

# Nucleus Decay Oscillations, Quantum Nonlinearity and Emergent Gravity

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**Abstract.** Several experimental groups observed periodic modulations of nucleus decay constants which amplitudes are of the order  $10^{-3}$  and periods of one year, 24 hours or about one month. We argue that such deviations from radioactive decay law can be described in nonlinear quantum mechanics framework, in which decay process obeys to nonlinear Schrödinger equation with Doeblin-Goldin terms. Corrections to Hamiltonian of quantum system interaction with gravitation field considered, it's shown that they correspond to some emergent gravity models, in particular, bilocal gravity theory. It's shown that modified proposed model predicts decay parameter variations under influence of Sun gravity similar to experimental results for  $^{214}\text{Po}$   $\alpha$ -decay life-time oscillations.

## 1. Introduction

Natural radioactivity law is one of most fundamental laws of modern physics, in accordance with it, nuclear decay parameters are time-invariant and practically independent of environment [1]. However, some recent experiments have reported the evidence of periodic modulations of nuclear  $\alpha$ - and  $\beta$ -decay parameters of the order of  $10^{-3}$  and with typical periods of one year, one day or about one month [2-8]. Possible mechanism of such decay parameter oscillations is still unclear, explanations proposed until now don't look convincing [3]. Therefore, it's sensible to start the effect analysis from reconsideration of nuclear decay fundamentals. In this paper, these oscillation effects studied in the framework of quantum-mechanical theory of unstable system decay. It follows from our analysis that standard quantum formalism can't explain the observed parameter oscillations. However, it will be argued that nonlinear modifications of quantum mechanics, extensively studied in last years [9-12], presumably can describe the decay parameter variations with the similar features. It's shown that Sun gravity influence in such models correspond to some emergent gravity theories, in particular, bilocal field models [14].

First results, indicating the essential deviations from exponential  $\beta$ -decay rate dependence, were obtained during the precise measurement of  $^{32}\text{Si}$  isotope life-time [2]. Sinusoidal annual oscillations with the amplitude  $6 \cdot 10^{-4}$  relative to total decay rate and maxima located at the end of February were registered during 5 years of measurements. Since then, the annual oscillations of  $\beta$ -decay rate for different heavy nuclei from Ba to Ra were reported, for most of them the oscillation amplitude is of the order  $5 \cdot 10^{-4}$  with its maximum on the average at mid-February [3]. Some other  $\beta$ -decay experiments exclude any decay constant modulations as large as reported ones [4]. Annual oscillations of  $^{238}\text{Pu}$   $\alpha$ -decay rate with the amplitude of the order  $10^{-3}$  also were reported [5]. Life-time of short-living  $\alpha$ -decayed isotope  $^{214}\text{Po}$  was



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measured directly, the annual and daily oscillations with amplitude of the order  $9 \cdot 10^{-4}$ , with annual maxima at mid-March and daily maxima around 6 a.m. were found during three years of measurements [6]. Specific effects related to nucleus decay oscillations also described in the literature. Annual and daily oscillation periods were found in the studies of  $^{239}\text{Pu}$   $\alpha$ -decay statistics [8,9]. Small oscillations of decay electron energy spectra with period 6 months were found in Tritium  $\beta$ -decay [7].

Until now theoretical discussion of these results had quite restricted character. In particular, oscillations of  $\beta$ -decay rate was hypothesized as anomalous interaction of Sun neutrino flux with nuclei or seasonal variations of fundamental constants [3]. Yet, neither of these hypothesis can't explain  $\alpha$ -decay parameter oscillations of the same order, because nucleus  $\alpha$ -decay should be insensitive to neutrino flux or other electro-weak processes. Really,  $\alpha$ - and  $\beta$ -decays stipulated by nucleon strong and weak interactions correspondingly. Therefore, observations of parameter oscillations for both decay modes supposes that some universal mechanism independent of particular type of nuclear interactions induces the decay parameter oscillations.

Nowadays, the most universal microscopic theory is quantum mechanics (QM), so it's worth to start the study of these effects from reminding quantum-mechanical description of nucleus decay [15]. In its framework, radioactive nucleus treated as the metastable quantum state, evolution of such states was the subject of many investigations and its principal features are now well understood [16]. However, due to serious mathematical difficulties, precise calculations of decay processes aren't possible, and due to it, hence simple semi-qualitative models are used in practical calculations. In particular, some decays modes of heavy nuclei can be effectively described as the quantum tunneling of decay products through the potential barrier constituted by nuclear shell and nucleus Coulomb field. The notorious example, is Gamow theory of  $\alpha$ -decay which describes successfully its main features, as well as some other decay modes [17,18]. However, in its standard formulation, Gamow theory excludes any significant variations of decay parameters under influence of Sun gravity and similar factors. In this paper, it's argued that such influence can appear, if one applies for  $\alpha$ -decay description the nonlinear modification of standard QM, which developed extensively in the last years [10,11]. In particular, we shall use Doebner-Goldin nonlinear QM model for nucleus decay description [12,13]. In its framework, influence of Sun gravity corresponds to emergent gravity theory with fundamental bilocal scalar field [14]. Basing on its ansatz, Gamow  $\alpha$ -decay theory with nonlinear Hamiltonian will be constructed, and its model calculations compared with experimental results for  $^{214}\text{Po}$   $\alpha$ -decay life-time variations [6].

## 2. Nonlinear QM model

Interest to nonlinear evolution equations can be dated back to the early days of quantum physics, but at that time they appeared in effective theories describing collective effects [15]. Now it's acknowledged also that nonlinear corrections to standard QM Hamiltonian can exist also at fundamental level [19,20]. Significant progress in the studies of such nonlinear QM formalism was achieved in the 80s, marked by the seminal papers of Bialynicki-Birula and Mycielcki (BM), and Weinberg [10,11]. Since then, many variants of nonlinear QM were considered in the literature (see [13] and refs. therein). Some experimental tests of QM nonlinearity were performed, but they didn't have universal character, rather they tested Weinberg and BM models only [20].

Currently, there are two different approaches to the nature of QM nonlinearity. The main and historically first one supposes that dynamical nonlinearity is universal and generic property of quantum systems [10,11]. In particular, it means that nonlinear evolution terms can influence their free motion, inducing soliton-like corrections to standard QM wave packet evolution [10]. Alternative concept of QM nonlinearity which can be called interactive, was proposed by Kibble, it postulates that free system evolution should be principally linear, so that nonlinear dynamics related exclusively to the system interactions with the fields or field self-interaction

[21]. Until now, detailed calculations of such nonlinear effects were performed only for hard processes of particle production in the strong fields [22,23], this formalism can't be applied directly to nonrelativistic processes, like nucleus decay. Due to it, to describe the interaction of metastable state with external field, we'll start from consideration of universal nonlinear models and develop their modification, which can incorporate the nonlinear particle-field and nucleus-field interactions at low energies.

In universal approach to QM nonlinearity, it supposed that particle evolution described by nonlinear Schrödinger equation of the form [15]

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{r}, t)\psi + F(\psi, \bar{\psi})\psi \quad (1)$$

where  $m$  is particle mass,  $V$  is external potential,  $F$  is arbitrary functional of system state. Currently, the most popular and elaborated nonlinear QM model of universal type is by Doebner and Goldin (DG) [12,13]. In its formalism, the simplest variant of nonlinear term is

$$F = \hbar^2\lambda\left(\nabla^2 + \frac{|\nabla\psi|^2}{|\psi|^2}\right) \quad (2)$$

where  $\lambda$  is imaginary constant. With the notation

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}, t)$$

we abbreviate (1) to  $i\hbar\partial_t\psi = H_0\psi + F\psi$ . In fact, general DG model describes six-parameter family of nonlinear Hamiltonians, but the action of all its nonlinear terms on realistic quantum systems is similar  $F$  of (2), hence for the start only this ansatz will be used in our calculations [13]. The choice of  $\lambda$  of (2) to be imaginary prompted by results of nonrelativistic current algebras [12], but they doesn't have mandatory character; below we'll consider  $F$  terms both with imaginary and real  $\lambda$  for  $|\lambda| \ll 1$ .

Main properties of eq. (1) for imaginary  $\lambda$  were studied in [12], they can be promptly extended on real  $\lambda$  and summarized for both cases as follows: (a) The probability is conserved. (b) The equation is homogeneous. (c) The equation is Euclidian- and time-translation invariant (for  $V = 0$ ). (d) Noninteracting particle subsystem remain uncorrelated (separation property). Distinct values of  $\lambda$  can occur for different particle species. (e) Writing  $\langle Q \rangle = \int \bar{\psi} \hat{Q} \psi d^3x$  for operator expectation value, in particular, since  $\int \bar{\psi} F \psi d^3x = 0$ , the energy functional for a solution of (1) is  $\langle i\hbar\partial_t \rangle = \langle H_0 \rangle$ . Considering Ehrenfest theorem for such Hamiltonian, it follows for  $\vec{p} = -i\hbar\nabla$  and imaginary  $\lambda$ ,

$$\frac{d}{dt} \langle \vec{r} \rangle = \frac{\langle \vec{p} \rangle}{m},$$

whereas for real  $\lambda$ ,

$$\frac{d}{dt} \langle \vec{r} \rangle = \frac{1 - 2\lambda m}{m} \langle \vec{p} \rangle,$$

For  $V = 0$ , plane waves  $\psi = \exp\left[i\left(\vec{k}_0 \cdot \vec{r} - \omega t\right)\right]$  with  $\omega = E\hbar$ ,  $|\vec{k}_0|^2 = 2mE/\hbar$  are solutions both for real and imaginary  $\lambda$ . For real  $\lambda$ , QM continuity equation for probability density  $\rho$  fulfilled, but density current acquires the form

$$\vec{j} = \frac{\hbar(1 - 2\lambda m)}{2mi} (\bar{\psi} \nabla \psi - \psi \nabla \bar{\psi})$$

For imaginary  $\lambda$  the continuity equation becomes of Fokker-Plank type [12].

As was mentioned above, simple quantum model of metastable state decay describes it as particle tunneling through the potential barrier with suitable parameters [15]. Its natural to expect that for small  $\lambda$  the tunneling mechanism doesn't change principally, and resulting states don't differ much from standard QM solutions. Hence such linear solutions can be treated as consistent approximations for incoming states to nonlinear solutions for the same system parameters. To illustrate the influence of nonlinear DG term on particle tunneling, consider 1-dimensional plane wave tunneling through the potential barrier. Suppose that rectangular barrier of the height  $V_0$  located between  $x = 0$  and  $x = a$ , and plane wave particle state with energy  $E < V_0$  spreads from  $x = -\infty$ . Long-living metastable states appear for small transmission coefficient  $D \rightarrow 0$ , which corresponds to barrier width  $a \rightarrow \infty$  for fixed  $E, V_0$ . For example, for  $^{238}\text{U}$   $\alpha$ -decay  $D \approx 10^{-37}$  [17]. We'll study solutions of equation (1), basing on its asymptotic in this limit. Standard QM stationary solution for  $x < 0$

$$\psi_0(x) = \exp(ikx) + A \exp(-ikx)$$

with  $k = \sqrt{2mE}/\hbar$ ; for  $\alpha \rightarrow \infty$  it gives  $|A| \rightarrow 1$ , i.e. nearly complete wave reflection from the barrier. Hence  $\psi_0$  can be decomposed as  $\psi_0 = \psi_\infty + \psi_d$  where the asymptotic state  $\psi_\infty = 2 \cos(kx - \alpha_0)$ ,  $\alpha_0 = \arctan \chi_0/k$  where

$$\chi_0 = \frac{1}{\hbar} \sqrt{2m(V_0 - E)} \quad (3)$$

Then,  $\psi_d \simeq A_d \exp(ikx)$  where  $|A_d| \simeq \exp(-\chi_0 a)$ , i.e. is exponentially small. In distinction, for nonlinear equation

$$E\psi = H_0\psi + F\psi \quad (4)$$

the incoming and reflected waves suffer rescattering, so stationary state  $\psi \neq \psi_0$  for  $x < 0$ . For real  $\lambda$ , the stationary solution can be obtained performing the nonlinear transformation of solution of adjoined linear equation [12,13]. Namely, for its real solution  $\eta(x)$

$$\psi = \eta^{\frac{1-2\Gamma}{1-4\Gamma}}$$

In particular, asymptotic solution for  $\alpha \rightarrow \infty$

$$\psi_N = 2[\cos(qx - \alpha)]^{\frac{1-2\Gamma}{1-4\Gamma}}$$

with  $\Gamma = \lambda m$ ,  $\alpha \approx \alpha_0$  and

$$q = \frac{1}{\hbar} \frac{\sqrt{2m(1-4\Gamma)}}{1-2\Gamma}.$$

Thus, asymptotic solution  $\psi_N$  differs from standard QM one, for finite  $a$  the correction to it can be taken to be equal to  $\psi_d$ , i.e.  $\psi \approx \psi_N + \psi_d$ . For imaginary  $\lambda$  the corresponding nonlinear transformation given in [12,13], however, for complete wave reflection from the barrier the consistent asymptotic solution for  $\psi_N$  doesn't exist, because  $\psi_N$  phase singularities appear at its nodes. In this case, the linear QM solution  $\psi_\infty(x)$  for the same system parameters can be used as its approximation. For  $x > a$ , both for real and imaginary  $\lambda$ , the solution is  $\psi^+(x) = C^+ \exp(ikx)$ ,  $C^+$  value calculated below.

To describe the tunneling, its necessary first to find solution for  $0 < x < a$  with  $\psi(a) \rightarrow 0$  for  $a \rightarrow \infty$ , which is main term of tunneling state. For real  $\lambda$ , such solution of eq. (1) is  $\psi_1(x) = B_1 \exp(-\chi x)$  where

$$\chi = \frac{1}{\hbar} \sqrt{\frac{2m(V_0 - E)}{1-4\Gamma}}$$

In the linear QM formalism, for  $a \rightarrow \infty$ , it follows that

$$B_1 = -\frac{2k(k - i\chi_0)}{k^2 + \chi_0^2}$$

For small  $|\lambda|$  this formulae can be used with good accuracy also in nonlinear case substituting in it  $\chi_0$  of (3) by  $\chi$ . Analogously to standard QM, another solution, which describes the secondary term, is  $\psi_2(x) = B_2 \exp(\chi x)$ , yet  $|B_2| \sim \exp(-2\chi a)$ , so  $\psi_2$  is exponentially small in comparison with  $\psi_1$ . Therefore, transmission coefficient can be estimated with good accuracy accounting only main term  $\psi_1$ , it gives  $D_1 = |B_1|^2 \exp(-2\chi a)$ , so that  $D_1$  directly depends on  $\lambda$ .

Due to dynamics nonlinearity, the superpositions of two terms, in general, aren't solutions. Analytic solutions, which correspond to such superpositions, exist in two cases only, defined by  $b_s = B_2/B_1$  ratio. First, for imaginary  $b_s$  the solution is  $\psi = \psi_1 + \psi_2$ ; second, for  $b_s$  real

$$\psi(x) = B_1^{\frac{1-2\Gamma}{1-4\Gamma}} [\exp(-\chi_b x) + b_s \exp(\chi_b x)]^{\frac{1-2\Gamma}{1-4\Gamma}}$$

where

$$\chi_b = \frac{1}{\hbar} \frac{\sqrt{2m(V_0 - E)(1 - 4\Gamma)}}{1 - 2\Gamma} \quad (5)$$

Other solutions, corresponding to complex  $b_s$ , can be approximated as the linear interpolation between these two solutions. In the linear approximation

$$B_2 = B_1 \frac{\chi^2 - k^2 + 2ik\chi}{\chi^2 + k^2} \exp(-2\chi a)$$

For typical  $\alpha$ -decay parameters  $E \approx V_0/2$ , it correspond to  $\chi$ ,  $k$  values such that  $\chi \approx k$ . Therefore,  $b_s$  can be taken to be imaginary, and resulting  $\psi$  will be  $\psi_{1,2}$  superposition. In this case, transmission coefficient  $D_s = 2D_1$ . To extend the solution subspace, one can substitute real  $\lambda$  by complex  $\lambda' = \lambda_1 + i\lambda_2$  with  $|\lambda_2| \ll |\lambda_1|$ , such correction doesn't change  $D_s$  significantly.

For imaginary  $\lambda$ , the main term  $\psi_1 = B_1 \exp(-\chi_0 \varpi x)$  where

$$\varpi = \frac{1 + 2m\lambda}{\sqrt{1 + 4m^2|\lambda|^2}}$$

Transmission coefficient for main term is equal to  $D_1 = |B_1|^2 \exp(-2\chi_0 v x)$  where  $v = \text{Re}\varpi$ . It supposes that  $D_1$  dependence on  $\lambda$  is less pronounced than for real  $\lambda$ . Then, secondary term  $\psi_2 = B_2 \exp(-\chi_0 \varpi x)$ . Both for real and imaginary  $\lambda$ ,  $C^+ = \psi(a) \exp(-ika)$ , which defines  $\psi = \psi^+$  for  $x > a$ .

It's noteworthy that considered nonlinear Hamiltonian term  $F$  influences mainly the transitions between degenerate states, as property (e) demonstrates. Due to it, tunneling transmission coefficients and related decay rates are sensitive to the presence of nonlinear terms in evolution equation. Therefore, measurements of such process parameters can be important method of quantum nonlinearity search.

### 3. $\alpha$ -decay oscillation model

Gamow theory of nucleus  $\alpha$ -decay supposes that in initial nuclei state, free  $\alpha$ -particle already exists inside the nucleus, but its total energy  $E$  is smaller than maximal height of potential barrier constituted by nuclear forces and Coulomb potential [17]. Hence  $\alpha$ -particle can leave nucleus volume only via quantum tunneling through this barrier. For real nucleus, barrier potential isn't rectangular, but has complicated form described by some function  $V(r)$  defined experimentally

[17,18]. In this case, to calculate transmission rate in our model, WKB approximation for Hamiltonian of (1) used [15]; its applicability to our nonlinear Hamiltonian can be easily checked. The calculations described here only for real  $\lambda$ , for imaginary  $\lambda$  they are similar. In this ansatz, 3-dimensional  $\alpha$ -particle wave function reduced to  $\psi = \frac{1}{r} \exp(i\sigma/\hbar)$ ; function  $\sigma(r)$  can be decomposed in  $\hbar$  order  $\sigma = \sigma_0 + \sigma_1 + \dots$  [15]. Given  $\alpha$ -particle energy  $E$ , one can find the distances  $R_0$   $R_1$  from nucleus centre at which  $V(R_{0,1}) = E$ . Then, for our nonlinear Hamiltonian the equation for  $\sigma_0$  is

$$\left(\Lambda - \frac{1}{2m}\right) \left(\frac{\partial\sigma_0}{\partial r}\right)^2 = E - V(r) \quad (6)$$

where  $\Lambda = 2\lambda$  for  $R_0 \leq r \leq R_1$ ,  $\Lambda = 0$  for  $r < R_0$ ,  $r > R_1$ . Its solution for  $R_0 \leq r \leq R_1$  can be written as

$$\psi = \frac{1}{r} \exp\left(\frac{i\sigma_0}{\hbar}\right) = \frac{C_r}{r} \exp\left[-\frac{1}{\hbar} \int_{R_0}^r p(\epsilon) d\epsilon\right]$$

where  $C_r$  is normalization constant,

$$p(r) = \frac{1}{\hbar} \sqrt{\frac{2m(V(r) - E)}{1 - 4\Gamma}}$$

where  $\Gamma = m\lambda$ . Account of higher order  $\sigma$  terms doesn't change transmission coefficient which is equal to

$$D = \exp\left[-\frac{2}{\hbar} \int_{R_0}^{R_1} p(\epsilon) d\epsilon\right] = \exp\left[-\frac{\phi}{\sqrt{1 - 4\Gamma}}\right] \quad (7)$$

here  $\phi$  is constant, whereas  $\Gamma$  can change in time. For imaginary  $\lambda$  the calculations result in the same  $D$  ansatz, but with

$$p(r) = \frac{1}{\hbar} \sqrt{\frac{2m(V - E)}{1 + 4m^2|\lambda|^2}}$$

To calculate nucleus life-time,  $D$  should be multiplied by the number of  $\alpha$ -particle kicks into nucleus potential wall per second [17]. It's worth to notice that the relative state of remnant nucleus and  $\alpha$ -particle is maximally entangled [19].

To study decay parameter variations in external field, we'll suppose now that nonlinear Hamiltonian term  $F$  depends on external field  $A$ . In our model, such field is gravity, characterized usually by its potential  $U(\vec{R}, t)$ . In this case,  $U$  should be accounted in evolution equation twice. First,  $mU$  should be added to  $H_0$ , so that it changed to  $H'_0 = H_0 + mU$ ; second, nonlinear  $H$  term  $F$  can depend on  $U$  or some its derivative. For minimal modification of DG model we'll assume that for  $F$  ansatz of (2) its possible dependence on external field is restricted to parameter  $\lambda$  dependence:  $\lambda = f(U)$ , so now  $\lambda$  isn't constant, but the function of  $\vec{R}$  and  $t$ . It supposed also that  $f \rightarrow 0$  for  $U \rightarrow 0$ , so that free particle evolution is linear.

Considered model doesn't permit to derive  $\lambda$  dependence on Sun gravity, but it can be obtained from its comparison with experimental results for  $^{214}\text{Po}$   $\alpha$ -decay. We'll suppose that  $\lambda$  is function of potential  $U(\vec{R}_n, t)$  where  $\vec{R}_n$  is nucleus coordinate in Sun reference frame (SRF). As follows from eq. (7) for small  $\lambda$

$$D \approx (1 + 2\phi\Gamma) \exp(-\phi)$$

For  $^{214}\text{Po}$  decay, its life-time  $\tau_0 = 16.4 \cdot 10^{-6}$  sec, model estimate gives  $\phi \approx 60$ . For annual variation  $\tau$  the best fit for 3 year exposition has main harmonics

$$\tau_a(t) = \tau_0 [1 + A_a \sin \omega_a (t + \varphi_a)]$$

where  $t$  defined in days,  $A_a = 9.8 \cdot 10^{-4}$ ,  $\omega_a = 2\pi/365$ ,  $\phi = 174$  days [6]. Remind that Earth orbit is elliptic, the minimal distance from Sun is at about January 3 and maximal at about July 5, maximal/minimal orbit radius difference is about  $3 \cdot 10^{-2}$  [24]. Plainly, the minima and maxima of  $U$  time derivative  $\partial_t U$  will be located approximately in the middle between these dates, i.e. about April 5 and October 3, correspondingly. In general, this dependence described as

$$\partial_t U = K^a \sin \omega_a (t + \varphi_u)$$

here  $\varphi_u = 185$  days,  $K^a = 1.5 \text{ m}^2/\text{sec}$ , as the result, such model  $\varphi_u$  value in a good agreement with experimental  $\varphi_a$  value. Thereon, it means that the plausible data fit is  $\lambda(t) = g\partial_t U$ , where  $g$  is interaction constant, which can be found from the data for  $^{214}\text{Po}$  decay. It follows from the assumed equality of oscillation amplitudes  $A_a = 2\phi mgK^a$  that gives

$$g = 0.35 \cdot 10^{-8} \frac{\text{sec}^3}{\text{m}^2 \text{MeV}}$$

Another experimentally found harmonic corresponds to daily variations with best fit

$$\tau_d(t) = \tau_0 [1 + A_d \sin \omega_d (t + \varphi_d)]$$

where  $t$  defined in hours,  $A_d = 8.3 \cdot 10^{-4}$ ,  $\omega_d = 2\pi/24$ ,  $\varphi_d = 12$  hours [6]. Such oscillation can be attributed to variation of Sun gravity due to daily lab. rotation around Earth axe. It's easy to check that nucleus life-time dependence also coincides with  $\partial_t U$  time dependence with high precision. Really, it described as

$$\partial_t U = K^e \sin \omega_d (t + \varphi_e)$$

with  $\varphi_e = 12$  hours,  $K^e = 0.9 \text{ m}^2/\text{sec}$  [24]. It follows that  $A_d = 2\phi mgK^e$ ; if to substitute in this equality  $g$  value, calculated above, it gives  $A_d = 5.5 \cdot 10^{-4}$ , which is in a reasonable agreement with its experimental value. It's possible also that  $\tau_{a,d}$  can depend on some other orbit parameter or  $U$  derivative, in particular, on lab. velocity in SRF or some absolute reference frame [6]; such options will be considered elsewhere.

#### 4. Nonlinearity and Emergent Gravity

In this paper, we studied hypothetical nonlinear corrections to standard QM description of system interaction with external fields. It's notable that in nonrelativistic QM, the ansatz for system interaction with massless fields, such as gravity and electromagnetism, is stipulated, in fact, by Bohr quantum-to-classical correspondence principle [19,25]. However, in the last sixty years many new physical concepts were discovered, which presumably are independent of it. The illustrative example is, in our opinion, quantum chromodynamics, the theory derived just from experimental facts with no reference to correspondence principle [26]. In the same vein, there is no obvious prohibition on the existence of additional Hamiltonian terms for the system interaction with massless fields. Such terms can have strictly quantum origin and disappear in classical limit, their existence should be verified in dedicated experiments. To study their general features, we considered the simple nonrelativistic model, which includes the additional terms for the interaction of quantum systems with gravitational field. Account of these terms permits to describe with good accuracy annual and daily oscillations of  $^{214}\text{Po}$   $\alpha$ -decay parameters observed in the experiment.

It was argued earlier that QM nonlinearity violates relativistic causality for multiparticle systems, in particular, it permits superluminal signaling for EPR-Bohm pair states [27,28]. However, this conclusion was objected and still disputed [20]. Plainly, these arguments would be even more controversial, if nonlinear effects exist only inside the field volume. In particular, the metastable state in external field can be considered as open system, yet it was shown that superluminal signaling between such systems is impossible [29]. Moreover, heavy nucleus is strongly-bound system, so it's unclear whether it's possible to prepare entangled state of two  $\alpha$ -particles located initially inside two different nuclei. It was shown that in our model the standard relation between average system momentum and velocity can be violated and differ from particle mass. However, because in our model this ratio depends on external gravitational field, then for Sun gravity influence it can depend on time of day or year season, hence its tests demand special subtle experiments.

It was supposed by many authors previously that gravity is emergent (induced) theory and corresponds, in fact, to some nonlocal field theory, which can be effectively described as multilocal one (see [14] and refs. therein). It supposes that such field simultaneously possesses several multilocal modes  $\{\Phi_1, \dots, \Phi_n, \dots\}$ . It was shown that fundamental bilocal scalar field  $\Phi_2(\vec{r}_1, t_1, \vec{r}_2, t_2)$  reproduces classical gravity effects with good precision, so it can be plausible candidate for performing simplified description of multilocal field effects [14]. Simultaneously, bilocal field  $\Phi_2$  supposedly can interact with bilocal operators of massive fields, in particular, it can be nonrelativistic particle systems. As was discussed above, such interaction should not violate causality. This condition would be fulfilled if for the pair of separated quantum objects  $\Phi_2$  influences only their bilocal observables of EPR-Bohm type [19,27]. Notorious example of such observable is the difference of two fermion spin projections on the same coordinate axis. The same condition should be imposed on hypothetical  $n$ -local terms, they also should act only on corresponding  $n$ -local observables of EPR-Bohm type.

Denote as  $\vec{r}_1$  the coordinate of  $\alpha$ -particle,  $\vec{r}_2$  the coordinate of remnant nucleus centre of mass, and  $\vec{r}_s = \vec{r}_1 - \vec{r}_2$ . For considered  $\alpha$ -decay model, the joint state of remnant nucleus and  $\alpha$ -particle is entangled, their bilocal observable  $\vec{r}_s$  is of EPR-Bohm type. It's notable that it's similar to one of fundamental bilocal coordinates of bilocal field [14], which are

$$x_\mu^1 - x_\mu^2; \quad \frac{1}{2} (x_\mu^1 + x_\mu^2)$$

As follows from our analysis of  $^{214}\text{Po}$   $\alpha$ -decay for bilocal scalar field  $\Phi_2 \sim \partial_t U$ , its interaction with the system of two nonrelativistic particles can be described as

$$F \sim \partial_t U \left( \vec{R}_n \right) f \left( \frac{\partial}{\partial \vec{r}_s} \right)$$

In accordance with it, for D-G model ansatz the corresponding nonlinear term of nucleus Hamiltonian can be written as

$$F = k_b \partial_t U \left( \vec{R}_n \right) \left( \frac{\partial^2}{\partial \vec{r}_s^2} + \frac{1}{\psi^2} \left| \frac{\partial \psi}{\partial \vec{r}_s} \right|^2 \right)$$

where  $k_b$  is arbitrary constant and  $|\vec{r}_s| \ll |\vec{R}_n|$ . It means that under influence of Sun gravity  $\langle \vec{r}_s^2(t) \rangle$  can differ from its value for linear Gamow theory. Plainly,  $\partial \lambda_{\vec{R}}$  and  $\partial \lambda_t$  are quite small in comparison with nucleus size and its life-time and so related  $\lambda$  variations can be neglected. For large  $|\vec{r}_s|$  it can be supposed heuristically that  $\Phi_2$  symmetrized ansatz is equal

$$\Phi_2 = \left\{ \frac{\partial_t U(\vec{r}_1) \partial_t U(\vec{r}_2) [\partial_t U(\vec{r}_1) + \partial_t U(\vec{r}_2)]}{2} \right\}^{1/3} \quad (8)$$

It's worth to notice also that the role of state entanglement in quantum gravity extensively discussed now in connection with AdS/CFT holography theory [30].

As follows from equivalence principle [24], in lab. reference frame, located on Earth surface, Sun gravitation potential  $U'(\vec{R}_c) \approx 0$ , yet  $\partial_{\vec{r}} U' \neq 0$ . It means that nuclear decay process violate equivalence principle, however, some theories of emergent (induced) gravity predict that it can be violated in quantum processes [14,30]. In particular, it can occur in bilocal gravity model, in which gravity induced by scalar  $\Phi_2$  [14]. In addition, other results for  $^{214}\text{Po}$   $\alpha$ -decay seems to support such conclusion. Namely, beside described life-time oscillations, these data contains also harmonics with period 24 hours 50 minutes, which is equal to lunar day duration and so can be related to moon gravity effect [6].

Modern emergent gravity theories, first of all, holography models indicate that gravity to some extent can be similar to massless gauge theories, in particular, QED ( [14, 30] and refs. therein). Meanwhile, it's well known that in nonrelativistic limit there are some electromagnetic effects, like Casimir effect or Lamb shift, which can't be described by Schrödinger equation with classical electromagnetic interaction terms, but only via accounting higher order QED terms [26]. It can be hypothesized that the observed decay oscillations can have analogous origin corresponding to the nonzero infrared limit of some quantum gravity terms. Considered QM nonlinearity has universal character, so beside nucleus decays, such temporary variations under influence of Sun gravity can be observed, in principle, for other systems in which metastable states and tunneling play important role. In particular, it can be some chemical reactions, molecular absorption by solids and liquids, etc. It's worth to notice specially its possible role in some biological processes. Multiple results demonstrate the influence of moon-sidereal and moon-tide rhythms on plant and other organism development [31,32].

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