Neutrino oscillations in Stueckelberg semiclassical relativistic dynamics

L.P. Horwitz[a,b,c] **and I. Aharonovich**[b]

[a] School of Physics, Tel Aviv University, Ramat Aviv 69978, Israel

[b] Department of Physics, Bar Ilan University, Ramat Gan 52900, Israel

[c] Department of Physics, Ariel University Center of Samaria 40700, Israel

Abstract. The Stueckelberg formulation of a manifestly covariant relativistic classical and quantum mechanics is briefly reviewed and it is shown that in this framework a simple (semiclassical) model exists for the description of neutrino oscillations. The model is shown to be consistent with the field equations and the Lorentz force (developed here without and with spin by canonical methods) for Glashow-Salam-Weinberg type non-Abelian fields interacting with the leptons. We discuss a possible fundamental mechanism, in the context of a relativistic theory of spin for (first quantized) quantum mechanical systems, for CP violation. The model also predicts a possibly small "pull back,", *i.e.*, early arrival of a neutrino beam, for which the neutrino motion is almost everywhere within the light cone, a result which may emerge from future long baseline experiments designed to investigate neutrino transit times with significantly higher accuracy than presently available.

1. Introduction

In 1941 Stueckelberg [1] considered the possibility that in classical mechanics, a timelike particle worldline can, due to interaction, curve sufficiently in spacetime to evolve continuously into a particle moving backward in time (see Figure 1). This configuration was interpreted as representing particle-antiparticle annihilation in classical mechanics. He parametrized this curve with an invariant parameter τ, which may be thought of as a world time[1] associated with the evolution. The time t of the *event* moving on a worldline of the type of Figure 1, corresponding to its identification by Einstein as a coordinate of the Minkowski spacetime manifold, is understood to be an observable in the same sense as the space coordinates $\mathbf{x}$,*i.e.*, as the outcome of a *measurement*, and the motion of an event in the eight dimensional phase space $\{x^\mu, p^\mu\}$ is assumed to be governed by a Hamiltonian dynamics. The Hamiltonian for a freely moving event was taken by Stueckelberg to be

$$K = \frac{p^\mu p_\mu}{2M},\tag{1}$$

---

[1] In 1973, Piron and Horwitz [2] generalized Stueckelberg's theory to be applicable to many body systems by assuming τ to be a universal world time essentially identical to the time postulated by Newton. The classical Kepler problem was solved in [2] by replacing the two body nonrelativistic potential $V(r)$ by $V(\rho)$, where ρ is the invariant spacelike separation between the two particles, and Arshansky and Horwitz [3] solved the quantum bound state and scattering problems in this context; their method provides a solution, in principle, for any two body problem described nonrelativistically by a symmetric potential function $V(r)$.

where M is an intrinsic property of the event with dimension of mass; it may be thought of as the Galilean limiting mass of the particle [4]. The quantity $m^2 = -p^\mu p_\mu = E^2 - \mathbf{p}^2$ corresponds
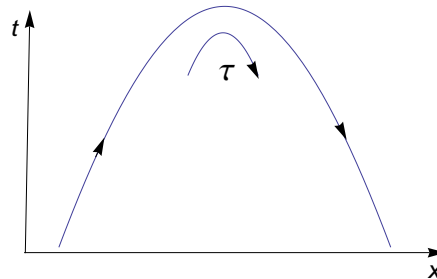
Figure 1. Basic Stueckelberg graph for classical pair annihilation. τ is the world time parameter along the curve.

to the actual measured mass of the associated particle. Since $t, \mathbf{x}$ are dynamical variables, $E, \mathbf{p}$ are also, and the theory is therefore intrinsically "off-shell".

The Hamilton equations are, quite generally,

$$\dot{x}^\mu = \frac{\partial K}{\partial p_\mu} \tag{2}$$

and

$$\dot{p}_\mu = -\frac{\partial K}{\partial x^\mu}, \tag{3}$$

where the dot corresponds to differentiation with respect to τ (thus, for the free particle, $d\mathbf{x}/dt = \mathbf{p}/E$, the standard relativistic definition of velocity).

Stueckelberg [1] wrote a Schrödinger type equation to describe the evolution of the corresponding quantum state for a general Hamiltonian function of the quantum variables x^μ, p^μ (which satisfy the commutation relations $[x^\mu, p^\nu] = ig^{\mu\nu}$, with metric signature $(-, +, +, +)$)

$$i\frac{\partial \psi_\tau(x)}{\partial \tau} = K\psi_\tau(x), \tag{4}$$

where we denote $x \equiv (t, \mathbf{x})$. Since the wavefunction is coherent in both space and time, the theory predicts interference in time [5] in the same way that the nonrelativistic quantum theory predicts interference in space. An experiment of this type has been done by Lindner *et al* [6] (this experiment was discussed in the context of this theory by Horwitz [7]). The extension of the theory to describe particles with spin [8,9] introduces a foliation of the quantum mechanical Hilbert space as well as the corresponding quantum field theory as a consequence of the spin-statistics relation [10] (Palacios *et al* [11] have proposed an experiment which could be able to detect the persistence of the entanglement of two particles with spin at different times, as is consistent with this theory).

This foliation, which can be put into correspondence with the orientation of the spacelike surfaces [10] forming the support of the complete set of local observables providing the representations of quantum fields [12], is involved in a fundamental way in CP (or T) conjugation, as we shall discuss below, and therefore intrinsically associated with the phenomenon of CP violation (see, for example, refs. [13] for a discussion of neutrino flavor oscillations and a comprehensive treatment of the CP violation problem). Similar considerations can be applied to CP and T violations in the K, B and D systems [14]. There have been many discussions in

the framework of quantum field theory [15], one of the earliest that of Weinberg [16], providing a mechanism through coupling with a scalar field. It will be of interest to investigate the relation between our observations in the framework of the first quantized theory with these studies.

Under a gauge transformation, for which ψ is replaced by $e^{i\Lambda}\psi$ (see, $e.g.$, Yang and Mills [17]), this equation remains of the same form if one replaces p^μ by $p^\mu - ea^\mu$, where e is here the coupling to the 5D fields, and $i\frac{\partial}{\partial\tau}$ by $i\frac{\partial}{\partial\tau} + ea^5$, with a^5 a new scalar field, and, under gauge transformation, the gauge compensation field a^α is replaced by $a^\alpha - \partial^\alpha\Lambda$, for $\alpha = 0, 1, 2, 3, 5$ [18]. In the following, we describe a model non-Abelian gauge theory, in which the fifth gauge field plays a fundamental role, for neutrino oscillations. We study the structure of a Lorentz invariant and gauge covariant Hamiltonian and Lagrangian for a particle in interaction with a non-Abelian gauge field, and the resulting field equations and Lorentz force, both without and with spin. We point out mechanisms in this structure that can lead to CP symmetry violation on a fundamental level.

2. The model
In the flavor oscillations of the neutrino system, interactions with the vector bosons of the Glashow-Salam-Weinberg (GSW) theory [19] which induce the transition can produce pair annihilation-creation events. In the framework of Stueckelberg theory, pair annihilation and creation events can be correlated, as shown in Figure 2, by following the world line.[2] The
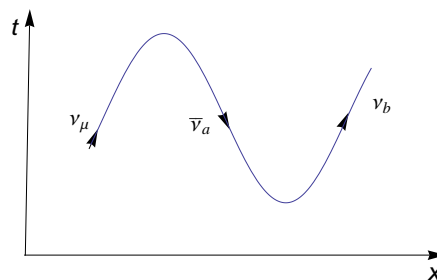
Figure 2. Annihilation-creation model for oscillation. ν_μ is the muon neutrino and ν_a, ν_b could be muon or electron neutrinos; the vector meson fields acting at the vertices can induce transitions.

methods of Feynman's original paper, based on a spacetime picture [20], closely related to Stueckelberg's earlier formulation, would admit such a construction as well. An "on-shell" version of our Figure 2 appears, with sharp vertices, in the first of ref. [20].

It may be noted from this figure that there is a net decrease in the time interval, possibly very small, observed for the particle to travel a certain distance. This mechanism is quite different from that discussed by Floyd [21], Matone [22], and Faraggi [23], discussing the phenomenon of early arrival as a purely quantum mechanical effect. The most recent experiments [24] have shown that the arrival times are consistent with light speed, $e.g.$, in the OPERA experient [24], over the 732 km distance form CERN to Gran Sasso, an arrival time of $6.5 \pm 7.4\{^{+8.3}_{-8.0}\}ns$ less than light speed arrival is reported, certainly consistent with light speed. [3]

---

[2] The curve shown in Figure 2 should be thought of as corresponding to the expectation values computed with the local density matrix associated with the gauge structured wave function of the neutrino beam.

[3] It has been observed in the Supernova 1987a that the neutrinos arrive about 3 hours before the light signal [25]. It has been argued that the light is delayed, for example, by gravitational effects on the virtual electron-positron pairs [26], but arguments of Lorentz invariance [27] and the apparently universal applicability of the geodesic behavior of light provide some difficulties for this view. On the other hand, an advanced arrival of the order

Suppose, for example, that such oscillations can occur twice during this transit [28] as in Figure 3. The particles (and antiparticles) have almost everywhere propagation speed less than
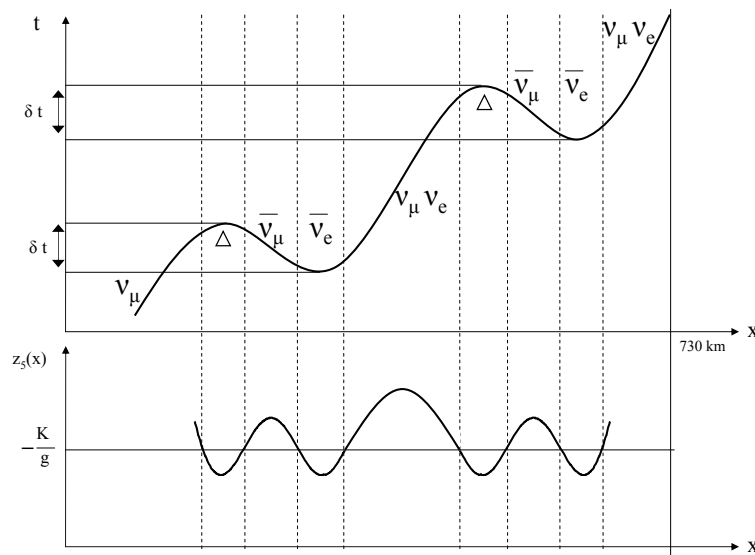
Figure 3. Two oscillation processes with sketch of required form of z^5 field, which must pass through the value K/g to admit passage through the light cone. δt indicates the "pull-back" time at each annihilation-creation event.

light velocity (except for the vertices, which we estimate, based on the Z, W lifetimes, to occur in about 10^{-22} seconds); it is clear from Figure 3 that an early arrival would *not* imply, in this model, that the neutrinos travel faster than light speed. The effect noted by Glashow and Cohen [29], indicating that $\hat{\text{C}}$erenkov radiation would be seen from faster than light neutrinos, would likely not be observed from the very short lived vertices, involving interaction with the W and Z fields, without sensitive detectors placed appropriately on the track [4]

A quantum mechanical counterpart of this model, in terms of Ehrenfest wave packets, is consistent with this construction. The derivation of the Landau-Peierls relation [31] $\Delta p \Delta t \geq \frac{\hbar}{2c}$ in the framework of the Stueckelberg theory [32] involves the assumption that the energy-momentum content of the propagating wave function contains predominantly components for

of 6.5 ns in each 730 km (consistent with this data) would result in approximately 3×10^3 hours early arrival. However, as we shall see below, the mechanism for the oscillations associated with such a "pull-back" involves the participation of the fifth field in an essential way, expected to fall off far from sources. One may estimate on the basis of a 3 hour early arrival the range of effectiveness of the fifth field, assuming an advance of 6.5 ns in each 730 km where effective. A simple estimate yields about 30 parsec(pc), as an effective size of the supernova. The Sun is about 10^4 pc from the center of the galaxy, so an effective range of about 30 pc is not unreasonable. This argument is certainly not a proof of a "pull back" ; it is meant to show that a small effect of this type could be consistent with the supernova 1987a data (see, moreover, further discussion in [25]).

[4] The neutrino arrivals detected at Gran Sasso appear to be almost certainly normal particles. The ICARUS detector (Antonello *et al* [30]) records no γ's or $e^+ e^-$ pairs which would be expected from $\hat{\text{C}}$erenkov radiation from faster than light speed neutrinos. Our model is consistent with the presence of neutrinos for which the total travel time is closely bounded by light velocity, since the "'pull-back" effect can be very small, limited (as in Figure 3) by the interval involved in the oscillation transitions.

which $\frac{p}{E} < 1$. Interactions, *e.g.*, at the vertices of the curve in Figure 3, can affect this distribution in such a way that, for some (small) interval of evolution, the wave packet can contain significant contributions to the expectation value of p/E much larger than unity, and thus the dispersion Δt in the Landau-Peierls relation can become very small without violating the uncertainty bound established by $< E/p >$. The interaction vertex may then be very sharp in t, admitting a precise manifestation of the deficit time intervals.

The upper part of Figure 3 shows schematically the orbit of a neutrino in spacetime during its transit, according to this theory, in which the first(annihilation) event results in the transition from a ν_μ to either a ν_μ or ν_e through interaction with a GSW boson (for this simple illustration we consider only the μ and e neutrinos, although there is no reason to exclude the τ neutrino) and the second (creation) event involves a transition from either of these states back to a ν_μ, ν_e state.

3. Formulation of the non-Abelian model
The gauge covariant form of the Stueckelberg Hamiltonian, valid for the non-Abelian case as well as for the Abelian, with coupling g to the 5D fields, is

$$K = \frac{(p^\mu - gz^\mu)(p_\mu - gz_\mu)}{2M} - gz^5(x), \tag{5}$$

where the z^μ fields are non-Abelian in the $SU(2)$ sector of the electroweak theory. Since, as we shall below, $\dot{x}^\mu$ is proportional to $p^\mu - gz^\mu$, the local expectation of the square of the "proper time" is proportional to that of the first term in the Hamiltonian. Therefore, see we see that the local expectation of z^5 must pass through that of the conserved value of $-K/g$ to admit passage of the orbit through the light cone. In the lower part of Figure 3, we have sketched a form for a smooth z^5 wave (in expectation value) that would satisfy this condition. Such a wave can be easily constructed as the superposition of a few harmonic waves with different wavelengths (originating in the spectral density of the neutrino wave functions [see Eq. (17)]).

The occurrence of such a superposition can be understood from the point of view of the structure of the $5D$ GSW fields. Working in the context of the first quantized theory, where the functions ψ belong to a Hilbert space $L^2(x, d^4x) \otimes d$, with d the dimensionality of the gauge fields ($d = 2$ corresponds to the Yang-Mills case [17] and the $SU(2)$ sector of the electroweak theory which we shall deal with here; our procedure for extracting the field equations and Lorentz force applies for any d), the field equations can be derived from the Lagrangian density (we consider the case of particles with spin in Sec. 5)

$$\begin{aligned}
\mathcal{L} = &\frac{1}{2}\mathrm{Tr}\big(i\frac{\partial\psi}{\partial\tau}\psi^\dagger - i\psi\frac{\partial\psi^\dagger}{\partial\tau}\big) \\
&- \frac{1}{2M}\mathrm{Tr}\big[(p^\mu - gz^\mu)\psi((p_\mu - gz_\mu)\psi)^\dagger\big] \\
&+ g\mathrm{Tr}(z^5\psi\psi^\dagger) - \frac{\lambda}{4}\mathrm{Tr}f^{\alpha\beta}f_{\alpha\beta},
\end{aligned} \tag{6}$$

where ψ is a vector valued function representing the algebraic action of the gauge field, and $\psi^\dagger$ is a 2-component (row) conjugate vector valued function; $\mathcal{L}$ is a local scalar function. The operation Tr corresponds to a trace over the algebraic indices of the fields (the dimensional parameter λ arises from the relation of these fields to the zero mode fields of the usual $4D$ theory [18]). For the variation of the field strengths we take δz^α to be general infinitesimal Hermitian algebra-valued functions. Extracting the coefficients of these variations, with the definition of the non-Abelian gauge invariant field strength tensor [17]

$$f^{\alpha\beta} = \partial^\alpha z^\beta - \partial^\beta z^\alpha - ig[z^\alpha, z^\beta], \tag{7}$$

one obtains the field equations

$$\lambda\big[\partial^\alpha f_{\beta\alpha} - ig[z^\alpha, f_{\beta\alpha}]\big] = j_\beta \tag{8}$$

where

$$j_\mu = \frac{ig}{2M}\{(\partial_\mu - igz_\mu)\psi\psi^\dagger - \psi((\partial_\mu - igz_\mu)\psi)^\dagger\}, \tag{9}$$

and

$$j_5 = g\psi\psi^\dagger \equiv \rho_5. \tag{10}$$

Let us now impose, as done by Yang and Mills [17], the subsidiary condition

$$\partial^\alpha z_\alpha = 0. \tag{11}$$

We then obtain from (8)

$$(-\partial_\tau^2 + \partial_t^2 - \nabla^2)z_\beta = j_\beta/\lambda + ig[z^\alpha, f_{\beta\alpha}], \tag{12}$$

where we have taken the $O(4,1)$ signature for the fifth variable τ. Representing $z_\beta(x,\tau)$ in terms of its Fourier transfrom $z_\beta(x,s)$, with

$$z_\beta(x,\tau) = \int ds\, e^{-is\tau} z_\beta(x,s), \tag{13}$$

one obtains

$$(s^2 + \partial_t^2 - \nabla^2)z_\beta(x,s) = j_\beta(x,s)/\lambda + ig\int d\tau\, e^{is\tau}[z^\alpha(x,\tau), f_{\beta\alpha}(x,\tau)], \tag{14}$$

providing a relation between the off-shell mass spectrum of the z_β field and the sources including the quantum mechanical current as well as the non-linear self-coupling of the fields.

Since the behavior of the z_5 field plays an essential role in the immediately applicable predictions of our model, consider the equation (14) for $\beta = 5$,

$$(s^2 + \partial_t^2 - \nabla^2)z_5(x,s) = j_5(x,s)/\lambda + ig\int d\tau\, e^{is\tau}[z^\nu(x,\tau), f_{5\nu}(x,\tau)]. \tag{15}$$

In a zeroth approximation, neglecting the nonlinear coupling term, we can study the equation

$$(s^2 + \partial_t{}^2 - \nabla^2)z_5(x,s) \cong j_5(x,s)/\lambda. \tag{16}$$

The source term is a convolution of the lepton wave functions in the Fourier space, so that

$$(s^2 + \partial_t{}^2 - \nabla^2)z_5(x,s) \cong \frac{g}{2\pi\lambda}\int ds'\,\psi(x,s')\psi^\dagger(x,s'-s). \tag{17}$$

The Fourier representation over s of the wave function corresponds to the set of probability amplitudes for finding the particle in the corresponding mass states; we expect these functions to peak in absolute value, in free motion, at the measured neutrino masses. There is therefore the possibility of several mass values contributing to the frequency of the spectrum of the z_5 field (the diagonal contributions contribute only to its zero mode, a massless radiative field of essentially zero measure). It is of interest to note that in order for the sources to give rise to a form for the z_5 field of the type illustrated in Figure 3, there must be at least three peaks in the mass distribution of the wave functions, corresponding to three families of neutrinos. This condition has been noted in a somewhat different context in the last of [13] (p. 37) and in other studies (for example refs.[33], discussing the three family structure).

4. The non-Abelian Lorentz force

We now turn to study the quantum trajectories of the particles with non-Abelian gauge interactions to further check the consistency of our model. The Heisenberg equations of motion are associated with expectation values for which the classical motion is a good approximation if the wave packets are fairly well localized.

From the Hamiltonian (5) one obtains

$$
\begin{aligned}
\dot{x}^\lambda &= i[K, x^\lambda] \\
&= \frac{1}{M}(p^\lambda - gz^\lambda),
\end{aligned}
\tag{18}
$$

of the same form as the classical result.

The second derivative is defined by

$$
\ddot{x}^\lambda = i[K, \dot{x}^\lambda] + \frac{\partial \dot{x}^\lambda}{\partial \tau},
\tag{19}
$$

where the last term is necessary because $\dot{x}^\lambda$ contains, according to (18), an explicit τ dependence which occurs in the fields z^λ. One then obtains (the Lorentz force for the non-Abelian case was also obtained, using an algebraic approach, in [34])

$$
\ddot{x}^\lambda = -\frac{g}{2M}\{\dot{x}^\mu, f^\lambda{}_\mu\} - \frac{g}{M}f^{5\lambda}.
\tag{20}
$$

Let us make here the crude approximation that was used in obtaining (16), *i.e.*, neglecting the nonlinear coupling to the spacetime components of the field. Then, (20) becomes, for the time component,

$$
\ddot{t} \cong -\frac{g}{M}\frac{\partial z^5}{\partial t}.
\tag{21}
$$

The rising z^5 field (Figure 3), before the first passage through the light cone, would imply a negative curvature, as required. This consistency persists through the whole process.

We further note that

$$
-\frac{ds^2}{d\tau^2} = \frac{2}{M}(K + gz^5),
\tag{22}
$$

so that

$$
\frac{d}{d\tau}\frac{ds^2}{d\tau^2} = -\frac{2g}{M}\frac{dz^5}{d\tau},
\tag{23}
$$

consistent as well with the form of Figure 3.

5. The Hamiltonian for the spin $\frac{1}{2}$ neutrinos

The original construction of Wigner [35] for the description of relativistic spin, for application to a relativistic quantum theory, as explained in refs. [8] and [9], has a fundamental difficulty in that the resulting Wigner rotation [35] depends on momentum. The action of the resulting little group would depend on this momentum vector and the expectation value of the operator x^μ, $< x^\mu >$ would not be covariant; x^μ acts as a derivative of the momentum and this would destroy the unitarity of the little group action. Inducing the representation on a timelike four vector n^μ [8][9] preserves the covariance of $< x^\mu >$, and also admits the possibility of linear superpositions over momenta preserving the definition of the spin, for example, the construction of wave packets in spacetime of definite spin. It also implies a foliation of the Fock space for identical particles with strong implications for many body systems as well as quantum field theory [10].

The wave functions, in coordinate or momentum space, now related in the usual way by Fourier transform, then transform , under a Lorentz transformation Λ, as

$$\hat{\psi}'_{\tau,\sigma}(x,n) = D(\Lambda,n)_{\sigma\sigma'}\hat{\psi}_{\tau,\sigma'}(\Lambda^{-1}x,\Lambda^{-1}n), \tag{24}$$

where $D(\Lambda,n)$ is the associated Wigner rotation represented here in a fundamental representation of $SL(2,C)$. There are two inequivalent representations of $SL(2,C)$; since the operator $\sigma^\mu p_\mu$ connects these two representations, and such an operator would certainly occur in almost any dynamical theory, the wave function must contain both fundamental representations. Within a simple transformation (see *e.g.*, [9],[10]), a vector-valued function combining the two forms of $L(n)\hat{\psi}(x)$ (where the $SL(2,C)$ matrix $L(n)$ acts to bring $(-1,0,0,0)$ to n^μ) of each type transforms like the Dirac wave function ψ, *i.e.*,

$$\psi'_\tau(x,n) = S(\Lambda)\psi_\tau(\Lambda^{-1}x,\Lambda^{-1}n), \tag{25}$$

where $S(\Lambda)$ is generated in the usual way by

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu,\gamma^\nu]. \tag{26}$$

The norm, constructed of the sum of the norms of the two two-component representations, is then given by[5]

$$\|\psi\|_n^2 = \int d^4x\, \bar{\psi}_\tau(x,n)\gamma\cdot n\psi_\tau(x,n) \tag{27}$$

for each n in the negative light cone. The complete positive definite norm is given by the integral $\int \frac{d^3\mathbf{n}}{n^0}$ over the full foliation (which is required to be convergent on the full Hilbert space).

Following the method of ref. [9] for the non-Abelian case, we find a Hamiltonian of the form

$$K = \frac{1}{2M}(p-gz)_\mu(p-gz)^\mu - \frac{g}{2M}f_{\mu\nu}\Sigma_n^{\mu\nu} - gz^5, \tag{28}$$

where

$$\Sigma_n^{\mu\nu} = \Sigma^{\mu\nu} + K^\mu n^\nu - K^\nu n^\mu \equiv \frac{i}{4}[\gamma_n^\mu,\gamma_n^\nu], \tag{29}$$

with

$$\gamma_n^\mu = \gamma_\lambda\pi_n^{\lambda\mu}; \tag{30}$$

the projection

$$\pi_n^{\lambda\mu} = g^{\lambda\mu} + n^\lambda n^\mu \tag{31}$$

plays an important role in the description of the dynamics in the induced representation. In (28), the existence of projections on each index in the spin coupling term implies that $f^{\mu\nu}$ can be replaced by $f_n^{\mu\nu}$ in this term, a tensor projected into the foliation subspace (see ref. [10] for discussion of the properties of this foliation).

The $\Sigma_n^{\mu\nu}$ generate a Lorentz covariant form of the usual Pauli algebra (the compact $SU(2)$ part of the Lorentz algebra), and the K^μ generate the non-compact part of the Lorentz algebra [9] (since $n^\mu\Sigma_n^{\mu\nu} = K^\mu n_\mu = 0$, there are just three independent K^μ and three $\Sigma_n^{\mu\nu}$).

---

[5] Note that in the Stueckelberg theory, the wave functions provide *local* probability amplitudes associated with particles, unlike the non-local properties pointed out by Newton and Wigner [36] of the Klein-Gordon or Dirac (on shell) functions [2].

The quantities K^μ and $\Sigma_n^{\mu\nu}$ satisfy the commutation relations [9]

$$
\begin{aligned}
[K^\mu, K^\nu] &= -i\Sigma_n^{\mu\nu} \\
[\Sigma_n^{\mu\nu}, K^\lambda] &= -i[(g^{\nu\lambda} + n^\nu n^\lambda)K^\mu - (g^{\mu\lambda} + n^\mu n^\lambda)K^\nu], \\
[\Sigma_n^{\mu\nu}, \Sigma_n^{\lambda\sigma}] &= -i[(g^{\nu\lambda} + n^\nu n^\lambda)\Sigma_n^{\mu\sigma} - (g^{\sigma\mu} + n^\sigma n^\mu)\Sigma_n^{\lambda\nu} \\
&\quad - (g^{\mu\lambda} + n^\mu n^\lambda)\Sigma_n^{\nu\sigma} + (g^{\sigma\nu} + n^\sigma n^\nu)\Sigma_n^{\lambda\nu}].
\end{aligned}
\tag{32}
$$

The last of (32) is the Lie algebra of $SU(2)$ in the spacelike surface orthogonal to n^μ. The three independent K^μ correspond to the non-compact part of the algebra which, along with the $\Sigma_n^{\mu\nu}$ provide a representation of the Lie algebra of the full Lorentz group. The covariance of this representation follows from

$$
S^{-1}(\Lambda)\Sigma_{\Lambda n}^{\mu\nu}S(\Lambda)\Lambda_\mu^\lambda\Lambda_\nu^\sigma = \Sigma_n^{\lambda\sigma}.
\tag{33}
$$

In the special frame for which $n^\mu = (-1, 0, 0, 0))$, Σ_n^{ij} become the Pauli matrices $\frac{1}{2}\sigma^k$ with (i, j, k) cyclic, and $\Sigma_n^{0j} = 0$. In this frame there is no direct electric type interaction with the spin in the minimal coupling model (28) (the theory admits a covariant form of electric coupling of electric dipole type [9]; we will not consider this structure here). We remark that γ^5 commutes with this Hamiltonian, and therefore there is a chiral decomposition (independently of the mass of the neutrinos) that would admit the usual construction of the $SU(2) \times U(1)$ electroweak gauge theory. The $SU(2)$ sector that we discuss below would then apply to the left handed leptons. The asymptotic (free) solutions also admit a (foliated) helicity decomposition [9].

We record here the properties of the wave functions of a particle with spin one half under the discrete symmetries C, P and T, obtained from the Stueckelberg-Schrödinger Eq. (4), with the evolution generator K given by (5)(computed for simplicity for real Abelian gauge fields) [9]:

$$
\begin{aligned}
\psi_{\tau n}^C &= C\gamma^0\psi_{-\tau n}^*(x) \\
\psi_{\tau n}^P(x) &= \gamma^0\psi_{\tau, -\mathbf{n}, n^0}(-\mathbf{x}, t), \\
\psi_{\tau n}^T &= i\gamma^1\gamma^3\psi_{-\tau, \mathbf{n}, -n^0}^*(\mathbf{x}, -t), \\
\psi_{\tau n}^{CP}(x) &= C\psi_{-\tau, -\mathbf{n}, n^0}(-\mathbf{x}, t) \\
\psi_{\tau n}^{CPT}(x) &= i\gamma^5\psi_{\tau, -n}(-\mathbf{x}, -t),
\end{aligned}
\tag{34}
$$

where $C = i\gamma^2\gamma^0$. The CPT conjugate wavefunction, according to its evolution in τ, moves backwards in spacetime relative to the motion of $\psi_{\tau n}$. For a wave packet with $E < 0$ components, which moves backwards in t as τ goes forward, it is the CPT conjugate wavefunction which moves forward with opposite charge, *i.e.*, the observed antiparticle.

We shall discuss the possibilities of CP violation provided by this structure below.

6. Lorentz force for spin $1/2$ particle with non-Abelian gauge interactions
As in (18), one obtains the particle velocity

$$
\dot{x}^\lambda = i[K, x^\lambda] = \frac{1}{M}(p^\lambda - gz^\lambda).
\tag{35}
$$

For the second derivative, from (19) and (32), we obtain

$$
\begin{aligned}
\ddot{x}^\lambda = &-\frac{g}{2M}\{f^{\lambda\mu}, \dot{x}_\mu\} - \frac{g}{M}f^{5\lambda} \\
&+ \frac{g}{2M^2}\partial^\lambda f^n_{\mu\nu}\Sigma_n^{\mu\nu} + \frac{ig^2}{2M^2}[f^n_{\mu\nu}, z^\lambda]\Sigma_n^{\mu\nu}.
\end{aligned}
\tag{36}
$$

The third term of (36) corresponds to a Stern-Gerlach type force. Note that we have included the subscript or superscript n to the quantities that are transverse in the foliation.

Under the assumption that the fields are not too rapidly varying, and again neglecting coupling to the spacetime components of the field z^α, we see that the acceleration of the time variable along the orbit may again be approximated by (21).

7. Non-Abelian field equations with spin

We are now in a position to write the Lagrangian for the full theory with spin. We take for the Lagrangian the form (6) with an additional term for the spin interaction and factors of $\gamma^0(\gamma \cdot n)$ to assure covariance, yielding under variation of $\psi^\dagger$ the Stueckelberg equation for ψ with Hamiltonian (28):

$$
\begin{aligned}
\mathcal{L}_n = {} & \frac{1}{2}\text{Tr}\big(i\frac{\partial \psi}{\partial \tau}\bar{\psi} - i\psi\frac{\partial \bar{\psi}}{\partial \tau}\big)(\gamma \cdot n) \\
& - \frac{1}{2M}\text{Tr}\big[(p^\mu - gz^\mu)\psi\overline{(p_\mu - gz_\mu)\psi}(\gamma \cdot n)\big] \\
& + g\text{Tr}(z^5\psi\bar{\psi}(\gamma \cdot n)) - \frac{\lambda}{4}\text{Tr}f^{\alpha\beta}f_{\alpha\beta} \\
& + \frac{g}{2M}\text{Tr}(f_{\mu\nu}\Sigma_n{}^{\mu\nu}\psi\bar{\psi}(\gamma \cdot n)).
\end{aligned}
\tag{37}
$$

Defining j_α as in (9), (10), but with the factor $\gamma^0\gamma \cdot n$, required for covariance, i.e.,

$$
j_{n\mu} = \frac{ig}{2M}\{(\partial_\mu - igz_\mu)\psi\bar{\psi} - \psi\overline{(\partial_\mu - igz_\mu)\psi}\}(\gamma \cdot n),
\tag{38}
$$

and

$$
j_{n5} = g\psi\bar{\psi}(\gamma \cdot n) \equiv \rho_n,
\tag{39}
$$

the variation of the Lagrangian with respect to the z-fields, where we have used the cyclic properties of matrices under the trace, yields, setting the coefficients of $\delta z^\nu, \delta z^5$ equal to zero, the field equations

$$
\lambda(\partial_\beta f^{5\beta} - ig[z_\beta, f^{5\beta}]) = \rho_n
\tag{40}
$$

and

$$
\begin{aligned}
& \lambda(\partial_\beta f^{\nu\beta} - ig[z_\beta, f^{\nu\beta}]) \\
& = j_n{}^\nu + \frac{g}{M}\Sigma_n{}^{\mu\nu}\{\partial_\mu\rho_n - ig[z_\mu, \rho_n]\}.
\end{aligned}
\tag{41}
$$

Eq. (41) corresponds to a Gordon type decomposition of the current, here projected into the foliation space (spacelike) orthogonal to n^μ. Note that the covariant derivative of ρ_n in the last term is also projected into the foliation space.

With the subsidiary condition $\partial^\beta z_\beta = 0$, as before, we may write the field equations as

$$
\lambda(-\partial^\beta\partial_\beta z^5 - ig[z_\mu, f^{5\mu}]) = \rho_n
\tag{42}
$$

and

$$
\lambda(-\partial^\beta\partial_\beta z^\nu - ig[z_\beta, f^{\nu\beta}]) = j_n{}^\nu + \frac{g}{M}\Sigma_n{}^{\mu\nu}\{\partial_\mu\rho_n - ig[z_\mu, \rho_n]\}.
\tag{43}
$$

Note that the spin coupling is not explicit in (42). Neglecting, as before, coupling to the spacetime components, one reaches the same conclusions for the approximate behavior of the z^5 field, i.e., as determined by Eq.(17) with $\psi^\dagger$ replaced by $\bar{\psi}\gamma \cdot n$. The latter reduces to the same expression for $n^\mu \to (-1, 0, 0, 0)$.

8. CP and T conjugation

The association of this timelike vector with the spacelike surfaces used by Schwinger and Tomonaga [12] for the quantization of field theories has been recently discussed [10]. These spacelike surfaces form the support of a complete set of commuting local observables on which the Hilbert space of states in constructed. It follows from the above properties of the wave functions for a particle with spin, that the CPT conjugate theory would be associated with the same spacelike surface, corresponding to $\pm n^\mu$. However, the CP conjugate, taking $\mathbf{n} \to -\mathbf{n}$ and $n^0 \to n^0$ refers to an entirely different spacelike surface (the time reversed states, for which $\mathbf{n} \to \mathbf{n}$ and $n^0 \to -n^0$ are associated with this spacelike surface as well, with reflected unit timelike vector). The equivalence of the physical processes described in these two frameworks would depend on the existence of an isometry (including both unitary and antiunitary transformations) changing the basis of the space from the set of local observables on the first spacelike surface to those defined on the conjugated surface as well as the equivalence of the physics evolving from it after the CP (or T) conjugation.

The spin coupling term in (28) contains the possibility of CP violation in generating a physics that is inequivalent on the new spacelike surface. The nonrelativistic quantum theory with Zeeman type $\sigma \cdot \mathbf{H}$ coupling is, of course, not invariant under T conjugation. Precisely the same situation is true in the corresponding relativistic equation (28); as we have pointed out, in the special frame in which $n^\mu = (-1, 0, 0, 0)$, the matrices $\Sigma_n^{\mu\nu}$ reduce to Pauli matrices. Under Lorentz transformation they still generate the algebra of $SU(2)$ in a fundamental representation, and therefore still contain the imaginary unit. Therefore, the physical evolution on the CP conjugate spacelike surface is not, in general, equivalent to the original evolution. For this phenomenon to occur, it is necessary that there be present an $f_{\mu\nu}$ field. In addition to self-interaction effects, for which the intrinsic CP violation can be expected to cancel, the Stueckelberg oscillation diagram of Figure 2 suggests the existence of fields present in the equations of motion of the second branch due to the proximity of the accelerated motion in the first branch, thus providing a fundamental mechanism for CP violation. A consequence of this structure is that the physics in the corresponding CP conjugated system of the quantum fields, evolving from the CP conjugate spacelike surface, could be inequivalent.

9. Conclusions

We have argued that, according to the derivation of the Landau-Peierls relation given in ref. [32], the vertices of the neutrino-antineutrino transitions may be very sharp, and provide for a rather precise "pull back" of the time interval. Significantly higher precision than available in the present experiments would be necessary to see such an effect.

We have worked out the equations describing the Lorentz forces and the field equations of the corresponding $(5D)$ non-Abelian gauge theory, with the help of Stueckelberg type Hamiltonians both for the spinless case and for the case of relativistic particles with spin in interaction with such a nonabelian gauge field, and have shown that the conclusions reached are, in lowest approximation, consistent with our simple model. We emphasize that, in the framework of the Stueckelberg model, the dynamics of the fifth gauge field, modulated by the particle mass spectrum contained in the wave function (as in Eq.(17)), plays an essential role for the oscillation process.

The presence of spin, described in the relativistic framework of Wigner [35], as in refs. [9][10], introduces a foliation in the Hilbert space and in the structure of the fields, both classical and quantum. Since, in Tomonaga-Schwinger quantization of the fields, the spacelike surface constructed to define a complete set of local observables is characterized by being orthogonal to the timelike vector n of the foliation [10], the actions of the discrete CP or T transformations change the basis for the construction of the Hilbert space to essentially different spacelike surfaces. Along with the form of the spin coupling term in (28), this suggests a model for

CP or T violation on the first quantized level. Further consequences of this foliation will be explored elsewhere.

We furthermore remark that our model would be applicable to the K, B and D systems [14] as well, manifested by the quark gluon interactions in their substructure. This possibility is under study.

References
[1] E. C. G. Stueckelberg, *Helv. Phys. Acta* 14, 322-323,588-594 (1941); 15, 23-27 (1942).
[2] L. P. Horwitz and C. Piron, *Helv. Phys. Acta* 46, 316-326 (1973).
[3] R. I. Arshansky and L. P. Horwitz, *Jour. Math. Phys.* 30 66,380 (1989).
[4] L. P. Horwitz, W. C. Schieve and C. Piron, *Ann. Phys.* 137, 306-340 (1981).
[5] L. P. Horwitz and Y. Rabin, *Lett. al Nuovo Cim* 17 501 (1976).
[6] F. Lindner, *et al Phys. Rev. Lett.* 95, 040401 (2005).
[7] L. Horwitz, *Phy. Lett. A* 355, 1-6 (2006).
[8] L. Horwitz, C. Piron and F. Reuse, *Helv. Phys. Acta* 48, 546 (1975)
[9] R. Arshansky and L. P. Horwitz, *J. Phys. A: Math. Gen.* 15, L659-662 (1982).
[10] L. P. Horwitz, *Jour. of Phys. A: Math. Theor.* 46 035305 (2013).
[11] A. Palacios, T. N. Rescigno, and C. W. McCurdy, *Phys. Rev. Lett.* 103, 253001-1-253001-4 (2009).
[12] S. Tomonaga, *Prog. Theor. Phys.* 1, 27 (1946), J. Schwinger, Phys. Rev. 74 1439 (1948).
      P. Weiss , Proc. Roy. Soc. A169
      102 (1938), J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, Springer-Verlag (1976).
[13] S. M. Bilenky and B. Pontecorvo, *Phys. Rep.* (Phys. Lett. C) 41, 225 (1978);
      H. Nunokawa, S. Parke and W. F. J. Valle, *Prog. Part. Nucl. Phys.* 60, 338 (2008), arXiv:0710.0554 (2007).
[14] B. Kayser, *The frequency of neutral meson and neutrino oscillation*, SLAC-PUB-7123 (2005)
      Neutrino Physics SLAC Summer Institute on Particle Physics, Aug. 2-13 (2004)
      T. Aaltonen *et al*, *Phys. Rev. D*85, 012009 (2012)
      See also, J. P. Lees *et al*, *Phys. Rev. Lett.* 109, 211801 (2012), arXiv:1207.5832
      B. Bhattacharya, M. Gronau, and J. L. Rosner, *Phys. Rev. D* 85 054014 (2012), arXiv:1207.0761.
[15] For example, E. Sassaroli,*Neutrino flavor mixing and oscillation in Field theory*, arXiv:hep-ph/9805480 (1998)
      Fayyazuddin and Riazuddin, *Modern Introduction to Particle Physics*, World Scientific, Singapore (2012)
      R. N. Mohapatra, *Unification and Supersymmetry*, Springer, New York (1986).
[16] S. Weinberg, *Phys. Rev. Lett.* 37 657(1976)
      R. D. Peccei and H. R. Quinn, *Phys. Rev. D* 16, 1791 (1977).
[17] C. N. Yang and R. Mills , *Phys. Rev.* 96, 191-195 (1954).
[18] D. Saad, L. P. Horwitz and R. I. Arshansky, *Found. Phys.* 19, 1125-1149 (1989)
      I. Aharonovich and L. P. Horwitz, *Jour. Math. Phys.* 47, 122902-1-12290226 (2006), 51, 052903-1-052903-27
          (2010), 52, 082901-1-08290111 (2011), 53, 032902-1-032902-29 (2012)
      Eur. Phys. Lett. 97, 60001-p1-60001-p3 (2012).
[19] S. Glashow, J. Iliopoulis, and L. Maini, *Phys. Rev. D*2, 1285-1292 (1970)
      A. Salam, *Elementary Particle Theory: Relativistic Groups and Analyticity*, 8th Nobel Symposium, 367-377,
          ed. Svartholm N, Almquist and Wiksell, Stockholm (1968)
      S. Weinberg, *Phys. Rev. Lett.* 19, 1264-1266 (1967).
[20] R. P. Feynman, Phys. Rev. 76, 749-784, 769-789 (1949)
      J. Schwinger, *Phys. Rev.* 82, 664-679 (1951).
[21] E. R. Floyd, *Opera superluminal neutrinos per quantum trajectories* arXiv: 1112.4779 (2012).
[22] M. Matone, *Superluminality and a curious phenomenon in the relativistic Hamilton-Jacobi equation* arXiv:
          1109.6631 (2011).

[23] A. Faraggi, *Opera data and the equivalence postulate of quantum mechanics*, arXiv:1110.1857 (2011).

[24] T. Adam *et al*,Jour. of High Energy Physics 10 093 (2012), arXiv:1109.4897
Agafonova N Yu *et al*, *Phys. Rev. Lett.* 109 070801 (2012) arXiv:1208.1392.

[25] W. D. Arnett, J. N. Bahcall, R. P. Kirschner, and S. E. Woosley, *Ann. Rev. Astron. Astrophys* 27, 629 (1989).

[26] J. D. Franson, *Reduced photon velocities in the OPERA neutrino experiment and Supernova 1978a*, arXiv:1111.6986 (2012).

[27] Personal communication, S. Weinberg.

[28] See, for example, J. W. F. Valle, *Journal of Physics: Conference Series* 56, 473-505 (2006).

[29] A. G. Cohen and S. L. Glashow, *Phys. Rev. Lett.* 107, 181803 (2011); arXiv:1109.6562 (2011).

[30] M. Antonello, *et al*, arXiv: 1110.3763 (2011).

[31] L. Landau and R. Peierls, *Z. Phys.* 69, 56-69 (1931).

[32] R. Arshansky and L. P. Horwitz, *Found. of Phys.* 15, 701-715 (1984).

[33] For example, G. L. Fogli, E. Lisi and G. Scioscia, *Accelerator reactor neutrinos oscillation experiments in a simple three-generation framework*, IASSNS-AST 95/28, BARI-TH/219-95, arXiv hep-ph/9506350 (1995)
A. Bandyopadahyay, S. Choubey, S. Goswami and K. Kamales, *Phys. Rev. D* 65 073031 (2002) and arXiv:hep-ph/0110307 (2002). See also ref [14].

[34] M. C. Land, N. Shnerb and L. P. Horwitz, *Jour. Math. Phys.* 36, 3263-3288 (1995).

[35] E. Wigner, *Ann. of Math.* 40, 149-204 (1939).

[36] T. D. Newton and E. Wigner, *Rev. Mod. Phys.* 21, 400-406(1949).