

PERTURBATIVE STABILITY OF THE QCD ANALYSIS OF DIS DATA

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We perform pQCD analysis of the existing DIS data for charged leptons with account of corrections up to the NNLO. The parton distributions, value of strong coupling constant, and high-twist terms are extracted and their stability with respect to account of the NNLO corrections is analyzed. All the quantities are generally stable within their experimental errors. Obtained value of the strong coupling constant is $\alpha_s^{\text{NNLO}}(M_Z) = 0.1143 \pm 0.0014(\text{exp})$ with a guess $\alpha_s^{\text{NNNLO}}(M_Z) \sim 0.113$.

Perturbative method is a powerful tool of the modern quantum field theory. In particular, analysis of the deep-inelastic-scattering (DIS) data in terms of perturbative QCD (pQCD) allows for quantitative description of this process and extraction of parton distribution functions (PDFs) together with value of the strong coupling constant α_s , which can be used for calculation of the cross sections for other processes with hadronic beams and targets and check of self-consistency of Standard Model. The common practice for analysis of such kind is to take into account only the $O(\alpha_s)$ (or next-to-leading-order (NLO)) corrections to the DIS cross sections since the next-to-next-to-leading-order (NNLO) corrections has not been completely calculated yet. Meanwhile the value of α_s is rather large at the values of transferred momentum Q typical for existing DIS data and the higher-order (HO) correction may have impact on the value of extracted quantities. Particular important is to know the HO PDFs that is motivated by the need to calculate precise value of the Higgs boson production cross section since these calculations require account of the HO QCD corrections¹ and corresponding HO PDFs as consistent input. Account of the HO corrections is also important for reducing the total uncertainty in the value of α_s determined from the existing DIS data since the theoretical error due to HO corrections dominates the error in value of α_s obtained in the NLO approximation². Besides, account of HO corrections is necessary for clarification of nature of the $1/Q^2$ terms, which are apparent in the NLO analysis of DIS data. These terms correspond to the high-twist (HT) contributions, but also can be simulated by the HO contributions since they are proportional to factors $[\alpha_s(Q)]^n$

and also fall with Q . If such simulation does take place, the HT terms observed in NLO should decrease with account of the HO corrections (see Ref.³ and references therein).

The NNLO corrections to the DIS coefficient functions have been calculated about decade ago, but substantial progress in calculation of the corrections to the anomalous dimensions has been achieved only recently with estimation of their Mellin moments up to 14-th⁴. This input reduces uncertainties in the values of corresponding splitting functions up to $O(\%)$ in the region of x covered by existing DIS data that would provide reasonable level of theoretical uncertainties due to incomplete knowledge of the NNLO evolution kernel in the analysis of these data⁵. We perform the QCD fit to the existing data on DIS of charged leptons off proton/deuteron targets⁶ with account of the available QCD corrections to the coefficient and splitting functions up to the NNLO. Details of the analysis are published elsewhere⁷. We fitted PDFs, value of α_s and the twist-4 contributions, which are included to the fitted structure functions $F_{2,L}$ in additive form: $F_{2,L} = F_{2,L}^{\text{LT,TMC}} + H_{2,L}(x)/Q^2$, where $F_{2,L}^{\text{LT,TMC}}$ are the leading-twist terms with account of the target mass corrections and $H(x)$ are parameterized in the piece-linear form with x -spacing equal to 0.1. The principal feature of our analysis is quantitative estimation of the experimental errors in fitted parameters. These errors give natural scale for estimation of stability of the fit with respect to different corrections: Effect with the magnitude less than corresponding experimental error cannot be considered as significant since at this level it can be simulated by fluctuation of the data.

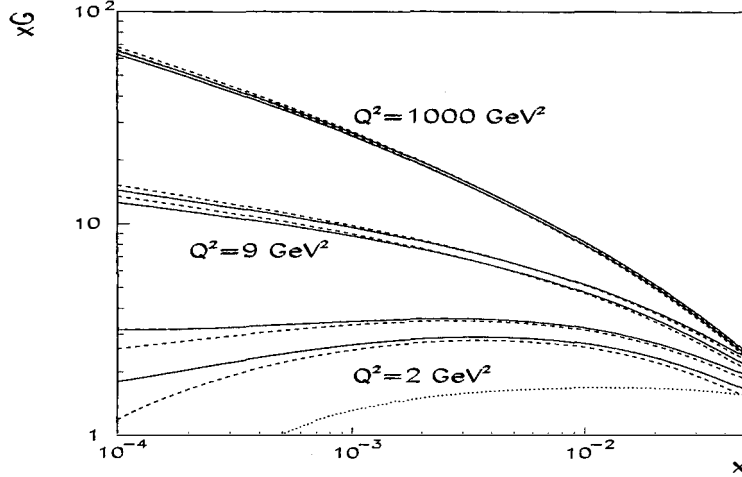


Figure 1: Experimental error bands for the gluon distributions obtained in our NNLO (full), NLO (dashes), and in the NNLO MRST analysis (dots).

The NNLO PDFs obtained in our analysis are generally comparable to the NLO ones within their experimental errors. Perturbative stability of the gluon distribution at small x is demonstrated in Fig.1. For comparison we give in the same figure the gluon distribution obtained in the recent NNLO analysis of Ref.⁸. The latter is smaller than ours and gets negative at $x \sim 10^{-4}$ at low Q^2 , contrary to ours.

The values of α_s obtained in different pQCD orders are given in Table 1. The associated renormalization scale (RS) error is calculated as the shift of α_s under variation of the QCD RS

Table 1: Values of $\alpha_s(M_Z)$ obtained in different orders of pQCD.

LO	$0.1301 \pm 0.0026(\text{exp}) \pm 0.0149(\text{RS})$
NLO	$0.1171 \pm 0.0015(\text{exp}) \pm 0.0033(\text{RS})$
NNLO	$0.1143 \pm 0.0014(\text{exp}) \pm 0.0009(\text{RS})$

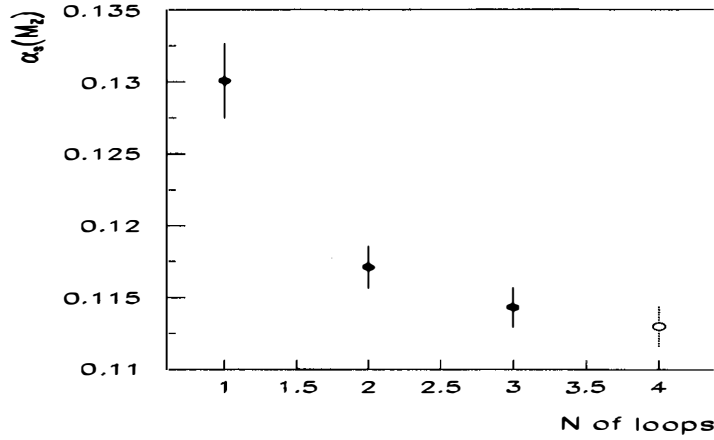


Figure 2: Values of $\alpha_s(M_Z)$ and their experimental errors obtained in different orders of pQCD (full symbols). Open symbol shows extrapolation to the NNNLO.

from Q^2 to $4Q^2$. This shift is regularly considered as the uncertainty due to inaccount of the HO corrections. Such estimate is very crude since the the range of variation of the RS is conventional and no possible x -dependence of the RS is taken into account. Nevertheless, comparing the RS errors to the corresponding change of $\alpha_s(M_Z)$ with the orders of pQCD one can convince that they coincide with good accuracy. Extrapolation of this regularity to the NNNLO illustrated in Fig:2 leads to the estimate $\alpha_s^{\text{NNNLO}}(M_Z) \sim 0.113$. The theoretical error in α_s , which is dominated by the RS error is larger than the experimental error in the NLO. In the NNLO the RS error gets smaller than experimental one. Probably this is valid for the NNNLO too and thus both the central value and the error in α_s extracted from the existing DIS data in NNNLO would be the about the same as in NNLO.

The HT terms are also stable with respect to the NNLO corrections (see Fig.3). They do decrease from LO to NLO and from NLO to NNLO, but in the second case the shift is smaller and is comparable to the experimental error. The magnitude of the H_2^p at $x = 0.6$ and $Q^2 = 5 \text{ GeV}^2$ is $\sim 10\%$ of the LT term. At lower Q^2 corresponding to the resonance region the relative contribution of the HT terms is even larger⁹. The value of H_2^p is comparable to the experimental error in F_2^p up to $Q^2 \sim 20 \text{ GeV}^2$, i.e. an analysis of these data without account of the HT terms is biased if one does not apply very stringent cut in Q^2 . This result is in disagreement with the conclusion of Ref.¹⁰ that the HT term in F_2 vanishes in the NNLO. Meanwhile this disagreement may be insignificant since the analysis of Ref.¹⁰ is based on the model-dependent determination of the HT terms and quality of the data description is poor ($\chi^2/\text{NDP}=1375/926$ versus $2521/2274$ in our fit). Note also that the HT term in the nonsinglet part of F_2^p obtained in the NNLO analysis of Ref.¹¹ does not vanish. In the NNLO analysis of Ref.³ vanishing of the HT term in the neutrino-nucleon structure function xF_3 was observed, but

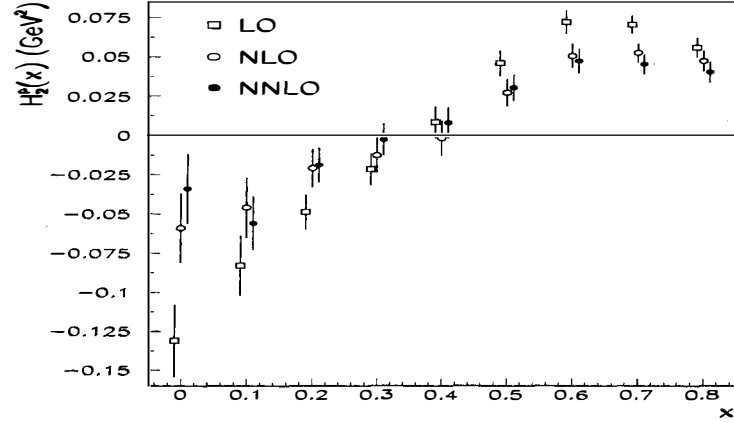


Figure 3: The twist-4 contribution to the proton structure function F_2^p obtained in the different orders of pQCD.

in this case we see no disagreement with our results since error in H_3 obtained in Ref.³ is about order of magnitude larger than magnitude of H_2 obtained in our analysis. If the magnitudes of H_2 and H_3 are not very different, the conclusive study of the latter is possible only if it is determined with the precision $O(0.01)$ GeV^2 , which can be achieved in experiments with luminosities typical for the proposed neutrino factories¹².

In conclusion, we observe relative perturbative stability of the QCD analysis of existing data on DIS of charged leptons: The change of PDFs, α_s , and HT terms due to NNLO corrections is generally of the order of the experimental errors in these quantities.

Acknowledgments

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