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PAPER

Can spacetime superposition alleviate gravitationally induced quantum decoherence?

Changjing Zeng[1,4], **Qianqian Liu**[1,2,4], **Cuihong Wen**[1,3,*] **and Jieci Wang**[1,*]

[1] Department of Physics, and Collaborative Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha, Hunan 410081, People's Republic of China
[2] Hunan Key Laboratory for Stellar and Interstellar Physics, and School of Physics and Optoelectronics, Xiangtan University, Xiangtan, Hunan 411105, People's Republic of China
[3] College of Information Science and Engineering, Hunan Normal University, Changsha, Hunan 410081, People's Republic of China
[4] These authors contributed equally to this work.
[*] Authors to whom any correspondence should be addressed.

E-mail: cuihongwen@hunnu.edu.cn and jcwang@hunnu.edu.cn

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Abstract

As a combination of the microscopic structure of spacetime and the principle of quantum superposition, the study of spacetime superposition provides a fundamental bottom-up approach to a comprehensive understanding of relativity and quantum theory. In this paper, we study how quantum gravitational effects generated by the superposition of the black hole's masses affect the many-body entanglement for Dirac fields. The main obstacle to performing quantum information processing tasks near a classical black hole is the inevitable entanglement degradation caused by the Unruh–Hawking thermal bath. Fortunately, here we find that the many-body quantum system near a black hole with superposing masses exhibits more entanglement compared to those in the classical black hole background. The superposition properties of spacetime are found to provide additional quantum resources for mitigating gravitationally induced quantum decoherence and improving the efficiency of quantum information tasks in curved spacetime. In addition, the greatest quantum advantage for the recovery of entanglement in the curved spacetime can be obtained by preparing an optimal initial state.

1. Introduction

The cosmic singularity, naked singularities, and the black hole information paradox are among the pressing issues in modern physics, as they imply the incompleteness of general relativity [1–3]. This has spurred physicists' pursuit of a comprehensive theory of gravity, known as quantum gravity theory, aimed at explaining the quantum behavior and microscopic structure of spacetime [4–10]. Over the past few decades, various theories have emerged attempting to establish a self-consistent theory of quantum gravity, with string theory [11, 12] and loop quantum gravity [13] being prominent among them. To gain a deeper understanding of the superposition nature of gravity, recent researchers have employed physical apparatus to define spacetime events and delve into the superposition of spacetime. This includes introducing the concept of quantum reference frames to study gravitational redshift and time dilations in relativistic quantum systems [14], revealing the superposition nature of gravity by studying the gravitational entanglement induced between two massive particles [15], and the violation of classical causal order induced by the superpositions of massive bodies [16]. This series of research efforts have deepened our understanding of the superposition characteristics of gravity and provided a strong support for the development of quantum gravity theory.

General relativity successfully predicts the existence of black holes, and the detection of gravitational waves by the LIGO detector from a binary black hole merger system provides indirect evidence of the presence of black holes [17]. The extreme gravitational environments generated by black holes make them primary candidates for studying quantum gravity effects [18]. The combination of general relativity and quantum information theory gives birth to the field of relativistic quantum information [19–29], which can give out some new understanding of quantum information in relativistic settings. Within the framework of relativistic quantum information, the spacetime structure induced by a black hole, leads to fundamental decoherence in quantum systems, inevitably reducing the performance of quantum protocols near black holes [19–21]. Recently, Foo *et al* investigated the quantum gravity effects produced by a black hole in a superposition of masses on the transition probabilities of a detector [30], which provides an operational framework for the superposition of spacetime. Foo *et al* proposed that the mass superposition of black holes could be utilized as a resource of quantum communication [31]. In addition, it was found that the detector's response exhibits discontinuous resonances at rational ratios of the superposed periodic length scale in the quantum superposed Minkowski spacetime [32]. These studies provide crucial insights into quantum effects in the gravitational environment of black holes. On the other hand, the existence of quantum entanglement is widely considered one of the defining properties of quantum mechanics [33]. Quantum entanglement has been recognized as a foundational resource for quantum computing, quantum cryptography, quantum teleportation, and quantum communication [34–38]. We are therefore motivated by the fundamental question of how the many-body quantum entanglement is changed under the influence of quantum gravitational effects arising from the superposition of spacetime.

In this paper, we study the behaviors of many-body entanglement for a Greenberger–Horne–Zeilinger (GHZ) state near the event horizon of a black hole with superposition mass. The initial state involves freely falling observers Alice and Bob, along with an observer Charlie in uniform acceleration near the event horizon. The observer outside the black hole cannot obtain information about the whole spacetime, resulting in the gravitationally induced quantum decoherence. To explore this scenario in the context of quantum gravity, we assume that the black hole is in a superposition of two different masses, leading to a superposition of spacetimes, causing the initial state to undergo a superposition of quantum channels. We find that the many-body quantum systems near a black hole with superposing masses exhibit more entanglement compared to those in classical black hole spacetime. The quantum properties of spacetime are found to provide additional quantum resources to alleviate gravitationally induced quantum decoherence and improve the efficiency of quantum information tasks in the curved spacetime.

The organization of the paper is as follows. In section 2 we analyze the degradation of many-body entanglement for GHZ states in the background of a classical black hole. In section 3 we discuss how the superposition nature of the spacetime background affects the many-body entanglement for the Dirac field. In the final section, we will summarize our results. Throughout this paper we use $G = c = \hbar = \kappa_B = 1$.

2. Entanglement degradation near the classical black hole event horizon

The Dirac equation in a general background spacetime can be expressed as [39]

$$[\gamma^a e_a{}^\mu (\partial_\mu + \Gamma_\mu)]\,\Psi = 0, \tag{1}$$

where γ^a are the Dirac matrices, the four-vectors $e_a{}^\mu$ represent the inverse of the tetrad $e^a{}_\mu$ defined by $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$ with $\eta_{ab} = \mathrm{diag}(-1,1,1,1)$, and $\Gamma_\mu = \frac{1}{8}[\gamma^a,\gamma^b] e_a{}^\nu e_{b\nu;\mu}$ are the spin connection coefficients. The Schwarzschild black hole is the simplest static classical black hole, and its metric is [22, 40]

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{R}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{R}\right)^{-1}\mathrm{d}R^2 + R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\Psi^2\right), \tag{2}$$

where M is the mass of the black hole. In the background of Schwarzschild spacetime, by substituting the the metric into the Dirac equation equation (1), we can obtain the positive frequency outgoing waves in the exterior and interior regions of the black hole event horizon

$$\Psi_{\mathbf{k}}^{\mathrm{out}+} = \mathcal{G} e^{-i\omega\mathcal{U}},\, \Psi_{\mathbf{k}}^{in+} = \mathcal{G} e^{i\omega\mathcal{U}}, \tag{3}$$

where $\mathcal{G}$ is a 4-component Dirac spinor, and $\mathcal{U} = t - R_*$, with $R_* = R + 2M\ln\frac{R-2M}{2M}$. The positive frequency wave modes $\Psi_{\mathbf{k}}^{\mathrm{out}+}$ and $\Psi_{\mathbf{k}}^{\mathrm{in}+}$ represent Schwarzschild modes, which are only analytically continuous outside and inside the event horizon of the classic black hole, respectively.

In terms of these modes, the Dirac field Ψ can be expanded as

$$\Psi = \int \mathrm{d}\mathbf{k}\left[\hat{a}_{\mathbf{k}}^{\mathrm{out}}\Psi_{\mathrm{out},\mathbf{k}}^{+} + \hat{b}_{-\mathbf{k}}^{\mathrm{out}\dagger}\Psi_{\mathrm{out},\mathbf{k}}^{-} + \hat{a}_{\mathbf{k}}^{\mathrm{in}}\Psi_{\mathrm{in},\mathbf{k}}^{+} + \hat{b}_{-\mathbf{k}}^{\mathrm{in}\dagger}\Psi_{\mathrm{in},\mathbf{k}}^{-}\right], \tag{4}$$

where $\hat{a}_{\mathbf{k}}^{\mathrm{out}}$ and $\hat{b}_{\mathbf{k}}^{\mathrm{out}\dagger}$ are the fermion annihilation and antifermion creation operators acting on the state of the exterior region, and $\hat{a}_{\mathbf{k}}^{\mathrm{in}}$ and $\hat{b}_{\mathbf{k}}^{\mathrm{in}\dagger}$ are the fermion annihilation and antifermion creation operators acting on the interior vacuum of the classical black hole spacetime.

To obtain a set of modes which are analytic everywhere in the spacetime manifold, we construct a different set of operators that are linear combinations of [43, 44]

$$\begin{aligned} \tilde{c}_{\mathbf{k},R} &= \cos r \hat{a}_{\mathbf{k}}^{\text{out}} - \sin r \hat{b}_{-\mathbf{k}}^{\text{in}\dagger}, \\ \tilde{c}_{\mathbf{k},L} &= \cos r \hat{a}_{\mathbf{k}}^{\text{in}} - \sin r \hat{b}_{-\mathbf{k}}^{\text{out}\dagger}, \\ \tilde{c}_{\mathbf{k},R}^{\dagger} &= \cos r \hat{a}_{\mathbf{k}}^{\text{out}\dagger} - \sin r \hat{b}_{-\mathbf{k}}^{\text{in}}, \\ \tilde{c}_{\mathbf{k},L}^{\dagger} &= \cos r \hat{a}_{\mathbf{k}}^{\text{in}\dagger} - \sin r \hat{b}_{-\mathbf{k}}^{\text{out}}, \end{aligned} \tag{5}$$

where $\cos r = \left(\mathrm{e}^{-\frac{\omega}{T}} + 1\right)^{-\frac{1}{2}}$, and T is Hawking temperature of the classical black hole. We denote the constructed modes with subscripts L and R by 'left' and 'right' modes, respectively. After properly normalizing the state vector, the Kruskal vacuum is found to be $|0\rangle_K = \bigotimes_{\mathbf{k}} |0_{\mathbf{k}}\rangle_K = \bigotimes_{\mathbf{k}} |0_{\mathbf{k}}\rangle_R \otimes |0_{\mathbf{k}}\rangle_L$, where $|0_{\mathbf{k}}\rangle_R$ and $|0_{\mathbf{k}}\rangle_L$ are annihilated by the annihilation operators $\tilde{c}_{\mathbf{k},R}$ and $\tilde{c}_{\mathbf{k},L}$. The vacuum state $|0_{\mathbf{k}}\rangle_K$ for a single frequency mode $\mathbf{k}$ is given by

$$\begin{aligned} |0_{\mathbf{k}}\rangle_K = \cos^2 r|0000\rangle - \sin r \cos r|0011\rangle \\ + \sin r \cos r|1100\rangle - \sin^2 r|1111\rangle, \end{aligned} \tag{6}$$

where $|mnm'n'\rangle = |m_{\mathbf{k}}\rangle_{\text{out}}^{+} |n_{-\mathbf{k}}\rangle_{\text{in}}^{-} |m'_{-\mathbf{k}}\rangle_{\text{out}}^{-} |n'_{\mathbf{k}}\rangle_{\text{in}}^{+}$, $\{|n_{-\mathbf{k}}\rangle_{\text{in}}^{-}\}$ and $\{|n_{\mathbf{k}}\rangle_{\text{out}}^{+}\}$ are the orthonormal bases for the inside and outside region of the classical black hole, and the $\{+,-\}$ superscript on the kets is used to indicate the particle and antiparticle vacuum states. The first excited state for the single frequency fermion mode is found to be

$$\begin{aligned} |1_{\mathbf{k}}\rangle_K^{+} &= \left[q_R \left(\tilde{c}_{\mathbf{k},R}^{\dagger} \otimes I_L\right) + q_L \left(I_R \otimes \tilde{c}_{\mathbf{k},L}^{\dagger}\right)\right] |0_{\mathbf{k}}\rangle_K \\ &= q_R \left[\cos r|1000\rangle - \sin r|1011\rangle\right] \\ &\quad + q_L \left[\sin r|1100\rangle + \cos r|0001\rangle\right], \end{aligned} \tag{7}$$

with $|q_R|^2 + |q_L|^2 = 1$. The operator $\tilde{c}_{\mathbf{k},R}^{\dagger}$ in equation (5) indicates the creation of two particles, i.e. a fermion in the exterior vacuum and an antifermion in the interior vacuum of the classical black hole. Similarly, the create operator $\tilde{c}_{\mathbf{k},L}^{\dagger}$ means that an antifermion and an fermion are created outside and inside the event horizon of the classical black hole. In this work, we set $|q_R| = 1$, which means that all the particles move toward the black hole exteriors while all the antiparticles move to the inside region, i.e. only particles can be detected as Hawking radiation.

Initially, three observers Alice, Bob and Charlie, share a tripartite GHZ state

$$|\psi\rangle_{ABC} = \alpha\,|0\rangle_A\,|0\rangle_B\,|0\rangle_C + \sqrt{1-\alpha^2}\,|1\rangle_A\,|1\rangle_B\,|1\rangle_C, \tag{8}$$

where α is the initial state parameter changing from 0 to 1. We assume that Alice and Bob freely fall into the event horizon and will define the vacuum and excited states in the Kruskal coordinates. Charlie remains stationary in a short distance above the event horizon, and will define the vacuum state in the Schwarzschild base. Then the final state under the influence of the classical black hole can be rewritten as

$$\begin{aligned} |\psi\rangle_{ABC_IC_{II}} =& \alpha \cos r|0\rangle_A\,|0\rangle_B\,|0\rangle_{C_I}\,|0\rangle_{C_{II}} + \alpha \sin r\,|0\rangle_A\,|0\rangle_B\,|1\rangle_{C_I}\,|1\rangle_{C_{II}} \\ &+ \sqrt{1-\alpha^2}\,|1\rangle_A\,|1\rangle_B\,|1\rangle_{C_I}\,|0\rangle_{C_{II}}. \end{aligned} \tag{9}$$

Because the outer and inner regions of the classical black hole are causally disconnected, Charlie cannot access modes inside the black hole event horizon. By tracing out the inaccessible mode C_{II}, we can obtain the density matrix

$$\begin{aligned} \xi\left(\rho_{ABC_I}\right) =& \alpha^2 \cos^2 r|000\rangle\langle 000| + \alpha^2 \sin^2 r|001\rangle\langle 001| + \left(1-\alpha^2\right)|111\rangle\langle 111| \\ &+ \alpha \cos r\sqrt{1-\alpha^2}\left(|000\rangle\langle 111| + |111\rangle\langle 000|\right). \end{aligned} \tag{10}$$

In this paper we employ the π-tangle [45, 46] as a measure of the entanglement for the tripartite state, which can be defined as

$$\pi_{\beta\gamma\eta} = \frac{1}{3}\left(\pi_\beta + \pi_\gamma + \pi_\eta\right), \tag{11}$$

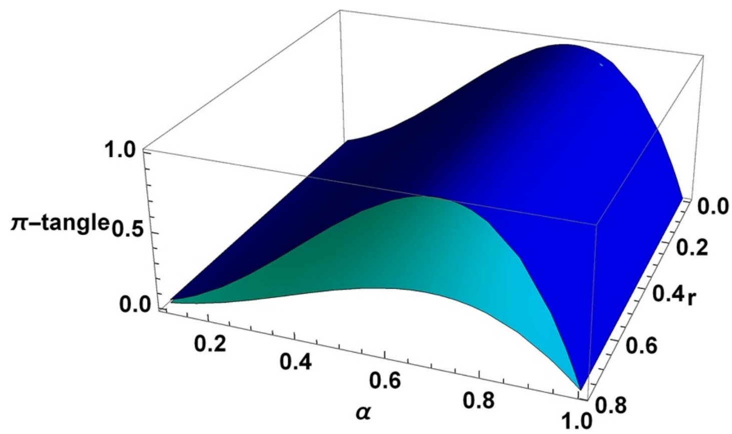

Figure 1. The entanglement $\pi_{\rho_{ABC}}$ for the state ρ_{ABC} in flat spacetime (above) and the entanglement $\pi_{\xi(\rho_{ABC_I})}$ for the state ρ_{ABC_I} (below) as functions of the squeezing factor r and the initial state parameter α.

where

$$\begin{aligned}\pi_\beta &= \mathcal{N}^2_{\beta(\gamma\eta)} - \mathcal{N}^2_{\beta\gamma} - \mathcal{N}^2_{\beta\eta},\\ \pi_\gamma &= \mathcal{N}^2_{\gamma(\beta\eta)} - \mathcal{N}^2_{\gamma\beta} - \mathcal{N}^2_{\gamma\eta},\\ \pi_\eta &= \mathcal{N}^2_{\eta(\beta\gamma)} - \mathcal{N}^2_{\eta\beta} - \mathcal{N}^2_{\eta\gamma},\end{aligned} \tag{12}$$

are the residual entanglement. The $\mathcal{N}_{\beta\gamma}$ is referred to as two-tangle, and it is the negativity of the mixed-state density matrix $\rho_{\beta\gamma} = \mathrm{Tr}_\eta(\rho_{\beta\gamma\eta})$, defined as $\mathcal{N}_{\beta\gamma} = \left\|\rho^{T_\beta}_{\beta\gamma}\right\| - 1$. The term $\mathcal{N}^2_{\beta(\gamma\eta)}$ is one-tangle , defined as $\mathcal{N}_{\beta(\gamma\eta)} = \left\|\rho^{T_\beta}_{\beta\gamma\eta}\right\| - 1$. And T_β denotes the partial transpose of $\rho_{\beta\gamma}$ and $\rho_{\beta\gamma\eta}$ for β subsystem. Here $\|\mathcal{A}\| - 1$ is actually equal to twice the sum of the absolute values of the negative eigenvalues of operator $\mathcal{A}$ [45, 47]. Therefore, one-tangle and two-tangle can be computed by

$$\mathcal{N}_{\beta(\gamma\eta)} = 2\sum_{i=1}^{n}\left|\lambda^{(-)}_{\beta(\gamma\eta)}\right|^i, \mathcal{N}_{\beta\gamma} = 2\sum_{i=1}^{n}\left|\lambda^{(-)}_{\beta\gamma}\right|^i, \tag{13}$$

where $\left[\lambda^{(-)}_{A(BC)}\right]^i$ and $\left[\lambda^{(-)}_{AB}\right]^i$ represent all negative eigenvalues of the partial transpose matrix.

On the other hand, if the black hole is in a classical mass mixture, the joint state of Alice, Rob and Charlie will experience a mixing channel [31]

$$\begin{aligned}\rho_{Av} &= \frac{1}{2}\left[\xi_{11}(\rho_{ABC_I}) + \xi_{22}(\rho_{ABC_I})\right]\\ &= \frac{1}{2} Tr_{C_{II}}\left(|\psi_1\rangle_{ABC_IC_{II}}\langle\psi_1| + |\psi_2\rangle_{ABC_IC_{II}}\langle\psi_2|\right)\\ &= \frac{1}{2}\Big[\alpha^2\left(\mathbb{C}_1^2 + \mathbb{C}_2^2\right)|000\rangle\langle000| + \alpha^2\left(\mathbb{S}_1^2 + \mathbb{S}_2^2\right)|001\rangle\langle001|\\ &\quad + 2\left(1-\alpha^2\right)|111\rangle\langle111| + \alpha\left(\mathbb{C}_1 + \mathbb{C}_2\right)\sqrt{1-\alpha^2}\left(|000\rangle\langle111| + |111\rangle\langle000|\right)\Big],\end{aligned} \tag{14}$$

where $|lmn\rangle = |l\rangle_A|m\rangle_B|n\rangle_{C_I}$, $\mathbb{C}_i = \cos r_i$, $\mathbb{S}_i = \sin r_i$, and

$$\begin{aligned}|\psi_1\rangle_{ABC_IC_{II}} =& \alpha\mathbb{C}_1|0\rangle_A|0\rangle_B|0\rangle_{C_I}|0\rangle_{C_{II}} + \alpha\mathbb{S}_1|0\rangle_A|0\rangle_B|1\rangle_{C_I}|1\rangle_{C_{II}}\\ &+ \sqrt{1-\alpha^2}|1\rangle_A|1\rangle_B|1\rangle_{C_I}|0\rangle_{C_{II}},\\ |\psi_2\rangle_{ABC_IC_{II}} =& \alpha\mathbb{C}_2|0\rangle_A|0\rangle_B|0\rangle_{C_I}|0\rangle_{C_{II}} + \alpha\mathbb{S}_2|0\rangle_A|0\rangle_B|1\rangle_{C_I}|1\rangle_{C_{II}}\\ &+ \sqrt{1-\alpha^2}|1\rangle_A|1\rangle_B|1\rangle_{C_I}|0\rangle_{C_{II}}.\end{aligned} \tag{15}$$

The entanglement difference between the initial state ρ_{ABC} and the $\xi(\rho_{ABC_I})$ after the influence of the classical black hole is plotted in figure 1. We find that, regardless of the value of the initial state parameter α, the entanglement difference monotonously decrease as the acceleration parameter r increases. This means that the tripartite system is influenced by the quantum thermal effect near the event horizon of the classical black hole, leading to a loss of entanglement in the initial state. It is well known that the entanglement degradation will affect the performance of any information processing tasks. For this reason, the thermal bath of the classical black hole will give rise to a fundamental decoherence of quantum systems and a degradation of quantum communication protocols that take place near the black hole's event horizon. Form a relativistic quantum information perspective, the observer Charlie outside the black hole cannot obtain information about the whole spacetime, leading to the loss of quantum entanglement and gravitationally induced quantum decoherence.

3. Many-body entanglement in the spacetime with mass superposition

In this section, we study the behaviors of many-body entanglement in the spacetime with mass superposition. Herein, we assume that the black hole is in a superposition of masses m_1 and m_2, and that the black hole is a supermassive black hole or in a state of equilibrium, allowing us to ignore the evaporation of the black hole. Specifically, Charlie has a quantum machine that can access a control bit. If the control bit is in state $|0\rangle_d$, nothing is done, but if it is in state $|1\rangle_d$, a mass $m = m_2 - m_1$ is thrown into the black hole. At this point, the mass of the black hole becomes m_2. In other words, at the beginning, the black hole is in a single mass state $|0\rangle_m$, and the control qubit is in a superposition state $\left(|0\rangle_d + |1\rangle_d\right)/\sqrt{2}$. The black hole and the control qubit subsequently become an entangled state. If the control qubit is in state $|0\rangle_d$, the black hole remains in state $|0\rangle_m$. However, if the control qubit is in state $|1\rangle_d$, the black hole corresponds to being in state $|1\rangle_m$.

In our model, the black hole mass is in a superimposed state, leading to a superposition of event horizons, which implies that the quantum channel spacetime is also superimposed. The state $|\psi\rangle_{ABC}$ undergoes a superposition of two channels and becomes entangled with the black hole and the control bit

$$|\psi\rangle_{ABC_IC_{II}md} = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle_{ABC_IC_{II}}|0\rangle_m|0\rangle_d + |\psi_2\rangle_{ABC_IC_{II}}|1\rangle_m|1\rangle_1\right), \tag{16}$$

where $|0\rangle_m$ and $|1\rangle_m$ denote the two states of the black hole with masses m_1 and m_2, respectively. By tracing out the wave modes C_{II} from the causally disconnected regions, we can obtain the density operator

$$\begin{aligned}\rho_{ABC_Imd} =& \frac{1}{2}\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)\otimes|00\rangle_{md\,md}\langle 00| + \xi_{22}\left(\rho_{\psi_{ABC_I}}\right)\otimes|11\rangle_{md\,md}\langle 11|\right.\\ &\left.+\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)\otimes|00\rangle_{md\,md}\langle 11| + \xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\otimes|11\rangle_{md\,md}\langle 00|\right],\end{aligned} \tag{17}$$

where

$$\xi_{ij}\left(\rho_{\psi_{ABC_I}}\right) := \mathrm{Tr}_{C_{II}}\left[|\psi_i\rangle_{ABC_IC_{II}}\langle\psi_j|\right], \tag{18}$$

with $i,j \in \{1,2\}$.

We assume that Charlie can perform a projective measurement on the black hole and the control qubit in basis $\{|+\rangle_m, |-\rangle_m\}$ and $\{|+\rangle_d, |-\rangle_d\}$, where $|+\rangle := (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle := (|0\rangle - |1\rangle)/\sqrt{2}$. It is worth mentioning that we currently lack a complete theory of gravity, so it is unclear what the physical implementation of such a measurement on the black hole would be. Additionally, it is uncertain whether a projective measurement can be performed on a black hole in a superposition of masses. To overcome this limit, we assume that Charlie's machine raises the state to a higher energy mode and accesses the control bit again. If the control bit is in state $|0\rangle_d$, the machine throws a mass $m = m_2 - m_1$ into the black hole. If the control bit is in state $|1\rangle_d$, it does nothing. After the operation is completed, the mass state of the black hole will be unaffected by the control qubit, remaining in state $|1\rangle_d$. In this way, the black hole becomes disentangled from the Alice, Bob and Charlie's subsystems. Then the density matrix ρ_{ABC_Imd} can be rewritten as $|1\rangle_{m\,m}\langle 1|\otimes\rho_{ABC_Id}$, where ρ_{ABC_Id} is

$$\begin{aligned}\rho_{ABC_Id} =& \frac{1}{2}\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)\otimes|0\rangle_{c\,c}\langle 0| + \xi_{22}\left(\rho_{\psi_{ABC_I}}\right)\otimes|1\rangle_{c\,c}\langle 1|\right.\\ &\left.+\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)\otimes|0\rangle_{c\,c}\langle 1| + \xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\otimes|1\rangle_{c\,c}\langle 0|\right].\end{aligned} \tag{19}$$

After Charlie measuring the control bit on the base $\{|+\rangle_d, |-\rangle_d\}$, we can get

$$\begin{aligned}\rho^{+}_{ABC_I} =& {}_d\langle +|\rho_{ABC_Id}|+\rangle_d/\mathrm{Tr}\left[{}_d\langle +|\rho_{ABC_Id}|+\rangle_c\right]\\ =& \frac{\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)}{Tr\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\right]},\end{aligned} \tag{20}$$

and

$$\rho^{-}_{ABC_I} = {}_d\langle -|\rho_{ABC_I d}|-\rangle_d/\mathrm{Tr}\left[{}_d\langle -|\rho_{ABC_I d}|-\rangle_c\right]$$
$$= \frac{\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)}{Tr\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\right]}. \tag{21}$$

The joint state of Alice, Bob and Charlie can be written as

$$\rho^{+}_{ABC_I} = \frac{1}{2K}\Big[\alpha^2(\mathbb{C}_1+\mathbb{C}_2)^2|000\rangle\langle 000|+\alpha^2(\mathbb{S}_1+\mathbb{S}_2)^2|001\rangle\langle 001|$$
$$+2\alpha\sqrt{1-\alpha^2}(\mathbb{C}_1+\mathbb{C}_2)(|000\rangle\langle 111|+|111\rangle\langle 000|)$$
$$+4\left(1-\alpha^2\right)|111\rangle\langle 111|\Big], \tag{22}$$

where

$$K=\alpha^2(\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2-1)+2, \tag{23}$$

and

$$\rho^{-}_{ABC_I} = \frac{1}{2Z}\left[\alpha^2(\mathbb{C}_1-\mathbb{C}_2)^2|000\rangle\langle 000|+\alpha^2(\mathbb{S}_1-\mathbb{S}_2)^2|001\rangle\langle 001|\right], \tag{24}$$

where

$$Z=\alpha^2(1-\mathbb{C}_1\mathbb{C}_2-\mathbb{S}_1\mathbb{S}_2). \tag{25}$$

It can be observed that $\rho^{-}_{ABC_I}=\frac{1}{2C}|0\rangle\langle 0|\otimes|0\rangle\langle 0|\otimes\alpha^2\left((\mathbb{C}_1-\mathbb{C}_2)^2|0\rangle\langle 0|+(\mathbb{S}_1-\mathbb{S}_2)^2|1\rangle\langle 1|\right)$ is a separable state. This means that there is no entanglement between Alice, Bob, and Charlie in this case. The probabilities of Charlie measuring $|+\rangle_d$ and $|-\rangle_d$ are

$$p_+ = \mathrm{Tr}\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\right]/4$$
$$=K/2, \tag{26}$$

and

$$p_- = \mathrm{Tr}\left[\xi_{11}\left(\rho_{\psi_{ABC_I}}\right)+\xi_{22}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{12}\left(\rho_{\psi_{ABC_I}}\right)-\xi_{21}\left(\rho_{\psi_{ABC_I}}\right)\right]/4$$
$$=Z/2. \tag{27}$$

For state $\hat{\rho}^{+}_{ABC_I}$, taking different values of α will result in different outcomes for π-tangle, we have

$$\pi_{1\rho^{+}_{ABC_I}} = \left[\frac{24(\mathbb{C}_1+\mathbb{C}_2)^2}{(7+\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2)^2}+\left(\frac{\sqrt{32(\mathbb{C}_1+\mathbb{C}_2)^2+(\mathbb{S}_1+\mathbb{S}_2)^4}-(\mathbb{S}_1+\mathbb{S}_2)^2}{2(7+\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2)}\right)^2\right]/3,$$
$$\pi_{2\rho^{+}_{ABC_I}} = \left[\frac{8(\mathbb{C}_1+\mathbb{C}_2)^2}{(3+\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2)^2}+\left(\frac{\sqrt{16(\mathbb{C}_1+\mathbb{C}_2)^2+(\mathbb{S}_1+\mathbb{S}_2)^4}-(\mathbb{S}_1+\mathbb{S}_2)^2}{2(3+\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2)}\right)^2\right]/3,$$
$$\pi_{3\rho^{+}_{ABC_I}} = \left[\frac{56(\mathbb{C}_1+\mathbb{C}_2)^2}{(9+7\mathbb{C}_1\mathbb{C}_2+7\mathbb{S}_1\mathbb{S}_2)^2}+\left(\frac{\sqrt{112(\mathbb{C}_1+\mathbb{C}_2)^2+49(\mathbb{S}_1+\mathbb{S}_2)^4}-7(\mathbb{S}_1+\mathbb{S}_2)^2}{2(9+7\mathbb{C}_1\mathbb{C}_2+7\mathbb{S}_1\mathbb{S}_2)}\right)^2\right]/3, \tag{28}$$

where $\pi_{1\rho^{+}_{ABC_I}}$, $\pi_{2\rho^{+}_{ABC_I}}$ and $\pi_{3\rho^{+}_{ABC_I}}$ correspond to $\alpha=1/2$, $\alpha=1/\sqrt{2}$, and $\alpha=\sqrt{7/8}$ with the probabilities of $p_{1+}=(\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2+7)/8$, $p_{2+}=(\mathbb{C}_1\mathbb{C}_2+\mathbb{S}_1\mathbb{S}_2+3)/4$ and $p_{3+}=(7\mathbb{C}_1\mathbb{C}_2+7\mathbb{S}_1\mathbb{S}_2+9)/16$, respectively.

After the multiple measurements on the control qubit and keeping the measurement results, Charlie can obtain the average entanglement of the GHZ state in the spacetime with mass superposition, which is $\mathbf{E}_{Av}(\rho_\psi)=p_{i+}\pi_{i\rho^{+}_{ABC_I}}$. After some calculations, we find that with the increase of α, the overall average entanglement increases and then decreases. When $\alpha=1/\sqrt{2}$, the overall average entanglement of the many-body state reaches its maximum value. In the limit $r_1=r_2\to 0$, the average entanglement

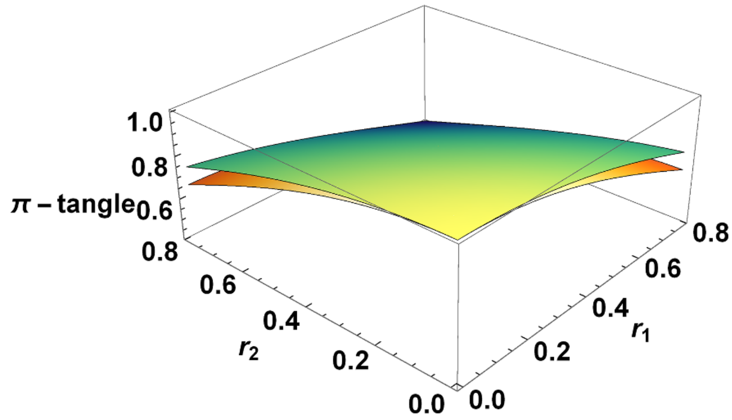

Figure 2. The entanglement under the influence of spacetime with mass superposition (above) and the final entanglement in the classical black hole spacetime (below) as functions of the squeezing factors r_1 and r_2. The initial state parameter is fixed as $\alpha=\sqrt{\frac{1}{2}}$.

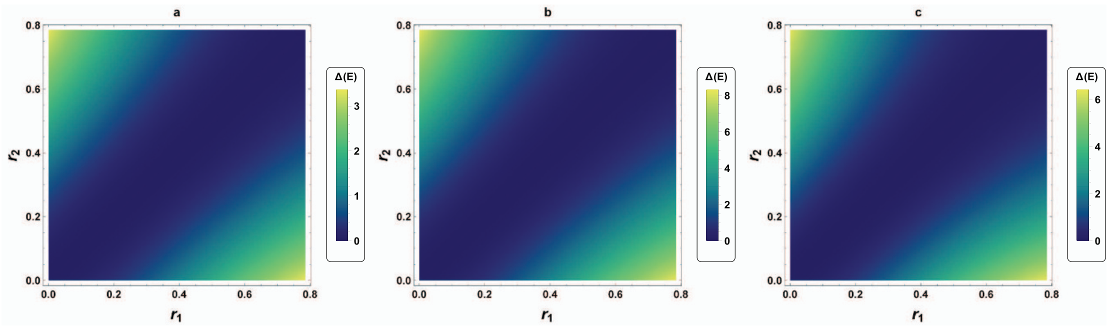

Figure 3. The percentage enhance of entanglement for the black hole mass in quantum superposition over the entanglement in the classic black hole case as a function of the mixture parameter r_1 and r_2. The plots a, b and c correspond to $\alpha=\frac{1}{2}$, $\alpha=\sqrt{\frac{1}{2}}$ and $\alpha=\sqrt{\frac{7}{8}}$, respectively.

$\mathbf{E}_{A\nu}(\rho_\psi)\to 1$. Moreover, regardless of the value of α, the average entanglement decreases with the increase of acceleration parameters r_1 and r_2.

In the classical mixed case, the entangled states of Alice, Rob and Charlie near the black hole is given in equation (14). Similarly, by using equations (11)–(13) , we can also obtain the quantum entanglement $\mathbf{E}(\rho_{A\nu})=\pi_{i\rho_{A\nu}}\,(i=1,2,3)$ for the state $\rho_{A\nu}$. To further understand its physical significance, we compare the average entanglement obtained for the black hole with mass superposition to the entanglement for the black hole which is in a classical mass mixture

$$\Delta\mathbf{E}=\mathbf{E}_{A\nu}(\rho_\psi)-\mathbf{E}(\rho_{A\nu})=p_{i+}\pi_{i\rho^{+}_{ABC_I}}-\pi_{i\rho_{A\nu}}. \quad (29)$$

As shown in figure 2, compared with the classical black hole case, the many-body state near a black hole with superimposed mass can reserve more quantum entanglement. The average entanglement of the many-body quantum state is enhanced in the spacetime with mass superposition for the Dirac field. The superposition properties of spacetime are found to provide additional quantum resources to alleviate the gravitationally induced quantum decoherence and enhance the amount of quantum resources in the curved spacetime. In addition, the greater the difference between the parameters r_1 and r_2, the more pronounced the enhancement of entanglement becomes. This means that the superposition of black hole states with different masses can regain the entanglement degradation caused by the spacetime effect of the classical black hole.

From figure 3, we can see that for different state parameter α, the maximum enhancement of entanglement are obviously different from each other. The entanglement degradation under the gravitationally induced decoherence is also influenced by the type of initial state. That is to say, the greatest quantum advantage for the recovering of entanglement in the curved spacetime can be obtained by preparing an optimal initial state. As we known, the effect of Hawking effect on the quantum state of field can be described as a quantum channel. Here we find that the average entanglement of the many-body quantum state of Dirac field affected by the superposition channel is always more than that of the classical mixture channel, regardless of the value of the initial state parameter. This means that the superposition properties of spacetime can alleviate the quantum decoherence induced by the Unruh–Hawking channel. Considering that quantum entanglement is the most important resource in quantum system, we can safely arrive to the conclusion that the superposition nature of black holes can improve the efficiency of quantum information tasks in the curved spacetime.

4. Conclusions

We have investigated how the superposition nature of black hole spacetime influences the many-body entanglement for Dirac fields. It has been observed that the quantum thermal effect induced by the classical spacetime inevitably leads to the degradation of entanglement in the quantum system, reducing the performance of quantum communication protocols in the curved spacetime. It has been found that the many-body quantum systems near the black hole with superposing masses exhibit more entanglement compared to those in classical black hole spacetime. This suggests that the superposition properties of spacetime can provide additional quantum resources to alleviate gravitationally induced quantum decoherence and improve the efficiency of quantum information tasks in the curved spacetime. It has been also shown that the greatest quantum advantage for the recovering of entanglement in the curved can be obtained by preparing an optimal initial state. The analysis in this paper provides a 'bottom-up' view of the quantum nature induced by mass superposition, which can be treated as an indication of the superposition nature of gravity and therefore is of theoretical significance.

Data availability statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study. The data that support the findings of this study are available upon reasonable request from the authors.

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Appendix. A black holes in a superposition of masses

We speculate that black holes are in a state of mass superposition based on two main assumptions. The most natural way that if the original matter that collapsed to form the black hole was in a state of mass superposition, then the resulting black hole would also be in a state of mass superposition. In addition, the Hawking radiation of the black hole will reduces the mass of the original black hole, which naturally leads to the uncertainty of the mass of the black hole due to its entangled with the emitted radiation [48]. However, for simplicity, we have ignored the process of black hole evaporation in this discussion.

Another possibility is that Rob dropped an object that is in a superposition of energies or masses into the black hole. Consider a scenario where Rob uses a physical detector to detect the object after it has been thrown into the black hole. For all practical purposes, Rob would think that the machine had fallen into the black hole and would observe the event horizon expanding as a result. That is, once Rob no longer receives signals from the machine, he would infer that the mass of the black hole has increased. If he is standing very close to the event horizon, this process may last for a short time. Thus, in a practical sense, throwing an object with a superposition of energy into a black hole can be seen as putting the black hole into a state of mass superposition.

ORCID iD

Jieci Wang https://orcid.org/0000-0001-5072-3096

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