ASYMPTOTIC FREEDOM AND QCD: A PROBLEMATIC COEXISTENCE

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ABSTRACT: By two completely independent methods one obtains definite indications that the long distance behaviour (confinement) of QCD is incompatible with Asymptotic Freedom and the validity of perturbative OCD at short distances. Some considerations upon the impact of the above results on our understanding of QCD are also presented.

1.INTRODUCTION

Apart from lattice calculations which are still at the beginning, and whose continuum limit is yet to be clarified, our understanding of QCD is mainly based on the analysis of perturbative QCD (PQCD) and its application to the large class of high energy phenomena that are sensitive to short distance (light cone) physics.

So far it has been taken as an almost [a)] unchallanged dogma that this way of proceeding is the correct one, and the reasonable phenomenology arising from the application of PQCD to short-distance physics has been almost [a)] universally considered as a confirmation of the strong foundations upon which the dogma stands.

Experience has shown that it does not seem likely that a clarification on the fundamental issue of the relevance of PQCD may come from the phenomenology of high energy short-distance physics, due to the severe interpretational obstacle that hadronization poses to any analysis that tries to be as objective as possible.

In order to overcome this impasse we [b)] decided to approach this problem in a completely different way. It is obvious that to assess the validity of PQCD-or better of Asymptotic Freedom (AF) that justifies the use of perturbation theory even in a world where color charges are supposed to be confined-it is necessary to be able to perform a non-perturbative analysis.

Lattice Gauge Theories are clearly a very good framework for carrying out such analysis, but in order to overcome the deficiencies which still affect these calculations (such as finite volume effects, limited β-ranges, etc.) we thought it important to have also an independent, analytical tool to look into this problem. As we shall see, it turns out that this new tool, based on gauge invariant gaussian functionals, proves also very useful to direct the search of

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a) I belong to the very small minority that was never too impressed by the possibility of using PQCD, without having assessed its compatibility with confined QCD.
b) I am refering to my collaborators M.Consoli (see Ref.1) and P.Colangelo, L.Cosmai and M.Pellicoro (see Ref.2,3).

meaningful quantities in lattice Monte Carlo (MC) calculations.

I am now going to describe briefly our calculations as well as the background problematics, and present strong arguments for the invalidity of AF, and hence of perturbation theory, in a non-abelian gauge theory in the unbroken phase.

2. THE DISCOVERY OF "AF/2"

Quite a long time ago, in 1977, G.K. Savvidy made an important discovery. By using the effective potential method to lowest order (one-loop approximation) he found that there exist configurations of the non abelian gauge field, characterized by a non-zero expectation value of the magnetic field H, whose energy density is lower than the energy density of the perturbative ground state (H = 0), and the difference is given by [in SU(2)]

$$\Delta E = E(H) - E(0) = H^2/2 \left(1 - \frac{11}{24\pi^2} g^2 \ln\left(\frac{\Lambda^2}{gH}\right)\right) + O(g^2H^2) \tag{2.1}$$

whose minimum lies at

$$gH^*_s = \frac{\Lambda^2}{e^{\frac{1}{2}}} \exp - \frac{24\pi^2}{11 g^2(\Lambda)} , \tag{2.2}$$

and its value is:

$$\Delta E(H^*_s) = - \frac{11}{96\pi^2} (gH^*_s)^2, \tag{2.3}$$

where g is the bare coupling constant, i.e. the coupling constant at the cut-off Λ, $g(\Lambda)$.

These results are very good news for both AF and confinement. Indeed the existence of a "gluon condensate" characterized by the magnetic field H* is generally thought to be a characteristic signal for confinement and a non-vanishing string tension, while (2.2) is precisely what the perturbative β-function predicts for the evolution of the coupling constant. Eq.(2.2) can in fact be rewritten as

$$g^2(\Lambda) = \frac{24\pi^2}{11 \ln(\Lambda^2/gH^*_s e^{\frac{1}{2}})} , \tag{2.4}$$

which is just the well known 1-loop expression for the running coupling constant; gH^*_s fixes the scale and can be identified with Λ^2_{QCD}.

However one year later, in 1978, Nielsen and Olesen[5] realized that the

Savvidy's state, although lower in energy than the perturbative ground state, is unstable. Such instability is due to the fact that the one-loop effective potential exhibits also an imaginary part, that was overlooked by Savvidy.

Actually the reason for the appearance of an imaginary part, as pointed out by Nielsen and Olesen, is rather simple. The energy density of the Savvidy's vacuum is proportional to the sum of the energies of the "gluon" -modes in an external magnetic field. The problem of a relativistic particle in a homogeneous magnetic field H was solved a long time ago by Landau who showed that the energy levels are labelled by a continuous eigenvalue p_3 , the momentum along the direction of the magnetic field, and a discrete one n, the quantum number of a harmonic oscillator which describes the motion in the plane transverse to the magnetic field. Such levels are given by

$$E_n^2 (p_3) = p_3^2 + gH(2n+1)+2g H S_z ,$$

where the last term is nothing but the Zeeman-splitting associated with a particle with anomalous gyromagnetic ratio g=2 (S_z is the spin component along the magnetic field). Setting now in (2.5) n=0 and $S_z=-1$, one sees that $E_o(p_3)$ becomes imaginary when $p_3^2 < gH$. This is where the instability comes from.

Nielsen and Olesen realized also that most likely the instability was an artifact of the one-loop approximation, and that a better approximation which treated the "unstable mode" (n=0, $S_z=-1$, $p_3^2 < gH$) non-perturbatively would no doubt lead to a stable configuration of lower energy density than the perturbative ground state. It is certainly to be regretted that their remarkable insight was not supported by a theoretical tool precise enough to give unambiguous numerical results; for in this case they would certainly have discovered the strange phenomenon that we have called AF/2.

What is then AF/2 ? It is the very simple fact that when analyzed properly, Savvidy's vacuum is in fact a stable mininum, and the magnetic field is related to the cut-off Λ and to the bare coupling constant $g(\Lambda)$ by the relation:

$$gH_U^* = \frac{\Lambda^2}{e^{\frac{1}{2}}} \exp - \frac{12\pi^2}{11\, g^2(\Lambda)} \qquad (2.6)$$

instead of (2.2). This leads to a running coupling constant

$$g^2(\Lambda) = \frac{12\ \pi^2}{11\ln(\Lambda^2/gH_U^* e^{\frac{1}{2}})} \tag{2.7}$$

which differs by a factor of 2 from the AF-prediction (2.4).

Let us see briefly how this comes about. The technique we have utilized is a well known variational technique based on gaussian wave-functionals, modified such as to obey the Gauss law.

Without entering the rather subtle calculational details I give here only a brief sketch of the steps leading to (2.6) and (2.7)[c].

i. We choose the timelike gauge $A_0^\alpha(\vec{x}) = 0$ [α=1,2,3] and set

$$A_i^\alpha(\vec{x}) = f_i^\alpha(\vec{x}) + \eta_i^\alpha(\vec{x}) , \tag{2.8}$$

i.e. expand the gauge-field around a classical configuration $f_i^\alpha(\vec{x})$, and describe the field configuration by a wave functional

$$\psi\,[A] = \Gamma[A]\ G\,[\eta] \tag{2.9}$$

where

$$G[\eta] = (\det G)^{-1/4} \exp - \frac{1}{4} \int_x \int_y \eta_i^\alpha(\vec{x}) G^{-1}{}_{ij}^{\alpha\beta}(\vec{x},\vec{y})\eta_j^\beta(\vec{y}) \tag{2.10}$$

is a Gaussian functional and $\Gamma[A]$ is determined by gauge invariance only. It is also convenient to choose a transverse gauge, i.e. require that the inverse propagator $\tilde{G}^{-1}$, obeys the transversality condition:

$$D^{\alpha\beta}_j(\vec{x})\ G^{-1\beta\beta'}_{jj'}(\vec{x},\vec{y}) = 0, \tag{2.11}$$

where

$$D^{\alpha\beta}{}_i(\vec{x}) = \delta^{\alpha\beta}\partial_j + g\ \varepsilon^{\alpha\beta\gamma}\ f_j^\gamma(\vec{x}). \tag{2.12}$$

ii. We introduce variational parameters λ_N, by writing

$$G_{ij}^{\alpha\beta}(\vec{x},\vec{y}) = \sum_N \frac{1}{2\lambda_N} \psi^\alpha{}_i(\vec{x})_N\ \psi_j^\beta(\vec{y})_N^* , \tag{2.13}$$

where $\psi_i^\alpha(\vec{x})_N$ are a complete set of eigenfunctions of the operator appearing in the quadratic part of the Hamiltonian [Landau levels] obeying the transversality conditions (2.11,12).

iii. We write the Hamiltonian density as:

$$\mathcal{H}(\vec{x}) = \frac{1}{2} \sum_\alpha (\vec{E}_\alpha^2 + \vec{B}_\alpha^2), \tag{2.14}$$

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c) Actually even though some of the steps in Ref.1 are incorrect, its final results are perfectly valid.

where

$$E_i^\alpha(\vec{x}) = \frac{1}{i}\frac{\delta}{\delta\, \eta_i^\alpha(\vec{x})} \tag{2.14'}$$

and

$$B_i^\alpha(\vec{x}) = \varepsilon_{ijk}\,(\partial_j A_k(\vec{x}) + \frac{g}{2}\,\varepsilon^{\alpha\beta\gamma}A_j^\beta(\vec{x})A_k^\gamma(\vec{y})\,). \tag{2.14''}$$

iv. We choose for $f_i^\alpha(\vec{x})$, a homogeneous magnetic field in the z-direction and along the third isospin axis, i.e. modulo a gauge trf. We write

$$f_i^k(\vec{x}) = \delta^{\alpha 3}\,\delta_{i2}\,x_1\,H. \tag{2.15}$$

With these ingredients it is straightforward to evaluate to lowest order in g^2 the energy density of the state $|\psi>$, and obtain for the energy difference $\Delta E(H)$ the following expression:

$$\Delta E(H) \overset{\sim}{=} \frac{H^2}{2} - \frac{11}{48\pi^2}\,g^2H^2\ln(\Lambda^2/gH) - H^2/2(u-u^2/2) \tag{2.16}$$

where

$$u = \frac{g^2}{4\pi^2}\int_{-1}^{+1}\frac{d\xi}{\xi(x)}, \tag{2.17}$$

and

$$\xi(x) = \frac{1}{\sqrt{gH}}\,\lambda_o\,(x\sqrt{gH}). \tag{2.18}$$

Comparing (2.16) with (2.1) we notice the extra piece $-H^2/2\ (u-u^2/2)$, which arises from the stabilization of the Savvidy's unstable mode occurring in our calculation. By minimizing (2.16) with respect to u we find that the extra-piece yields precisely $-H^2/4$, thus cancelling half of the classical energy $H^2/2$, i.e. we get

$$\Delta E(H) = \frac{H^2}{4} - \frac{11}{48\pi^2}\,g^2H^2\ln(\Lambda^2/gH) + O(g^2H^2). \tag{2.19}$$

Minimization with respect to H now yields (2.6) and (2.7) i.e. AF/2.

Actually to prove that the higher order corrections to (2.19) are $O(g^2H^2)$ is a rather non-trivial task, for a simple analysis of the renormalization of the unstable mode's contribution would lead us to expect a term $O(g^2H_u^2\ln\Lambda^2/gH)$ thus spoiling our central results.

A careful analysis of the renormalization properties of the unstable mode shows however [6)] that the leading logarithmic contribution $O(g^2\ln\Lambda^2/gH)$ simply modifies u appearing in (2.16) to $u_R = u(1-\frac{11}{24\pi^2}\,g^2\ln\Lambda^2/gH)$, thus completely justifying (2.19).

3. AF/2 IN LATTICE GAUGE THEORIES

The oddity of AF/2, which as we shall see in a while has devastating consequences on PQCD , has prompted us to find out whether such a behaviour was also born out by lattice calculations. To this aim we have analyzed two observables:[2,3] the SU(2) internal energy (average-plaquette) and the SU(3) string tension.

We believe that for both quantities we have found unambiguous evidence for AF/2. Let's see how.

(i) SU(2) internal energy

One defines the lattice gauge theory a' la Wilson[7], i.e. writing the action [in SU(N)] as

$$S = \frac{2N}{g^2} \sum_P \left(1 - \frac{1}{N} \operatorname{Tr} U_p\right), \tag{3.1}$$

where U_p is the product of 4 SU(N) transformations associated with the links of a given plaquette. Thus the internal energy, that we wish to analyze, is given by [in SU(2)]

$$E(\beta) = \langle 1 - 1/2 \operatorname{Tr} U_p \rangle , \tag{3.2}$$

where $\beta = 4/g^2$.

By now standard MC techniques we have computed (3.2)[2,3] for several values of β in the interval [2.2,14] on a rather large euclidean lattice of $(16)^4$ sites.

Let's see how can we relate the calculated values of $E(\beta)$ to a non-perturbative "gluon condensate"

$$W(\beta) \xrightarrow[\text{large}\,\beta]{} \langle \frac{\alpha}{\pi} :F^{\alpha}_{\mu\nu}(0)F^{\alpha}_{\mu\nu}(0): \rangle \exp - A\beta \quad . \tag{3.3}$$

Note that if AF is correct, A can be computed and is given by

$$A_{AF} = \frac{12\pi^2}{11} \quad ; \tag{3.4}$$

viceversa if AF/2 is valid A is half the value (3.4). Following the authors of Ref.8 we write

$$E(\beta) = \sum_{k\geq 1} c_k/\beta^k + W(\beta) \quad , \tag{3.5}$$

where the series $\sum_{k\geq 1} c_k/\beta_k$ corresponds to the perturbative expansion, whose first two coefficients have been calculated [$c_1 = 3/4$, $c_2 = .152 \pm .001$ [8]].

In order to determine the unknown perturbative coefficients (c_k,k>3) we look at our MC dara for large values of β and optimize the χ^2 of a fit of the type (3.5). As for W(β) we try a one-parameter exponential parametrization according to AF [Eq.(3.4)] or AF/2. In Fig.1 we report the comparison between the AF and AF/2 predictions obtained by allowing one extra term (c_3) in the perturbative expansion.

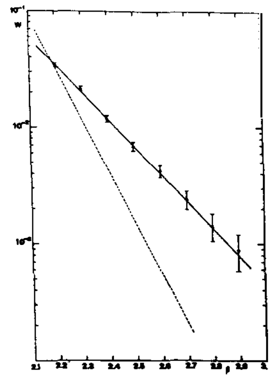

FIG.1 The two parameter- AF/2 fit of the gluon condensate (continuous line) compared with AF (dashed line).

A careful analysis[2,3] shows that AF is still in trouble by adding two more tems (c_4,c_5), and that continuing the addition of perturbative terms, though leading to acceptable χ^2's, produces a negative value for the condensate, which has however unpleasant phenomenological consequences. We may then conclude that accurate MC data on the average plaquette in SU(2) definitely favour AF/2 over AF.

(ii) SU(3) String tension

Wilson showed where to look for confinement in a pure gauge theory; just analyze the behaviour of large "Wilson loops"

$$W\,[C] = \mathrm{Tr}(P \exp i \oint_C dx^\mu A^a_u(x)\lambda^a/2), \tag{3.6}$$

where C is an arbitrary closed contour, and P denotes ordering of the colour transformations along the path C. If the theory is confined one should detect that for large loops W[C] ~ exp (- σArea C), σ is then the string tension, otherwise W[C] is expected to drop off as the perimeter of C.

In Lattice gauge theories what one does is to analyze the quantities:

$$X_{IJ}(\beta) = -\ln \frac{W(I,J)\,W(I-1,J-1)}{W(I,J-1)W(I-1,J)} , \tag{3.7}$$

where W(I,J) is a rectangular Wilson loop with sides containing I and J sites respectively, for different β-values. From general arguments one expects for X_{IJ} a form of the type

$$X_{IJ}(\beta) = C_o(\beta)+C_1(\beta)\,\frac{1}{I(I-1)}+\frac{1}{J(J-1)} + C_2(\beta)\,\frac{1}{I(I-1)J(J-1)}+O(\frac{1}{I^2J},\frac{1}{J^2I}) \tag{3.8}$$

and a non-vanishing $C_o(\beta)$ is the expected signal of confinement, and is associated with the string tension as

$$C_o(\beta) = \sigma a^2 \tag{3.9}$$

where a is the lattice spacing. According to AF, for large β and in SU(3)

$$a(\beta) \to \frac{1}{\Lambda_o} \exp - \frac{4\pi^2}{33}\beta , \tag{3.10}$$

with Λ_o a fixed scale parametter. AF/2 on the other hand implies in the exponential (3.10) a β-coefficient a factor of two smaller.

By utilizing the accurate MC data published by different groups,[9,10] we obtain the results shown in Fig.2. Again AF/2 fares definitely better than AF.

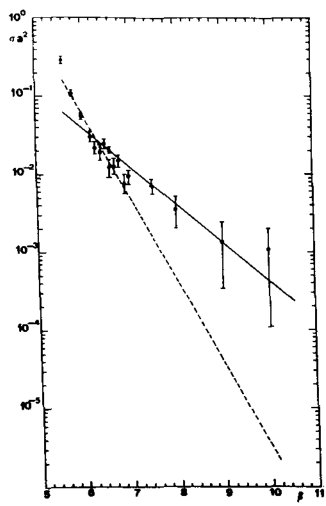

FIG.2 σa^2 as a function of $\beta = 6/g^2$, (●) are from Ref.9, (▲) from Ref.10. The continuous line is the AF/2 prediction, while the dashed line represents AF.

4. THE IMPACT ON OUR UNDERSTANDING OF QCD

I have indicated that two completely different methods strongly suggest that in QCD:

i. Short distances are well described by perturbation theory and AF;

ii. long distances (confinement) are instead controlled by non-perturbative effects which scale according to AF/2.

It is this difformity between short and long distances that turns out to be fatal for PQCD. Let's see why.

Suppose that PQCD is a good representation of short distance physics; the real question is then how short are short distances? In order to answer it we must set a scale, i.e. we must relate the bare-coupling constant $g(\Lambda)$, the ultraviolet cut-off Λ and a fixed scale μ (of the order of 1 GeV) in the only way compatible with PQCD, i.e.

$$g^2(\Lambda) = \frac{48\pi^2}{11N} \frac{1}{\log(\Lambda^2/\mu^2)}, \tag{4.1}$$

the AF-formula for SU(N). But if (4.1) holds, we see from (2.6) that the condensate gH^*_{μ}, as well as the string tension, diverge like $\mu\Lambda$; an obvious nonsense. If on the other hand $g^2(\Lambda)$ is related to μ and Λ by AF/2 (i.e. according to (2.7)) the confinement parameters remain finite, but then short distance physics becomes free, i.e. the perturbative phase gets decoupled from the confined phase. Indeed in the perturbative phase the gluon-coupling constant at the finite scale $g^2(\mu)$ is related to the bare coupling constant (at scale Λ) by the renormalization group equation

$$1/g^2(\mu) = 1/g^2(\Lambda) - \frac{11\ N}{48\pi^2} \ \ell n(\Lambda^2/\mu^2), \tag{4.2}$$

and by imposing the AF/2 -scaling law one gets

$$1/g^2(\mu) = \frac{11\ N}{24\ \pi^2} \ \ell n(\Lambda^2/\mu^2) \underset{\Lambda\to\infty}{\longrightarrow} \infty \tag{4.3}$$

i.e. $\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi^2}$ vanishes at any finite scale μ. Actually according to (4.3) the physics at short distances is better than asymptotically free, it is just free, a fact that I have suggested more than a decade ago [11] and has been at the basis of a rather successful line of research, alternative to PQCD.[12].

Thus the collapse of PQCD implied by AF/2 does not mean that freedom at short distances, the feature of PQCD that most contributed to its centrality in modern high energy physics, gets lost in confined QCD, quite the contrary it emerges in a new and more powerful way.

Where do we go from here? It is clear that much more work and thought is needed before we can draw definite conclusions. However it appears that the central issue of all investigations that will depart from the results discussed in this talk will be to see how our stateswhich clearly violate both rotational and isospin invariance will evolve into the real ground state which apparently violates neither. In any case the fact that on our states the direction of the magnetic field introduces a preferred direction in the dynamics of the gauge-fields appears a strong indication that the ideas of Anisotropic Chromodynamics (ACD)[13] might prove useful in clearing the road to understanding the real QCD.

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