\,a_3 a_4 a} + \frac{t^2}{s - M_\mathrm{s}^2 + i M_\mathrm{s} \Gamma^{J=2}_{C^*}}\, d^{\,a_1 a_2 0}\, d^{\,a_3 a_4 0} \right] . \end{aligned} \tag{A.45}$$

A squared amplitude is averaged over helicities and gauge indices in the initial state, and summed over them in the final state,

$$\begin{aligned} &\left| \mathcal{M}_{1\mathrm{st}}(g_1, g_2 \to g_3, g_4) \right|^2 \\ &= \frac{1}{2^2} \frac{1}{(N^2-1)^2} \sum_{a_1, a_2, a_3, a_4} \Big\{ \left| \mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^- \to g_3^-, g_4^-) \right|^2 + \left| \mathcal{M}_{1\mathrm{st}}(g_1^+, g_2^+ \to g_3^+, g_4^+) \right|^2 \\ &\qquad + \left| \mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to g_3^-, g_4^+) \right|^2 + \left| \mathcal{M}_{1\mathrm{st}}(g_1^+, g_2^- \to g_3^+, g_4^-) \right|^2 \\ &\qquad + \left| \mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to g_3^+, g_4^-) \right|^2 + \left| \mathcal{M}_{1\mathrm{st}}(g_1^+, g_2^- \to g_3^-, g_4^+) \right|^2 \Big\} . \end{aligned} \tag{A.46}$$

Here, we do not have to consider an interference effect between processes with exchanges of g^* and C^*, $gg \to g^* \to gg$ and $gg \to C^* \to gg$. Group factors in eqs.(A.43), (A.44) and (A.45) are squared for each product of scattering processes,

- $(gg \to g^* \to gg)^2$

$$\frac{1}{(N^2-1)^2} \sum_{a_1,a_2,a_3,a_4}^{N^2-1} \sum_{a,b}^{N^2-1} d^{\,a_1 a_2 a}\, d^{\,a_3 a_4 a}\, d^{\,a_1 a_2 b}\, d^{\,a_3 a_4 b} = \frac{1}{N^2-1} \frac{(N^2-4)^2}{16^2 N^2}\,, \tag{A.47}$$

- $(gg \to C^* \to gg)^2$

$$\frac{1}{(N^2-1)^2} \sum_{a_1,a_2,a_3,a_4}^{N^2-1} \left(d^{\,a_1 a_2 0}\right)^2 \left(d^{\,a_3 a_4 0}\right)^2 = \frac{1}{8^2 N^2}\,. \tag{A.48}$$

- $(gg \to g^* \to gg) \times (gg \to C^* \to gg)$

$$\frac{1}{(N^2-1)^2} \sum_{a_1,a_2,a_3,a_4}^{N^2-1} \sum_{a}^{N^2-1} d^{\,a_1 a_2 0}\, d^{\,a_3 a_4 0}\, d^{\,a_1 a_2 a}\, d^{\,a_3 a_4 a} = \frac{1}{(N^2-1)^2} \frac{1}{8N} \sum_{a_1,a_3,a}^{N^2-1} d^{\,a_1 a_1 a}\, d^{\,a_3 a_3 a}\,, \tag{A.49}$$

where, from the definition of the completely symmetric symbol of eq.(A.23),

$$\sum_{a_1}^{N^2-1} d^{\,a_1 a_1 a} = \sum_{a_1}^{N^2-1} \mathrm{Tr}\big(T^{a_1} T^{a_1} T^{a}\big) = \frac{N^2-1}{2N}\,\mathrm{Tr}(T^a) = 0\,. \tag{A.50}$$

Using parity conservation, the squared two gluon scattering amplitude is given by

$$\begin{aligned} &\big|\mathcal{M}_{1\mathrm{st}}(gg \to gg)\big|^2 \\ &= \frac{8g^4}{M_\mathrm{s}^4} \frac{1}{N^2} \left\{ \frac{(N^2-4)^2}{4(N^2-1)} \left[\frac{M_\mathrm{s}^8}{(s-M_\mathrm{s}^2)^2 + (M_\mathrm{s}\Gamma_{g^*}^{J=0})^2} + \frac{u^4+t^4}{(s-M_\mathrm{s}^2)^2 + (M_\mathrm{s}\Gamma_{g^*}^{J=2})^2} \right] \right. \\ &\qquad \left. + \left[\frac{M_\mathrm{s}^8}{(s-M_\mathrm{s}^2)^2 + (M_\mathrm{s}\Gamma_{C^*}^{J=0})^2} + \frac{u^4+t^4}{(s-M_\mathrm{s}^2)^2 + (M_\mathrm{s}\Gamma_{C^*}^{J=2})^2} \right] \right\}. \end{aligned} \tag{A.51}$$

A.1.2 Second string excited gluon exchanges

Next, let us consider the two gluon scattering process with an exchange of a second string excited gluon. Focusing on the second s-channel pole for $n = 2$ in s-channel pole expansions of eq.(A.11), the string form factor functions are

$$\begin{aligned} V_t &= -\frac{1}{2M_\mathrm{s}^4} \frac{su}{t} \frac{1}{s-2M_\mathrm{s}^2} \big(u + M_\mathrm{s}^2\big)\big(u + 2M_\mathrm{s}^2\big)\,, \\ V_u &= -\frac{1}{2M_\mathrm{s}^4} \frac{st}{u} \frac{1}{s-2M_\mathrm{s}^2} \big(t + M_\mathrm{s}^2\big)\big(t + 2M_\mathrm{s}^2\big)\,, \end{aligned} \tag{A.52}$$

and in the approximation of $s \simeq 2M_\mathrm{s}^2$,

$$V_t \simeq -\frac{u}{s-2M_\mathrm{s}^2} \left(-\frac{u}{s} + \frac{t}{s}\right) \quad , \quad V_u \simeq \frac{t}{s-2M_\mathrm{s}^2} \left(-\frac{u}{s} + \frac{t}{s}\right). \tag{A.53}$$

In this approximation, the two gluon scattering amplitudes with the second string excited gluon exchanges are obtained from eqs.(A.14), (A.15) and (A.16),

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^- \to g_3^-, g_4^-) \simeq 8\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \left(-\frac{u}{s} + \frac{t}{s} \right) {\rm Tr}\big([T^{a_1}, T^{a_2}][T^{a_3}, T^{a_4}] \big) \,, \tag{A.54}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^-, g_4^+) \simeq 8\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \left(-\frac{u}{s} \right)^2 \left(-\frac{u}{s} + \frac{t}{s} \right) {\rm Tr}\big([T^{a_1}, T^{a_2}][T^{a_3}, T^{a_4}] \big) \,, \tag{A.55}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^+, g_4^-) \simeq 8\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \left(-\frac{t}{s} \right)^2 \left(-\frac{u}{s} + \frac{t}{s} \right) {\rm Tr}\big([T^{a_1}, T^{a_2}][T^{a_3}, T^{a_4}] \big) \,. \tag{A.56}$$

Angular dependences in eqs.(A.54), (A.55) and (A.56) are factorized into the Wigner d-functions for $J = 1$, $J = 2$ and $J = 3$,

$$\begin{aligned} d^{\,J=1}_{\,0,0}(\theta) &= -\frac{u}{s} + \frac{t}{s} = \cos\theta \,, \\ d^{\,J=3}_{\,\pm 2, \pm 2}(\theta) &= \left(\frac{1 + \cos\theta}{2} \right)^2 \big(3\cos\theta - 2 \big) \,, \\ d^{\,J=3}_{\,\pm 2, \mp 2}(\theta) &= \left(\frac{1 - \cos\theta}{2} \right)^2 \big(3\cos\theta + 2 \big) \,, \end{aligned} \tag{A.57}$$

and $d^{\,J=2}_{\,\pm 2, \pm 2}(\theta)$ and $d^{\,J=2}_{\,\pm 2, \mp 2}(\theta)$ of eq.(A.22). In the process of $g^- g^- \to g^- g^-$ of eq.(A.54), the second string excited gluon with spin $J = 1$ is exchanged, while in the processes of $g^- g^+ \to g^- g^+$ and $g^- g^+ \to g^+ g^-$, those with spin $J = 2$ and $J = 3$ are exchanged, as follows,

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^- \to g_3^-, g_4^-) \simeq -4\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \, d^{\,J=1}_{\,0,0}(\theta) \sum_a f^{a_1 a_2 a} f^{a_3 a_4 a} \,, \tag{A.58}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^-, g_4^+) \simeq -4\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \left[\frac{2}{3} d^{\,J=2}_{\,-2,-2}(\theta) + \frac{1}{3} d^{\,J=3}_{\,-2,-2}(\theta) \right] \sum_a f^{a_1 a_1 a} f^{a_3 a_4 a} \,, \tag{A.59}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^+, g_4^-) \simeq -4\, g^2 M_{\rm s}^2 \, \frac{1}{s - 2M_{\rm s}^2} \left[-\frac{2}{3} d^{\,J=2}_{\,-2,+2}(\theta) + \frac{1}{3} d^{\,J=3}_{\,-2,+2}(\theta) \right] \sum_a f^{a_1 a_2 a} f^{a_3 a_4 a} \,. \tag{A.60}$$

Note that there are excitations of $SU(3)_C$ gluons only, as the second string excited gluons exchanged in these processes. That is because the completely anti-symmetric symbol with $U(1)$ component, $f^{a_1 a_2 0}$, vanish. The completely anti-symmetric symbol follows that $i f^{a_1 a_2 a_3} = 2{\rm Tr}\big([T^{a_1}, T^{a_2}] T^{a_3} \big)$.

widths of second excited gluons Comparing eqs.(A.54), (A.55) and (A.56) to eq.(A.32),

$$\begin{aligned} F_{g^{**}}{}^{J=1\,a}_{\pm1,\pm1;\,a_3,a_4} &= \quad 2\,gM_{\rm s}\,f^{a_3a_4a}\,, \\ F_{g^{**}}{}^{J=2\,a}_{\pm1,\mp1;\,a_3,a_4} &= \pm\frac{2\sqrt{2}}{\sqrt{3}}\,gM_{\rm s}\,f^{a_3a_4a}\,, \\ F_{g^{**}}{}^{J=3\,a}_{\pm1,\mp1;\,a_3,a_4} &= \quad \frac{2}{\sqrt{3}}\,gM_{\rm s}\,f^{a_3a_4a}\,. \end{aligned} \tag{A.61}$$

Decay widths of $g^{**} \to gg$ for $J=1$, $J=2$ and $J=3$ are

$$\begin{aligned} \Gamma^{J=1\,a}_{g^{**}\to gg} &= \frac{1}{2}\,\frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times1+1}\sum_{a_3,a_4}\frac{1}{2}\left\{\left|F_{g^{**}}{}^{J=1\,a}_{+1,+1;\,a_3,a_4}\right|^2+\left|F_{g^{**}}{}^{J=1\,a}_{-1,-1;\,a_3,a_4}\right|^2\right\} \\ &= \frac{g^2\sqrt{2}M_{\rm s}}{16\pi}\,\frac{1}{3}\sum_{a_3,a_4} f^{a_3a_4a}\,f^{a_3a_4a}\,, \end{aligned} \tag{A.62}$$

$$\begin{aligned} \Gamma^{J=2\,a}_{g^{**}\to gg} &= \frac{1}{2}\,\frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times2+1}\sum_{a_3,a_4}\left\{\left|F_{g^{**}}{}^{J=2\,a}_{+1,-1;\,a_3,a_4}\right|^2+\left|F_{g^{**}}{}^{J=2\,a}_{-1,+1;\,a_3,a_4}\right|^2\right\} \\ &= \frac{g^2\sqrt{2}M_{\rm s}}{12\pi}\,\frac{1}{5}\sum_{a_3,a_4} f^{a_3a_4a}\,f^{a_3a_4a}\,, \end{aligned} \tag{A.63}$$

$$\begin{aligned} \Gamma^{J=3\,a}_{g^{**}\to gg} &= \frac{1}{2}\,\frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times3+1}\sum_{a_3,a_4}\left\{\left|F_{g^{**}}{}^{J=3\,a}_{+1,-1;\,a_3,a_4}\right|^2+\left|F_{g^{**}}{}^{J=3\,a}_{-1,+1;\,a_3,a_4}\right|^2\right\} \\ &= \frac{g^2\sqrt{2}M_{\rm s}}{24\pi}\,\frac{1}{7}\sum_{a_3,a_4} f^{a_3a_4a}\,f^{a_3a_4a}\,, \end{aligned} \tag{A.64}$$

where a group factor is averaged over an initial gauge index,

$$\frac{1}{N^2-1}\sum_{a}^{N^2-1}\sum_{a_3,a_4}^{N^2-1} f^{a_3a_4a}\,f^{a_3a_4a} = N\,. \tag{A.65}$$

Widths of the second string excited gluon g^{**} with $J=1$, $J=2$ and $J=3$ for the process of $g^{**}\to gg$ are

$$\Gamma^{J=1}_{g^{**}\to gg} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{3}\,\frac{N}{4} \quad , \quad \Gamma^{J=2}_{g^{**}\to gg} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{5}\,\frac{N}{3} \quad , \quad \Gamma^{J=3}_{g^{**}\to gg} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{7}\,\frac{N}{6}\,. \tag{A.66}$$

squared amplitudes with second excited gluon exchanges The amplitudes of eqs.(A.58), (A.59) and (A.60) are rewritten by softening the s-channel pole into the Breit-Wigner form with the widths,

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^- \to g_3^-, g_4^-) \to -4\, g^2 M_{\rm s}^2 \, \frac{1}{2M_{\rm s}^2} \, \frac{-u+t}{s - 2M_{\rm s}^2 + i\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=1}} \, \sum_a f^{a_1 a_2 a} \, f^{a_3 a_4 a} \,, \quad \text{(A.67)}$$

$$\begin{aligned}
&\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^-, g_4^+) \\
&\to -4\, g^2 M_{\rm s}^2 \, \frac{1}{8M_{\rm s}^6} \left[\, \frac{2}{3} \, \frac{2M_{\rm s}^2\, u^2}{s - 2M_{\rm s}^2 + i\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=2}} + \frac{1}{3} \, \frac{u^2(-u+5t)}{s - 2M_{\rm s}^2 + i\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=3}} \right] \sum_a f^{a_1 a_2 a} \, f^{a_3 a_4 a}
\end{aligned} \quad \text{(A.68)}$$

$$\begin{aligned}
&\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to g_3^+, g_4^-) \\
&\to -4\, g^2 M_{\rm s}^2 \, \frac{1}{8M_{\rm s}^6} \left[-\frac{2}{3} \, \frac{2M_{\rm s}^2\, t^2}{s - 2M_{\rm s}^2 + i\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=2}} + \frac{1}{3} \, \frac{-t^2(-t+5u)}{s - 2M_{\rm s}^2 + i\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=3}} \right] \sum_a f^{a_1 a_2 a} \, f^{a_3 a_4 a} \,,
\end{aligned} \quad \text{(A.69)}$$

since in $d^{\,J=3}_{\pm 2, \pm 2}(\theta)$ and $d^{\,J=3}_{\pm 2, \mp 2}(\theta)$ of eq.(A.57), $3\cos\theta - 2 = (-u+5t)/s$ and $3\cos\theta + 2 = (-t+5u)/s$, respectively.

A squared amplitude of the two gluon scattering process with the second excited gluon exchange is calculated as well as eq.(A.46). Since the group factor in eqs.(A.54), (A.55) and (A.56) is squared,

$$\frac{1}{(N^2-1)^2} \sum_{a_1,a_2,a_3,a_4}^{N^2-1} \sum_{a,b}^{N^2-1} f^{a_1 a_2 a} \, f^{a_3 a_4 a} \, f^{a_1 a_2 b} \, f^{a_3 a_4 b} = \frac{N^2}{N^2-1} \,, \quad \text{(A.70)}$$

the squared amplitude is given by

$$\big|\mathcal{M}_{\rm 2nd}(gg \to gg)\big|^2 = \big|\mathcal{M}_{\rm 2nd}(gg \to gg)\big|^2_{\rm non-int.} + \big|\mathcal{M}_{\rm 2nd}(gg \to gg)\big|^2_{\rm int.} \,, \quad \text{(A.71)}$$

where

$$\begin{aligned}
&\big|\mathcal{M}_{\rm 2nd}(gg \to gg)\big|^2_{\rm non-int.} \\
&= \frac{g^4}{8M_{\rm s}^8} \, \frac{N^2}{N^2-1} \left\{ \frac{16M_{\rm s}^8(-u+t)^2}{(s-2M_{\rm s}^2)^2 + (\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=1})^2} \right. \\
&\qquad \left. + \left[\frac{4}{9} \, \frac{4M_{\rm s}^4(u^4+t^4)}{(s-2M_{\rm s}^2)^2 + (\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=2})^2} + \frac{1}{9} \, \frac{u^4(-u+5t)^2 + t^4(-t+5u)^2}{(s-2M_{\rm s}^2)^2 + (\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=3})^2} \right] \right\} ,
\end{aligned} \quad \text{(A.72)}$$

$$\begin{aligned}
&\big|\mathcal{M}_{\rm 2nd}(gg \to gg)\big|^2_{\rm int.} \\
&= \frac{g^4}{4M_{\rm s}^8} \, \frac{N^2}{N^2-1} \left[\frac{2}{9} \, \frac{(s-2M_{\rm s}^2)^2 + 2M_{\rm s}^2\Gamma_{g^{**}}^{J=2}\Gamma_{g^{**}}^{J=3}}{(s-2M_{\rm s}^2)^2 + (\sqrt{2}M_{\rm s}\Gamma_{g^{**}}^{J=2})^2} \, \frac{2M_{\rm s}^2\left(u^4(-u+5t) + t^4(-t+5u)\right)}{(s-2M_{\rm s}^2)^2 + (\sqrt{2}M_{\rm s}^2\Gamma_{g^{**}}^{J=3})^2} \right] .
\end{aligned} \quad \text{(A.73)}$$

The last term of eq.(A.73) is an interference effect between processes with exchanges of the second string excited gluons with $J=2$ and $J=3$.

A.2 Two gluons and two quarks amplitudes

A.2.1 $qg \to qg$ amplitudes

The open string amplitudes with external gluons and quarks at disk-level are calculated in ref.[13] as

$$\mathcal{M}(q_1^-, g_2^-, \overline{q}_3^+, g_4^+) = 2g^2 \frac{\langle 12\rangle^2}{\langle 14\rangle\langle 34\rangle}\left[\frac{s}{t}V_s\left(T^{a_2}T^{a_4}\right)_{\alpha_3\alpha_1} + \frac{u}{t}V_u\left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}\right], \tag{A.74}$$

$$\mathcal{M}(q_1^-, g_2^+, \overline{q}_3^+, g_4^-) = 2g^2 \frac{\langle 14\rangle^2}{\langle 12\rangle\langle 32\rangle}\left[\frac{u}{t}V_u\left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1} + \frac{s}{t}V_s\left(T^{a_2}T^{a_4}\right)_{\alpha_3\alpha_1}\right], \tag{A.75}$$

where q_1^-, $g_2^\mp$, $\overline{q}_3^+$ and $g_4^\pm$ in eqs.(A.74) and (A.75) are incoming gluons and quarks. The latter of eq.(A.75) is obtained by a replacement of $2 \leftrightarrow 4$ in eq.(A.74). Amplitudes with replacements of $3 \leftrightarrow 4$ in eqs.(A.74) and (A.75) can also be obtained,

$$\mathcal{M}(q_1^-, g_2^-, g_3^+, \overline{q}_4^+) = 2g^2 \frac{\langle 12\rangle^2}{\langle 13\rangle\langle 43\rangle}\left[\frac{s}{u}V_s\left(T^{a_2}T^{a_3}\right)_{\alpha_4\alpha_1} + \frac{t}{u}V_t\left(T^{a_3}T^{a_2}\right)_{\alpha_4\alpha_1}\right], \tag{A.76}$$

$$\mathcal{M}(q_1^-, g_2^+, g_3^-, \overline{q}_4^+) = 2g^2 \frac{\langle 13\rangle^2}{\langle 12\rangle\langle 42\rangle}\left[\frac{t}{u}V_t\left(T^{a_3}T^{a_2}\right)_{\alpha_4\alpha_1} + \frac{s}{u}V_s\left(T^{a_2}T^{a_3}\right)_{\alpha_4\alpha_1}\right], \tag{A.77}$$

Here, similarly to the previous section, we consider two-body scattering processes of quark and gluon, $q_1, g_2 \to q_3, g_4$ or $q_1, g_2 \to g_3, q_4$ (1, 2 refer to incoming particles, while 3, 4 refer to outgoing ones). In these processes, kinetic factors of eqs.(A.74), (A.75), (A.76) and (A.77) are

$$\begin{aligned} \frac{\langle 12\rangle^2}{\langle 14\rangle\langle 34\rangle} &= \sqrt{-\frac{s}{u}} \quad , \quad \frac{\langle 14\rangle^2}{\langle 12\rangle\langle 32\rangle} = \sqrt{-\frac{u}{s}}\,, \\ \frac{\langle 12\rangle^2}{\langle 13\rangle\langle 43\rangle} &= -\sqrt{-\frac{s}{t}} \quad , \quad \frac{\langle 13\rangle^2}{\langle 12\rangle\langle 42\rangle} = -\sqrt{-\frac{t}{s}}\,. \end{aligned} \tag{A.78}$$

first string excited quark exchanges The quark and gluon scattering process exchanges a string excited quark in s-channel. First, we focus on the first s-channel pole, that is a first string excited quark. Using the string form factor functions in the limit of $s \simeq M_s^2$ of eq.(A.18),

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, g_2^- \to q_3^-, g_4^-) \simeq -2g^2 M_s^2 \frac{1}{s - M_s^2}\sqrt{-\frac{u}{s}} \quad \left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}, \tag{A.79}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, g_2^+ \to q_3^-, g_4^+) \simeq -2g^2 M_s^2 \frac{1}{s - M_s^2}\left(-\frac{u}{s}\right)^{\frac{3}{2}}\left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}. \tag{A.80}$$

Similarly, amplitudes obtained by replacements of $t \leftrightarrow u$ are

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, g_2^- \to g_3^-, q_4^-) \simeq +2g^2 M_s^2 \frac{1}{s - M_s^2}\sqrt{-\frac{t}{s}} \quad \left(T^{a_3}T^{a_2}\right)_{\alpha_4\alpha_1}, \tag{A.81}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, g_2^+ \to g_3^+, q_4^-) \simeq +2g^2 M_s^2 \frac{1}{s - M_s^2}\left(-\frac{t}{s}\right)^{\frac{3}{2}}\left(T^{a_2}T^{a_3}\right)_{\alpha_4\alpha_1}. \tag{A.82}$$

Angular dependences in eqs.(A.79), (A.80), (A.81) and (A.82) can be decomposed into the Wigner d-functions of $J = 1/2$ and $J = 3/2$,

$$\begin{aligned} d^{\,J=1/2}_{\pm1/2,\,\pm1/2}(\theta) &= \quad\sqrt{-\frac{u}{s}} = \quad \cos\frac{\theta}{2}\,, \\ d^{\,J=1/2}_{\pm1/2,\,\mp1/2}(\theta) &= \mp\sqrt{-\frac{t}{s}} = \mp\sin\frac{\theta}{2}\,, \\ d^{\,J=3/2}_{\pm3/2,\,\pm3/2}(\theta) &= \quad\left(-\frac{u}{s}\right)^{\frac{3}{2}} = \quad \cos^3\frac{\theta}{2}\,, \\ d^{\,J=3/2}_{\pm3/2,\,\mp3/2}(\theta) &= \mp\left(-\frac{t}{s}\right)^{\frac{3}{2}} = \mp\sin^3\frac{\theta}{2}\,. \end{aligned} \tag{A.83}$$

It is found that in the processes of $q_1, g_2 \to q_3, g_4$ or $q_1, g_2 \to g_3, q_4$, the first string excited quark with $J = 1/2$ and $J = 3/2$ are exchanged. The amplitudes with an exchange of the first string excited quark can be rewritten as,

$$\mathcal{M}_{\rm 1st}(q_1^-, g_2^- \to q_3^-, g_4^-) \simeq -2g^2 M_{\rm s}^2\, \frac{1}{s - M_{\rm s}^2}\, d^{\,J=1/2}_{+1/2,\,+1/2}(\theta) \sum_\alpha \left(T^{a_4}\right)_{\alpha_3\alpha} \left(T^{a_2}\right)_{\alpha\alpha_1}\,, \tag{A.84}$$

$$\mathcal{M}_{\rm 1st}(q_1^-, g_2^+ \to q_3^-, g_4^+) \simeq -2g^2 M_{\rm s}^2\, \frac{1}{s - M_{\rm s}^2}\, d^{\,J=3/2}_{-3/2,\,-3/2}(\theta) \sum_\alpha \left(T^{a_4}\right)_{\alpha_3\alpha} \left(T^{a_2}\right)_{\alpha\alpha_1}\,, \tag{A.85}$$

$$\mathcal{M}_{\rm 1st}(q_1^-, g_2^- \to g_3^-, q_4^-) \simeq -2g^2 M_{\rm s}^2\, \frac{1}{s - M_{\rm s}^2}\, d^{\,J=1/2}_{+1/2,\,-1/2}(\theta) \sum_\alpha \left(T^{a_3}\right)_{\alpha_4\alpha} \left(T^{a_2}\right)_{\alpha\alpha_1}\,, \tag{A.86}$$

$$\mathcal{M}_{\rm 1st}(q_1^-, g_2^+ \to g_3^+, q_4^-) \simeq +2g^2 M_{\rm s}^2\, \frac{1}{s - M_{\rm s}^2}\, d^{\,J=3/2}_{-3/2,\,+3/2}(\theta) \sum_\alpha \left(T^{a_3}\right)_{\alpha_4\alpha} \left(T^{a_2}\right)_{\alpha\alpha_1}\,, \tag{A.87}$$

widths of first string excited quarks Then matrix elements of the first string excited quark q^*, $F_{q^*}{}^{J\,\alpha}_{\lambda_i,\lambda_j;\,\alpha_i,a_j}$, can be extracted by comparing to eq.(A.32),

$$\begin{aligned} F_{q^*}{}^{J=1/2}_{\pm1/2,\pm1;\,\alpha_3,A_4} = F_{q^*}{}^{J=3/2}_{\pm1/2,\mp1;\,\alpha_3,A_4} &= \quad\sqrt{2}\,gM_{\rm s}\left(T^{A_4}\right)_{\alpha\alpha_3}\,, \\ F_{q^*}{}^{J=1/2}_{\pm1,\pm1/2;\,A_3,\alpha_4} = F_{q^*}{}^{J=3/2}_{\pm1,\mp1/2;\,A_3,\alpha_4} &= \mp\sqrt{2}\,gM_{\rm s}\left(T^{A_3}\right)_{\alpha\alpha_4}\,. \end{aligned} \tag{A.88}$$

Using the above results, widths of $q^* \to qG$ for $J = 1/2$ and $J = 3/2$ are

$$\begin{aligned} \Gamma^{J=1/2\,\alpha}_{q^*\to qG} &= \frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times\frac{1}{2}+1} \sum_{\alpha_3,A_4} \frac{1}{2}\left\{\left|F_{q^*}{}^{J=1/2\,\alpha}_{+1/2,+1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^*}{}^{J=1/2\,\alpha}_{-1/2,-1;\,\alpha_3,A_4}\right|^2\right\} \\ &= \frac{g^2 M_{\rm s}}{8\pi}\,\frac{1}{2} \sum_{\alpha_3,A_4} \left(T^{A_4}\right)_{\alpha_3\alpha} \left(T^{A_4}\right)_{\alpha\alpha_3}\,, \end{aligned} \tag{A.89}$$

$$\begin{aligned} \Gamma^{J=3/2\,\alpha}_{q^*\to qG} &= \frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times\frac{3}{2}+1} \sum_{\alpha_3,A_4} \frac{1}{2}\left\{\left|F_{q^*}{}^{J=3/2\,\alpha}_{+1/2,-1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^*}{}^{J=3/2\,\alpha}_{-1/2,+1;\,\alpha_3,A_4}\right|^2\right\} \\ &= \frac{g^2 M_{\rm s}}{8\pi}\,\frac{1}{4} \sum_{\alpha_3,A_4} \left(T^{A_4}\right)_{\alpha_3\alpha} \left(T^{A_4}\right)_{\alpha\alpha_3}\,, \end{aligned} \tag{A.90}$$

A group factor eqs.(A.89) and (A.90) is averaged over a gauge index α of q^* for each decay process into g or C,

- $q^* \to qg$

$$\frac{1}{N}\sum_{\alpha,\alpha_3}^{N}\sum_{a_4}^{N^2-1}\left(T^{a_4}\right)_{\alpha_3\alpha}\left(T^{a_4}\right)_{\alpha\alpha_3} = \frac{1}{N}\sum_{a_4}^{N^2-1}\mathrm{Tr}\left(T^{a_4}T^{a_4}\right) = \frac{N^2-1}{2N}\,, \tag{A.91}$$

- $q^* \to qC$

$$\frac{1}{N}\sum_{\alpha,\alpha_3}^{N}\left(T^{0}\right)_{\alpha_3\alpha}\left(T^{0}\right)_{\alpha\alpha_3} = \frac{1}{N}\,\mathrm{Tr}\left(T^{0}T^{0}\right) = \frac{1}{2N}\,. \tag{A.92}$$

Total widths of the first string excited quarks q^* with $J = 1/2$ and $J = 3/2$ are

$$\Gamma_{q^*}^{J=1/2} = \frac{g^2 M_{\mathrm{s}}}{4\pi}\,\frac{1}{2}\,\frac{N}{4} \quad , \quad \Gamma_{q^*}^{J=3/2} = \frac{g^2 M_{\mathrm{s}}}{4\pi}\,\frac{1}{4}\,\frac{N}{4}\,. \tag{A.93}$$

squared amplitudes with first string excited quark exchanges A squared amplitude of the process of $q_1, g_2 \to q_3, g_4$ is calculated,

$$\begin{aligned} &\left|\mathcal{M}_{\mathrm{1st}}(q_1, g_2 \to q_3, g_4)\right|^2 \\ =&\frac{1}{2^2}\,\frac{1}{N(N^2-1)}\sum_{\alpha_1,a_2,\alpha_3,a_4}\Big\{\left|\mathcal{M}_{\mathrm{1st}}(q_1^-, g_2^- \to q_3^-, g_4^-)\right|^2 + \left|\mathcal{M}_{\mathrm{1st}}(q_1^+, g_2^+ \to q_3^+, g_4^+)\right|^2 \\ &\qquad + \left|\mathcal{M}_{\mathrm{1st}}(q_1^-, g_2^+ \to q_3^-, g_4^+)\right|^2 + \left|\mathcal{M}_{\mathrm{1st}}(q_1^+, g_2^- \to q_3^+, g_4^-)\right|^2\Big\}\,, \end{aligned} \tag{A.94}$$

and that of the process of $q_1, g_2 \to g_3, q_4$ is similar. Since the group factor of eqs.(A.84), (A.85), (A.86) and (A.87) is squared as follows,

$$\frac{1}{N(N^2-1)}\sum_{\alpha_1,\alpha_3}^{N}\sum_{a_2,a_4}^{N^2-1}\left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}\left(T^{a_2}T^{a_4}\right)_{\alpha_1\alpha_3} = \frac{N^2-1}{4N^2}\,, \tag{A.95}$$

the squared amplitudes of the processes become

$$\left|\mathcal{M}_{\mathrm{1st}}(qg \to qg)\right|^2 = \frac{2g^4}{M_{\mathrm{s}}^2}\,\frac{N^2-1}{4N^2}\left[\frac{M_{\mathrm{s}}^4(-u)}{(s-M_{\mathrm{s}}^2)^2 + (M_{\mathrm{s}}\Gamma_{q^*}^{J=1/2})^2} + \frac{(-u)^3}{(s-M_{\mathrm{s}}^2)^2 + (M_{\mathrm{s}}\Gamma_{q^*}^{J=3/2})^2}\right], \tag{A.96}$$

$$\left|\mathcal{M}_{\mathrm{1st}}(qg \to gq)\right|^2 = \frac{2g^4}{M_{\mathrm{s}}^2}\,\frac{N^2-1}{4N^2}\left[\frac{M_{\mathrm{s}}^4(-t)}{(s-M_{\mathrm{s}}^2)^2 + (M_{\mathrm{s}}\Gamma_{q^*}^{J=1/2})^2} + \frac{(-t)^3}{(s-M_{\mathrm{s}}^2)^2 + (M_{\mathrm{s}}\Gamma_{q^*}^{J=3/2})^2}\right]. \tag{A.97}$$

second string excited quark exchanges Next, let us consider the quark and gluon scattering amplitude with an exchange of a second string excited quark. Inserting the string form factor functions in the limit of $s \simeq 2M_s^2$ of eq.(A.53) into the amplitudes,

$$\mathcal{M}_{2nd}(q_1^-, g_2^- \to q_3^-, g_4^-) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2} \sqrt{-\frac{u}{s}} \left(-\frac{u}{s}+\frac{t}{s}\right) \left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}, \tag{A.98}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^+ \to q_3^-, g_4^+) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2} \left(-\frac{u}{s}\right)^{\frac{3}{2}} \left(-\frac{u}{s}+\frac{t}{s}\right) \left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1}, \tag{A.99}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^- \to g_3^-, q_4^-) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2} \sqrt{-\frac{t}{s}} \left(-\frac{u}{s}+\frac{t}{s}\right) \left(T^{a_3}T^{a_2}\right)_{\alpha_4\alpha_1}, \tag{A.100}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^+ \to g_3^+, q_4^-) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2} \left(-\frac{t}{s}\right)^{\frac{3}{2}} \left(-\frac{u}{s}+\frac{t}{s}\right) \left(T^{a_2}T^{a_3}\right)_{\alpha_4\alpha_1}. \tag{A.101}$$

Angular dependences on eqs.(A.98), (A.99), (A.100) and (A.101) are decomposed into the Wigner d-functions,

$$\begin{aligned} d^{J=3/2}_{\pm 1/2,\pm 1/2}(\theta) &= \cos\frac{\theta}{2}\left(\frac{3\cos\theta-1}{2}\right), \\ d^{J=3/2}_{\pm 1/2,\mp 1/2}(\theta) &= \mp\sin\frac{\theta}{2}\left(\frac{3\cos\theta+1}{2}\right), \\ d^{J=5/2}_{\pm 3/2,\pm 3/2}(\theta) &= \cos^3\frac{\theta}{2}\left(\frac{5\cos\theta-3}{2}\right), \\ d^{J=5/2}_{\pm 3/2,\mp 3/2}(\theta) &= \mp\sin^3\frac{\theta}{2}\left(\frac{5\cos\theta+3}{2}\right), \end{aligned} \tag{A.102}$$

and $d^{J=1/2}_{\pm 1/2,\pm 1/2}(\theta)$ and $d^{J=3/2}_{\pm 3/2,\pm 3/2}(\theta)$ of eq.(A.83). Thus, the second string excited quarks which are exchanged in the processes have $J=1/2$, $J=3/2$ and $J=5/2$, as follows

$$\mathcal{M}_{2nd}(q_1^-, g_2^- \to q_3^-, g_4^-) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2}\left[\frac{1}{3}d^{J=1/2}_{+1/2,+1/2}(\theta)+\frac{2}{3}d^{J=3/2}_{+1/2,+1/2}(\theta)\right]\sum_\alpha \left(T^{a_4}\right)_{\alpha_3\alpha}\left(T^{a_2}\right)_{\alpha\alpha_1}, \tag{A.103}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^+ \to q_3^-, g_4^+) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2}\left[\frac{3}{5}d^{J=3/2}_{-3/2,-3/2}(\theta)+\frac{2}{5}d^{J=5/2}_{-3/2,-3/2}(\theta)\right]\sum_\alpha \left(T^{a_4}\right)_{\alpha_3\alpha}\left(T^{a_2}\right)_{\alpha\alpha_1}, \tag{A.104}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^- \to g_3^-, q_4^-) \simeq +4g^2 M_s^2 \frac{1}{s-2M_s^2}\left[-\frac{1}{3}d^{J=1/2}_{+1/2,-1/2}(\theta)+\frac{2}{3}d^{J=3/2}_{+1/2,-1/2}(\theta)\right]\sum_\alpha \left(T^{a_3}\right)_{\alpha_4\alpha}\left(T^{a_2}\right)_{\alpha\alpha_1}, \tag{A.105}$$

$$\mathcal{M}_{2nd}(q_1^-, g_2^+ \to g_3^+, q_4^-) \simeq -4g^2 M_s^2 \frac{1}{s-2M_s^2}\left[-\frac{3}{5}d^{J=3/2}_{-3/2,+3/2}(\theta)+\frac{2}{5}d^{J=5/2}_{-3/2,+3/2}(\theta)\right]\sum_\alpha \left(T^{a_3}\right)_{\alpha_4\alpha}\left(T^{a_2}\right)_{\alpha\alpha_1}, \tag{A.106}$$

widths of second string excited quarks Matrix elements of the second string excited quark q^{**}, $F_{q^{**}}{}^{J\,\alpha}_{\lambda_i,\lambda_j;\,\alpha_i,a_j}$, can be extracted as

$$\begin{aligned}
F_{q^{**}}{}^{J=1/2\,\alpha}_{\pm1/2,\pm1;\,\alpha_3,A_4} &= \frac{2}{\sqrt{3}}\,gM_{\rm s}\left(T^{A_4}\right)_{\alpha\alpha_3} \quad , \quad F_{q^{**}}{}^{J=1/2\,\alpha}_{\pm1,\pm1/2;\,A_3,\alpha_4} = \mp\frac{2}{\sqrt{3}}\,gM_{\rm s}\left(T^{A_3}\right)_{\alpha\alpha_4} \,, \\
F_{q^{**}}{}^{J=3/2\,\alpha}_{\pm1/2,\pm1;\,\alpha_3,A_4} &= \frac{2\sqrt{2}}{\sqrt{3}}\,gM_{\rm s}\left(T^{A_4}\right)_{\alpha\alpha_3} \quad , \quad F_{q^{**}}{}^{J=3/2\,\alpha}_{\pm1,\pm1/2;\,A_3,\alpha_4} = \pm\frac{2\sqrt{2}}{\sqrt{3}}\,gM_{\rm s}\left(T^{A_3}\right)_{\alpha\alpha_4} \,, \\
F_{q^{**}}{}^{J=3/2\,\alpha}_{\pm1/2,\mp1;\,\alpha_3,A_4} &= \frac{2\sqrt{3}}{\sqrt{5}}\,gM_{\rm s}\left(T^{A_4}\right)_{\alpha\alpha_3} \quad , \quad F_{q^{**}}{}^{J=3/2\,\alpha}_{\pm1,\mp1/2;\,A_3,\alpha_4} = \mp\frac{2\sqrt{3}}{\sqrt{5}}\,gM_{\rm s}\left(T^{A_3}\right)_{\alpha\alpha_4} \,, \\
F_{q^{**}}{}^{J=5/2\,\alpha}_{\pm1/2,\mp1;\,\alpha_3,A_4} &= \frac{2\sqrt{2}}{\sqrt{5}}\,gM_{\rm s}\left(T^{A_4}\right)_{\alpha\alpha_3} \quad , \quad F_{q^{**}}{}^{J=5/2\,\alpha}_{\pm1,\mp1/2;\,A_3,\alpha_4} = \pm\frac{2\sqrt{2}}{\sqrt{5}}\,gM_{\rm s}\left(T^{A_3}\right)_{\alpha\alpha_4} \,.
\end{aligned} \tag{A.107}$$

Then widths of $q^{**} \to qG$ for $J = 1/2$, $J = 3/2$ and $J = 5/2$ are

$$\begin{aligned}
\Gamma^{J=1/2\,\alpha}_{q^{**}\to qG} &= \frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times\frac{1}{2}+1}\sum_{\alpha_3,A_4}\frac{1}{2}\left\{\left|F_{q^{**}}{}^{J=1/2\,\alpha}_{+1/2,+1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^{**}}{}^{J=1/2\,\alpha}_{-1/2,-1;\,\alpha_3,A_4}\right|^2\right\} \\
&= \frac{g^2\sqrt{2}M_{\rm s}}{24\pi}\,\frac{1}{2}\sum_{\alpha_3,A_4}\left(T^{A_4}\right)_{\alpha_3\alpha}\left(T^{A_4}\right)_{\alpha\alpha_3} \,,
\end{aligned} \tag{A.108}$$

$$\begin{aligned}
\Gamma^{J=3/2\,\alpha}_{q^{**}\to qG} &= \frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times\frac{3}{2}+1}\sum_{\alpha_3,A_4}\frac{1}{2}\left\{\left|F_{q^{**}}{}^{J=3/2\,\alpha}_{+1/2,+1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^{**}}{}^{J=3/2\,\alpha}_{-1/2,-1;\,\alpha_3,A_4}\right|^2\right. \\
&\qquad\qquad \left. + \left|F_{q^{**}}{}^{J=3/2\,\alpha}_{+1/2,-1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^{**}}{}^{J=3/2\,\alpha}_{-1/2,+1;\,\alpha_3,A_4}\right|^2\right\} \\
&= \frac{g^2\sqrt{2}M_{\rm s}}{8\pi}\left(\frac{2}{3}+\frac{3}{5}\right)\frac{1}{4}\sum_{\alpha_3,A_4}\left(T^{A_4}\right)_{\alpha_3\alpha}\left(T^{A_4}\right)_{\alpha\alpha_3} \,,
\end{aligned} \tag{A.109}$$

$$\begin{aligned}
\Gamma^{J=5/2\,\alpha}_{q^{**}\to qG} &= \frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times\frac{5}{2}+1}\sum_{\alpha_3,A_4}\frac{1}{2}\left\{\left|F_{q^{**}}{}^{J=5/2\,\alpha}_{+1/2,-1;\,\alpha_3,A_4}\right|^2 + \left|F_{q^{**}}{}^{J=5/2\,\alpha}_{-1/2,+1;\,\alpha_3,A_4}\right|^2\right\} \\
&= \frac{g^2\sqrt{2}M_{\rm s}}{20\pi}\,\frac{1}{6}\sum_{\alpha_3,A_4}\left(T^{A_4}\right)_{\alpha_3\alpha}\left(T^{A_4}\right)_{\alpha\alpha_3} \,.
\end{aligned} \tag{A.110}$$

Since a group factor of eqs.(A.108), (A.109) and (A.110) is in eqs.(A.91) and (A.92) for each decay process, total widths of the second string excited quarks q^{**} with $J = 1/2$, $J = 3/2$ and $J = 5/2$ are

$$\Gamma^{J=1/2}_{q^{**}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{2}\,\frac{N}{12} \quad , \quad \Gamma^{J=3/2}_{q^{**}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{4}\,\frac{19N}{60} \quad , \quad \Gamma^{J=5/2}_{q^{**}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{6}\,\frac{N}{10} \,. \tag{A.111}$$

squared amplitudes with second string excited quark exchanges Softening the s-channel pole into the Breit-Wigner form, the amplitudes are rewritten as

$$\begin{aligned}
&\mathcal{M}_{\rm 2nd}(q_1^-, g_2^- \to q_3^-, g_4^-) \\
&\to -4g^2M_{\rm s}^2\,\frac{1}{2\sqrt{2}M_{\rm s}^3}\left[\,\frac{1}{3}\,\frac{2M_{\rm s}^2\sqrt{-u}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=1/2}_{q^{**}}} + \frac{2}{3}\,\frac{\sqrt{-u}(-u+2t)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}}}\right]\left(T^{a_4}T^{a_2}\right)_{\alpha_3\alpha_1} \,,
\end{aligned} \tag{A.112}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-, g_2^+ \to q_3^-, g_4^+)$$
$$\to - 4g^2 M_{\rm s}^2 \, \frac{1}{4\sqrt{2}M_{\rm s}^5}\left[\;\frac{3}{5}\,\frac{2M_{\rm s}^2(-u)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2}}+\frac{2}{5}\,\frac{(-u)^{3/2}(-u+4t)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=5/2}}\right]\big(T^{a_4}T^{a_2}\big)_{\alpha_3\alpha_1}\,, \tag{A.113}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-, g_2^- \to g_3^-, q_4^-)$$
$$\to + 4g^2 M_{\rm s}^2 \, \frac{1}{2\sqrt{2}M_{\rm s}^3}\left[-\frac{1}{3}\,\frac{-2M_{\rm s}^2\sqrt{-t}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=1/2}}+\frac{2}{3}\,\frac{\sqrt{-t}(-t+2u)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2}}\right]\big(T^{a_4}T^{a_2}\big)_{\alpha_3\alpha_1}\,, \tag{A.114}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-, g_2^+ \to g_3^+, q_4^-)$$
$$\to - 4g^2 M_{\rm s}^2 \, \frac{1}{4\sqrt{2}M_{\rm s}^5}\left[-\frac{3}{5}\,\frac{2M_{\rm s}^2(-t)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2}}+\frac{2}{5}\,\frac{-(-t)^{3/2}(-t+4u)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=5/2}}\right]\big(T^{a_4}T^{a_2}\big)_{\alpha_3\alpha_1}\,, \tag{A.115}$$

since in $d^{\,J=3/2}_{\pm1/2,\pm1/2}(\theta)$ and $d^{\,J=3/2}_{\pm3/2,\mp3/2}(\theta)$ of eq.(A.102), $(3\cos\theta-1)/2 = (-u+2t)/s$ and $(3\cos\theta+1)/2 = -(-t+2u)/s$, while in $d^{\,J=5/2}_{\pm3/2,\pm3/2}(\theta)$ and $d^{\,J=5/2}_{\pm3/2,\mp3/2}(\theta)$, $(5\cos\theta-3)/2 = (-u+4t)/s$ and $(5\cos\theta+3)/2 = -(-t+4u)/s$.

A squared amplitudes of the processes of $qg\to qg$ and $qg\to gq$ are

$$\begin{aligned}\big|\mathcal{M}_{\rm 2nd}(qg\to qg)\big|^2 &= \big|\mathcal{M}_{\rm 2nd}(qg\to qg)\big|^2_{\rm non-int.}+\big|\mathcal{M}_{\rm 2nd}(qg\to qg)\big|^2_{\rm int.}\,,\\ \big|\mathcal{M}_{\rm 2nd}(qg\to gq)\big|^2 &= \big|\mathcal{M}_{\rm 2nd}(qg\to gq)\big|^2_{\rm non-int.}+\big|\mathcal{M}_{\rm 2nd}(qg\to gq)\big|^2_{\rm int.}\,,\end{aligned} \tag{A.116}$$

where

$$\begin{aligned}&\big|\mathcal{M}_{\rm 2nd}(qg\to qg)\big|^2_{\rm non-int.}\\ =&\frac{g^4}{4M_{\rm s}^6}\,\frac{N^2-1}{4N^2}\Bigg\{\left[\;\frac{1}{9}\;\frac{16M_{\rm s}^8(-u)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=1/2})^2}+\;\frac{4}{9}\;\frac{4M_{\rm s}^4(-u)(-u+2t)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2})^2}\right]\\ &\qquad+\left[\frac{9}{25}\;\frac{4M_{\rm s}^4(-u)^3}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2})^2}+\frac{4}{25}\;\frac{(-u)^3(-u+4t)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=5/2})^2}\right]\Bigg\}\,,\end{aligned} \tag{A.117}$$

$$\begin{aligned}&\big|\mathcal{M}_{\rm 2nd}(qg\to qg)\big|^2_{\rm int.}\\ =&\frac{g^4}{2M_{\rm s}^6}\,\frac{N^2-1}{4N^2}\Bigg\{\left[\;\frac{2}{9}\;\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma_{q^{**}}^{J=1/2}\Gamma_{q^{**}}^{J=3/2}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=1/2})^2}\;\frac{8M_{\rm s}^6\,(-u)(-u+2t)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma_{q^{**}}^{J=3/2})^2}\right]\\ &\qquad+\left[\frac{6}{25}\;\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma_{q^{**}}^{J=3/2}\Gamma_{q^{**}}^{J=5/2}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma_{q^{**}}^{J=3/2})^2}\;\frac{2M_{\rm s}^2\,(-u)^3(-u+4t)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma_{q^{**}}^{J=5/2})^2}\right]\Bigg\}\,,\end{aligned} \tag{A.118}$$

$$\begin{aligned}
&\left|\mathcal{M}_{\rm 2nd}(qg\to gq)\right|^2_{\rm non-int.}\\
=&\frac{g^4}{4M_{\rm s}^6}\,\frac{N^2-1}{4N^2}\Bigg\{\Bigg[\,\frac{1}{9}\,\frac{16M_{\rm s}^8(-t)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=1/2}_{q^{**}})^2}+\frac{4}{9}\,\frac{4M_{\rm s}^4(-t)(-t+2u)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}})^2}\Bigg]\\
&\qquad+\Bigg[\frac{9}{25}\,\frac{4M_{\rm s}^4(-t)^3}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}})^2}+\frac{4}{25}\,\frac{(-t)^3(-t+4u)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=5/2}_{q^{**}})^2}\Bigg]\Bigg\}\,.
\end{aligned} \tag{A.119}$$

$$\begin{aligned}
&\left|\mathcal{M}_{\rm 2nd}(qg\to gq)\right|^2_{\rm int.}\\
=&\frac{g^4}{2M_{\rm s}^6}\,\frac{N^2-1}{4N^2}\Bigg\{\Bigg[\,\frac{2}{9}\,\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=1/2}_{q^{**}}\Gamma^{J=3/2}_{q^{**}}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=1/2}_{q^{**}})^2}\,\frac{8M_{\rm s}^6\,(-t)(-t+2u)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}})^2}\Bigg]\\
&\qquad+\Bigg[\frac{6}{25}\,\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=3/2}_{q^{**}}\Gamma^{J=5/2}_{q^{**}}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3/2}_{q^{**}})^2}\,\frac{2M_{\rm s}^2\,(-t)^3(-t+4u)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}^2\Gamma^{J=5/2}_{q^{**}})^2}\Bigg]\Bigg\}\,.
\end{aligned} \tag{A.120}$$

Interference terms of eqs.(A.118) and (A.120) are interferences between the processes with exchanges of the second string excited quarks with $J = 1/2$ and $J = 3/2$, and, $J = 3/2$ and $J = 5/2$, respectively.

A.2.2 $gg \to q\overline{q}$ and $q\overline{q} \to gg$ amplitudes

The following string amplitudes with external gluons and quarks are obtained by some replacements in eq.(A.74),

$$\mathcal{M}(g_1^-,g_2^+,q_3^+,\overline{q}_4^-) = 2g^2\frac{\langle 32\rangle^2}{\langle 31\rangle\langle 41\rangle}\left[\frac{u}{s}\,V_u\left(T^{a_1}T^{a_2}\right)_{\alpha_4\alpha_3}+\frac{t}{s}\,V_t\left(T^{a_2}T^{a_1}\right)_{\alpha_4\alpha_3}\right], \tag{A.121}$$

$$\mathcal{M}(g_1^-,g_2^+,q_3^-,\overline{q}_4^+) = 2g^2\frac{\langle 31\rangle^2}{\langle 32\rangle\langle 42\rangle}\left[\frac{t}{s}\,V_t\left(T^{a_2}T^{a_1}\right)_{\alpha_4\alpha_3}+\frac{u}{s}\,V_u\left(T^{a_1}T^{a_2}\right)_{\alpha_4\alpha_3}\right], \tag{A.122}$$

$$\mathcal{M}(q_1^-,\overline{q}_2^+,g_3^+,g_4^-) = 2g^2\frac{\langle 14\rangle^2}{\langle 13\rangle\langle 23\rangle}\left[\frac{u}{s}\,V_u\left(T^{a_3}T^{a_4}\right)_{\alpha_2\alpha_1}+\frac{t}{s}\,V_t\left(T^{a_4}T^{a_3}\right)_{\alpha_2\alpha_1}\right], \tag{A.123}$$

$$\mathcal{M}(q_1^-,\overline{q}_2^+,g_3^-,g_4^+) = 2g^2\frac{\langle 13\rangle^2}{\langle 14\rangle\langle 24\rangle}\left[\frac{t}{s}\,V_t\left(T^{a_4}T^{a_3}\right)_{\alpha_2\alpha_1}+\frac{u}{s}\,V_u\left(T^{a_3}T^{a_4}\right)_{\alpha_2\alpha_1}\right]. \tag{A.124}$$

We consider two-body scattering processes of gluons and quarks, $g_1, g_2 \to q_3, \overline{q}_4$ and $q_1, \overline{q}_2 \to g_3, g_4$. In these processes, kinetic factors of eqs.(A.121), (A.122), (A.123) and (A.124) are

$$\begin{aligned}
\frac{\langle 32\rangle^2}{\langle 31\rangle\langle 41\rangle} &= \sqrt{\frac{u}{t}}\quad,\quad \frac{\langle 31\rangle^2}{\langle 32\rangle\langle 42\rangle} = -\sqrt{\frac{t}{u}}\,,\\
\frac{\langle 14\rangle^2}{\langle 13\rangle\langle 23\rangle} &= -\sqrt{\frac{u}{t}}\quad,\quad \frac{\langle 13\rangle^2}{\langle 14\rangle\langle 24\rangle} = \sqrt{\frac{t}{u}}\,.
\end{aligned} \tag{A.125}$$

first string excited gluon exchanges The gluons-to-quarks or quarks-to-gluons scattering process exchanges the string excited gluons. First, focusing on the first s-channel pole which is the first string excited gluon and using the string form factor functions in the limit of $s \simeq M_s^2$ in eq.(A.17),

$$\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^-, \overline{q}_4^+) \simeq \ \ 2g^2 M_s^2 \frac{1}{s - M_s^2} \sqrt{-\frac{t}{s}} \left(-\frac{u}{s}\right)^{\frac{3}{2}} \{T^{a_1}, T^{a_2}\}_{\alpha_4 \alpha_3}, \tag{A.126}$$

$$\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^+, \overline{q}_4^-) \simeq -2g^2 M_s^2 \frac{1}{s - M_s^2} \sqrt{-\frac{u}{s}} \left(-\frac{t}{s}\right)^{\frac{3}{2}} \{T^{a_1}, T^{a_2}\}_{\alpha_4 \alpha_3}, \tag{A.127}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^-, g_4^+) \simeq -2g^2 M_s^2 \frac{1}{s - M_s^2} \sqrt{-\frac{t}{s}} \left(-\frac{u}{s}\right)^{\frac{3}{2}} \{T^{a_3}, T^{a_4}\}_{\alpha_2 \alpha_1}, \tag{A.128}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^+, g_4^-) \simeq \ \ 2g^2 M_s^2 \frac{1}{s - M_s^2} \sqrt{-\frac{u}{s}} \left(-\frac{t}{s}\right)^{\frac{3}{2}} \{T^{a_3}, T^{a_4}\}_{\alpha_2 \alpha_1}. \tag{A.129}$$

Since angular dependences of eqs.(A.126), (A.127), (A.128) and (A.129) are factorized into the Wigner d-functions,

$$\begin{aligned} d^{\,J=2}_{\pm 2, \pm 1}(\theta) = d^{\,J=2}_{\mp 1, \mp 2}(\theta) &= \mp 2 \sin\frac{\theta}{2} \cos^3\frac{\theta}{2}, \\ d^{\,J=2}_{\pm 2, \mp 1}(\theta) = d^{\,J=2}_{\pm 1, \mp 2}(\theta) &= \mp 2 \cos\frac{\theta}{2} \sin^3\frac{\theta}{2}, \end{aligned} \tag{A.130}$$

it is found that the first string excited gluons with $J = 2$ are exchanged in the processes. The amplitudes become

$$\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^-, \overline{q}_4^+) \simeq \ \ 4g^2 M_s^2 \frac{1}{s - M_s^2} d^{\,J=2}_{-2, -1}(\theta) \sum_A d^{\,a_1 a_2 A} (T^A)_{\alpha_4 \alpha_3}, \tag{A.131}$$

$$\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^+, \overline{q}_4^-) \simeq -4g^2 M_s^2 \frac{1}{s - M_s^2} d^{\,J=2}_{-2, +1}(\theta) \sum_A d^{\,a_1 a_2 A} (T^A)_{\alpha_4 \alpha_3}, \tag{A.132}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^-, g_4^+) \simeq \ \ 4g^2 M_s^2 \frac{1}{s - M_s^2} d^{\,J=2}_{-1, -2}(\theta) \sum_A d^{\,a_3 a_4 A} (T^A)_{\alpha_2 \alpha_1}, \tag{A.133}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^+, g_4^-) \simeq \ \ 4g^2 M_s^2 \frac{1}{s - M_s^2} d^{\,J=2}_{-1, +2}(\theta) \sum_A d^{\,a_3 a_4 A} (T^A)_{\alpha_2 \alpha_1}. \tag{A.134}$$

widths of first string excited gluons In order to extract a matrix element of $F_{G^*}{}^{J=2\,A}_{\pm 1/2, \mp 1/2;\, \alpha_3, \alpha_4}$ from eqs.(A.131) and (A.132), using the result of the matrix element of $F_{G^*}{}^{J=2\,A}_{\pm 1, \mp 1;\, a_1, a_2}$ in eq.(A.33),

$$F_{G^*}{}^{J=2\,A}_{\pm 1/2, \mp 1/2;\, \alpha_3, \alpha_4} = \pm \frac{1}{\sqrt{2}} g M_s \, (T^A)_{\alpha_4 \alpha_3}. \tag{A.135}$$

The widths of $G^* \to q\bar{q}$ for $J=2$ are

$$\begin{aligned}\Gamma^{J=2\,A}_{G^*\to q\bar{q}} &= \frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times 2+1}\sum_{\alpha_3,\alpha_4}\left\{\left|F_{G^*}{}^{J=2\,A}_{+1/2,-1/2;\,\alpha_3,\alpha_4}\right|^2+\left|F_{G^*}{}^{J=2\,A}_{-1/2,+1/2;\,\alpha_3,\alpha_4}\right|^2\right\}\\ &= \frac{g^2 M_{\rm s}}{16\pi}\,\frac{1}{5}\,{\rm Tr}(T^A T^A)\,.\end{aligned} \tag{A.136}$$

A group factor of eq.(A.136) is averaged over a gauge index A of G^* for each decay process of g^* and C^*,

- $g^* \to q\bar{q}$

$$\frac{1}{N^2-1}\sum_a^{N^2-1}{\rm Tr}(T^a T^a) = \frac{1}{N^2-1}\,\frac{N^2-1}{2N}\,{\rm Tr}(\mathbb{I}_N) = \frac{1}{2}\,, \tag{A.137}$$

- $C^* \to q\bar{q}$

$${\rm Tr}(T^0 T^0) = \frac{1}{2N}\,{\rm Tr}(\mathbb{I}_N) = \frac{1}{2}\,. \tag{A.138}$$

Thus, widths of the first string excited gluons, g^* and C^*, with $J=2$ for the process of $G^* \to q\bar{q}$ are

$$\Gamma^{J=2}_{g^*\to q\bar{q}} = \frac{g^2 M_{\rm s}}{4\pi}\,\frac{1}{5}\,\frac{N_f}{8} \quad , \quad \Gamma^{J=2}_{C^*\to q\bar{q}} = \frac{g^2 M_{\rm s}}{4\pi}\,\frac{1}{5}\,\frac{N_f}{8}\,, \tag{A.139}$$

where N_f is the number of quark flavor. Total widths of g^* and C^* with $J=2$ are obtained from eqs.(A.41), (A.42) and (A.139),

$$\Gamma^{J=2}_{g^*} = \frac{g^2 M_{\rm s}}{4\pi}\,\frac{1}{5}\left(\frac{N}{2}+\frac{N_f}{8}\right) \quad , \quad \Gamma^{J=2}_{C^*} = \frac{g^2 M_{\rm s}}{4\pi}\,\frac{1}{5}\left(N+\frac{N_f}{8}\right). \tag{A.140}$$

squared amplitudes with first string excited gluon exchanges The amplitude of eqs.(A.131), (A.132), (A.133) and (A.134) are written as

$$\begin{aligned}&\mathcal{M}_{\rm 1st}(g_1^-, g_2^+ \to q_3^-, \bar{q}_4^+)\\ &\to\; 8\,g^2 M_{\rm s}^2\,\frac{1}{M_{\rm s}^4}\left[\frac{\sqrt{-t}(-u)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{g^*}}\sum_a d^{\,a_1a_2a}(T^a)_{\alpha_4\alpha_3}+\frac{\sqrt{-t}(-u)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{C^*}}\,d^{\,a_1a_20}(T^0)_{\alpha_4\alpha_3}\right],\end{aligned} \tag{A.141}$$

$$\begin{aligned}&\mathcal{M}_{\rm 1st}(g_1^-, g_2^+ \to q_3^+, \bar{q}_4^-)\\ &\to -\,8\,g^2 M_{\rm s}^2\,\frac{1}{M_{\rm s}^4}\left[\frac{\sqrt{-u}(-t)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{g^*}}\sum_a d^{\,a_1a_2a}(T^a)_{\alpha_4\alpha_3}+\frac{\sqrt{-u}(-t)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{C^*}}\,d^{\,a_1a_20}(T^0)_{\alpha_4\alpha_3}\right],\end{aligned} \tag{A.142}$$

$$\begin{aligned}&\mathcal{M}_{\rm 1st}(q_1^-, \bar{q}_2^+ \to g_3^-, g_4^+)\\ &\to\; 8\,g^2 M_{\rm s}^2\,\frac{1}{M_{\rm s}^4}\left[\frac{-\sqrt{-t}(-u)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{g^*}}\sum_a d^{\,a_3a_4a}(T^a)_{\alpha_2\alpha_1}+\frac{-\sqrt{-t}(-u)^{3/2}}{s-M_{\rm s}^2+iM_{\rm s}\Gamma^{J=2}_{C^*}}\,d^{\,a_3a_40}(T^0)_{\alpha_2\alpha_1}\right],\end{aligned} \tag{A.143}$$

$$
\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^+, g_4^-)
$$
$$
\to \; 8\, g^2 M_\mathrm{s}^2 \, \frac{1}{M_\mathrm{s}^4}\left[\frac{\sqrt{-u}(-t)^{3/2}}{s - M_\mathrm{s}^2 + i M_\mathrm{s}\Gamma^{J=2}_{g^*}} \sum_a d^{\,a_3 a_4 a} (T^a)_{\alpha_2\alpha_1} + \frac{\sqrt{-u}(-t)^{3/2}}{s - M_\mathrm{s}^2 + i M_\mathrm{s}\Gamma^{J=2}_{C^*}}\, d^{\,a_3 a_4 0}(T^0)_{\alpha_2\alpha_1} \right]. \tag{A.144}
$$

Squared amplitudes of the processes of $g_1, g_2 \to q_3, \overline{q}_4$ and $q_1, \overline{q}_2 \to g_3, g_4$ are calculated,

$$
\begin{aligned}
&\left|\mathcal{M}_{1\mathrm{st}}(g_1, g_2 \to q_3, \overline{q}_4)\right|^2 \\
&= \frac{1}{2^2}\,\frac{N_f}{(N^2-1)^2} \sum_{a_1,a_2,\alpha_3,\alpha_4} \Big\{ \left|\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^-, \overline{q}_4^+)\right|^2 + \left|\mathcal{M}_{1\mathrm{st}}(g_1^+, g_2^- \to q_3^+, \overline{q}_4^-)\right|^2 \\
&\qquad + \left|\mathcal{M}_{1\mathrm{st}}(g_1^-, g_2^+ \to q_3^+, \overline{q}_4^-)\right|^2 + \left|\mathcal{M}_{1\mathrm{st}}(g_1^+, g_2^- \to q_3^-, \overline{q}_4^+)\right|^2 \Big\},
\end{aligned} \tag{A.145}
$$

$$
\begin{aligned}
&\left|\mathcal{M}_{1\mathrm{st}}(q_1, \overline{q}_2 \to g_3, g_4)\right|^2 \\
&= \frac{1}{2^2}\,\frac{1}{N^2} \sum_{\alpha_1,\alpha_2,a_3,a_4} \Big\{ \left|\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^-, g_4^+)\right|^2 + \left|\mathcal{M}_{1\mathrm{st}}(q_1^+, \overline{q}_2^- \to g_3^+, g_4^-)\right|^2 \\
&\qquad + \left|\mathcal{M}_{1\mathrm{st}}(q_1^-, \overline{q}_2^+ \to g_3^+, g_4^-)\right|^2 + \left|\mathcal{M}_{1\mathrm{st}}(q_1^+, \overline{q}_2^- \to g_3^-, g_4^+)\right|^2 \Big\}.
\end{aligned} \tag{A.146}
$$

Since group factors of eqs.(A.141) and (A.142) are squared for each product scattering processes,

- $(gg \to g^* \to q\overline{q})^2$

$$
\frac{1}{(N^2-1)^2} \sum_{a_1,a_2}^{N^2-1} \sum_{a,b}^{N^2-1} d^{\,a_1 a_2 a}\, d^{\,a_1 a_2 b}\, \mathrm{Tr}\big(T^a T^b\big) = \frac{1}{(N^2-1)^2}\,\frac{1}{2} \sum_{a_1,a_2,a}^{N^2-1} \big(d^{\,a_1 a_2 a}\big)^2 = \frac{1}{N^2-1}\,\frac{N^2-4}{32N}, \tag{A.147}
$$

- $(q\overline{q} \to g^* \to gg)^2$

$$
\frac{1}{N^2} \sum_{a_3,a_4}^{N^2-1} \sum_{a,b}^{N^2-1} d^{\,a_3 a_4 a}\, d^{\,a_3 a_4 b}\, \mathrm{Tr}\big(T^a T^b\big) = \frac{N^2-1}{N^2}\,\frac{N^2-4}{32N}, \tag{A.148}
$$

- $(gg \to C^* \to q\overline{q})^2$

$$
\frac{1}{(N^2-1)^2} \sum_{a_1,a_2}^{N^2-1} d^{\,a_1 a_2 0}\, d^{\,a_1 a_2 0}\, \mathrm{Tr}\big(T^0 T^0\big) = \frac{1}{N^2-1}\,\frac{1}{16N}, \tag{A.149}
$$

- $(q\overline{q} \to C^* \to gg)^2$

$$
\frac{1}{N^2} \sum_{a_3,a_4}^{N^2-1} d^{\,a_3 a_4 0}\, d^{\,a_3 a_4 0}\, \mathrm{Tr}\big(T^0 T^0\big) = \frac{N^2-1}{N^2}\,\frac{1}{16N}, \tag{A.150}
$$

- $(gg \to g^* \to q\overline{q}) \times (gg \to C^* \to q\overline{q})$ or $(q\overline{q} \to g^* \to gg) \times (q\overline{q} \to C^* \to gg)$

$$
\sum_{a_1,a_2}^{N^2-1} \sum_{a}^{N^2-1} d^{\,a_1 a_2 a}\, d^{\,a_1 a_2 0}\, \mathrm{Tr}\big(T^a T^0\big) = \frac{1}{4N} \sum_{a_1,a}^{N^2-1} d^{\,a_1 a_1 a}\, \mathrm{Tr}\big(T^a\big) = 0, \tag{A.151}
$$

the squared amplitudes are

$$\left|\mathcal{M}_{\rm 1st}(gg\to q\overline{q})\right|^2 = \frac{2g^4}{M_{\rm s}^4}\,\frac{N_f}{N(N^2-1)}\left[\frac{N^2-4}{2}\,\frac{tu^3+ut^3}{(s-M_{\rm s}^2)^2+(M_{\rm s}\Gamma^{J=2}_{g^*})^2}+\frac{tu^3+ut^3}{(s-M_{\rm s}^2)^2+(M_{\rm s}\Gamma^{J=2}_{C^*})^2}\right]. \tag{A.152}$$

$$\left|\mathcal{M}_{\rm 1st}(q\overline{q}\to gg)\right|^2 = \frac{2g^4}{M_{\rm s}^4}\quad\frac{N^2-1}{N^3}\quad\left[\frac{N^2-4}{2}\,\frac{tu^3+ut^3}{(s-M_{\rm s}^2)^2+(M_{\rm s}\Gamma^{J=2}_{g^*})^2}+\frac{tu^3+ut^3}{(s-M_{\rm s}^2)^2+(M_{\rm s}\Gamma^{J=2}_{C^*})^2}\right]. \tag{A.153}$$

second string excited gluon exchanges Next, we consider the two-body scattering amplitudes of gluons and quarks, with the second string excited gluon exchanges. From the string form factor functions in the limit of $s\simeq 2M_{\rm s}^2$ of eq.(A.53),

$$\mathcal{M}_{\rm 2nd}(g_1^-,g_2^+\to q_3^-,\overline{q}_4^+) \simeq 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\,\sqrt{-\frac{t}{s}}\left(-\frac{u}{s}\right)^{\frac{3}{2}}\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_1},T^{a_2}\right]_{\alpha_4\alpha_3}, \tag{A.154}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-,g_2^+\to q_3^+,\overline{q}_4^-) \simeq -4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\,\sqrt{-\frac{u}{s}}\left(-\frac{t}{s}\right)^{\frac{3}{2}}\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_1},T^{a_2}\right]_{\alpha_4\alpha_3}, \tag{A.155}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to g_3^-,g_4^+) \simeq -4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\,\sqrt{-\frac{t}{s}}\left(-\frac{u}{s}\right)^{\frac{3}{2}}\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_3},T^{a_4}\right]_{\alpha_2\alpha_1}, \tag{A.156}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to g_3^+,g_4^-) \simeq 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\,\sqrt{-\frac{u}{s}}\left(-\frac{t}{s}\right)^{\frac{3}{2}}\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_3},T^{a_4}\right]_{\alpha_2\alpha_1}. \tag{A.157}$$

Angular dependences in eqs.(A.126), (A.127), (A.128) and (A.129) are factorized into the Wigner d-functions of $J=2$ and $J=3$,

$$\begin{aligned} d^{\,J=3}_{\,\pm2,\pm1}(\theta) = d^{\,J=3}_{\,\mp1,\mp2}(\theta) &= \mp 2\sqrt{\frac{5}{2}}\,\sin\frac{\theta}{2}\,\cos^3\frac{\theta}{2}\left(\frac{3\cos\theta-1}{2}\right), \\ d^{\,J=3}_{\,\pm2,\mp1}(\theta) = d^{\,J=3}_{\,\pm1,\mp2}(\theta) &= \mp 2\sqrt{\frac{5}{2}}\,\cos\frac{\theta}{2}\,\sin^3\frac{\theta}{2}\left(\frac{3\cos\theta+1}{2}\right). \end{aligned} \tag{A.158}$$

Therefore, it is found that the second string excited gluons with $J=2$ and $J=3$ are exchanged, as follows,

$$\mathcal{M}_{\rm 2nd}(g_1^-,g_2^+\to q_3^-,\overline{q}_4^+) \simeq 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[\frac{1}{6}\,d^{\,J=2}_{\,-2,-1}(\theta)+\frac{1}{3}\sqrt{\frac{2}{5}}\,d^{\,J=3}_{\,-2,-1}(\theta)\right]\sum_a if^{a_1a_2a}\left(T^a\right)_{\alpha_4\alpha_3}, \tag{A.159}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-,g_2^+\to q_3^+,\overline{q}_4^-) \simeq -4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[-\frac{1}{6}\,d^{\,J=2}_{\,-2,+1}(\theta)+\frac{1}{3}\sqrt{\frac{2}{5}}\,d^{\,J=3}_{\,-2,+1}(\theta)\right]\sum_a if^{a_1a_2a}\left(T^a\right)_{\alpha_4\alpha_3}, \tag{A.160}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to g_3^-,g_4^+) \simeq 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[\frac{1}{6}\,d^{\,J=2}_{\,-1,-2}(\theta)+\frac{1}{3}\sqrt{\frac{2}{5}}\,d^{\,J=3}_{\,-1,-2}(\theta)\right]\sum_a if^{a_3a_4a}\left(T^a\right)_{\alpha_2\alpha_1}, \tag{A.161}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to g_3^+,g_4^-) \simeq \quad 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[-\frac{1}{6}\,d^{\,J=2}_{\,-1,+2}\,(\theta)+\frac{1}{3}\sqrt{\frac{2}{5}}\,d^{\,J=3}_{\,-1,+2}\,(\theta)\right]\sum_a if^{a_3a_4a}\big(T^a\big)_{\alpha_2\alpha_1}\,. \tag{A.162}$$

widths of second string excited gluons From the above amplitudes, we can extract matrix elements of the second string excited gluons, $F_{g^{**}}{}^{J=2,3\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4}$, using the result of the matrix element of $F_{g^{**}}{}^{J=2,3\,a}_{\pm1,\mp1;\,a_3,a_4}$ of eq.(A.61),

$$F_{g^{**}}{}^{J=2\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4} = +\,i\,\frac{1}{\sqrt{6}}\,gM_{\rm s}\,\big(T^a\big)_{\alpha_4\alpha_3} \quad , \quad F_{g^{**}}{}^{J=3\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4} = \pm\,i\,\frac{2\sqrt{2}}{\sqrt{15}}\,gM_{\rm s}\,\big(T^a\big)_{\alpha_4\alpha_3}\,. \tag{A.163}$$

The widths of $g^{**}\to q\overline{q}$ for $J=2$ are

$$\begin{aligned}\Gamma^{J=2\,a}_{g^{**}\to q\overline{q}} &= \frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times2+1}\sum_{\alpha_3,\alpha_4}\left\{\left|F_{g^{**}}{}^{J=2\,a}_{+1/2,-1/2;\,\alpha_3,\alpha_4}\right|^2+\left|F_{g^{**}}{}^{J=2\,a}_{-1/2,+1/2;\,\alpha_3,\alpha_4}\right|^2\right\}\\ &= \frac{g^2\sqrt{2}M_{\rm s}}{96\pi}\,\frac{1}{5}\,{\rm Tr}\big(T^aT^a\big)\,,\end{aligned} \tag{A.164}$$

$$\begin{aligned}\Gamma^{J=3\,a}_{g^{**}\to q\overline{q}} &= \frac{1}{16\pi\sqrt{2}M_{\rm s}}\,\frac{1}{2\times3+1}\sum_{\alpha_3,\alpha_4}\left\{\left|F_{g^{**}}{}^{J=3\,a}_{+1/2,-1/2;\,\alpha_3,\alpha_4}\right|^2+\left|F_{g^{**}}{}^{J=3\,a}_{-1/2,+1/2;\,\alpha_3,\alpha_4}\right|^2\right\}\\ &= \frac{g^2\sqrt{2}M_{\rm s}}{30\pi}\,\frac{1}{7}\,{\rm Tr}\big(T^aT^a\big)\,.\end{aligned} \tag{A.165}$$

Using the result of the group factor averaged over a gauge index a of g^{**} of eq.(A.137), widths of the second string excited gluons, g^{**}, with $J=2$ and $J=3$ for the process of $g^{**}\to q\overline{q}$ are

$$\Gamma^{J=2}_{g^{**}\to q\overline{q}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{5}\,\frac{N_f}{48} \quad , \quad \Gamma^{J=3}_{g^{**}\to q\overline{q}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{7}\,\frac{N_f}{15}\,. \tag{A.166}$$

Total widths of g^{**} with $J=2$ and $J=3$ are obtained from eqs.(A.66) and (A.210),

$$\Gamma^{J=2}_{g^{**}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{5}\left(\frac{N}{3}+\frac{N_f}{48}\right) \quad , \quad \Gamma^{J=3}_{g^{**}} = \frac{g^2\sqrt{2}M_{\rm s}}{4\pi}\,\frac{1}{7}\left(\frac{N}{6}+\frac{N_f}{15}\right). \tag{A.167}$$

squared amplitudes with second string excited gluon exchanges The amplitudes of eqs.(A.159), (A.160), (A.161) and (A.162) are rewritten as

$$\begin{aligned}&\mathcal{M}_{\rm 2nd}(g_1^-,g_2^+\to q_3^-,\overline{q}_4^+)\\ \to\quad &4g^2M_{\rm s}^2\,\frac{1}{8M_{\rm s}^6}\left[\;\frac{1}{6}\,\frac{2M_{\rm s}^2\cdot2\sqrt{-t}(-u)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}}}+\frac{1}{3}\,\frac{2\sqrt{-t}(-u)^{3/2}(-u+2t)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}}}\right]\sum_a if^{a_1a_2a}\big(T^a\big)_{\alpha_4\alpha_3}\,,\end{aligned} \tag{A.168}$$

$$\mathcal{M}_{\rm 2nd}(g_1^-, g_2^+ \to q_3^+, \overline{q}_4^-)$$
$$\to -4g^2 M_{\rm s}^2 \frac{1}{8M_{\rm s}^6}\left[-\frac{1}{6}\frac{2M_{\rm s}^2\cdot 2\sqrt{-u}(-t)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}}}+\frac{1}{3}\frac{-2\sqrt{-u}(-t)^{3/2}(-t+2u)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}}}\right]\sum_a if^{a_1a_2a}\big(T^a\big)_{\alpha_4\alpha_3}, \tag{A.169}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-, \overline{q}_2^+ \to g_3^-, g_4^+)$$
$$\to 4g^2 M_{\rm s}^2 \frac{1}{8M_{\rm s}^6}\left[\frac{1}{6}\frac{-2M_{\rm s}^2\cdot 2\sqrt{-t}(-u)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}}}+\frac{1}{3}\frac{-2\sqrt{-t}(-u)^{3/2}(-u+2t)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}}}\right]\sum_a if^{a_3a_4a}\big(T^a\big)_{\alpha_2\alpha_1}, \tag{A.170}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-, \overline{q}_2^+ \to g_3^+, g_4^-)$$
$$\to 4g^2 M_{\rm s}^2 \frac{1}{8M_{\rm s}^6}\left[-\frac{1}{6}\frac{2M_{\rm s}^2\cdot 2\sqrt{-u}(-t)^{3/2}}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}}}+\frac{1}{3}\frac{-2\sqrt{-u}(-t)^{3/2}(-t+2u)}{s-2M_{\rm s}^2+i\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}}}\right]\sum_a if^{a_3a_4a}\big(T^a\big)_{\alpha_2\alpha_1}. \tag{A.171}$$

Since the group factors are squared for each product of scattering processes,

- $(gg \to g^{**} \to q\overline{q})^2$

$$\frac{1}{(N^2-1)^2}\sum_{a_1,a_2}^{N^2-1}\sum_{a,b}^{N^2-1} f^{a_1a_2a}\, f^{a_1a_2b}\,{\rm Tr}\big(T^aT^b\big) = \frac{1}{(N^2-1)^2}\frac{1}{2}\sum_{a_1,a_2,a}^{N^2-1}\big(f^{a_1a_2a}\big)^2 = \frac{1}{N^2-1}\frac{N}{2}, \tag{A.172}$$

- $(q\overline{q} \to g^{**} \to gg)^2$

$$\frac{1}{N^2}\sum_{a_1,a_2}^{N^2-1}\sum_{a,b}^{N^2-1} f^{a_1a_2a}\, f^{a_1a_2b}\,{\rm Tr}\big(T^aT^b\big) = \frac{1}{N^2}\frac{1}{2}\sum_{a_1,a_2,a}^{N^2-1}\big(f^{a_1a_2a}\big)^2 = \frac{N^2-1}{N^2}\frac{N}{2}, \tag{A.173}$$

the squared amplitudes are

$$\begin{aligned}\big|\mathcal{M}_{\rm 2nd}(gg\to q\overline{q})\big|^2 &= \big|\mathcal{M}_{\rm 2nd}(gg\to q\overline{q})\big|^2_{\rm non-int.} + \big|\mathcal{M}_{\rm 2nd}(gg\to q\overline{q})\big|^2_{\rm int.},\\ \big|\mathcal{M}_{\rm 2nd}(q\overline{q}\to gg)\big|^2 &= \big|\mathcal{M}_{\rm 2nd}(q\overline{q}\to gg)\big|^2_{\rm non-int.} + \big|\mathcal{M}_{\rm 2nd}(q\overline{q}\to gg)\big|^2_{\rm int.},\end{aligned} \tag{A.174}$$

where

$$\big|\mathcal{M}_{\rm 2nd}(gg\to q\overline{q})\big|^2_{\rm non-int.}$$
$$= \frac{g^4}{8M_{\rm s}^8}\frac{N_f\cdot N}{2(N^2-1)}\left[\frac{1}{9}\frac{4M_{\rm s}^4\big(tu^3+ut^3\big)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}+\frac{4}{9}\frac{tu^3(-u+2t)^2+ut^3(-t+2u)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right], \tag{A.175}$$

$$\big|\mathcal{M}_{\rm 2nd}(gg\to q\overline{q})\big|^2_{\rm int.}$$
$$= \frac{g^4}{4M_{\rm s}^8}\frac{N_f\cdot N}{2(N^2-1)}\left[\frac{2}{9}\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=2}_{g^{**}}\Gamma^{J=3}_{g^{**}}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}\frac{2M_{\rm s}^2\big(tu^3(-u+2t)+ut^3(-t+2u)\big)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right], \tag{A.176}$$

$$
\begin{aligned}
&\left|\mathcal{M}_{\rm 2nd}(q\bar{q}\to gg)\right|^2_{\rm non-int.}\\
&=\frac{g^4}{8M_{\rm s}^8}\quad\frac{N^2-1}{2N}\quad\left[\frac{1}{9}\,\frac{4M_{\rm s}^4\big(tu^3+ut^3\big)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}+\frac{4}{9}\,\frac{tu^3(-u+2t)^2+ut^3(-t+2u)^2}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right],
\end{aligned}
\tag{A.177}
$$

$$
\begin{aligned}
&\left|\mathcal{M}_{\rm 2nd}(q\bar{q}\to gg)\right|^2_{\rm int.}\\
&=\frac{g^4}{4M_{\rm s}^8}\quad\frac{N^2-1}{2N}\quad\left[\frac{2}{9}\,\frac{(s-2M_{\rm s}^2)^2+2M_{\rm s}^2\Gamma^{J=2}_{g^{**}}\Gamma^{J=3}_{g^{**}}}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=2}_{g^{**}})^2}\,\frac{2M_{\rm s}^2\big(tu^3(-u+2t)+ut^3(-t+2u)\big)}{(s-2M_{\rm s}^2)^2+(\sqrt{2}M_{\rm s}\Gamma^{J=3}_{g^{**}})^2}\right].
\end{aligned}
\tag{A.178}
$$

widths of first string excited gluons with J=1 A fact that the first string excited gluons have $J=0$, $J=1$ and $J=2$ is verified, by counting of physical degrees of freedom in ref.[38]. However, as it is found from eqs.(A.128) and (A.129), the string excited gluon with $J=1$ cannot be accessible in the process of $q\bar{q}\to gg$. In order to calculate a width of the string excited gluon with $J=1$, we identify it with an intermediated state in scattering processes of $\widetilde{g}\widetilde{g}\to\widetilde{g}\widetilde{g}$ and $q\bar{q}\to\widetilde{g}\widetilde{g}$, where $\widetilde{g}$ denotes a gluino which is a superpartner of gluon.

Using the supersymmetric Ward identity[74], amplitudes with external four gluinos are shown as

$$
\mathcal{M}(\widetilde{g}_1^-,\widetilde{g}_2^+,\widetilde{g}_3^+,\widetilde{g}_4^-)=\frac{\langle 23\rangle}{\langle 14\rangle}\,\mathcal{M}(g_1^-,g_2^+,g_3^+,g_4^-)\,,
\tag{A.179}
$$

$$
\mathcal{M}(\widetilde{g}_1^-,\widetilde{g}_2^+,\widetilde{g}_3^-,\widetilde{g}_4^+)=\frac{\langle 24\rangle}{\langle 13\rangle}\,\mathcal{M}(g_1^-,g_2^+,g_3^-,g_4^+)\,.
\tag{A.180}
$$

If we consider two-gluino scattering processes, $\widetilde{g}_1,\widetilde{g}_2\to\widetilde{g}_3,\widetilde{g}_4$, kinetic factors of eqs.(A.179) and (A.179) are

$$
\frac{\langle 23\rangle}{\langle 14\rangle}=-1\quad,\quad\frac{\langle 24\rangle}{\langle 13\rangle}=+1\,.
\tag{A.181}
$$

Let us consider two-gluino scattering amplitudes with the first string excited gluon exchanges. Inserting the string form factor functions in the limit of $s\simeq M_{\rm s}^2$ of eq.(A.18) into them, from eqs.(A.20) and (A.21),

$$
\mathcal{M}_{\rm 1st}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\simeq\ \ 4\,g^2M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left(-\frac{u}{s}\right)^2{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big)\,,
\tag{A.182}
$$

$$
\mathcal{M}_{\rm 1st}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\simeq-4\,g^2M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left(-\frac{t}{s}\right)^2{\rm Tr}\big(\{T^{a_1},T^{a_2}\}\{T^{a_3},T^{a_4}\}\big)\,.
\tag{A.183}
$$

Since angular dependences in eqs.(A.182) and (A.183) are decomposed into the Wigner d-

functions of $J = 1$ and $J = 2$,

$$\begin{aligned} d^{\,J=1}_{\pm1,\pm1}(\theta) &= \frac{1+\cos\theta}{2}\,, \\ d^{\,J=1}_{\pm1,\mp1}(\theta) &= \frac{1-\cos\theta}{2}\,, \\ d^{\,J=2}_{\pm1,\pm1}(\theta) &= \frac{1+\cos\theta}{2}\,\big(2\cos\theta-1\big)\,, \\ d^{\,J=2}_{\pm1,\mp1}(\theta) &= \frac{1-\cos\theta}{2}\,\big(2\cos\theta+1\big)\,, \end{aligned} \tag{A.184}$$

the two-gluino scattering amplitudes become

$$\mathcal{M}_{1\mathrm{st}}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+) \simeq \;\; 32\,g^2M_\mathrm{s}^2\,\frac{1}{s-M_\mathrm{s}^2}\left[\frac{3}{4}\,d^{\,J=1}_{-1,-1}(\theta)+\frac{1}{4}\,d^{\,J=2}_{-1,-1}(\theta)\right]\sum_A d^{\,a_1a_2A}\,d^{\,a_3a_4A}\,, \tag{A.185}$$

$$\mathcal{M}_{1\mathrm{st}}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-) \simeq -32\,g^2M_\mathrm{s}^2\,\frac{1}{s-M_\mathrm{s}^2}\left[\frac{3}{4}\,d^{\,J=1}_{-1,+1}(\theta)-\frac{1}{4}\,d^{\,J=2}_{-1,+1}(\theta)\right]\sum_A d^{\,a_1a_2A}\,d^{\,a_3a_4A}\,. \tag{A.186}$$

Extracting matrix elements of the first string excited gluons from the amplitudes of eqs.(A.185) and (A.185),

$$F_{G^*}{}^{J=1\,A}_{\pm1/2,\mp1/2;\,a_3,a_4} = \mp\,2\sqrt{6}\,gM_\mathrm{s}\,d^{\,a_3a_3A} \quad , \quad F_{G^*}{}^{J=2\,A}_{\pm1/2,\mp1/2;\,a_3,a_4} = 2\sqrt{2}\,gM_\mathrm{s}\,d^{\,a_3a_3A}\,. \tag{A.187}$$

On the other hand, using another of the supersymmetric Ward identity, amplitudes with external two gluinos and two quarks are

$$\mathcal{M}(q_1^-,\overline{q}_2^+,\widetilde{g}_3^+,\widetilde{g}_4^-) = \frac{[24]}{[23]}\,\mathcal{M}(q_1^-,\overline{q}_2^+,g_3^+,g_4^-)\,, \tag{A.188}$$

$$\mathcal{M}(q_1^-,\overline{q}_2^+,\widetilde{g}_3^-,\widetilde{g}_4^+) = \frac{[23]}{[24]}\,\mathcal{M}(q_1^-,\overline{q}_2^-,g_3^-,g_4^+)\,, \tag{A.189}$$

If we consider two-body scattering processes of quarks and gluinos, $q_1, \overline{q}_2 \to \widetilde{g}_3, \widetilde{g}_4$, kinetic factors of eqs.(A.188) and (A.189) are

$$\frac{[24]}{[23]} = -\sqrt{\frac{t}{u}} \quad , \quad \frac{[23]}{[24]} = -\sqrt{\frac{u}{t}}\,. \tag{A.190}$$

Let us consider scattering amplitudes of quarks into gluinos, with the first string excited gluon exchanges. From eqs.(A.128) and (A.129),

$$\mathcal{M}_{1\mathrm{st}}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+) \simeq \;\; 2g^2M_\mathrm{s}^2\,\frac{1}{s-M_\mathrm{s}^2}\left(-\frac{t}{s}\right)\left(-\frac{u}{s}\right)\left\{T^{a_3},T^{a_4}\right\}_{\alpha_2\alpha_1}, \tag{A.191}$$

$$\mathcal{M}_{1\mathrm{st}}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-) \simeq -2g^2M_\mathrm{s}^2\,\frac{1}{s-M_\mathrm{s}^2}\left(-\frac{u}{s}\right)\left(-\frac{t}{s}\right)\left\{T^{a_3},T^{a_4}\right\}_{\alpha_2\alpha_1}. \tag{A.192}$$

Since angular dependences of eqs.(A.191) and (A.192) are decomposed into the Wigner d-functions of eq.(A.184), the amplitudes are

$$\mathcal{M}_{\rm 1st}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\simeq\ \ 8\,g^2M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left[\frac{1}{4}\,d^{\,J=1}_{\,-1,-1}(\theta)-\frac{1}{4}\,d^{\,J=2}_{\,-1,-1}(\theta)\right]\sum_A d^{\,a_3a_4A}\left(T^A\right)_{\alpha_2\alpha_2}, \tag{A.193}$$

$$\mathcal{M}_{\rm 1st}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\simeq -8\,g^2M_{\rm s}^2\,\frac{1}{s-M_{\rm s}^2}\left[\frac{1}{4}\,d^{\,J=1}_{\,-1,+1}(\theta)+\frac{1}{4}\,d^{\,J=2}_{\,-1,+1}(\theta)\right]\sum_A d^{\,a_3a_4A}\left(T^A\right)_{\alpha_2\alpha_1}. \tag{A.194}$$

Using the results of the matrix elements of G^* of eq.(A.187),

$$F_{G^*\ \pm1/2,\mp1/2;\,\alpha_3,\alpha_4}^{\ J=1\,A}=\mp\frac{1}{\sqrt{6}}\,gM_{\rm s}\left(T^A\right)_{\alpha_4\alpha_3}\quad,\quad F_{G^*\ \pm1/2,\mp1/2;\,\alpha_3,\alpha_4}^{\ J=2\,A}=-\frac{1}{\sqrt{2}}\,gM_{\rm s}\left(T^A\right)_{\alpha_4\alpha_3} \tag{A.195}$$

a width of the first string excited gluon with $J=1$ for the process of $G^*\to q\overline{q}$ is

$$\begin{aligned}\Gamma^{J=1\,A}_{G^*\to q\overline{q}}&=\frac{1}{16\pi M_{\rm s}}\,\frac{1}{2\times1+1}\sum_{\alpha_3,\alpha_4}\left\{\left|F_{G^*\ +1/2,-1/2;\,\alpha_3,\alpha_4}^{\ J=1\,A}\right|^2+\left|F_{G^*\ -1/2,+1/2;\,\alpha_3,\alpha_4}^{\ J=1\,A}\right|^2\right\}\\&=\frac{g^2M_{\rm s}}{48\pi}\,\frac{1}{3}\,{\rm Tr}\big(T^AT^A\big)\,.\end{aligned} \tag{A.196}$$

Since the group factor is $\frac{1}{2}$ in eqs.(A.137) and (A.138) for both g^* and C^*,

$$\Gamma^{J=1}_{g^*\to q\overline{q}}=\frac{g^2M_{\rm s}}{4\pi}\,\frac{1}{3}\,\frac{N_f}{24}\quad,\quad\Gamma^{J=1}_{C^*\to q\overline{q}}=\frac{g^2M_{\rm s}}{4\pi}\,\frac{1}{3}\,\frac{N_f}{24}\,. \tag{A.197}$$

widths of second string excited gluons with J=1 Next, consider two-gluino scattering amplitudes and quarks into gluinos scattering amplitudes with second string excited gluon exchanges. The amplitudes are

$$\mathcal{M}_{\rm 2nd}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\simeq -8\,g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left(-\frac{u}{s}\right)^2\left(-\frac{u}{s}+\frac{t}{s}\right){\rm Tr}\big([T^{a_1},T^{a_2}][T^{a_3},T^{a_4}]\big)\,, \tag{A.198}$$

$$\mathcal{M}_{\rm 2nd}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\simeq\ \ 8\,g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left(-\frac{t}{s}\right)^2\left(-\frac{u}{s}+\frac{t}{s}\right){\rm Tr}\big([T^{a_1},T^{a_2}][T^{a_3},T^{a_4}]\big)\,. \tag{A.199}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\simeq\ \ 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left(-\frac{t}{s}\right)\left(-\frac{u}{s}\right)\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_3},T^{a_4}\right]_{\alpha_2\alpha_1}, \tag{A.200}$$

$$\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\simeq -4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left(-\frac{u}{s}\right)\left(-\frac{t}{s}\right)\left(-\frac{u}{s}+\frac{t}{s}\right)\left[T^{a_3},T^{a_4}\right]_{\alpha_2\alpha_1}. \tag{A.201}$$

Since angular dependences of eqs.(A.198), (A.199), (A.200) and (A.201) are decomposed into the Wigner d-functions of $J = 1$, $J = 2$ and $J = 3$,

$$
\begin{aligned}
d^{\,J=3}_{\pm1,\pm1}(\theta) &= \frac{1+\cos\theta}{2}\left(\frac{15}{4}\big(\cos\theta-1\big)^2 + \ 5\big(\cos\theta-1\big)+1\right),\\
d^{\,J=3}_{\pm1,\mp1}(\theta) &= \frac{1-\cos\theta}{2}\left(\frac{15}{4}\big(\cos\theta-1\big)^2 + 10\big(\cos\theta-1\big)+6\right),
\end{aligned}
\tag{A.202}
$$

the amplitudes become

$$
\begin{aligned}
&\mathcal{M}_{\rm 2nd}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\\
&\simeq \ \ 4\,g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[\ \ \frac{9}{20}\,d^{\,J=1}_{-1,-1}(\theta)+\frac{5}{12}\,d^{\,J=2}_{-1,-1}(\theta)+\frac{2}{15}\,d^{\,J=3}_{-1,-1}(\theta)\right]\sum_a f^{a_1a_2a}\,f^{a_3a_4a},
\end{aligned}
\tag{A.203}
$$

$$
\begin{aligned}
&\mathcal{M}_{\rm 2nd}(\widetilde{g}_1^-,\widetilde{g}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\\
&\simeq -\,4\,g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[-\frac{9}{20}\,d^{\,J=1}_{-1,+1}(\theta)+\frac{5}{12}\,d^{\,J=2}_{-1,+1}(\theta)-\frac{2}{15}\,d^{\,J=3}_{-1,+1}(\theta)\right]\sum_a f^{a_1a_2a}\,f^{a_3a_4a},
\end{aligned}
\tag{A.204}
$$

$$
\begin{aligned}
&\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^-,\widetilde{g}_4^+)\\
&\simeq \ \ 4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[\ \ \frac{1}{20}\,d^{\,J=1}_{-1,-1}(\theta)+\frac{1}{12}\,d^{\,J=2}_{-1,+1}(\theta)-\frac{2}{15}\,d^{\,J=3}_{-1,+1}(\theta)\right]\sum_a i f^{a_3a_4a}\,\big(T^a\big)_{\alpha_2\alpha_1},
\end{aligned}
\tag{A.205}
$$

$$
\begin{aligned}
&\mathcal{M}_{\rm 2nd}(q_1^-,\overline{q}_2^+\to\widetilde{g}_3^+,\widetilde{g}_4^-)\\
&\simeq -\,4g^2M_{\rm s}^2\,\frac{1}{s-2M_{\rm s}^2}\left[-\frac{1}{20}\,d^{\,J=1}_{-1,-1}(\theta)+\frac{1}{12}\,d^{\,J=2}_{-1,+1}(\theta)+\frac{2}{15}\,d^{\,J=3}_{-1,+1}(\theta)\right]\sum_a i f^{a_3a_4a}\,\big(T^a\big)_{\alpha_2\alpha_1}.
\end{aligned}
\tag{A.206}
$$

Therefore, matrix elements of g^{**} are

$$
\begin{aligned}
F_{g^{**}}{}^{J=1\,a}_{\pm1/2,\mp1/2;\,a_3,a_4} &= \ \ \frac{3}{\sqrt{5}}\,gM_{\rm s}\,f^{a_3a_4a},\\
F_{g^{**}}{}^{J=2\,a}_{\pm1/2,\mp1/2;\,a_3,a_4} &= \mp\sqrt{\frac{5}{3}}\,gM_{\rm s}\,f^{a_3a_4a},\\
F_{g^{**}}{}^{J=3\,a}_{\pm1/2,\mp1/2;\,a_3,a_4} &= \ \ \frac{2\sqrt{2}}{\sqrt{15}}\,gM_{\rm s}^2\,f^{a_3a_4a},
\end{aligned}
\tag{A.207}
$$

$$
\begin{aligned}
F_{g^{**}}{}^{J=1\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4} &= \ \ i\,\frac{1}{3\sqrt{5}}\,gM_{\rm s}\,\big(T^a\big)_{\alpha_4\alpha_3},\\
F_{g^{**}}{}^{J=2\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4} &= \mp\,i\,\frac{1}{\sqrt{15}}\,gM_{\rm s}\,\big(T^a\big)_{\alpha_4\alpha_3},\\
F_{g^{**}}{}^{J=3\,a}_{\pm1/2,\mp1/2;\,\alpha_3,\alpha_4} &= -\,i\,\frac{2\sqrt{2}}{\sqrt{15}}\,gM_{\rm s}\,\big(T^a\big)_{\alpha_4\alpha_3}.
\end{aligned}
\tag{A.208}
$$

Then a width of the second string excited gluon with $J=1$ for the process of $g^{**} \to q\bar{q}$ is

$$\begin{aligned} \Gamma^{J=1\,a}_{g^{**}\to q\bar{q}} &= \frac{1}{16\pi\sqrt{2}M_s}\,\frac{1}{2\times 1+1}\sum_{\alpha_3,\alpha_4}\left\{\left|F_{g^{**}}{}^{J=1\,a}_{+1/2,-1/2;\,\alpha_3,\alpha_4}\right|^2+\left|F_{g^{**}}{}^{J=1\,a}_{-1/2,+1/2;\,\alpha_3,\alpha_4}\right|^2\right\} \\ &= \frac{g^2\sqrt{2}M_s}{16\pi\cdot 45}\,\frac{1}{3}\,\mathrm{Tr}\big(T^aT^a\big)\,, \end{aligned} \tag{A.209}$$

that is,

$$\Gamma^{J=1}_{g^{**}\to q\bar{q}} = \frac{g^2\sqrt{2}M_s}{4\pi}\,\frac{1}{3}\,\frac{N_f}{360}\,, \tag{A.210}$$

and a total width of g^{**} with $J=1$ is from eq.(A.66),

$$\Gamma^{J=1}_{g^{**}} = \frac{g^2\sqrt{2}M_s}{4\pi}\,\frac{1}{3}\left(\frac{N}{4}+\frac{N_f}{360}\right). \tag{A.211}$$

B Standard spinor products

Spinor products associated to momenta $\mathbf{k}_i$ and $\mathbf{k}_j$ are defined by

$$\begin{aligned} \langle ij\rangle &\equiv \overline{u}(\mathbf{k}_i,-)\,u(\mathbf{k}_j,+) = \overline{v}(\mathbf{k}_i,+)\,v(\mathbf{k}_j,-)\,, \\ [ij] &\equiv \overline{u}(\mathbf{k}_i,+)\,u(\mathbf{k}_j,-) = \overline{v}(\mathbf{k}_i,-)\,v(\mathbf{k}_j,+)\,. \end{aligned} \tag{B.1}$$

$u(\mathbf{k},\lambda)$ and $v(\mathbf{k},\lambda)$ are Dirac spinor wave functions of massless fermions, as eigenstates of helicity λ. The massless fermion has a four-momentum $k^\mu = (k,\mathbf{k})$ with $\mathbf{k} = |\mathbf{k}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, where $|k| = |\mathbf{k}|$, θ and ϕ are polar and azimuthal angles in the polar coordinate system. In the standard representation, the Dirac spinor wave functions can be chosen as follows,

$$u(\mathbf{k},+) = v(\mathbf{k},-) = \sqrt{k}\begin{pmatrix}\begin{pmatrix} e^{-i\phi/2}\cos\theta/2 \\ e^{i\phi/2}\sin\theta/2\end{pmatrix} \\ \begin{pmatrix} e^{-i\phi/2}\cos\theta/2 \\ e^{i\phi/2}\sin\theta/2\end{pmatrix}\end{pmatrix}\quad,\quad u(\mathbf{k},-) = v(\mathbf{k},+) = \sqrt{k}\begin{pmatrix}\begin{pmatrix} -e^{-i\phi/2}\sin\theta/2 \\ e^{i\phi/2}\cos\theta/2\end{pmatrix} \\ -\begin{pmatrix} -e^{-i\phi/2}\sin\theta/2 \\ e^{i\phi/2}\cos\theta/2\end{pmatrix}\end{pmatrix}. \tag{B.2}$$

When $k>0$, the spinor products $\langle ij\rangle$ are

$$\langle ij\rangle = \sqrt{-2k_i\cdot k_j}\,\frac{\sqrt{2}\big[-\sin\frac{\theta_i-\theta_j}{2}\cos\frac{\phi_i-\phi_j}{2} - i\sin\frac{\theta_i+\theta_j}{2}\sin\frac{\phi_i-\phi_j}{2}\big]}{\sqrt{1-\sin\theta_i\sin\theta_j\cos(\phi_i-\phi_j)-\cos\theta_i\cos\theta_j}}\,, \tag{B.3}$$

$$[ij] = \sqrt{-2k_i\cdot k_j}\,\frac{\sqrt{2}\big[+\sin\frac{\theta_i-\theta_j}{2}\cos\frac{\phi_i-\phi_j}{2} - i\sin\frac{\theta_i+\theta_j}{2}\sin\frac{\phi_i-\phi_j}{2}\big]}{\sqrt{1-\sin\theta_i\sin\theta_j\cos(\phi_i-\phi_j)-\cos\theta_i\cos\theta_j}}\,, \tag{B.4}$$

and has important properties

$$\big|\langle ij\rangle\big| = \big|[ij]\big| = \sqrt{-2k_i\cdot k_j}\quad,\quad \langle ij\rangle = -\langle ji\rangle\quad,\quad [ij] = -[ji]\,. \tag{B.5}$$

For example, consider a two-body scattering process $1,2\to 3,4$ (1,2 refer to incoming particles, while 3,4 to outgoing particles). In the center-of-mass frame, momenta of the incoming particles can be set along the z axis, and that of the outgoing ones can be set in the x-z plane. Suppose that a scattering angle in this process is θ, polar and azimuthal angles of each particle momentum, θ_i and ϕ_i, can be taken

$$
\begin{aligned}
k_1^\mu &= k(1,0,0,1) &\Rightarrow&\quad (\theta_1,\phi_1) = (0,0)\,,\\
k_2^\mu &= k(1,0,0,-1) &\Rightarrow&\quad (\theta_2,\phi_2) = (\pi,0)\,,\\
k_3^\mu &= k(1,\sin\theta,0,\cos\theta) &\Rightarrow&\quad (\theta_3,\phi_3) = (\theta,0)\,,\\
k_4^\mu &= k(1,-\sin\theta,0,-\cos\theta) &\Rightarrow&\quad (\theta_4,\phi_4) = (\theta+\pi,0)\,.
\end{aligned}
\tag{B.6}
$$

In the process, spinor products are given as

$$
\langle 12\rangle = \langle 34\rangle = \sqrt{s} \quad , \quad \langle 13\rangle = \langle 24\rangle = \sqrt{-t} \quad , \quad \langle 14\rangle = \langle 32\rangle = \sqrt{-u}\,, \tag{B.7}
$$

where s, t and u are the Mandelstam variables,

$$
\begin{aligned}
s &= -(k_1+k_2)^2 = -2k_1\cdot k_2 = 4k^2\,,\\
t &= -(k_1-k_3)^2 = +2k_1\cdot k_3 = -s\,\frac{1-\cos\theta}{2}\,,\\
u &= -(k_1-k_4)^2 = +2k_1\cdot k_4 = -s\,\frac{1+\cos\theta}{2}\,.
\end{aligned}
\tag{B.8}
$$

C Calculation of four-point open string amplitudes

C.1 Four-point open string amplitudes at disk-level

String amplitudes are correlation functions of the corresponding vertex operators on a given topology of the two-dimensional world-sheet. In this thesis, we use the results of four-point open string amplitudes at tree-level, namely, four-point correlation functions of the open string vertex operators on a boundary of a two-dimensional disk D_2. The S-matrix on D_2 is

$$
S_{D_2}(k_1;k_2;k_3;k_4) = e^{-\lambda}\int_{-\infty}^{\infty} dz_4\,\mathrm{Tr}\Big\langle {}^\star_\star c(z_1)\mathcal{V}(z_1){}^\star_\star\,{}^\star_\star c(z_2)\mathcal{V}(z_2){}^\star_\star\,{}^\star_\star c(z_3)\mathcal{V}(z_3){}^\star_\star\,{}^\star_\star \mathcal{V}(z_4){}^\star_\star\Big\rangle + (k_2\leftrightarrow k_3)\,, \tag{C.1}
$$

where ${}^\star_\star\ {}^\star_\star$ denotes the boundary conformal normal ordering, $c(z_i)$ is the c ghost field and $\mathcal{V}(z_i)$ is the vertex operator, and the trace is for Chan-Paton factors of the vertex operators. We can fix three positions z_1, z_2 and z_3 of the vertex operators, for example, $z_1=0$, $z_2=1$ and $z_3=\infty$. There are six cyclic inequivalent orderings of the four vertex operators, depending on the position of $\mathcal{V}(z_4)$,

$$
\begin{aligned}
S_{D_2}(k_1;k_2;k_3;k_4) = e^{-\lambda}\Big\{&\int_{-\infty}^{0} dz_4\,\mathrm{Tr}\Big\langle {}^\star_\star \mathcal{V}(z_4){}^\star_\star\,{}^\star_\star c(z_1)\mathcal{V}(z_1){}^\star_\star\,{}^\star_\star c(z_2)\mathcal{V}(z_2){}^\star_\star\,{}^\star_\star c(z_3)\mathcal{V}(z_3){}^\star_\star\Big\rangle\\
&+\int_{0}^{1} dz_4\,\mathrm{Tr}\Big\langle {}^\star_\star c(z_1)\mathcal{V}(z_1){}^\star_\star\,{}^\star_\star \mathcal{V}(z_4){}^\star_\star\,{}^\star_\star c(z_2)\mathcal{V}(z_2){}^\star_\star\,{}^\star_\star c(z_3)\mathcal{V}(z_3){}^\star_\star\Big\rangle\\
&+\int_{1}^{\infty} dz_4\,\mathrm{Tr}\Big\langle {}^\star_\star c(z_1)\mathcal{V}(z_1){}^\star_\star\,{}^\star_\star c(z_2)\mathcal{V}(z_2){}^\star_\star\,{}^\star_\star \mathcal{V}(z_4){}^\star_\star\,{}^\star_\star c(z_3)\mathcal{V}(z_3){}^\star_\star\Big\rangle\Big\}\\
&+(k_2\leftrightarrow k_3)\,.
\end{aligned}
\tag{C.2}
$$

Note that in order to cancel the background $\beta\gamma$ ghost number -2 on the disk, the vertex operators are chosen to have the appropriate ghost picture and the total picture must be -2.

C.2 Four gluon amplitudes

The gluon vertex operator with the picture -1 is

$$\mathcal{V}^{(-1)}_{g^a}(z,e,k) = g_A\, T^a e_\mu \psi^\mu(z) e^{-\phi(z)} e^{ik\cdot X(z)}\,, \tag{C.3}$$

and that with the picture 0 is

$$\mathcal{V}^{(0)}_{g^a}(z,e,k) = \frac{g_A}{\sqrt{2\alpha'}}\, T^a e_\mu \big[i\dot{X}^\mu(z) + 2\alpha' k\cdot\psi(z)\psi^\mu(z)\big] e^{ik\cdot X(z)}\,, \tag{C.4}$$

where e_μ is the polarization vector. The picture corresponds to an exponent coefficient of the holomorphic scalar field $\phi(z)$, and $e^{-\phi(z)}$ is the vertex operator of the $\beta\gamma$ NS vacuum $|0\rangle^{\beta\gamma}_{\rm NS}$ in eq.(1.66).[33] This comes from the fact that the $\beta\gamma$ ghosts are bosonized using ϕ and anticommutative fields η and ξ,

$$\beta \cong e^{-\phi}\partial\xi\,, \qquad \gamma \cong e^{\phi}\eta\,. \tag{C.5}$$

Then, the S-matrix is

$$S_{D_2}(\{a_i,e_i,k_i\}) = e^{-\lambda}\int_{-\infty}^{\infty} dz_4\, \mathrm{Tr}\Big\langle {}^\star_\star c(z_1)\mathcal{V}^{(-1)}_{g^{a_1}}(z_1,e_1,k_1){}^\star_\star\, {}^\star_\star c(z_2)\mathcal{V}^{(-1)}_{g^{a_2}}(z_2,e_2,k_2){}^\star_\star \times {}^\star_\star c(z_3)\mathcal{V}^{(0)}_{g^{a_3}}(z_3,e_3,k_3){}^\star_\star\, {}^\star_\star \mathcal{V}^{(0)}_{g^{a_4}}(z_4,e_4,k_4){}^\star_\star \Big\rangle + (k_2 \leftrightarrow k_3)\,. \tag{C.6}$$

The correlation functions of operators are calculated as follows,

$$\Big\langle \prod_{i=1}^{4} {}^\star_\star e^{ik_i\cdot X(z_i)}{}^\star_\star \Big\rangle = iC^X_{D_2}(2\pi)^p\delta^p(\textstyle\sum_i k_i)\displaystyle\prod_{\substack{i,j=1\\ i<j}}^{4} |z_{ij}|^{2\alpha' k_i\cdot k_j}\,, \tag{C.7}$$

$$\Big\langle \prod_{i=1}^{3} {}^\star_\star e^{ik_i\cdot X(z_i)}{}^\star_\star\, {}^\star_\star \dot{X}^\mu(z_4) e^{ik_4\cdot X(z_4)}{}^\star_\star \Big\rangle = -2i\alpha'\sum_{i=1}^{3}\frac{k_i^\mu}{z_{4i}}\Big\langle \prod_{i=1}^{4} {}^\star_\star e^{ik_i\cdot X(z_i)}{}^\star_\star \Big\rangle\,, \tag{C.8}$$

$$\Big\langle \prod_{i=1}^{2} {}^\star_\star e^{ik_i\cdot X(z_i)}{}^\star_\star \prod_{j=3}^{4} {}^\star_\star \dot{X}^{\mu_j}(z_j) e^{ik_j\cdot X(z_j)}{}^\star_\star \Big\rangle = \left[(-2i\alpha')^2\prod_{j=3}^{4}\sum_{i=1,i\neq j}^{4}\frac{k_i^{\mu_j}}{z_{ji}} - 2\alpha'\frac{\eta^{\mu_3\mu_4}}{(z_{34})^2}\right]\Big\langle \prod_{i=1}^{4} {}^\star_\star e^{ik_i\cdot X(z_i)}{}^\star_\star \Big\rangle\,, \tag{C.9}$$

$$\big\langle c(z_1)c(z_2)c(z_3)\big\rangle = C^g_{D_2} z_{12}z_{13}z_{23}\,, \tag{C.10}$$

$$\big\langle {}^\star_\star e^{-\phi(z_1)}{}^\star_\star\, {}^\star_\star e^{-\phi(z_2)}{}^\star_\star \big\rangle = \frac{1}{z_{12}}\,, \tag{C.11}$$

[33]The vertex operator of the $\beta\gamma$ Ramond vacuum $|0\rangle^{\beta\gamma}_{\rm R}$ in eq.(1.67) is $e^{-\phi(z)/2}$.

$$\langle \psi^\mu(z_1)\psi^\nu(z_2)\rangle = \frac{\eta^{\mu\nu}}{z_{12}}\,, \tag{C.12}$$

$$\langle \psi^\mu(z_1)\psi^\nu(z_2) {}^\star_\star \psi^\sigma\psi^\rho(z_3){}^\star_\star\rangle = \frac{1}{z_{13}z_{23}}\big(-\eta^{\mu\sigma}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\sigma}\big)\,, \tag{C.13}$$

$$\begin{aligned}\Big\langle \psi^{\mu_1}(z_1)\psi^{\mu_2}(z_2)\prod_{i=3}^{4} {}^\star_\star\psi^{\nu_i}\psi^{\mu_i}(z_i){}^\star_\star\Big\rangle = {}& \frac{1}{z_{13}z_{24}z_{34}}\big[-\eta^{\mu_1\nu_3}(-\eta^{\mu_2\nu_4}\eta^{\mu_3\mu_4} + \eta^{\mu_3\nu_4}\eta^{\mu_2\mu_4})\\ &\qquad + \eta^{\mu_1\mu_3}(-\eta^{\mu_2\nu_4}\eta^{\mu_4\nu_3} + \eta^{\mu_2\mu_4}\eta^{\nu_3\nu_4})\big]\\ &+ \frac{1}{z_{23}z_{14}z_{34}}\big[+\eta^{\mu_2\nu_3}(-\eta^{\mu_1\nu_4}\eta^{\mu_3\mu_4} + \eta^{\mu_1\mu_4}\eta^{\mu_3\nu_4})\\ &\qquad - \eta^{\mu_2\mu_3}(-\eta^{\mu_1\nu_4}\eta^{\mu_4\nu_3} + \eta^{\mu_1\mu_4}\eta^{\nu_3\nu_4})\big]\\ &+ \frac{1}{z_{12}(z_{34})^2}\,\eta^{\mu_1\mu_2}\big(-\eta^{\mu_3\mu_4}\eta^{\nu_3\nu_4} + \eta^{\mu_4\nu_3}\eta^{\mu_3\nu_4}\big)\,,\end{aligned} \tag{C.14}$$

where $C^X_{D_2}$ and $C^g_{D_2}$ are constants and $z_{ij} \equiv z_i - z_j$. The normalization of the gauge boson vertex operator g_A has the following relation with the gauge coupling constant on the Dp-brane, $g_A = \sqrt{2\alpha'}\, g_{\mathrm{D}p}$, and the universal factor $C_{D_2} \equiv e^{-\lambda}C^X_{D_2}C^g_{D_2}$ is determined so that $C_{D_2} = 1/g^2_{\mathrm{D}p}\alpha'^2$.

Since the S-matrix is described by the invariant amplitude $\mathcal{M}$ as $S = i(2\pi)^p\delta^p(\sum_i k_i)\mathcal{M}$, the disk-level open string amplitude with external four gluons is

$$\begin{aligned}\mathcal{M}(g_1,g_2,g_3,g_4) = 4g^2\,\mathcal{K}(e_1,e_2,e_3,e_4)\bigg[&\frac{1}{su}\,V_t\,\mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_3}T^{a_4} + T^{a_1}T^{a_4}T^{a_3}T^{a_2}\big)\\ &+ \frac{1}{ut}\,V_s\,\mathrm{Tr}\big(T^{a_1}T^{a_4}T^{a_2}T^{a_3} + T^{a_1}T^{a_3}T^{a_2}T^{a_4}\big)\\ &+ \frac{1}{ts}\,V_u\,\mathrm{Tr}\big(T^{a_1}T^{a_2}T^{a_4}T^{a_3} + T^{a_1}T^{a_3}T^{a_4}T^{a_2}\big)\bigg]\,,\end{aligned} \tag{C.15}$$

where g is the gauge coupling constant of strong interaction, $\mathcal{K}(e_1,e_2,e_3,e_4)$ is the kinematical factor defined as

$$\begin{aligned}\mathcal{K}(e_1,e_2,e_3,e_4) = e_{1\mu}e_{2\nu}e_{3\rho}e_{4\sigma}\big[&-tu\,\eta^{\mu\nu}\eta^{\rho\sigma} - us\,\eta^{\mu\rho}\eta^{\nu\sigma} - st\,\eta^{\mu\sigma}\eta^{\nu\rho}\\ &+ 2u\,(\eta^{\mu\nu}k_1^\rho k_2^\sigma + \eta^{\mu\rho}k_1^\nu k_3^\sigma + \eta^{\nu\sigma}k_2^\mu k_4^\rho + \eta^{\rho\sigma}k_3^\mu k_4^\nu)\\ &+ 2s\,(\eta^{\mu\rho}k_3^\nu k_1^\sigma + \eta^{\mu\sigma}k_4^\nu k_1^\rho + \eta^{\nu\rho}k_3^\mu k_2^\sigma + \eta^{\nu\sigma}k_4^\mu k_2^\rho)\\ &+ 2t\,(\eta^{\mu\nu}k_2^\rho k_1^\sigma + \eta^{\mu\sigma}k_1^\nu k_4^\rho + \eta^{\nu\rho}k_2^\mu k_3^\sigma + \eta^{\rho\sigma}k_3^\nu k_4^\mu)\big]\,,\end{aligned} \tag{C.16}$$

and V_s, V_t and V_u are functions of the Mandelstam variables which are defined in Appendix A.

C.3 Two gluon and two quarks amplitudes

The quark and antiquark vertex operators with the picture $-1/2$ are,

$$\mathcal{V}^{(-1/2)}_{q_{\alpha\beta}}(z,u,k) = g_\psi T_{\alpha\beta}u^\lambda\Theta_\lambda(z)e^{-\phi(z)/2}\,\Xi^{a\cap b}(z)e^{ik\cdot X(z)}\,, \tag{C.17}$$

$$\mathcal{V}^{(-1/2)}_{\bar{q}_{\alpha\beta}}(z,\overline{u},k) = g_\psi T_{\alpha\beta}\overline{u}_{\dot\lambda}\Theta^{\dot\lambda}(z)e^{-\phi(z)/2}\,\overline{\Xi}^{a\cap b}(z)e^{ik\cdot X(z)}\,, \tag{C.18}$$

where u^λ and $\overline{u}_{\dot\lambda}$ are the fermionic wave functions, and $\Theta(z)$ is the spin field,

$$\Theta(z) = \prod_{a=0}^{1} \exp\left[is_a H^a(z)\right] . \tag{C.19}$$

The spin field is the vertex operator of the R sector ground state $|\{s_a\};k\rangle_{\mathrm{R}}$ with spin $s_a = \pm\frac{1}{2}$. This comes from the fact that in the R sector, the fermionic operator ψ^μ is bosonized using the holomorphic scalar field H^a,

$$e^{\pm iH^a} \cong \begin{cases} \frac{1}{\sqrt{2}}\left(\pm\psi^0 + \psi^1\right) & a=0 \\ \frac{1}{\sqrt{2}}\left(\psi^{2a} \pm i\psi^{2a+1}\right) & a=1,\ldots,4\,. \end{cases} \tag{C.20}$$

The vertex operators (C.17) and (C.18) correspond to an open string originating from a D-brane intersection (a,b), and they connect two segments of the disk boundary, associated with stacks of D-branes a and b. The field $\Xi^{a\cap b}$ in eq.(C.17) is the fermionic boundary changing operator. In intersecting D-brane models, the intersections are characterized by angles θ_{ab}. Then, $\Xi^{a\cap b}$ and $\overline{\Xi}^{a\cap b}$ can be expressed in terms of bosonic and fermionic twist fields σ and s [73],

$$\Xi^{a\cap b}(z) = \prod_{j=2}^{4} \sigma_{\theta^j_{ab}}(z) s_{\theta^j_{ab}}(z)\,, \qquad \overline{\Xi}^{a\cap b}(z) = \prod_{j=2}^{4} \sigma_{-\theta^j_{ab}}(z) s_{-\theta^j_{ab}}(z)\,, \tag{C.21}$$

where the spin fields

$$s_{\theta^j}(z) = \exp\left[i(\theta^j - \tfrac{1}{2})H^j(z)\right] , \qquad s_{-\theta^j}(z) = \exp\left[-i(\theta^j - \tfrac{1}{2})H^j(z)\right] . \tag{C.22}$$

The S-matrix including two gluons and quarks is

$$\begin{aligned} &S_{D_2}(\{a_i, e_i, k_i; \alpha_i, \beta_i, u_i, k_i\}) \\ &= e^{-\lambda} \int_{-\infty}^{\infty} dz_2 \, \mathrm{Tr}\Big\langle {}^\star_\star c(z_1)\mathcal{V}^{(0)}_{g^{a_1}}(z_1,e_1,k_1){}^\star_\star\, {}^\star_\star \mathcal{V}^{(-1)}_{g^{a_2}}(z_2,e_2,k_2){}^\star_\star \\ &\qquad \times {}^\star_\star c(z_3)\mathcal{V}^{(-1/2)}_{q_{\alpha_3\beta_3}}(z_3,u_3,k_3){}^\star_\star\, {}^\star_\star c(z_4)\mathcal{V}^{(-1/2)}_{\overline{q}_{\alpha_4\beta_4}}(z_4,\overline{u}_4,k_4){}^\star_\star \Big\rangle + (k_2 \leftrightarrow k_3)\,. \end{aligned} \tag{C.23}$$

The correlation functions are calculated as follows,

$$\left\langle {}^\star_\star e^{-\phi(z_1)}{}^\star_\star\, {}^\star_\star e^{-\phi(z_3)/2}{}^\star_\star\, {}^\star_\star e^{-\phi(z_4)/2}{}^\star_\star \right\rangle = \frac{1}{z_{13}^{1/2} z_{14}^{1/2} z_{34}^{1/4}}\,, \tag{C.24}$$

$$\left\langle \psi^\mu(z_1)\Theta_\lambda(z_3)\Theta^{\dot\lambda}(z_4)\right\rangle = \frac{1}{\sqrt{2}}\frac{1}{z_{13}^{1/2} z_{14}^{1/2}} \left(\sigma^\mu\varepsilon\right)_\lambda^{\ \dot\lambda}\,. \tag{C.25}$$

The normalization of the fermion vertex operators g_ψ has the relation with the string coupling, $g_\psi = \sqrt{2\alpha'}\,\alpha'^{1/4} g_{\mathrm{s}}$, and the universal factor is determined so that $C_{D_2} = 1/g_{\mathrm{s}}^2\alpha'^2$.

Then, the disk-level open string amplitude with external two gluons and quarks is

$$\mathcal{M}(g_1, g_2, q_3, \overline{q}_4) = 2g^2\, \mathcal{K}(e_1, e_2, u_3, \overline{u}_4) \left[\frac{1}{u}\, V_t \left(T^{a_1} T^{a_2}\right)_{\alpha_3 \alpha_4} + \frac{1}{t}\, V_u \left(T^{a_2} T^{a_1}\right)_{\alpha_3 \alpha_4} \right], \tag{C.26}$$

where $\mathcal{K}(e_1, e_2, u_3, \overline{u}_4)$ is the kinematical factor defined as

$$\mathcal{K}(e_1, e_2, u_3, \overline{u}_4) = e_{1\mu} e_{2\nu} \left\{ \frac{1}{2}\, k_{1\rho} (u_3 \sigma^\rho \overline{\sigma}^\mu \sigma^\nu \overline{u}_4) + \left[\delta^\nu_\rho k^\mu_3 + \frac{t}{s} (\delta^\mu_\rho k^\nu_1 - \delta^\nu_\rho k^\mu_2 - \eta^{\mu\nu} k_{1\rho}) \right] (u_3 \sigma^\rho \overline{u}_4) \right\}. \tag{C.27}$$

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