

Editorial

Symmetry protected topological phases of matter

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One problem of paramount importance in physics over various fields such as elementary particle physics, condensed matter physics, and statistical mechanics has been to make classification of phases of matter, including nontrivial structures of the vacuum. The triumphant history of theory on the phase classification and the phase transition can be traced back to Landau, which is based on the spontaneous breaking of symmetry. It is known, however, that other kinds of phases exist that cannot be characterized by conventional arguments of symmetry breaking, namely, topological phases. Topological phases are gapped phases at zero temperature without symmetry breaking. Examples of the topological phases include a quantum phase in a fractional quantum Hall system, where the ground state exhibits degeneracy, and an anyon, a quasi-particle with nontrivial statistical nature, neither boson nor fermion, emerges as a low-energy excitation. In contemporary physics of the 21st century, it has been revealed that, among various topological states, there exists a nontrivial ground state that does not have ground state degeneracy and yet can be distinguished from a trivial state. This nontrivial ground state, i.e., a topological insulator, has been recognized first in free electron systems [1–3] and later extended to more general quantum many-body systems with interactions including bosonic (spin) systems. Now, such a generalized situation is called a symmetry protected topological (SPT) phase [4–7]. From the point of a modern view of the SPT phase, nontrivial states, such as the Haldane state proposed in antiferromagnetic quantum spin systems in the 1980’s [8] and the ground state in the Affleck-Kennedy-Lieb-Tasaki model [9], belong to the SPT phase. A remarkable feature of the SPT phase is that, if the system has a surface, gapless surface states appear and they are stable against disturbances which do not break the symmetry. One way to understand these surface states is the quantum anomaly on the surface. That is, as long as the surface is concerned, the quantum anomaly arises, but the whole system has no quantum anomaly, and the symmetry is preserved due to cancellation from the bulk contribution. This canceling mechanism of the quantum anomaly has been long known as the anomaly inflow in elementary particle physics [10]. In fact, the classification of the SPT phases can be identified as the classification of the quantum anomaly through the bulk-boundary correspondence. This type of the quantum anomaly closely related to the SPT phase can provide us with a useful theoretical device to constrain phases that can or cannot be realized under the ’t Hooft anomaly

matching condition [11–13]. It is quite difficult to scrutinize nonperturbatively interacting systems, but the 't Hooft anomaly matching condition can be applied to those systems, and possible phases can be constrained. Nowadays, our understanding of the SPT phases has been so developed that applications of the anomaly matching condition can cover systems with finite groups and/or extending objects [14–16]. In this way, the concept of the topological phases has been essential by now in a wide range of physics, particularly in elementary particle physics, condensed matter physics, and statistical mechanics, as mentioned in the beginning. This special section is a collection of papers related to the topological aspects of physics including the topological phases. We would like to express our sincere gratitude to all the authors who made excellent contributions.

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