Structure of ground state band of even-even actinide nuclides

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Introduction

The ground state band (GSB) in even-even nuclei refers to sequence of lowest rotational states connected with E2 transitions and built over first I= 0^+ energy state. In many nuclei specially that are close to mid-shell in actinide regions, exhibit regular rotational character in their ground-state band [1]. However, some departure from the rigid rotor behavior could be seen as the level energies of the ground-state band increase with spin. The deviation/departure are due to centrifugal stretching, band mixing and rotational alignments [2]. In present work, we have investigated the rotational structure of GSBs of even-even nuclei in actinide mass region which exhibit a regular energy spacing and forming a rotational ladder like structure with successive increase in level energies using Nuclear Softness Model (NSM) [3].

Model and Methodology

In experiments, the measured quantity is the transition energies i.e. $\Delta E_{I,\ I-2}$ and if we assume even-even nuclei a perfect rigid rotor then the quantum mechanical expression for its rotational energy can be expressed as:

$$E(I)=\frac{\hbar^2}{2\Im}I\left(I+1\right) \qquad (1)$$

where $\Im$ denotes the nuclear moment of inertia and I is the total nuclear angular momentum. In case of even-even nucleus, the nuclear wavefunction vanishes for odd-I, hence only even spins with positive parity are allowed in a rotational band. But we know that the even-even nucleus is not a strict rigid rotor as some vibration with centrifugal stretching and rotational alignments of nucleon angular momentum are expected and hence some deviation from rigid rotor behaviour arises at moderate to high spins. Some of the observed deviation can be answered with addition of vibrational contribution through a term having I^2 dependency as:

$$E(I)=AI\left(I+1\right)-BI^2\left(I+1\right)^2 \qquad (2)$$

The values of A and B can be obtained by minimizing the Chi-square deviation between experimental and calculated energies. However, this formula also has its limitation and less successful in the domains where coupling of rotation and vibration are strong and nuclei are either pure vibrator or gamma-soft. In present work, we explored the applicability of relatively more complete three parameter formula, which incorporate one additional term related with second order perturbation in rotation-vibration coupling. The explicit form of the three-parameter formula used in present paper is written as :

$$E\left(I\right)=AI\left(I+1\right)-BI^2\left(I+1\right)-aI^3\left(I+1\right)^3 \quad (3)$$

Here, the parameters A, a and B in above equation can be determined by minimizing Chi-square deviation among experimental and theoretical data. In this thesis work, the Chi-square deviation using Levenberg-Marquardt Algorithm which is a well-established algorithm to perform nonlinear regression and optimization of physical model parameters This method is an iterative method and we performed the computation using a script written in Origin. Sometime this method fails to find convergence, in such cases we switch to other minimization algorithm such as Simplex minimization.

Results and Discussions

For present work, we have extracted the experimental data of GSB observed of even-even actinide nuclei falling in mass region 220≤A≤230. The experimental data is suitable for present NSM calculations if the band under discussion possess at least 04 experimental energy levels and the band should not possess sharp back-bending or band-crossing. Based on the set criteria, we found total 16 GSBs that are suitable to perform present NSM calculations. But in this paper, we have presented the calculations of 10 GSBs observed in ^{222}Ra, ^{224}Ra, ^{224}Th, ^{226}Ra, ^{226}Th, ^{228}Th, ^{230}Rn, ^{230}Th, ^{230}U, ^{228}Ra nuclides. In order to show the goodness of present calculations, we plotted the variation of energy vs spin using NSM calculations for even-even $^{226-228}$Th nuclides in Figure 1. It is evidenced from this figure, that present NSM calculations with optimized parameter as listed in Table-1 successfully reproduced the experimental energies well within experimental errors.

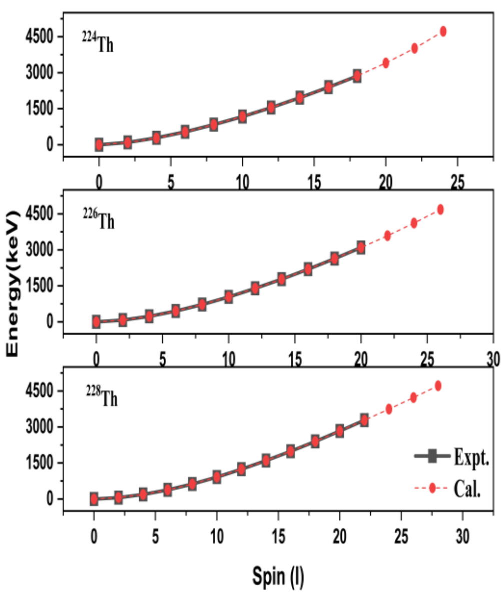

Fig. 1: Comparison of the experimental data [4,5,6] with optimized trajectory for $^{226-228}$Th nuclides

On the basis of good agreement between optimized level spectra and experimental data also supported by small χ^2 deviation, we proposed the location of 09 energy levels which are yet to observed experimentally. Presently estimated energy location will be helpful to future experimental investigation. Inspired from present model results, we have optimized the values of parameters *A*, *B* and *a* for ^{222}Ra, ^{224}Ra, ^{226}Ra, ^{230}Rn, ^{230}Th, ^{230}U and ^{228}Ra nuclides and listed in Table 1. We have also planned to investigate the variation of inertia parameter A (which in inverse of moment of inertia) within isotonic and isotopic chains and will also explore the applicability and reliability of present model calculations in super-heavy territory.

Table 1: Values of optimized parameters for actinide reion.

Nuclide	Optimized Parameters		
	A(keV)	**B(eV)**	**a(eV)**
$^{222}_{88}Ra_{134}$	16.86 (42)	-15.8 (18)	782 (57)
$^{224}_{88}Ra_{136}$	13.15 (19)	-5.65 (4)	411 (18)
$^{224}_{90}Th_{134}$	16.40 (25)	-15.2 (13)	734 (38)
$^{226}_{88}Ra_{138}$	12.10 (1)	-6.52 (1)	409.2 (20)
$^{226}_{90}Th_{136}$	12.72 (3)	-5.84 (14)	389.6 (44)
$^{228}_{90}Th_{138}$	10.24 (1)	-2.09 (1)	218.3 (23)
$^{230}_{88}Rn_{142}$	10.13 (3)	8.9 (19)	186.5 (55)
$^{230}_{90}Th_{140}$	9.24 (30)	9.6 (87)	143 (33)
$^{230}_{92}U_{138}$	9.11 (2)	8.41 (82)	141.7 (29)
$^{228}_{88}Ra_{140}$	11.23(1)	-2.14(0)	252.62(9)

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