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9 Fundamental Nonlocality

A. Kleppe*

SACT, Stensgt. 40 B, Oslo, Norway

Abstract. According to the Random Dynamics approach the structured physical reality that we observe emerges from a fundamentally chaotic and (almost) random primal layer. Properties that we take for granted, like space and time, causality and locality, are all derived. A mild nonlocality is however believed to remain even after the introduction of locality, but this (classical) nonlocality is very different from the quantum nonlocality discussed by Einstein, Rosen, Podolsky, as well as by John Bell. How to get to grips with the classical nonlocality of Random Dynamics, and how is it related to quantum nonlocality?

9.1 Introduction

Locality is a property that is mostly taken for granted in field theory. Perhaps that is the reason why it actually means so seldom is discussed at great length.

We usually think of locality in terms of information being localized, and propagating from one spacetime point to another by at most the speed of light. Nonlocality, on the other hand, refers to a situation where information can spread out ”instantaneously” over a large distance.

In the Random Dynamics approach[1], locality is not perceived as fundamental. The reason is that the primal Random Dynamics ”world machinery” $\mathcal{M}$ is a very general, random mathematical structure which merely contains non-identical elements and some set-theoretical notions. From this $\mathcal{M}$, differentiability and a concept of distance (geometry), as well as space and time, Lorentz invariance, locality, and eventually all other physical concepts, are to be derived.

But even after locality is derived, some smeared out left-over nonlocal effects remain, showing up in coupling constants (which feel an average over spacetime, and also depend on such averages). This remaining (mild) nonlocality is moreover supported by the Multiple Point Principle (MPP)[2].

In a nonlocal theory, the degrees of freedom are functions of more than one spacetime point. This allows for making predictions in a noncausal way, i.e. to get information about parts of the Universe that are at a spacelike interval from ourselves.

At our everyday, classical level, reality is however convincingly ”local”, which is reflected in that

- The Laws of Nature - the equations of physics - are local.
- Special relativity advocates locality.
- The continuity equation tells that there are no jumps!

Interactions are thus generically local, taking place in one spacetime point, implying that one spatio-temporal site x_μ is assigned to each degree of freedom. This is closely related to the idea of causation being $local$: A can influence B provided B is ”within reach”.

* e-mail: astri.kleppe@gmail.com

In a local theory things thus occur locally, and the action can be factorized; $S = S_1 + S_2 + ...$, where each S_j depends on the fields in limited regions of x_μ-spacetime.

This is intuitively graspable, locality seems to be a quite manageable and transparent concept. It is however not such a simple business to establish the conditions for having locality; one should not forget that Isaac Newton struggled with this notion, and locality was in reality only restored with relativity.

The question of the conditions for locality has recently been addressed by Don Bennett and Holger Bech Nielsen, who in the context of their model for explaining the phenomenologically observed spacetime flatness[3], state that to get locality we need:

- reparametrization invariance.
- spontaneous breakdown of translational invariance by new fields.

9.2 Nonlocality in Random Dynamics and the Multiple Point Principle scheme

One can imagine a scenario where nature is nonlocal at a fundamental level, yet effectively local at larger scales. the Multiple Point Principle scheme even postulates nonlocality at extremely long distances, as long as the long-distance nonlocality is invariant under diffeomorphisms or reparametrization.

In a case of extreme nonlocality, a term in the Lagrangian could depend on many space-time points, whereby interactions would occur between all space-time points at once, and the action would no longer be a sum of terms depending on limited space-time regions.

The nonlocality of Random Dynamics and MPP does however not appear in the action, but rather as signals propagated at once all over space and time: *"a mild form of nonlocality consisting of an interaction that is the same between any pair of points in spacetime independent of the distance between these points"*[4].

This means a nonlocality which cannot transmit (information carrying) wave pulses, while it *can* be used to transmit (unmodulated flat) waves which carry no information.

An interaction which is the same between the fields at any pair of spacetime points would however not be perceived as a nonlocal effect (regardless of the distance between the points), but rather as a background phenomenon which so to speak appears in the constants of nature. This is the point of view which is advocated in the Multiple Point Principle scenario. It is argued that since in a local theory, the dynamical (bare) physical constants can only depend on local spacetime points, there is only an (indirect) dependency on the past, but not on the future. How then can a bare cosmological constant at the stage of the Big Bang be finetuned to obtain the tiny value of the dressed cosmological constant of today - it seems to need some kind of advanced fortunetelling anticipating the coming stages of the universe. This can be solved precisely by assuming that the strict principle of locality is broken, with a nonlocality that admits a certain degree of the needed "clairvoyance", i.e. influence from the future. This nonlocality is precisely the "mild", classical nonlocality of Random Dynamics and the MPP scheme.

One may however still stumble over the concept of "an interaction that is the same between any pair of points in spacetime independent of the distance between these points".

9.2.1 Interpreting nonlocality

Locality is expressed in terms of a Lagrangian or an action that depends on one spacetime point, while a nonlocal action is a function of several spacetime points. In the local theory interactions are well localized and causality is well defined, while in the nonlocal case what is near and what is distant becomes blurred, since there can in principle exist connections over very large distances.

Intuitively, physical *distance* is a notion connected to motion, to the transportation from point A to point B. When it's fast to travel between A and B, the distance is small. So when there are correlations between events in two separated points A and B, one may say that the distance between A and B (from a correlation perspective) is small or even irrelevant. A situation where the interaction is the same between any pair of points in spacetime is more extreme.

What is an interaction that is independent of the distance? One can imagine a space with "distance independent" distances - an example being the metric space M with a discrete metric d such that if $x, y \in M$, the distance

$$d(x,y) = \begin{cases} 1 \text{ if } x \neq y \\ 0 \text{ if } x = y \end{cases} \tag{9.1}$$

Another example of a space with "distance independence" is an N-dimensional space where N is very large. In such a space the *expected distance* between a pair of points that are independently selected at random, is approximately $\sqrt{N/6}$. That is, the expected distance between two random points is about the same irrespectively which random points we choose. This result is valid for any bounded N-space (and if the Universe is unbounded, we can always cut out a bounded subspace that will contain our pairs of points).

But it's hard to imagine that the postulated interaction which is "the same between any pair of points in spacetime independent of the distance between these points" should indicate that we live in a N-dimensional space, and instead of overinterpreting geometry, one may question what in this connection is meant by 'interaction'. In the usual understanding, an interaction transmits (energy and) information between two spacetime points A and B. In order to carry information one needs a wave packet (with a beginning and an end) which will always be a superposition of many wavelengths (each with its own phase velocity). For a packet the frequency is some function of the wave length: $\omega = \omega(\lambda)$, and the velocity of the packet (the group velocity) is $d\omega/d\lambda < c$ (which for a real plane wave is zero, since $\omega(\lambda)$ is a constant).

If one instead imagines an (ideal) plane wave with no beginning and no end, which travels between two separated spacetime points A and B with arbitrarily large (i.e. $\gg c$) (phase) velocity (which poses no phenomenological problems, since such an unmodulated wave cannot carry any information between two different points), it cannot be used to create causal events between A and B. This is how the interaction appearing in the definition of the mild nonlocality of Random Dynamics and MPP should be understood, i.e. not an interaction in the causal sense but rather in the sense that a plane wave can "leave" from A and subsequently "arrive" at B, and since "leaving" and "arriving" occur simultaneously one can establish "interactions" (that do not convey information of course) by arbitrarily assigning "departures" and "arrivals" to any pair of spacetime points.

9.3 Quantum nonlocality

The mild nonlocality postulated by Random Dynamics and the Multiple Point Principle, is obviously not the same as the quantum nonlocality which is at hand when measurements on two or more distant quantum systems turn out to be correlated in a way that defies classical description.

Such quantum mechanical effects, interpreted as nonlocal, or non-separable, are the Aharohnov-Bohm effect[5] or the nonlocality discussed by Einstein, Rosen and Podolsky (EPR)[6]. While the Aharonov-Bohm effect implies nonlocality in the equations of motion, the nonlocality discussed by EPR, concerns nonlocal correlations.

When Einstein, Rosen and Podolsky in 1935 criticized the idea of the wave function as a complete description of the system, the argument was that the wave function doesn't capture some local, real properties that are part of the physical (quantum) reality. They argued that the wave function doesn't tell the whole story, and therefor there must be some more - hidden - variables. They in reality stated that quantum theory needs these additional variables to restore causality and locality.

The EPR demand that quantum mechanics should be a complete theory is not a requirement for a deterministic theory, but for a theory such that there is locality and separability for composite systems. Each separate component should thus be characterized by its own separate properties, and it should be impossible to alter the properties on a distant system by acting on a local system.

The classical example brought up by EPR is a spin zero particle which decays into two entangled spin 1/2 particles A and B. When a measurement performed on A results in a definite outcome a, a subsequent measurement on B will have an outcome which is complementary to a. Since there is supposedly no interaction between the states, the anti-correlation of the two outcomes gives the impression of originating from some pre-existing determined values of the measurement results. The situation appears to involve local, real properties that are nevertheless not captured by the wave equation. There seems to be some information that is unaccounted for by quantum theory, which therefore according to EPR, is incomplete.

So is there some kind of information flux between A and B, or are there some hidden variables that are somehow at hand, like a pool of information from which reality serves itself? By formulating the requirements made by Einstein, Rosen and Podolsky as probability constraints, John Bell[7] in 1964 developed a strategy which offers a testable difference between the predictions of quantum theory and the predictions of local hidden variable theories. He showed that the probability constraints are equivalent to the requirement that statistical correlations between separated systems should be reducible to a common cause, and with the assumption of Einsteinian locality and the assumption of physical realism (in the sense that particle properties, i.e. spin, mass, position, etc, are taken to be 'real'), he derived a joint probability distribution for measurements on two separate particles, expressed as an inequality demonstrating that the particles cannot be as strongly correlated as predicted by quantum mechanics.

In Bell's own words, *"in a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that a theory could not be Lorentz invariant"*.

Experimental evidence[8] based on Bell's inequality implies that the local realism favoured by Einstein yields predictions that disagree with those of quantum mechanical theory, thus ruling out hidden variable theories. That is, no physical theory of local hidden variables can reproduce all of the predictions of quantum mechanics.

Bell concluded that we must either accept nonlocality or abandon local realism. In a many-world interpretation, however, the observed correlations do not demand the introduction of nonlocality, since measurements are then allowed to have non-unique outcomes. Thus Bell's conclusion is maybe not absolutely conclusive, and both Bell's assumptions and his conclusion are indeed subject of ongoing discussions[9].

One problem with Bell's reasoning is that it assumes that the measurements performed at each site can be chosen independently of each other and of the hidden variable that is presumeably being measured. In order for the argument for his inequality to follow, Bell assumed counterfactual definiteness, namely that it is meaningful to speak about what the result of the experiment would have been if different choices had been made. In a deterministic theory the measurements chosen by the experimenters at each site, are predetermined by the laws of physics. From a deterministic perspective it thus doesn't make sense to speak about what would have happened if different measurements had been made. The chosen measurements can moreover be determined in advance, and the outcomes of the measurement at one site can be affected by the measurement performed at the other, without information traveling faster than the speed of light.

One way to evade Bell's theorem is therefore to assume superdeterminism, a term describing a class of completely deterministic theories. Since counterfactual definiteness does not apply to deterministic theories, in a (hypothetical) superdetemined theory his assumption is overthrown, and therefore his entire reasoning.

In 1985, John Bell discussed superdeterminism in a BBC interview[10]:

There is a way to escape the inference of superluminal speeds and spooky action at a distance. But it involves absolute determinism in the universe, the complete absence of free will. Suppose the world is super-deterministic, with not just inanimate nature running on behind-the-scenes clockwork, but with our behavior, including our belief that we are free to choose to do one experiment rather than another, absolutely predetermined, including the "decision" by the experimenter to carry out one set of measurements rather than another, the difficulty disappears. There is no need for a faster than light signal to tell particle A what measurement has been carried out on particle B, because the universe, including particle A, already "knows" what that measurement, and its outcome, will be.

9.3.1 Joint measurability

Another issue[12] that has been addressed in the discussions about Bell's theorem, is Bell's assumption of joint measurability, the idea that two properties can be measured without mutual interference.

In a classical theory with hidden variables, with two sites A and B where measurements are performed, Bell introduced a parameter (or strategy) λ, which locally characterizes the measurement outcomes for each system. The local separability postulated by EPR reads

$$P(a, b|A, B, \lambda) = P(a|A, \lambda)P(b|B, \lambda) \tag{9.2}$$

where $P(a|A, \lambda)$ is the probability that given λ, the outcome of the measurement on A is a. Suppose λ has values running over a set λ_j and each λ_j has a probability $\rho(\lambda_j)$ of being selected. Then

$$P(a, b|A, B) = \Sigma_{j=1}^{k} P(a, b|A, B, \lambda_j)\rho(\lambda_j) \tag{9.3}$$

is the joint probability for the measurement results. The correlator $\Sigma_{a,b}P(a, b|A, B)$ represents the expectation that the measurements on A and B are correlated.

If the measurements on the first site can have two outcomes A and A', and the outcomes on the second site are B and B', one form of Bell's inequality[11] states that in a classical theory (i.e. any theory of hidden variables), a certain combination of correlations $E(A, B) + E(A, B') + E(A', B) - E(A', B')$ is limited by

$$-2 \leq E(A, B) + E(A, B') + E(A', B) - E(A', B') \leq 2 \tag{9.4}$$

If these correlations were independent, the absolute value of the sum could be as much as 4; the mathematical formalism of quantum mechanics however predicts a maximum value of $2\sqrt{2}$. The enigma is why the predicted value isn't maximal. It may be due to relativistic causality[13]. If that is the case, nonlocality and causality might determine all of quantum mechanics. This leads to the notion of axiomatic nonlocality.

9.4 Axiomatic nonlocality

The Aharanov-Bohm effect and the EPR paradox both arise within nonrelativistic quantum theory. Locality on the other hand, is closely tied up with relativity, since a local variable only causes events within its future light cone, just as it can only be caused by events in its past light cone. It is thus conceivable that nonlocality is brought about by quantum mechanics and relativity taken together[13].

Special relativity is as non-nonlocal and causal as anything can be - causality literally resides within the walls of the light cone. Quantum theory, on the other hand, doesn't altogether satisfy the locality principle, but however nonlocal quantum correlations may be, they still preserve relativistic causality in the sense that they cannot be used to transmit signals, i.e. no measurement results are so correlated that they allow signaling between two distant systems. So even if relativity and nonlocality together seem *almost* impossible, it looks like quantum mechanics somehow reconcile them.

One way of examining the relation between quantum mechanics and relativity is to consider the "level underneath", and formulate an axiomatic basis for quantum theory in analogy with the axioms underlying relativity. Special relativity can be deduced from two axioms:

- the equivalence of two inertial reference frames
- the constancy of the speed of light

Imagining an analogous axiomatic basis for quantum physics, we can attempt to deduce quantum theory from:

- relativistic causality
- nonlocality

By formulating nonlocality as an axiom, one no longer has to explain it; on the other hand, quantum indeterminacy and limits on measurements may now appear as a consequence of the presence of a nonlocal action.

If nonlocality is accepted as physical reality, relativity however implies causal ambiguity. In the EPR system, when something has an effect on A, the wavefunction of B should be 'simultaneously' effected. But in relativity, simultaneity is an ambiguous concept, the succession of events will for example in the case of the EPR experiment, depend on the chosen reference frame (in some frame the measurement on A precedes the measurement on B, while in another frame the course of events is reversed).

The attempts to derive a theory from axiomatic causality and nonlocality moreover lead to the conclusion that quantum mechanics is not the only theory which emerges from the demand of simultaneous causality and nonlocality. Quantum theory is only one of a class of theories consistent with axiomatic causality and nonlocality, and it is not even the most nonlocal theory. It has been argued that quantum mechanics not only reconciles relativity and nonlocality, but it might also be the *unique* theory combining them.

9.5 Quantum information

Physical information is limited by all kinds of constraints. The transmission of signals is limited by the speed of light; and erasure of information is a dissipative process involving compression of phase space, and therefore irreversible.

It was Szilard who in 1929 invented the concept of a bit of information, the acquisition of one bit being associated with the entropy $\Delta S = k\ln 2$, since it involves choosing from two possibilities, 0 or 1.

The corresponding unit of quantum information is the qubit, which is a vector in a 2-dimensional complex vector space with inner product. In hommage to the bit concept, the elements of an ON basis in this space are called $|0>$ and $|1>$; and a normalized vector can be represented as $|\psi>= a|0>+b|1>$, $|a|^2+|b|^2=1$, where a, b are complex. One can perform measurements that project $|\psi>$ onto the basis ($|0>$, $|1>$), with non-deterministic outcomes: the probability of obtaining $|0>$ or $|1>$ is $|a|^2$ and $|b|^2$, respectively.

The quantum state of N qubits can be expressed as a vector in a space of dimension 2^N, and with an ON basis for this space where each qubit has a definite value $|0>$ or $|1>$, the N-qubit quantum state is $|1000110...0110>$.

One difference between classical and quantum information is the no cloning theorem: it is impossible to clone a quantum state, because quantum information cannot be copied with perfect fidelity. *If* it were possible to perfectly clone a quantum state, we could defy Heisenberg by measuring an observable of the copy without disturbing the original. That acquiring quantum information causes a disturbance is thus connected with the no cloning theorem.

9.5.1 Quantum computation and nonlocality

A quantum computer cannot do anything that a classical computer cannot do, but it does everything much much faster.

Quantum computers operate with probabilistic algorithms, meaning that if we run the same program twice we probably obtain different results, because of the randomness of the quantum measurement process. In order to describe the whole N-qubit quantum state, one might try a probabilistic classical algorithm, in which the outcome is not uniquely determined by the input. One may hope for a local simulation in which each qubit has a definite value at each time step, and each quantum gate can act on the qubits in various possible ways.

But Bell's theorem precisely addresses the impossibility of such a project: there is no local probabilistic algorithm that can reproduce the conclusions of quantum mechanics.

The reason is how quantum information is organized. Think of a 3N -qubit quantum system where $N \gg 1$. Choose a random state S of the 3N -system, and then divide the system into three subsystems, each with N qubits.

Send the subsystems to different locations in the world, say Paris, Copenhagen and Bled, and then investigate S by making measurements. We imagine having several copies of S to measure on, the only restriction being that the measurements are limited to be carried out within a subsystem - in Paris, in Copenhagen or in Bled; no collective measurements outside of the subsystems' boundaries are allowed. Then for a typical state of the 3N -qubit system, the measurements will tell almost nothing about S. Almost all the information that distinguishes one state from another resides in the nonlocal correlations between measurement outcomes in subsystems. These are the nonlocal correlations which Bell recognized as an essential part of the physical description.

If we choose a state for the 3N qubits randomly, we almost always find that the entropy of each subsystem is very close to $S = N - 2^{(N+1)}$. N is thus the maximal possible value of the entropy, corresponding to the case in which the subsystems carries no exponentially small amount of information when looking at each subsystem separately.

So measurements reveal very little information if we don't consider how the measurement results in Paris, Copenhagen and Bled were correlated with each other. The correlations are in a way part of a 'collective' measurement, and with knowledge of the correlations we can in principle completely reconstruct the state.

Nonlocal correlations are however very fragile and tend to rapidly decay. One reason is that the quantum system is in contact with a 'heat bath', namely its surrounding. Interactions between a quantum system and its environment establish nonlocal correlations between them, and the quantum information that was initially encoded in a device with time become encoded in the correlations between the device and its surrounding; and then the information is in reality lost.

An example is Schrödinger's cat: the state $|cat >= (|dead > +|alive >)$ is in principle possible, but it is seldom observed, because it is so extremely unstable.

Once the state $|cat >$ is prepared, the quantum information encoded in the superposition of $|dead >$ and $|alive >$ will immediately be transferred to correlations between $|cat >$ and the environment, and becomes completely inaccessible. To measure on $|cat >$ inevitably means to project it onto the state $|alive >$ or the state $|dead >$.

It was actually Schrödinger who coined the term entanglement to describe this peculiar connection between quantum systems[14]: *When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled.*

...

"Another way of expressing the peculiar situation is: the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts, even though they may be entirely separate and therefore virtually capable of being best possibly known, i.e., of possessing, each of them, a representative of its own. The lack of knowledge is by no means due to the interaction being insufficiently known – at least not in the way that it could possibly be known more completely – it is due to the interaction itself."

9.6 Nonlocality in Random Dynamics

In Random Dynamics, nonlocality is perceived as the fundamental state of affairs. The notion of axiomatic quantum nonlocality avocates a similar approach. In the course of the Random Dynamics derivation of physics from the fundamental set $\mathcal{M}$, we should therefore take into account the idea of quantum mechanics being the unique theory that encompasses both nonlocality and relativity, indeed reconciling them.

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