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Abstract

The real challenge of the proposed 2-TeV muon-muon collider storage ring has proved to lie in the design of the interaction region. This paper will present some of the critical issues involved in the design of an extreme low-beta interaction region and compare existing designs.

In order to achieve the high luminosities required in a muon-muon collider and to overcome the hour-glass effect, the bunch length of the muons has to be extremely short, within 3 mm (rms), or, equivalently, 10 ps. Therefore, the time slip per turn of a muon particle must be much less than 10 ps. This implies for the proposed 2-TeV muon-muon collider [1], which entails a ring with a 1 km radius, that the phase-slip factor η must have an absolute value much less than 1×10^{-4}. For this reason, an isochronous ring is preferred. Using the concept of the *flexible momentum-compaction lattices* proposed by Lee, Ng, and Trbojevic [2], the design of an isochronous ring is no more difficult than designing a conventional FODO lattice. The real challenge lies in the construction of an interaction region (IR) — one which will propagate a realistic muon beam with the defined peak-to-peak momentum spread of $\pm 0.5\%$ and a normalized emittance of 25π mm-mrad (rms) in both transverse planes.

Because of the short bunch length and symmetric beam requirement, the low-beta values in the IR must be 3 mm in both horizontal and vertical directions. For this reason, the design of the IR becomes nontrivial. A number of complications arise, but the most serious is that an insertion with such low minimum betas results in chromaticities which are many orders of magnitude larger than acceptable (compared with the arcs of the ring, for example). The chromaticities must be corrected locally by families of strong sextupoles, which must be carefully positioned so that the nonlinearity can be canceled as much as possible. A prototype of such an IR has been proposed by Dunham, Napoly, [3] and Gallardo [4], and is plotted in Fig. 1. The design is based on the idea of Brinkmann [5], whose basic approach is to flank either side of the interaction point (IP) with four high-beta bumps in regions of high dispersion. With two bumps exhibiting a high β_x and two with high β_y, sextupoles can be placed to correct the horizontal and vertical chomaticities, respectively. Both the horizontal and vertical beta bumps are individually separated by phase advances

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of $\Delta\phi_x \approx \pi$ and $\Delta\phi_y \approx \pi$, resulting in a cancellation of most of the nonlinear chromatic terms. However, the cancellation of nonlinearity can never be exact. According to our calculation (SYNCH) [7], the second-order amplitude-dependent tune spreads due to the sextupoles are $\alpha_{xx} = -0.564 \times 10^9$ m^{-1}, $\alpha_{xy} = -0.931 \times 10^8$ m^{-1}, and $\alpha_{yy} = -0.315 \times 10^{10}$ m^{-1}, where the α's are defined as

$$\Delta\nu_x = \alpha_{xx}\frac{\epsilon_x}{\pi} + \alpha_{xy}\frac{\epsilon_y}{\pi}\,,$$

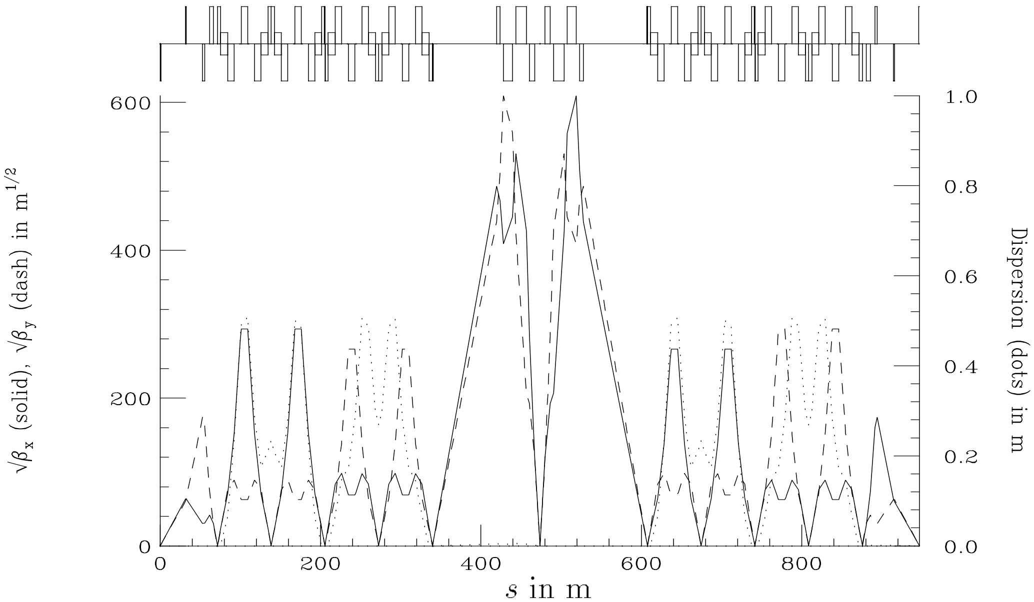
$$\Delta\nu_y = \alpha_{xy}\frac{\epsilon_x}{\pi} + \alpha_{yy}\frac{\epsilon_y}{\pi}\,,$$

with ϵ_x and ϵ_y denoting the *unnormalized* transverse emittances in m. For a normalized rms emittance of 25π mm-mrad, we expect the 2-TeV muon bunch to have ϵ_x and ϵ_y as large as $8 \times 10^{-9}\pi$ m (95%). In the present design, the second-order tune spreads are too large by two to three orders of magnitude, even though the nonlinearity cancellations mentioned did successfully reduce the tune spreads by three to four orders of magnitude. Although the locations of the beta bumps have been carefully designed at odd multipoles of $\pi/2$ from the IP in order to cancel higher-order chromatic aberrations, we think that the reduction of the amplitude-dependent tune spread is of primary importance. The total IR is about 1 km long, and, with betatron functions which shoot up to 400 km, the uncorrected natural chromaticities [6] are $\xi_x = -6593$ and $\xi_y = -6594$. These are tabulated in Table I in line I.

Table I: Comparison of different designs for the IR.

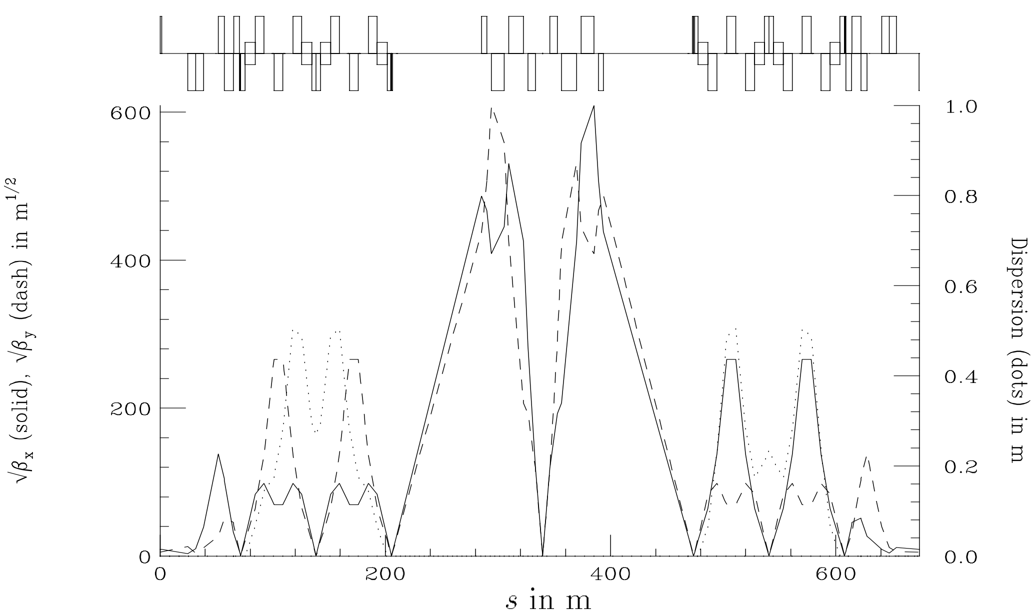
	Length	Natural Chrom.		Sex. Strengths		Tune spread Coefficients (m^{-1})		
	(m)	ξ_x	ξ_y	(m^{-2})		α_{xx}	α_{xy}	α_{yy}
I	947	−6593	−6594	0.338	−0.662	−0.564E9	−0.931E8	−0.315E10
II	674	−5134	−5134	0.596	−1.165	−0.710E9	−0.144E9	−0.392E10
III	540	−4034	−3705	0.355	−0.644	−0.407E9	−0.595E8	−0.197E10
IV	526	−4978	−4651	0.463	−0.853	−0.632E9	−0.102E9	−0.314E10

We attempted initially to modify the IP and decrease its length by reducing the number of flanking high-beta bumps by two. Horizontal chromaticity was corrected on one side and vertical on the other with correction sextupoles that were almost doubled in strength from the initial design. The total length of the IR was correspondingly shortened to 674 m. Natural chromaticities were reduced to $\xi_x = -5135$ and $\xi_y = -5134$, with only a slight increase in the second-order tune spread coefficients. These results are tabulated in line II in Table I and the lattice is shown in Fig. 2. Note that in this design and those below, the two sides of the IR are matched to $\beta_x = 83.58$ m, $\beta_y = 28.76$ m, and $\alpha_x = \alpha_y = 0$, so that either FODO cells or flexible momentum-compaction



Dispersion max/min: 0.50759/−0.00575m, γ_t: (161.28, 0.00)
β_x max/min: 370902.56/ 0.00300m, ν_x: 5.99968, ξ_x : −6593, Module length: 946.8242m
β_y max/min: 370901.84/ 0.00300m, ν_y: 5.99968, ξ_y : −6594, Total bend angle: 0.240023 rad

Fig. 1. The Gallardo's IR (Design I), with 4 beta bumps on each side.



Dispersion max/min: 0.50422/−0.00002m, γ_t: (192.49, 0.00)
β_x max/min: 370903.38/ 0.00300m, ν_x: 4.42752, ξ_x : −5134, Module length: 674.2582m
β_y max/min: 370903.38/ 0.00300m, ν_y: 4.20707, ξ_y : −5134, Total bend angle: 0.120000 rad

Fig. 2. The Gallardo's IR (Design II), with 2 beta bumps on each side.

modules can be attached easily. On the other hand, Design I is matched to $\beta_x = \beta_y = 0.261$ m, and $\alpha_x = \alpha_y = 0$.

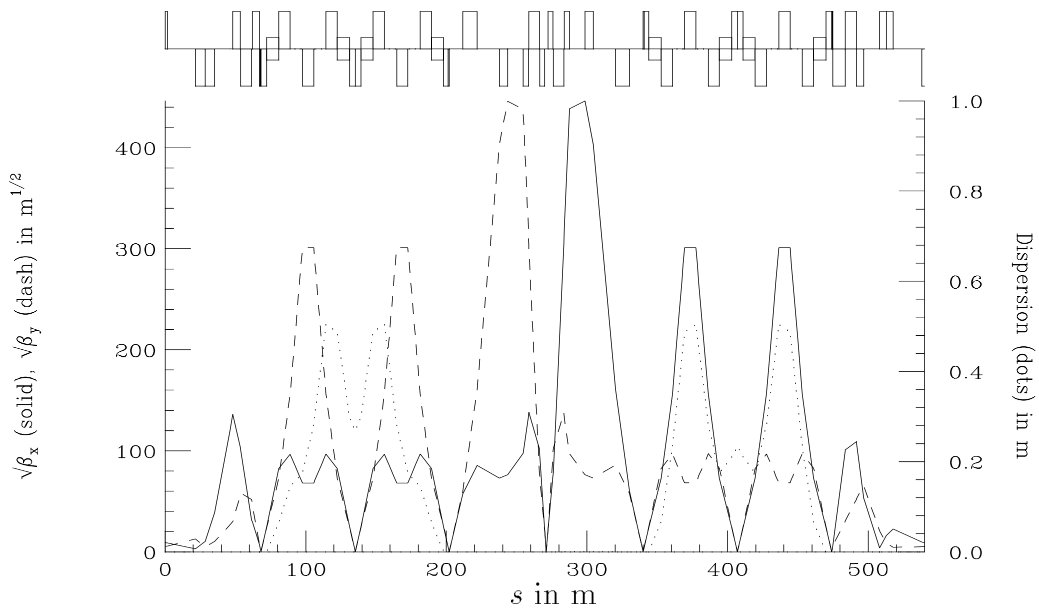
Gelfand [8] proposed an interaction region for the muon collider where the maximum betas are only about 200 km, but local chromaticity correction was not included. His IR was subsequently put into Design II to incorporate a needed chromaticity correction scheme. The resulting IR length was very much reduced, to 540 m, and the natural chromaticities were also lowered to $\xi_x = -4035$ and $\xi_y = -3705$. The sextupole strengths were roughly the same as in Design I, although the number of sextupoles was reduced by a factor of two. The second-order tune spreads were also smaller by roughly a factor of two. This example is in line III in Table I and the lattice is plotted in Fig. 3.

Trbojevic [9], in his low-beta design, has attained maximum betas of only 70 km. Again this IR region was transplanted into Design II. The length is comparable to that of Design III. However, due to the use of more quadrupoles near the IP, the natural chromaticities are slightly higher: $\xi_x = -4035$ and $\xi_y = -3705$, and so are the strengths of the correction sextupoles, and the tune spreads. This work is denoted by IV in Table I and the lattice is plotted in Fig 4.

Some tracking has been performed, and, although expected, the results are disappointing. For example, an on-momentum particle can pass through Design III only 100 times with a horizontal offset of less than 0.13 mm ($\epsilon_x < 2\times 10^{-10}\pi$ m), which is predictable from the calculated amplitude-dependent tune spreads. Various approaches to these problematic tune spreads are currently under consideration. For example, in all of the above designs, the horizontal sextupoles are positioned where the dispersions are $D = 0.46$ and 0.47 m, while the vertical sextupoles are positioned where the dispersions are $D = 0.17$ and 0.31 m. If the dispersion can be increased to 5 m, then the sextupoles strengths can be reduced by more than a factor of 10, and the tune spreads by at least 2 orders of magnitude. One thought is to add two dipoles 180° apart, enclosing the two high-beta bumps on each side of the IP. At present this idea is not possible to implement because either the ideal position is obstructed by a long quadrupole or the horizontal betatron function becomes too small. One could think of passing a dispersion wave through the whole IR, so that at the IP $D = 0$ and $D' \neq 0$. However, the allowable magnitude of D' is limited by the horizontal emittance, $\epsilon_x = 7.92\times 10^{-9}\pi$ m, of the muon bunch. At a distance s away from the IP, the horizontal beta function is given by

$$\beta_x = \beta_x^* + \frac{s^2}{\beta_x^*} \approx \frac{s^2}{\beta_x^*} \ ,$$

where $\beta_x^* = 3$ mm at the IP. Therefore, the horizontal amplitude of the bunch

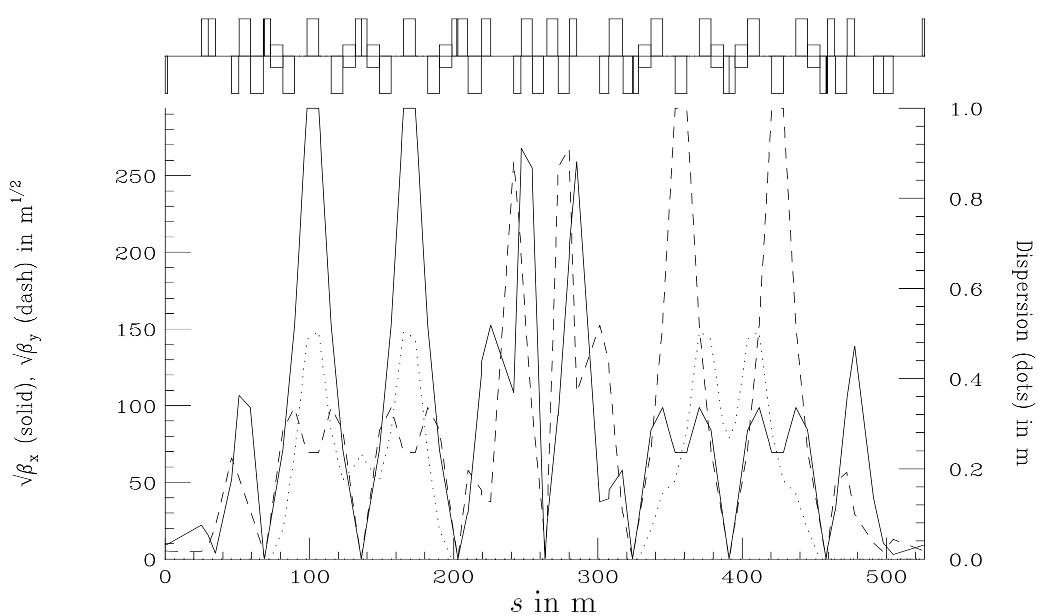


Dispersion max/min: 0.50437/−0.00009m, γ_t: (172.25, 0.00)

β_x max/min: 199279.08/ 0.00300m, ν_x: 4.43382, ξ_x : −4034, Module length: 539.9459m

β_y max/min: 198759.13/ 0.00300m, ν_y: 4.41958, ξ_y : −3705, Total bend angle: 0.120000 rad

Fig. 3. Design III with Gelfand's IP and 2 beta bumps on each side.



Dispersion max/min: 0.50427/ 0.00000m, γ_t: (170.07, 0.00)

β_x max/min: 86441.02/ 0.00300m, ν_x: 5.43411, ξ_x : −4978, Module length: 526.33571m

β_y max/min: 86419.80/ 0.00300m, ν_y: 5.40018, ξ_y : −4659, Total bend angle: 0.120008 rad

Fig. 4. Design IV with Trbojevic's IR and 2 beta bumps on each side.

is

$$x = \sqrt{\frac{\beta_x \epsilon_x}{\pi}} \approx s \sqrt{\frac{\epsilon_x}{\pi \beta_x^*}} \ .$$

There, the horizontal displacement of the beam due to momentum offset $\delta = 0.5\%$ is $D' s \delta$, which must be very much less than the horizontal bunch size x, or

$$D' \ll \sqrt{\frac{\epsilon_x}{\pi \beta_x^*}} \frac{1}{\delta} = 0.33 \ .$$

The horizontal (vertical) sextupoles are placed at a point with $\beta_x \approx 90$ km (5 km) which is an odd multiple of $\pi/2$ from the IP. The dispersion D generated by this dispersion wave will be given by

$$\sqrt{\beta_x^*} D' \Big|_{\mathrm{IP}} = \frac{D}{\sqrt{\beta}} \Big|_{\mathrm{sex.}} \ .$$

This implies that, at the horizontal (vertical) sextupoles, $D \ll 5.4$ m (1.3 m), which is still far from ideal. Another way to cancel the tune spreads is to introduce 3 sets of octupoles. This idea will be discussed elsewhere.

References

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[2] S.Y. Lee, K.Y. Ng, and D. Trbojevic, Phys. Rev. **E48**, 3040 (1993).

[3] B. Dunham and O. Napoly, "FFADA, Final Focus. Automatic Design and Analysis," CERN Report CLIC Note 222, (1994); O. Napoli, "CLIC Final Focus System: Upgraded Version with Increased Bandwidth and Error Analysis," CERN Report CLIC Note 227, (1994).

[4] J.C. Gallardo, private communication, 1995.

[5] R. Brinkmann, "Optimization of a Final Focus System for Large Momentum Bandwidth," DESY-M-90/14 (1990).

[6] SYNCH is not able to print chromaticities of these sizes, because the output fields are F10.5. Here we correct the chromaticities to zero with thin sextupoles and infer the natural chromaticities from the sextupoles.

[7] A.A. Garren, A.S. Kenney, E.D. Courant, and M.J. Syphers, "A User's Guide to SYNCH," Fermilab Report FN-420, 1985.

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[9] D. Trbojevic, private communication, 1995.