Strange cousin of $Z_c(4020/4025)$ as a tetraquark state*

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Abstract: Motivated by the analogous properties of $Z_c(3900/3885)$ and $Z_{cs}(3985/4000)$, we tentatively assign $Z_c(4020/4025)$ as the $A\bar{A}$-type hidden-charm tetraquark state with $J^{PC}=1^{+-}$, where A denotes the axialvector diquark states, and explore $A\bar{A}$-type tetraquark states without strange, with strange, and with hidden-strange via QCD sum rules in a consistent manner. We then explore the hadronic coupling constants in the two-body strong decays of tetraquark states without and with strange via QCD sum rules based on rigorous quark-hadron duality and acquire partial and total decay widths. The present calculations support assigning $Z_c(4020/4025)$ as the $A\bar{A}$-type tetraquark state with $J^{PC}=1^{+-}$, while the predictions for its strange cousin Z_{cs} state can be confronted with experimental data in the future.

Keywords: tetraquark state, QCD sum rules

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I. INTRODUCTION

In 2013, the BESIII Collaboration observed $Z_c^{\pm}(4025)$ in the $\pi^{\mp}$ recoil mass spectrum of the process $e^+e^-\to(D^*\bar{D}^*)^{\pm}\pi^{\mp}$, where the measured Breit-Wigner mass and width were $M=(4026.3\pm2.6\pm3.7)\,\text{MeV}$ and $\Gamma=(24.8\pm5.6\pm7.7)\,\text{MeV}$, respectively [1]. Two years later, the BESIII Collaboration observed its neutral partner $Z_c^0(4025)$ in the π^0 recoil mass spectrum of the process $e^+e^-\to(D^*\bar{D}^*)^0\pi^0$, where the measured Breit-Wigner mass and width were $M=(4025.5^{+2.0}_{-4.7}\pm3.1)\,\text{MeV}$ and $\Gamma=(23.0\pm6.0\pm1.0)\text{MeV}$, respectively [2]. The masses and widths of the charged structures $Z_c^{\pm}(4025)$ and neutral structure $Z_c^0(4025)$ were consistent with each other. Moreover, in 2013, the BESIII Collaboration observed $Z_c^{\pm}(4020)$ in the $\pi^{\pm}h_c$ mass spectrum of the process $e^+e^-\to\pi^+\pi^-h_c$, where the measured Breit-Wigner mass and width were $M=(4022.9\pm0.8\pm2.7)\,\text{MeV}$ and $\Gamma=(7.9\pm2.7\pm2.6)\,\text{MeV}$, respectively [3]. $Z_c(4020)$ and $Z_c(4025)$ were assigned to be the same particle by the Particle Data Group and listed in the Review of Particle Physics as $X(4020)$ [4], although the widths differed from each other considerably.

The spin and parity have not yet been measured. S-wave $D^*\bar{D}^*$ systems have the quantum numbers $J^{PC}=0^{++}$, 1^{+-}, and 2^{++}, S-wave $\pi^{\pm}h_c$ systems have the quantum numbers $J^{PC}=1^{--}$, P-wave $\pi^{\pm}h_c$ systems have the quantum numbers $J^{PC}=0^{++}$, 1^{+-}, and 2^{++}, and we can tentatively assign the quantum numbers $J^{PC}=1^{+-}$ for $Z_c(4020/4025)$. According to the nearby $D^*\bar{D}^*$ threshold, one may expect to assign $Z_c(4020/4025)$ as the tetraquark molecular state [5–12]. In the picture of tetraquark states, $Z_c(4020/4025)$ can be assigned as the $A\bar{A}$-type tetraquark state with $J^{PC}=1^{+-}$ [13–15], whereas $Z_c(3900)$ can be assigned as the $S\bar{A}-A\bar{S}$ type tetraquark state according to calculations via QCD sum rules [16], where S and A represent the scalar and axialvector diquark states, respectively.

In 2020, the BESIII Collaboration observed the $Z_{cs}^-(3985)$ structure in a K^+ recoil-mass spectrum with a significance of 5.3 σ in the processes $e^+e^-\to K^+(D_s^-D^{*0}+D_s^{*-}D^0)$ [17]. The measured Breit-Wigner mass and width were $M=3985.2^{+2.1}_{-2.0}\pm1.7\,\text{MeV}$ and $\Gamma=13.8^{+8.1}_{-5.2}\pm4.9\,\text{MeV}$, respectively [17]. In 2021, the LHCb Collaboration observed two new exotic states, $Z_{cs}^+(4000)$ and $Z_{cs}^+(4220)$, in the $J/\psi K^+$ mass spectrum of the process $B^+\to J/\psi\phi K^+$ [18]. The most significant state, $Z_{cs}^+(4000)$, had a Breit-Wigner mass and width of $M=4003\pm6^{+4}_{-14}\,\text{MeV}$ and $\Gamma=131\pm15\pm26\,\text{MeV}$, respectively, and the spin-parity $J^P=1^+$ [18]. Although we can reproduce the mass of $Z_{cs}(3985/4000)$ using QCD sum rules in the pictures of both the tetraquark and molecular states [19–26], direct calculations of the decay widths based on QCD sum rules support assigning $Z_{cs}(3985)$ and

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$Z_{cs}(4000)$ as the hidden-charm tetraquark state and molecular state with $J^{PC}=1^{+-}$, respectively. Alternatively, at least, $Z_{cs}(3985)$ may have a large diquark-antidiquark type Fock component, while $Z_{cs}(4000)$ may have a large color-singlet-color-singlet type Fock component [27].

$Z_c(3900/3885)$ and $Z_{cs}(3985/4000)$ are cousins and have analogous decay modes.

$$Z_c^{\pm}(3900)\to J/\psi\pi^{\pm},\; Z_{cs}^{+}(4000)\to J/\psi K^{+}, \tag{1}$$

$$Z_c^{\pm}(3885)\to (D\bar{D}^*)^{\pm},\; Z_{cs}^{-}(3985)\to D_s^- D^{*0}, D_s^{*-}D^0, \tag{2}$$

and we expect that $Z_c(4020/4025)$ also has strange cousins Z_{cs}, which have analogous decay modes. The Z_{cs} states may be observed in decays to final states, such as $D^*\bar{D}_s^*$, $D_s^*\bar{D}^*$, and h_cK. In this study, we tentatively assign $Z_c(4020/4025)$ as the $A\bar{A}$-type hidden-charm tetraquark state with $J^{PC}=1^{+-}$ and extend our previous study to investigate the mass and width of its strange cousin using QCD sum rules [20, 23, 27, 28]. The predictions can be confronted with experimental data in the future, which may contribute to disentangling the pictures of tetraquark and molecular states. As a byproduct, we obtain the mass of the hidden-strange/charm tetraquark state and the partial decay widths of $Z_c(4020/4025)$.

The article is arranged as follows. We derive QCD sum rules for the masses and pole residues of the $A\bar{A}$-type tetraquark states without strange, with strange, and with hidden-strange in Section II. In section III, we derive QCD sum rules for the hadronic coupling constants in the decays of the Z_c and Z_{cs} states. Section IV is reserved for our conclusion.

II. QCD SUM RULES FOR THE Z_c, Z_{cs}, and $Z_{cs\bar{s}}$ TETRAQUARK STATES WITH $J^{PC}=1^{+-}$

First, we present the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu\alpha\beta}(p)=\mathrm{i}\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_{\mu\nu}(x)J_{\alpha\beta}^{\dagger}(0)\right\}|0\rangle, \tag{3}$$

where $J_{\mu\nu}(x)=J_{\mu\nu}^{u\bar{d}}(x)$, $J_{\mu\nu}^{u\bar{s}}(x)$, and $J_{\mu\nu}^{s\bar{s}}(x)$,

$$\begin{aligned}
J_{\mu\nu}^{u\bar{d}}(x)=&\frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big\{u_j^T(x)C\gamma_\mu c_k(x)\bar{d}_m(x)\gamma_\nu C\bar{c}_n^T(x)\\
&-u_j^T(x)C\gamma_\nu c_k(x)\bar{d}_m(x)\gamma_\mu C\bar{c}_n^T(x)\Big\},\\
J_{\mu\nu}^{u\bar{s}}(x)=&\frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big\{u_j^T(x)C\gamma_\mu c_k(x)\bar{s}_m(x)\gamma_\nu C\bar{c}_n^T(x)\\
&-u_j^T(x)C\gamma_\nu c_k(x)\bar{s}_m(x)\gamma_\mu C\bar{c}_n^T(x)\Big\},\\
J_{\mu\nu}^{s\bar{s}}(x)=&\frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big\{s_j^T(x)C\gamma_\mu c_k(x)\bar{s}_m(x)\gamma_\nu C\bar{c}_n^T(x)\\
&-s_j^T(x)C\gamma_\nu c_k(x)\bar{s}_m(x)\gamma_\mu C\bar{c}_n^T(x)\Big\},
\end{aligned} \tag{4}$$

where i, j, k, m, and n are color indexes, and C is the charge conjugation matrix [15, 28]. We choose the currents $J_{\mu\nu}^{u\bar{d}}(x)$, $J_{\mu\nu}^{u\bar{s}}(x)$, and $J_{\mu\nu}^{s\bar{s}}(x)$ to explore the hidden-charm tetraquark states without strange, with strange, and with hidden-strange, respectively.

On the hadronic side, we explicitly isolate the ground state contributions of the hidden-charm tetraquark states with $J^{PC}=1^{+-}$ and 1^{--} and acquire the following results:

$$\begin{aligned}
\Pi_{\mu\nu\alpha\beta}(p)=&\frac{\lambda_Z^2}{M_Z^2-p^2}\Big(p^2g_{\mu\alpha}g_{\nu\beta}-p^2g_{\mu\beta}g_{\nu\alpha}\\
&-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\Big)\\
&+\frac{\lambda_Y^2}{M_Y^2-p^2}\Big(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha\\
&+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\Big)+\cdots,
\end{aligned} \tag{5}$$

where Z and Y denote the tetraquark states with $J^{PC}=1^{+-}$ and 1^{--}, respectively. The pole residues λ_Z and λ_Y are defined by

$$\begin{aligned}
\langle 0|\eta_{\mu\nu}(0)|Z(p)\rangle&=\lambda_Z\,\varepsilon_{\mu\nu\alpha\beta}\,\zeta^\alpha p^\beta,\\
\langle 0|\eta_{\mu\nu}(0)|Y(p)\rangle&=\lambda_Y\left(\zeta_\mu p_\nu-\zeta_\nu p_\mu\right),
\end{aligned} \tag{6}$$

the ζ_μ are the polarization vectors of the tetraquark states. We can rewrite the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in the form

$$\begin{aligned}
\Pi_{\mu\nu\alpha\beta}(p)=&\Pi_Z(p^2)\Big(p^2g_{\mu\alpha}g_{\nu\beta}-p^2g_{\mu\beta}g_{\nu\alpha}\\
&-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\Big)\\
&+\Pi_Y(p^2)\Big(-g_{\mu\alpha}p_\nu p_\beta-g_{\nu\beta}p_\mu p_\alpha\\
&+g_{\mu\beta}p_\nu p_\alpha+g_{\nu\alpha}p_\mu p_\beta\Big),
\end{aligned} \tag{7}$$

according to Lorentz covariance.

We project the components $\Pi_Z(p^2)$ and $\Pi_Y(p^2)$ by the tensors $P_{A,p}^{\mu\nu\alpha\beta}$ and $P_{V,p}^{\mu\nu\alpha\beta}$ to

$$\begin{aligned}
\widetilde{\Pi}_Z(p^2)&=p^2\Pi_Z(p^2)=P_{A,p}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p),\\
\widetilde{\Pi}_Y(p^2)&=p^2\Pi_Y(p^2)=P_{V,p}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p),
\end{aligned} \tag{8}$$

where

$$\begin{aligned} P_{A,p}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2}\right), \\ P_{V,p}^{\mu\nu\alpha\beta} &= \frac{1}{6}\left(g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2}\right) - \frac{1}{6}g^{\mu\alpha}g^{\nu\beta}. \end{aligned} \tag{9}$$

We accomplish operator product expansion up to the vacuum condensates of dimension 10 and take account of the vacuum condensates $\langle\bar{q}q\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle^2$, and $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s GG}{\pi}\rangle$, where $q = u$, d, or s, as in previous studies [14–16, 20, 23]. We project the components

$$\begin{aligned} \widetilde{\Pi}_Z(p^2) &= P_{A,p}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p), \\ \widetilde{\Pi}_Y(p^2) &= P_{V,p}^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p), \end{aligned} \tag{10}$$

on the QCD side. In the present study, we are only interested in the component $\widetilde{\Pi}_Z(p^2)$ as we investigate the axialvector tetraquark states. We take the truncations $n \le 10$ and $k \le 1$ in a consistent manner, and the operators of the orders $\mathcal{O}(\alpha_s^k)$ with $k > 1$ are discarded. The operators in the condensates $\langle g_s^3 GGG\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle^2$, and $\langle\frac{\alpha_s GG}{\pi}\rangle\langle\bar{q}g_s\sigma Gq\rangle$ are of the orders $\mathcal{O}(\alpha_s^{3/2})$, $\mathcal{O}(\alpha_s^2)$, and $\mathcal{O}(\alpha_s^{3/2})$, respectively, and play minor roles; hence, they can be safely ignored [12, 29].

We obtain the QCD spectral densities $\rho_Z(s)$ through the dispersion relation,

$$\rho_Z(s) = \frac{\mathrm{Im}\widetilde{\Pi}_Z(s)}{\pi}, \tag{11}$$

supposing quark-hadron duality below the continuum threshold s_0, and accomplish a Borel transform in regard to the variable $P^2 = -p^2$ to obtain the QCD sum rules

$$\tilde{\lambda}_Z^2 \exp\left(-\frac{M_Z^2}{T^2}\right) = \int_{4m_c^2}^{s_0} \mathrm{d}s\,\rho_Z(s)\exp\left(-\frac{s}{T^2}\right), \tag{12}$$

where $\tilde{\lambda}_Z = \lambda_Z M_Z$.

We differentiate Eq. (12) with respect to $\frac{1}{T^2}$, eliminate the re-defined pole residues $\tilde{\lambda}_Z$, and obtain QCD sum rules for the masses of the axialvector hidden-charm tetraquark states,

$$M_Z^2 = \frac{\int_{4m_c^2}^{s_0} \mathrm{d}s \frac{\mathrm{d}}{\mathrm{d}(-1/T^2)}\rho_Z(s)\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} \mathrm{d}s\,\rho_Z(s)\exp\left(-\frac{s}{T^2}\right)}. \tag{13}$$

We take the standard values of the vacuum condensates, $\langle\bar{q}q\rangle = -(0.24\pm0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, and $m_0^2 = (0.8\pm0.1)$ GeV^2 at the energy scale $\mu = 1\,\text{GeV}$ [30–32] and take the $\overline{MS}$ quark masses $m_c(m_c) = (1.275\pm0.025)\,\text{GeV}$ and $m_s(\mu = 2\,\text{GeV}) = (0.095\pm0.005)\,\text{GeV}$ from the Particle Data Group [4]. We set $m_q = m_u = m_d = 0$ and consider the energy-scale dependence of the input parameters,

$$\begin{aligned} \langle\bar{q}q\rangle(\mu) &= \langle\bar{q}q\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\ \langle\bar{s}s\rangle(\mu) &= \langle\bar{s}s\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\ \langle\bar{q}g_s\sigma Gq\rangle(\mu) &= \langle\bar{q}g_s\sigma Gq\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\ \langle\bar{s}g_s\sigma Gs\rangle(\mu) &= \langle\bar{s}g_s\sigma Gs\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\ m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}, \\ m_s(\mu) &= m_s(2\text{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{12}{33-2n_f}}, \\ \alpha_s(\mu) &= \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right] \end{aligned} \tag{14}$$

from the renormalization group equation, where $t = \log\frac{\mu^2}{\Lambda_{\rm QCD}^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$, and $\Lambda_{\rm QCD} = 210$ MeV, 292 MeV, and 332 MeV for the flavors $n_f = 5$, 4, and 3, respectively [4, 33]. We choose the flavor number $n_f = 4$ because there are u, d, s, and c quarks.

As in our previous studies, we acquire the acceptable energy scales of the QCD spectral densities for the hidden-charm tetraquark states according to the energy scale formula

$$\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2}, \tag{15}$$

with the effective c-quark mass $\mathbb{M}_c = 1.82\,\text{GeV}$ [11, 15, 34–35]. Furthermore, we consider the $SU(3)$ mass-breaking effects according to the modified energy scale formula

$$\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_c)^2} - k\mathbb{M}_s, \tag{16}$$

where $\mathbb{M}_s$ is the effective s-quark mass and fitted to be

0.2 GeV [23], and k is the number of valence s-quarks.

We search for suitable Borel parameters T^2 and continuum threshold parameters s_0 to satisfy the two criteria (pole or ground state dominance and convergence of operator product expansion) via trial and error. The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities, pole contributions, and contributions from the vacuum condensates of dimension 10 are shown in Table 1. From the table, we can clearly see that the modified energy scale formula is well satisfied. Then, we consider the uncertainties on the input parameters and acquire the masses and pole residues of the hidden-charm tetraquark states without strange, with strange, and with hidden-strange having quantum numbers $J^{PC}=1^{+-}$, which are also shown in Table 1. In Fig. 1, we plot the masses of Z_{cs} and $Z_{cs\bar{s}}$ with variations in the Borel parameters. As shown in the figure, platforms appear in the Borel windows, thus enabling reliable extraction of tetraquark masses.

The present prediction, $M_{Z_c}=(4.02\pm0.09)$ GeV (also in Ref. [28]), is consistent with the experimental values $M_{Z_c^\pm}=(4026.3\pm2.6\pm3.7)$ MeV, $M_{Z_c^\pm}=(4022.9\pm0.8\pm2.7)$ MeV, and $M_{Z_c^0}=(4025.5^{+2.0}_{-4.7}\pm3.1)$ MeV from the BESIII Collaboration [1–3], which supports assigning $Z_c(4020/4025)$ as the $J^{PC}=1^{+-}$ $A\bar{A}$-type tetraquark state. We cannot assign a hadron unambiguously with the mass alone; we must calculate the partial decay widths and total width to perform a more robust assignment.

III. DECAY WIDTHS OF THE Z_c AND Z_{cs} STATES WITH QCD SUM RULES

We investigate the two-body strong decays $Z_{cs}\to h_cK$, $J/\psi K$, and η_cK^* with the three-point correlation functions $\Pi_{\alpha\beta\mu\nu}(p,q)$, $\Pi^1_{\alpha\mu\nu}(p,q)$, and $\Pi^2_{\alpha\mu\nu}(p,q)$, respectively,

$$\begin{aligned}
\Pi_{\alpha\beta\mu\nu}(p,q)&=i^2\int d^4x d^4y\, e^{ipx}e^{iqy}\langle0|T\left\{J^{h_c}_{\alpha\beta}(x)J^K_5(y)J^{u\bar{s}\dagger}_{\mu\nu}(0)\right\}|0\rangle,\\
\Pi^1_{\alpha\mu\nu}(p,q)&=i^2\int d^4x d^4y\, e^{ipx}e^{iqy}\langle0|T\left\{J^{J/\psi}_{\alpha}(x)J^K_5(y)J^{u\bar{s}\dagger}_{\mu\nu}(0)\right\}|0\rangle,\\
\Pi^2_{\alpha\mu\nu}(p,q)&=i^2\int d^4x d^4y\, e^{ipx}e^{iqy}\langle0|T\left\{J^{\eta_c}_{5}(x)J^{K^*}_\alpha(y)J^{u\bar{s}\dagger}_{\mu\nu}(0)\right\}|0\rangle,
\end{aligned}\tag{17}$$

where the currents

$$\begin{aligned}
J^{h_c}_{\alpha\beta}(x)&=\bar{c}(x)\sigma_{\alpha\beta}c(x),\\
J^{J/\psi}_{\mu}(x)&=\bar{c}(x)\gamma_\mu c(x),\\
J^{K}_{5}(y)&=\bar{u}(y)i\gamma_5 s(y),\\
J^{\eta_c}_{5}(x)&=\bar{c}(x)i\gamma_5 c(x),\\
J^{K^*}_{\mu}(y)&=\bar{u}(y)\gamma_\mu s(y),
\end{aligned}\tag{18}$$

interpolate the mesons h_c, J/ψ, K, η_c, and K^*, respectively. With the simple substitution of $s\to d$, we obtain the corresponding ones for the Z_c tetraquark state.

Table 1. Borel parameters, continuum threshold parameters, energy scales, pole contributions, contributions from the vacuum condensates of dimension 10, and masses and pole residues for the axialvector tetraquark states.

	T^2/GeV^2	$\sqrt{s_0}/\text{GeV}$	μ/GeV	pole	$\lvert D(10)\rvert$	M_Z/GeV	$\tilde{\lambda}_Z/(10^{-2}\text{GeV}^5)$
Z_c	$3.3-3.7$	4.6 ± 0.1	1.7	$(40-59)$ %	$\ll1$ %	4.02 ± 0.09	3.00 ± 0.45
Z_{cs}	$3.4-3.8$	4.7 ± 0.1	1.7	$(41-60)$ %	$\ll1$ %	4.11 ± 0.08	3.49 ± 0.51
$Z_{cs\bar{s}}$	$3.5-3.9$	4.8 ± 0.1	1.7	$(42-61)$ %	$\ll1$ %	4.20 ± 0.09	4.00 ± 0.58

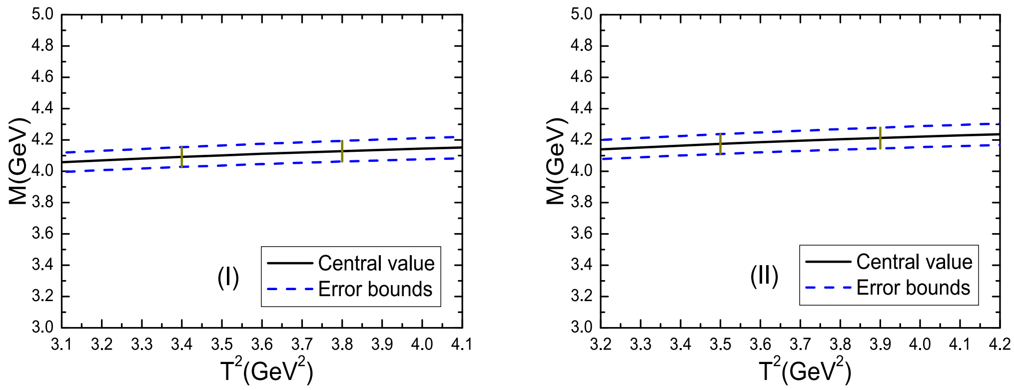

Fig. 1. (color online) Masses of the tetraquark states with variations in the Borel parameters T^2, where (I) and (II) correspond to Z_{cs} and $Z_{cs\bar{s}}$, respectively, and the regions between the two vertical lines are the Borel windows.

We insert a complete set of intermediate hadronic states having possible (non-vanishing) couplings with the current operators into the three-point correlation functions and explicitly isolate the ground state contributions.

$$\begin{aligned}\Pi^{\alpha\beta\mu\nu}(p,q) =&\lambda_K f_h\varepsilon^{\alpha\beta\alpha'\beta'}\xi_{\alpha'}p_{\beta'}\lambda_Z\varepsilon^{\mu\nu\mu'\nu'}\zeta^*_{\mu'}p'_{\nu'}\frac{-\mathrm{i}G_{ZhK}\varepsilon^{\rho\sigma\lambda\tau}p_\rho\xi^*_\sigma p'_\lambda\zeta_\tau}{(M_Z^2-p'^2)(M_h^2-p^2)(M_K^2-q^2)}\\&+\lambda_K f_h\varepsilon^{\alpha\beta\alpha'\beta'}\xi_{\alpha'}p_{\beta'}\lambda_Y\left(\zeta^{*\mu}p'^\nu-\zeta^{*\nu}p'^\mu\right)\frac{-G_{YhK}\xi^*\cdot\zeta}{(M_Y^2-p'^2)(M_h^2-p^2)(M_K^2-q^2)}\\&+\lambda_K f^T_{J/\psi}\left(\xi^\alpha p^\beta-\xi^\beta p^\alpha\right)\lambda_Z\varepsilon^{\mu\nu\mu'\nu'}\zeta^*_{\mu'}p'_{\nu'}\frac{-G_{ZJ/\psi K}\xi^*\cdot\zeta}{(M_Z^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}\\&+\lambda_K f^T_{J/\psi}\left(\xi^\alpha p^\beta-\xi^\beta p^\alpha\right)\lambda_Y\left(\zeta^{*\mu}p'^\nu-\zeta^{*\nu}p'^\mu\right)\frac{-\mathrm{i}G_{YJ/\psi K}\varepsilon^{\rho\sigma\lambda\tau}p_\rho\xi^*_\sigma p'_\lambda\zeta_\tau}{(M_Y^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}+\cdots,\end{aligned} \quad (19)$$

$$\begin{aligned}\Pi_1^{\alpha\mu\nu}(p,q) =&\lambda_K\lambda_{J/\psi}\xi^\alpha\lambda_Z\varepsilon^{\mu\nu\mu'\nu'}\zeta^*_{\mu'}p'_{\nu'}\frac{-G_{ZJ/\psi K}\xi^*\cdot\zeta}{(M_Z^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}\\&+\lambda_K\lambda_{J/\psi}\xi^\alpha\lambda_Y\left(\zeta^{*\mu}p'^\nu-\zeta^{*\nu}p'^\mu\right)\frac{-\mathrm{i}G_{YJ/\psi K}\varepsilon^{\rho\sigma\lambda\tau}p_\rho\xi^*_\sigma p'_\lambda\zeta_\tau}{(M_Y^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}+\cdots,\end{aligned} \quad (20)$$

$$\begin{aligned}\Pi_2^{\alpha\mu\nu}(p,q) =&\lambda_\eta\lambda_{K^*}\xi^\alpha\lambda_Z\varepsilon^{\mu\nu\mu'\nu'}\zeta^*_{\mu'}p'_{\nu'}\frac{-G_{Z\eta K^*}\xi^*\cdot\zeta}{(M_Z^2-p'^2)(M_\eta^2-p^2)(M_{K^*}^2-q^2)}\\&+\lambda_\eta\lambda_{K^*}\xi^\alpha\lambda_Y\left(\zeta^{*\mu}p'^\nu-\zeta^{*\nu}p'^\mu\right)\frac{-\mathrm{i}G_{Y\eta K^*}\varepsilon^{\rho\sigma\lambda\tau}q_\rho\xi^*_\sigma p'_\lambda\zeta_\tau}{(M_Y^2-p'^2)(M_\eta^2-p^2)(M_{K^*}^2-q^2)}+\cdots,\end{aligned} \quad (21)$$

where $\lambda_K=\frac{f_K M_K^2}{m_u+m_s}$, $\lambda_\eta=\frac{f_{\eta_c}M_{\eta_c}^2}{2m_c}$, $\lambda_{J/\psi}=f_{J/\psi}M_{J/\psi}$, $\lambda_{K^*}=f_{K^*}M_{K^*}$, $p'=p+q$, and the decay constants of the mesons h_c, J/ψ, K, η_c, and K^* are defined by

$$\begin{aligned}\langle 0|J^{h_c}_{\mu\nu}(0)|h_c(p)\rangle =&f_{h_c}\varepsilon_{\mu\nu\alpha\beta}\xi^\alpha p^\beta,\\ \langle 0|J^{h_c}_{\mu\nu}(0)|J/\psi(p)\rangle =&f^T_{J/\psi}\left(\xi_\mu p_\nu-\xi_\nu p_\mu\right),\\ \langle 0|J^{J/\psi}_{\mu}(0)|J/\psi(p)\rangle =&f_{J/\psi}M_{J/\psi}\xi_\mu,\\ \langle 0|J^{K^*}_{\mu}(0)|K^*(p)\rangle =&f_{K^*}M_{K^*}\xi_\mu,\\ \langle 0|J^{K}_{5}(0)|K(p)\rangle =&\frac{f_K M_K^2}{m_u+m_s},\\ \langle 0|J^{\eta_c}_{5}(0)|\eta_c(p)\rangle =&\frac{f_{\eta_c}M_{\eta_c}^2}{2m_c},\end{aligned} \quad (22)$$

where ξ are polarization vectors of h_c, J/ψ, and K^*, and the hadronic coupling constants are defined by

$$\begin{aligned}\langle h_c(p)K(q)|Z_{cs}(p')\rangle =&G_{ZhK}\varepsilon^{\rho\sigma\lambda\tau}p_\rho\xi^*_\sigma p'_\lambda\zeta_\tau,\\ \langle J/\psi(p)K(q)|Y_{cs}(p')\rangle =&G_{YJ/\psi K}\varepsilon^{\rho\sigma\lambda\tau}p_\rho\xi^*_\sigma p'_\lambda\zeta_\tau,\\ \langle h_c(p)K(q)|Y_{cs}(p')\rangle =&-\mathrm{i}G_{YhK}\xi^*\cdot\zeta,\\ \langle J/\psi(p)K(q)|Z_{cs}(p')\rangle =&-\mathrm{i}G_{ZJ/\psi K}\xi^*\cdot\zeta,\\ \langle \eta_c(p)K^*(q)|Z_{cs}(p')\rangle =&-\mathrm{i}G_{Z\eta K^*}\xi^*\cdot\zeta.\end{aligned} \quad (23)$$

The tensor structures in Eqs. (19)–(21) are sufficiently complex, and we must project the relevant components with suitable tensor operators,

$$\begin{aligned}&-\frac{2\mathrm{i}}{9}\left(p^2q^2-(p\cdot q)^2\right)\Pi_{h_cK}(p'^2,p^2,q^2)\\ &=P^{\alpha\beta\eta\theta}_{A,p}P^{\mu\nu\phi\omega}_{A,p'}\varepsilon_{\eta\theta\phi\omega}\Pi_{\alpha\beta\mu\nu}(p,q),\\ &-6\left(p^2+q^2+2p\cdot q\right)\Pi_{J/\psi K}(p'^2,p^2,q^2)=\varepsilon_{\mu\nu\alpha\sigma}p'^\sigma\Pi_1^{\alpha\mu\nu}(p,q)\\ &-6\left(p^2+q^2+2p\cdot q\right)\Pi_{\eta_cK^*}(p'^2,p^2,q^2)=\varepsilon_{\mu\nu\alpha\sigma}p'^\sigma\Pi_2^{\alpha\mu\nu}(p,q),\end{aligned} \quad (24)$$

where

$$\begin{aligned}\Pi_{h_cK}(p'^2,p^2,q^2) =&\frac{G_{ZhK}\lambda_K f_h\lambda_Z}{(M_Z^2-p'^2)(M_h^2-p^2)(M_K^2-q^2)}+\cdots,\\ \Pi_{J/\psi K}(p'^2,p^2,q^2) =&\frac{G_{ZJ/\psi K}\lambda_K\lambda_{J/\psi}\lambda_Z}{(M_Z^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}+\cdots,\\ \Pi_{\eta_cK^*}(p'^2,p^2,q^2) =&\frac{G_{Z\eta K^*}\lambda_{K^*}\lambda_\eta\lambda_Z}{(M_Z^2-p'^2)(M_\eta^2-p^2)(M_{K^*}^2-q^2)}+\cdots,\end{aligned} \quad (25)$$

which correspond to the two-body strong decays $Z_{cs}\to h_cK$, $J/\psi K$, and η_cK^*, respectively; the other components in Eqs. (19)–(21) have no contributions or contaminations. In Eq. (19), there are four channels, $Z_{cs}\to h_cK$, $Y_{cs}\to h_cK$, $Z_{cs}\to J/\psi K$, and $Y_{cs}\to J/\psi K$, which correspond to four different tensor structures and therefore four different components. We project the channel $Z_{cs}\to h_cK$ explicitly. In Eq. (20), there are two channels, $Z_{cs}\to J/\psi K$ and $Y_{cs}\to J/\psi K$, which correspond to two different tensor structures and therefore two different components. We project the channel $Z_{cs}\to J/\psi K$ explicitly. In Eq. (21), there are two channels, $Z_{cs}\to \eta_cK^*$ and $Y_{cs}\to \eta_cK^*$, which correspond to two different tensor structures and therefore two different components. We project the channel $Z_{cs}\to \eta_cK^*$ explicitly. The $\cdots$ in Eq. (25) represents the neglected contributions from the higher resonances and continuum states. According to the analysis in Refs. [27, 36–40], we can introduce the parameters C_{h_cK}, $C_{J/\psi K}$, and $C_{\eta_cK^*}$ to parametrize the higher resonance and continuum states involving the Z_{cs} channel,

$$\begin{aligned}\Pi_{h_cK}(p'^2,p^2,q^2)=&\frac{G_{ZhK}\lambda_Kf_h\lambda_Z}{(M_Z^2-p'^2)(M_h^2-p^2)(M_K^2-q^2)}+\frac{C_{h_cK}}{(M_h^2-p^2)(M_K^2-q^2)},\\ \Pi_{J/\psi K}(p'^2,p^2,q^2)=&\frac{G_{ZJ/\psi K}\lambda_K\lambda_{J/\psi}\lambda_Z}{(M_Z^2-p'^2)(M_{J/\psi}^2-p^2)(M_K^2-q^2)}+\frac{C_{J/\psi K}}{(M_{J/\psi}^2-p^2)(M_K^2-q^2)},\\ \Pi_{\eta_cK^*}(p'^2,p^2,q^2)=&\frac{G_{Z\eta K^*}\lambda_{K^*}\lambda_\eta\lambda_Z}{(M_Z^2-p'^2)(M_\eta^2-p^2)(M_{K^*}^2-q^2)}+\frac{C_{\eta_cK^*}}{(M_\eta^2-p^2)(M_{K^*}^2-q^2)}.\end{aligned}\tag{26}$$

Moreover, we perform Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the result

$$\begin{aligned}2\sqrt{2}J^{\mu\nu}_{u\bar{s}}=&\mathrm{i}\bar{s}u\,\bar{c}\sigma^{\mu\nu}c+\mathrm{i}\bar{s}\sigma^{\mu\nu}u\,\bar{c}c+\mathrm{i}\bar{s}c\,\bar{c}\sigma^{\mu\nu}u+\mathrm{i}\bar{s}\sigma^{\mu\nu}c\,\bar{c}u-\frac{\mathrm{i}}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\sigma_{\alpha\beta}c\,\bar{s}\mathrm{i}\gamma_5u\\ &-\bar{c}\mathrm{i}\gamma_5c\,\bar{s}\sigma^{\mu\nu}\gamma_5u-\bar{c}\sigma^{\mu\nu}\gamma_5u\,\bar{s}\mathrm{i}\gamma_5c-\bar{s}\mathrm{i}\gamma_5c\,\bar{c}\sigma^{\mu\nu}\gamma_5u+\mathrm{i}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\gamma^\alpha\gamma_5c\,\bar{s}\gamma^\beta u\\ &-\mathrm{i}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\gamma^\alpha c\,\bar{s}\gamma^\beta\gamma_5u+\mathrm{i}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\gamma^\alpha\gamma_5u\,\bar{s}\gamma^\beta c-\mathrm{i}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\gamma^\alpha u\,\bar{s}\gamma^\beta\gamma_5c,\end{aligned}\tag{27}$$

where the component $\frac{\mathrm{i}}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{c}\sigma_{\alpha\beta}c\,\bar{s}\mathrm{i}\gamma_5u$ leads to the correlation function

$$\begin{aligned}\widetilde{\Pi}_{\alpha\beta\mu\nu}(p,q)=&\frac{i^2\varepsilon_{\mu\nu\lambda\tau}}{4\sqrt{2}}\int\mathrm{d}^4x\mathrm{d}^4y\,\mathrm{e}^{\mathrm{i}px}\mathrm{e}^{\mathrm{i}qy}\langle0|T\left\{J^{h_c}_{\alpha\beta}(x)J^K_5(y)\,\bar{c}(0)\sigma^{\lambda\tau}c(0)\,\bar{u}(0)\mathrm{i}\gamma_5s(0)\right\}|0\rangle,\\ \to&\kappa\frac{i^2\varepsilon_{\mu\nu\lambda\tau}}{4\sqrt{2}}\int\mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}px}\langle0|T\left\{J^{h_c}_{\alpha\beta}(x)\,\bar{c}(0)\sigma^{\lambda\tau}c(0)\right\}|0\rangle\int\mathrm{d}^4y\,\mathrm{e}^{\mathrm{i}qy}\langle0|T\left\{J^K_5(y)\,\bar{u}(0)\mathrm{i}\gamma_5s(0)\right\}|0\rangle,\end{aligned}\tag{28}$$

and we introduce a parameter κ to represent the possible factorizable contributions on the hadron side as we choose the local currents. The conventional mesons and tetraquark states have average spatial sizes of the same order, and $J^{\mu\nu}_{u\bar{s}}(0)$ potentially couples to the tetraquark state rather than the two-meson scattering states; therefore, $\kappa\ll1$ [41]. However, such a term makes a contribution to the component $\Pi_{h_cK}(p'^2,p^2,q^2)$,

$$\frac{\widetilde{C}_{h_cK}}{(M_h^2-p^2)(M_K^2-q^2)},\tag{29}$$

where the coefficient $\widetilde{C}_{h_cK}$ can be absorbed into the coefficient C_{h_cK}. We can clearly see that the parameter C_{h_cK} is necessary, and the parameters $C_{J/\psi K}$ and $C_{\eta_cK^*}$ are implied in the same way.

We accomplish operator product expansion up to the vacuum condensates of dimension 5 and neglect the minor gluon condensate contributions [27, 36–40]. We then obtain the QCD spectral densities $\rho_{\rm QCD}(p'^2,s,u)$ through the double dispersion relation,

$$\Pi_{\rm QCD}(p'^2,p^2,q^2)=\int_{\Delta_s^2}^\infty\mathrm{d}s\int_{\Delta_u^2}^\infty\mathrm{d}u\frac{\rho_{\rm QCD}(p'^2,s,u)}{(s-p^2)(u-q^2)},\tag{30}$$

where Δ_s^2 and Δ_u^2 are the thresholds. On the hadron side, we obtain the hadronic spectral densities $\rho_H(s',s,u)$ through the triple dispersion relation,

$$\begin{aligned}\Pi_H(p'^2,p^2,q^2)=&\int_{\Delta_s'^2}^\infty\mathrm{d}s'\int_{\Delta_s^2}^\infty\mathrm{d}s\int_{\Delta_u^2}^\infty\mathrm{d}u\\ &\times\frac{\rho_H(s',s,u)}{(s'-p'^2)(s-p^2)(u-q^2)},\end{aligned}\tag{31}$$

according to Eq. (25), where $\Delta_s'^2$ are the thresholds. We match the hadron side with the QCD side below the continuum thresholds to acquire rigorous quark-hadron duality [36, 37],

$$\int_{\Delta_s^2}^{s_0} \mathrm{d}s \int_{\Delta_u^2}^{u_0} \mathrm{d}u \frac{\rho_{\rm QCD}(p'^2,s,u)}{(s-p^2)(u-q^2)} = \int_{\Delta_s^2}^{s_0} \mathrm{d}s \int_{\Delta_u^2}^{u_0} \mathrm{d}u \left[\int_{\Delta_s'^2}^{\infty} \mathrm{d}s' \frac{\rho_H(s',s,u)}{(s'-p'^2)(s-p^2)(u-q^2)}\right], \quad (32)$$

where s_0 and u_0 are the continuum thresholds. We first take the integral over $\mathrm{d}s'$ and introduce some unknown parameters, such as C_{h_cK}, $C_{J/\psi K}$, and $C_{\eta_c K^*}$, to parametrize contributions involving higher resonances and continuum states in the s' channel.

We set $p'^2=p^2$ in the correlation functions $\Pi(p'^2,p^2,q^2)$ and perform a double Borel transform in regard to the variables $P^2=-p^2$ and $Q^2=-q^2$. We then set the Borel parameters $T_1^2=T_2^2=T^2$ to obtain three QCD sum rules.

$$\begin{aligned}
&\frac{\lambda_{ZhK}G_{ZhK}}{M_Z^2-M_h^2}\left[\exp\left(-\frac{M_h^2}{T^2}\right)-\exp\left(-\frac{M_Z^2}{T^2}\right)\right]\exp\left(-\frac{M_K^2}{T^2}\right)+C_{h_cK}\exp\left(-\frac{M_h^2+M_K^2}{T^2}\right)\\
=&\frac{1}{64\sqrt{2}\pi^4}\int_{4m_c^2}^{s_h^0}\mathrm{d}s\int_0^{s_K^0}\mathrm{d}u\sqrt{1-\frac{4m_c^2}{s}}\left(1-\frac{4m_c^2}{s}\right)\exp\left(-\frac{s+u}{T^2}\right)+\frac{m_s[2\langle\bar qq\rangle-\langle\bar ss\rangle]}{48\sqrt{2}\pi^2T^2}\int_{4m_c^2}^{s_h^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\left(1-\frac{4m_c^2}{s}\right)\exp\left(-\frac{s}{T^2}\right)\\
&+\frac{m_s\langle\bar qGq\rangle}{96\sqrt{2}\pi^2T^2}\int_{4m_c^2}^{s_h^0}\mathrm{d}s\frac{1}{\sqrt{s\left(s-4m_c^2\right)}}\left(1-\frac{2m_c^2}{s}\right)\exp\left(-\frac{s}{T^2}\right)+\frac{m_s\langle\bar qGq\rangle}{96\sqrt{2}\pi^2T^2}\int_{4m_c^2}^{s_h^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\frac{1}{s}\exp\left(-\frac{s}{T^2}\right)\\
&+\frac{m_s\langle\bar qGq\rangle}{64\sqrt{2}\pi^2T^4}\int_{4m_c^2}^{s_h^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\left(1-\frac{4m_c^2}{s}\right)\exp\left(-\frac{s}{T^2}\right),
\end{aligned} \quad (33)$$

$$\begin{aligned}
&\frac{\lambda_{ZJ/\psi K}G_{ZJ/\psi K}}{M_Z^2-M_{J/\psi}^2}\left[\exp\left(-\frac{M_{J/\psi}^2}{T^2}\right)-\exp\left(-\frac{M_Z^2}{T^2}\right)\right]\exp\left(-\frac{M_K^2}{T^2}\right)+C_{J/\psi K}\exp\left(-\frac{M_{J/\psi}^2+M_K^2}{T^2}\right)\\
=&\frac{3}{128\sqrt{2}\pi^4}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\int_0^{s_K^0}\mathrm{d}u\sqrt{1-\frac{4m_c^2}{s}}\left[2um_c+m_s\left(s+2m_c^2\right)\left(\frac{2}{3}-\frac{u}{9s}\right)\right]\exp\left(-\frac{s+u}{T^2}\right)\\
&-\frac{\langle\bar qq\rangle+\langle\bar ss\rangle}{24\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\left(s+2m_c^2\right)\exp\left(-\frac{s}{T^2}\right)+\frac{m_sm_c[\langle\bar ss\rangle-2\langle\bar qq\rangle]}{16\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\exp\left(-\frac{s}{T^2}\right)\\
&+\frac{\langle\bar qGq\rangle+\langle\bar sGs\rangle}{576\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\frac{s+8m_c^2}{\sqrt{s\left(s-4m_c^2\right)}}\exp\left(-\frac{s}{T^2}\right)-\frac{\langle\bar qGq\rangle+\langle\bar sGs\rangle}{576\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\exp\left(-\frac{s}{T^2}\right)\\
&+\frac{m_sm_c\langle\bar qGq\rangle}{192\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\frac{1}{\sqrt{s\left(s-4m_c^2\right)}}\exp\left(-\frac{s}{T^2}\right)-\frac{m_sm_c\langle\bar qGq\rangle}{192\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\frac{1}{s}\exp\left(-\frac{s}{T^2}\right)\\
&-\frac{m_sm_c\langle\bar qGq\rangle}{16\sqrt{2}\pi^2T^2}\int_{4m_c^2}^{s_{J/\psi}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\exp\left(-\frac{s}{T^2}\right),
\end{aligned} \quad (34)$$

$$\begin{aligned}
&\frac{\lambda_{Z\eta K^*}G_{Z\eta K^*}}{M_Z^2-M_\eta^2}\left[\exp\left(-\frac{M_\eta^2}{T^2}\right)-\exp\left(-\frac{M_Z^2}{T^2}\right)\right]\exp\left(-\frac{M_{K^*}^2}{T^2}\right)+C_{\eta_cK^*}\exp\left(-\frac{M_\eta^2+M_{K^*}^2}{T^2}\right)\\
=&\frac{3}{128\sqrt{2}\pi^4}\int_{4m_c^2}^{s_{\eta_c}^0}\mathrm{d}s\int_0^{s_{K^*}^0}\mathrm{d}u\sqrt{1-\frac{4m_c^2}{s}}\left(\frac{10um_c}{9}+m_ss\right)\exp\left(-\frac{s+u}{T^2}\right)-\frac{\langle\bar qq\rangle+\langle\bar ss\rangle}{16\sqrt{2}\pi^2}\int_{4m_c^2}^{s_{\eta_c}^0}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\,s\exp\left(-\frac{s}{T^2}\right)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{m_s m_c\left[\langle\bar{s}s\rangle-6\langle\bar{q}q\rangle\right]}{48\sqrt{2}\pi^2}\int_{4m_c^2}^{s^0_{\eta_c}}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\exp\left(-\frac{s}{T^2}\right)+\frac{\langle\bar{q}Gq\rangle+\langle\bar{s}Gs\rangle}{576\sqrt{2}\pi^2}\int_{4m_c^2}^{s^0_{\eta_c}}\mathrm{d}s\frac{s+2m_c^2}{\sqrt{s\left(s-4m_c^2\right)}}\exp\left(-\frac{s}{T^2}\right)\\
&-\frac{\langle\bar{q}Gq\rangle+\langle\bar{s}Gs\rangle}{576\sqrt{2}\pi^2}\int_{4m_c^2}^{s^0_{\eta_c}}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\left(1-\frac{12s}{T^2}\right)\exp\left(-\frac{s}{T^2}\right)+\frac{m_s m_c\langle\bar{q}Gq\rangle}{96\sqrt{2}\pi^2}\int_{4m_c^2}^{s^0_{\eta_c}}\mathrm{d}s\frac{1}{\sqrt{s\left(s-4m_c^2\right)}}\exp\left(-\frac{s}{T^2}\right)\\
&+\frac{m_s m_c\langle\bar{s}Gs\rangle}{288\sqrt{2}\pi^2T^2}\int_{4m_c^2}^{s^0_{\eta_c}}\mathrm{d}s\sqrt{1-\frac{4m_c^2}{s}}\exp\left(-\frac{s}{T^2}\right),
\end{aligned}
\tag{35}
$$

where $\langle\bar{q}Gq\rangle=\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{s}Gs\rangle=\langle\bar{s}g_s\sigma Gs\rangle$, $\lambda_{ZhK}=\lambda_K f_h\lambda_Z$, $\lambda_{ZJ/\psi K}=\lambda_K\lambda_{J/\psi}\lambda_Z$, and $\lambda_{Z\eta K^*}=\lambda_{K^*}\lambda_\eta\lambda_Z$. We neglect the dependencies of the parameters C_{h_cK}, $C_{J/\psi K}$, and $C_{\eta_cK^*}$ on the Lorentz invariants p'^2, p^2, and q^2. Instead, we take them as free parameters and search for the best values to delete the contamination from high resonances and continuum states and hence acquire stable QCD sum rules. The corresponding hadronic coupling constants for the $Z_c(4020/4025)$ state can be obtained with the simple substitution of $s\to d$ and are treated in the same manner.

On the QCD side, we choose the flavor number $n_f=4$ and set the energy scale to be $\mu=1.3\,\mathrm{GeV}$, as in a previous study on the decays of $Z_{cs}(3985/4000)$ [27]. On the hadron side, we take the parameters as $M_K=0.4937\,\mathrm{GeV}$, $M_\pi=$ 0.13957 GeV, $M_{K^*}=$ 0.8917 GeV, $M_\rho=$ 0.77526 GeV, $M_{J/\psi}=$ 3.0969 GeV, $M_{\eta_c}=$ 2.9834 GeV, $M_{h_c}=$ 3.525 GeV [4], $f_K=$ 0.156 GeV, $f_\pi=$ 0.130 GeV [4], $f_{K^*}=$ 0.220 GeV, $f_\rho=$ 0.215 GeV, $\sqrt{s^0_K}=$ 1.0 GeV, $\sqrt{s^0_\pi}=$ 0.85 GeV, $\sqrt{s^0_{K^*}}=$ 1.3 GeV, $\sqrt{s^0_\rho}=$ 1.2 GeV [42], $f_{h_c}=$ 0.235 GeV, $f_{J/\psi}=$ 0.418 GeV, $f_{\eta_c}=$ 0.387 GeV [43], $\sqrt{s^0_{h_c}}=$ 4.05 GeV, $\sqrt{s^0_{J/\psi}}=$ 3.6 GeV, $\sqrt{s^0_{\eta_c}}=$ 3.5 GeV [4], $\frac{f_KM_K^2}{m_u+m_s}=-\frac{\langle\bar{q}q\rangle+\langle\bar{s}s\rangle}{f_K(1-\delta_K)}$, and $\frac{f_\pi M_\pi^2}{m_u+m_d}=-\frac{2\langle\bar{q}q\rangle}{f_\pi}$ from the Gell-Mann-Oakes-Renner relation $\delta_K=0.50$ [44].

In calculations, we fit the unknown parameters to be $C_{h_cK}=0.000064+0.000014\times T^2\,\mathrm{GeV}^4$, $C_{h_c\pi}=0.00006+0.000010\times T^2\,\mathrm{GeV}^4$, $C_{J/\psi K}=0.00335+0.000096\times T^2\,\mathrm{GeV}^7$, $C_{J/\psi\pi}=0.00305+0.000096\times T^2\,\mathrm{GeV}^7$, $C_{\eta_cK^*}=0.00368+0.00012\times T^2\,\mathrm{GeV}^7$, and $C_{\eta_c\rho}=0.00302+0.00012\times T^2\,\mathrm{GeV}^7$ to acquire flat Borel platforms with the interval $T^2_{\max}-T^2_{\min}=1\,\mathrm{GeV}^2$, where max and min represent the maximum and minimum values, respectively. The Borel windows are $T^2_{h_cK}=(4.0-5.0)\,\mathrm{GeV}^2$, $T^2_{h_c\pi}=(4.0-5.0)\,\mathrm{GeV}^2$, $T^2_{J/\psi K}=(4.3-5.3)\,\mathrm{GeV}^2$, $T^2_{J/\psi\pi}=(4.1-5.1)\,\mathrm{GeV}^2$, $T^2_{\eta_cK^*}=(3.9-4.9)\,\mathrm{GeV}^2$, and $T^2_{\eta_c\rho}=(3.9-4.9)\,\mathrm{GeV}^2$, where we add the subscripts h_cK, $h_c\pi\cdots$ to denote the corresponding decay channels. In the Borel windows, the uncertainties δG originating from the Borel parameters T^2 must be less than or approximately $0.01\,(\mathrm{GeV})$. Such a strict and powerful constraint plays a decisive role and works well, as in our previous studies [27, 36–40]. In Fig. 2, we plot the hadronic coupling constants $G_{Z_{cs}h_cK}$, $G_{Z_{cs}J/\psi K}$, $G_{Z_{cs}\eta_cK^*}$, $G_{Z_ch_c\pi}$, $G_{Z_cJ/\psi\pi}$, and $G_{Z_c\eta_c\rho}$ with variations in the Borel parameters. We can explicitly observe flat platforms, which enable reliable extraction of the hadronic coupling constants.

If we take the symbol ξ to represent the input parameters on the QCD side, then, for example, the uncertain-

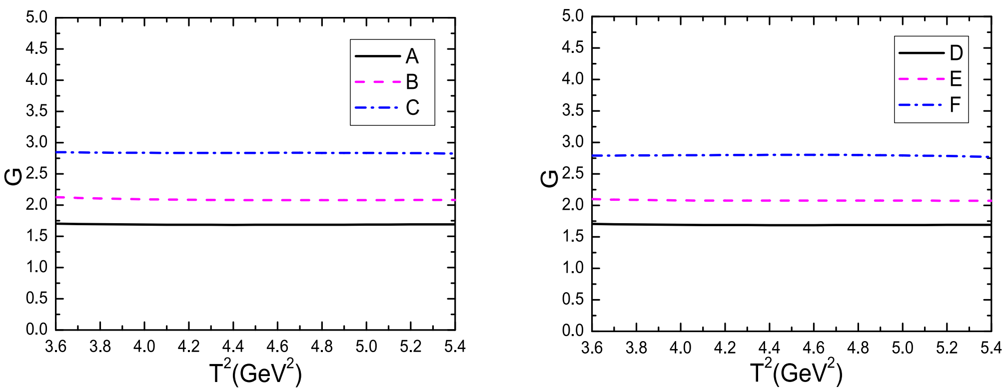

Fig. 2. (color online) Hadronic coupling constants with variations in the Borel parameters T^2, where A, B, C, D, E, and F correspond to $G_{Z_{cs}h_cK}$, $G_{Z_{cs}J/\psi K}$, $G_{Z_{cs}\eta_cK^*}$, $G_{Z_ch_c\pi}$, $G_{Z_cJ/\psi\pi}$, and $G_{Z_c\eta_c\rho}$, respectively.

ties $\bar{\xi} \to \bar{\xi} + \delta\xi$ result in the uncertainties $\bar{f}_{J/\psi}\bar{f}_K\bar{\lambda}_Z\bar{G}_{ZJ/\psi K} \to \bar{f}_{J/\psi}\bar{f}_K\bar{\lambda}_Z\bar{G}_{ZJ/\psi K} + \delta f_{J/\psi}f_K\lambda_Z G_{ZJ/\psi K}$ and $\bar{C}_{J/\psi K} \to \bar{C}_{J/\psi K} + \delta C_{J/\psi K}$, where

$$\delta f_{J/\psi}f_K\lambda_Z G_{ZJ/\psi K} = \bar{f}_{J/\psi}\bar{f}_K\bar{\lambda}_Z\bar{G}_{ZJ/\psi K} \times\left(\frac{\delta f_{J/\psi}}{\bar{f}_{J/\psi}} + \frac{\delta f_K}{\bar{f}_K} + \frac{\delta\lambda_Z}{\bar{\lambda}_Z} + \frac{\delta G_{ZJ/\psi K}}{\bar{G}_{ZJ/\psi K}}\right), \tag{36}$$

in which we add the index $^-$ to all the variables to denote the central values. In the case where the uncertainty $\delta C_{J/\psi K}$ is small enough to be ignored, error analysis is easy to perform by approximately setting $\frac{\delta f_{J/\psi}}{\bar{f}_{J/\psi}} = \frac{\delta f_K}{\bar{f}_K} = \frac{\delta\lambda_Z}{\bar{\lambda}_Z} = \frac{\delta G_{ZJ/\psi K}}{\bar{G}_{ZJ/\psi K}}$. However, if the uncertainty $\delta C_{J/\psi K}$ is considerable, it must be considered for every uncertainty $\delta\xi$. We must adjust $\delta C_{J/\psi K}$ via fine tuning with the help of trial and error according to the variation $\delta\xi$ to acquire enough flat platforms in the same region, as in the case of the central values $\bar{\xi}$ and $\bar{C}_{J/\psi K}$. This error analysis is difficult to perform. We typically set $\frac{\delta f_{J/\psi}}{\bar{f}_{J/\psi}} = \frac{\delta f_K}{\bar{f}_K} = \frac{\delta\lambda_Z}{\bar{\lambda}_Z} = 0$ to estimate the uncertainty $\delta G_{ZJ/\psi K}$; however, the validity of such an approximation is yet to be proved.

Now, let us methodically obtain the hadronic coupling constants according to above error analysis.

$$\begin{aligned} G_{Z_{cs}h_cK} &= 1.68 \pm 0.10, \\ G_{Z_{cs}J/\psi K} &= 2.08 \pm 0.08\,\text{GeV}, \\ G_{Z_{cs}\eta_c K^*} &= 2.84 \pm 0.09\,\text{GeV}, \\ G_{Z_c h_c\pi} &= 1.69 \pm 0.09, \\ G_{Z_c J/\psi\pi} &= 2.08 \pm 0.08\,\text{GeV}, \\ G_{Z_c\eta_c\rho} &= 2.80 \pm 0.09\,\text{GeV}, \end{aligned} \tag{37}$$

by setting

$$\delta f_{J/\psi}f_K\lambda_Z G_{ZJ/\psi K} = \bar{f}_{J/\psi}\bar{f}_K\bar{\lambda}_Z\bar{G}_{ZJ/\psi K}\frac{4\delta G_{ZJ/\psi K}}{\bar{G}_{ZJ/\psi K}}, \tag{38}$$

If we set

$$\delta f_{J/\psi}f_K\lambda_Z G_{ZJ/\psi K} = \bar{f}_{J/\psi}\bar{f}_K\bar{\lambda}_Z\bar{G}_{ZJ/\psi K}\frac{\delta G_{ZJ/\psi K}}{\bar{G}_{ZJ/\psi K}}, \tag{39}$$

the uncertainty $\delta G_{ZJ/\psi K}$ will be four times as large as that given in Eq. (37). Other uncertainties can be understood in the same way. According to Eq. (37), the $SU(3)$ breaking effects in the hadronic coupling constants are small.

It is then easy to obtain the partial decay widths by taking the relevant masses from the Particle Data Group [4],

$$\begin{aligned} \Gamma(Z_{cs} \to h_c K) &= 1.83 \pm 0.22\,\text{MeV}, \\ \Gamma(Z_{cs} \to J/\psi K) &= 8.05 \pm 0.62\,\text{MeV}, \\ \Gamma(Z_{cs} \to \eta_c K^*) &= 12.83 \pm 0.81\,\text{MeV}, \\ \Gamma(Z_c \to h_c\pi) &= 6.86 \pm 0.73\,\text{MeV}, \\ \Gamma(Z_c \to J/\psi\pi) &= 8.82 \pm 0.68\,\text{MeV}, \\ \Gamma(Z_c \to \eta_c\rho) &= 13.89 \pm 0.89\,\text{MeV}, \end{aligned} \tag{40}$$

and the total widths,

$$\begin{aligned} \Gamma_{Z_{cs}} &= 22.71 \pm 1.65\,(\text{or} \pm 6.60)\,\text{MeV}, \\ \Gamma_{Z_c} &= 29.57 \pm 2.30\,(\text{or} \pm 9.20)\,\text{MeV}, \end{aligned} \tag{41}$$

where the values in the brackets are obtained from Eq. (39). The prediction $\Gamma_{Z_c} = 29.57 \pm 2.30\,(\text{or} \pm 9.20)$ MeV is compatible with the upper bound of the experimental data $\Gamma = (24.8 \pm 5.6 \pm 7.7)\,\text{MeV}$ [1], $(23.0 \pm 6.0 \pm 1.0)\,\text{MeV}$ [2], and $(7.9 \pm 2.7 \pm 2.6)\,\text{MeV}$ [3] from the BESIII Collaboration and also supports assigning $Z_c(4020/4025)$ to be the $A\bar{A}$-type hidden-charm tetraquark states with $J^{PC} = 1^{+-}$. In the present study, we neglect the decays $Z_c(4020/4025) \to D^*\bar{D}^*$ and $Z_{cs} \to D^*\bar{D}_s^*$, $D_s^*\bar{D}^*$ because the Z_c and Z_{cs} states lie near the corresponding two-meson thresholds, and the available phase-spaces are small and even lead to the possible assignments of molecular states [5–12]. The most favorable channels are $Z_{cs} \to \eta_c K^*$ and $Z_c \to \eta_c\rho$ at present, even for $Z_c(4020/4025)$. The decay $Z_c(4020/4025) \to \eta_c\rho$ has not yet been observed, and observation of this channel may lead to a more robust assignment and shed light on the nature of Z_c states. We can search for the Z_{cs} state in the invariant mass spectra of h_cK, $J/\psi K$, $\eta_c K^*$, $D^*\bar{D}_s^*$, and $D_s^*\bar{D}^*$ in the future.

In the picture of diquark-antidiquark type tetraquark states, $Z_c(3900)$ and $Z_{cs}(3985)$ can be assigned tentatively as the $S\bar{A} - A\bar{S}$ type hidden-charm tetraquark states, and the hadronic coupling constants have the relations $|G_{ZD^*\bar{D}/ZD\bar{D}^*}| \ll |G_{ZJ/\psi\pi/Z\eta_c\rho}|$ and $|G_{ZD^*\bar{D}_s/ZD\bar{D}_s^*}| \ll |G_{ZJ/\psi K/Z\eta_c K^*}|$. Furthermore, the allowed phase-spaces in the decays to open-charm meson pairs are significantly smaller than those of decays to meson pairs involving charmonium. The contributions of decays to open-charm meson pairs to the total decay widths can be ignored [27, 36]. We expect that the conclusion holds in the present study for the $Z_c(4020/4025)$ and $Z_{cs}(4110)$ states and make a crude estimation of the partial decay widths, $\Gamma\left(Z_c \to D^*\bar{D}/D\bar{D}^*\right) < 1\,\text{MeV}$ and $\Gamma\left(Z_{cs} \to D^*\bar{D}_s/D\bar{D}_s^*\right) < 1$ MeV, based on the relations between the hadronic coupling constants obtained in Refs. [27, 36]; the contributions to the total widths from the decays to the final states

$D^*\bar{D}/D\bar{D}^*$ and $D^*\bar{D}_s/D\bar{D}_s^*$ are also ignored.

IV. CONCLUSION

In this article, we tentatively assign $Z_c(4020/4025)$ as the $A\bar{A}$-type hidden-charm tetraquark state with $J^{PC}=1^{+-}$ and construct $A\bar{A}$-type tensor currents to investigate the tetraquark states without strange, with strange, and with hidden-strange via QCD sum rules. We consider the contributions of the vacuum condensates up to dimension-10 in operator product expansion. Then, we resort to the modified energy scale formula $\mu=\sqrt{M^2_{X/Y/Z}-(2\mathbb{M}_c)^2}-k\mathbb{M}_s$ to account for the $SU(3)$ mass-breaking effects to choose suitable energy scales for the QCD spectral densities and obtain the tetraquark masses in a self-consistent manner. We introduce three-point correlation functions to investigate the hadronic coupling constants in the two-body strong decays of the tetraquark states without strange and with strange via QCD sum rules based on rigorous quark-hadron duality, which is a unique feature of our studies. The numerical results indicate that the $SU(3)$ breaking effects in the hadronic coupling constants are small. We then obtain the partial decay widths and total widths of the Z_c and Z_{cs} states and find that the total width Γ_{Z_c} is compatible with that of $Z_c(4020/4025)$ and also supports assigning $Z_c(4020/4025)$ as the $J^{PC}=1^{+-}A\bar{A}$-type tetraquark state. Further experimental data are required to achieve a more robust assignment because $Z_c(4020/4025)$ has not yet been observed in the $J/\psi\pi$ and $\eta_c\rho$ channels. In future, we may search for the strange cousin Z_{cs} in the $D^*\bar{D}_s^*$, $D_s^*\bar{D}^*$, h_cK, $J/\psi K$, and η_cK^* invariant mass spectra, the observation of which would shed light on the nature of Z_c states.

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