Gravitational wave science from space

Jonathan R. Gair

Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, UK
School of Mathematics, University of Edinburgh, Edinburgh, EH9 7JD, UK

E-mail: `j.gair@ed.ac.uk`

Abstract. The rich millihertz gravitational wave band can only be accessed with a space-based detector. The technology for such a detector will be demonstrated by the LISA Pathfinder satellite that is due to launch this year and ESA has selected gravitational wave detection from space as the science theme to be addressed by the L3 large mission to be launched around 2034. In this article we will discuss the sources that such an instrument will observe, and how the numbers of events and precision of parameter determination are affected by modifications to the, as yet not finalised, mission design. We will also describe some of the exciting scientific applications of these observations, to astrophysics, fundamental physics and cosmology.

1. Introduction

The millihertz gravitational wave band is expected to be very rich in gravitational wave sources. Binaries in which the largest member is a supermassive black hole (SMBH) with mass around one million solar masses radiate at millihertz frequencies. Such black holes are expected to reside in the centres of less massive galaxies and play an important role in the assembly of large scale structure, but these black holes are very difficult to observe electromagnetically. In addition, binaries of stellar compact remnants in the Milky Way with periods of around one hour radiate at millihertz frequencies. Unfortunately, this rich frequency band is not accessible to ground-based instruments. Seismic noise limits the sensitivity of ground-based interferometers such as Advanced LIGO and Virgo [1] to frequencies above ~ 10Hz, while pulsar timing arrays [2] are most sensitive at nanohertz frequencies and have very limited sensitivity above $\sim 1/\delta T$, where δT is the average time between consecutive observations, which is typically of the order of a week. To access this band therefore requires a later interferometer operating in space.

Proposals for a space-based gravitational wave detector have been discussed for two decades, in particular a concept for a Laser Interferometer Space Antenna (LISA) was extensively examined as a possible joint NASA/ESA project [3]. As a precursor to this, ESA commissioned a technology demonstration mission, LISA Pathfinder, to demonstrate the major technologies that will be required to make a space-based interferometer a success [4]. LISA Pathfinder will finally launch at the end of this year. The LISA project was ultimately not funded, due to NASA funding constraints. However, ESA has now selected *The Gravitational Universe* [5], a proposal for the detection of gravitational waves from space, as the science theme that will be addressed by its third large mission in this funding cycle (L3), to be launched in 2034. This science theme was based around a mission concept modified from the LISA design, which we refer to as evolved LISA (eLISA).

eLISA will consist of three satellites at the corners of an approximately equilateral triangle, in a heliocentric orbit at 1AU, but lagging behind the Earth in its orbit, and with laser beams passing in both directions between the satellites along the arms of the triangle. In the version of eLISA used to illustrate *The Gravitational Universe* proposal, the satellites were 1 million kilometres apart, the laser power was 0.7W, the telescope diameter was 25cm and the acceleration noise, that determines the instrument sensitivity at low frequencies, was taken equal to the original requirement for LISA. In addition, that eLISA configuration had only four laser links, i.e., the constellation consists of two different types of satellite – one "mother" satellite that has two lasers/telescopes and serves two arms, and two "daughter" satellites at the end of those arms, which have a single laser/telescope pointing toward the mother satellite, but no laser links with the other daughter. However, the final design of eLISA has not yet been fixed so it is important to understand how these various choices could influence the science outcome, so that the design can be optimised for maximal scientific return within the available budget. These studies are ongoing (see [6]), but we present a few initial results here. All parts of the mission design could in principle be varied, but we will only consider: a) the effect of changing from four links to six links, i.e., having laser links passing along two arms or three arms; b) the effect of varying the arm length from 1Gm to 2Gm or 5Gm; and c) the effect of changing the acceleration noise from the LISA design requirement to ten times worse than that. We change the telescope diameter and laser power in conjunction with changes (b) and (c), in such a way that the sensitivity at high frequencies is kept approximately constant. Longer arm lengths do impose certain restrictions on laser power to ensure sufficient photons reach the distant satellite, however we are not restricted to the choice made here and the effect of modifying the high-frequency sensitivity should be explored in the future. The relative sensitivity of the detectors formed under modifications (b) and (c) are shown in Figure 1. The effect of (a) is to allow the construction of two independent data streams, rather than one. The sensitivity of each will be as shown in Figure 1, but having two independent data streams increases the signal-to-noise ratio by approximately a factor of $\sqrt{2}$ and helps to improve parameter estimation. We will show how these different configurations influence the scientific outcome later in this article.

In the next section we will describe the different types of source expected for space-based gravitational wave detectors, including the expected event rate and precision of parameter estimation. In Section 3 we will describe the potential scientific implications of these observations and we will finish in Section 4 with a summary.

2. Sources for space-based gravitational wave detectors

2.1. Massive black hole mergers

It is well established observationally that most galaxies host a black hole in their centre [7]. These black holes are typically very massive, with masses in the range $10^4 M_\odot$–$10^{10} M_\odot$. Following mergers between their hosts, the black holes in the centres are expected to form binaries which then inspiral and merge through the emission of gravitational waves. Less massive galaxies tend to have less massive central black holes and undergo more frequent mergers, so we expect many of these gravitational wave systems to come from binaries with total mass of $10^4 M_\odot$–$10^7 M_\odot$, which radiate in the millihertz band. The properties of these black holes are poorly constrained by electromagnetic observations, so there are several viable models that predict the observed number densities of more massive black holes, but make different predictions in the eLISA range [5, 8]. This means that eLISA will provide currently unknown information about the properties of these black holes, but also means that predictions for the number of such systems that eLISA will see are rather uncertain. Light seed models, in which black holes grow from an initial population of $O(100 M_\odot)$ black holes, predict that the baseline eLISA design would see ~ 100 events in a two year mission [9]. Heavy seed models, in which black holes grow from seeds of $O(10^5 M_\odot)$, formed from direct collapse of massive dust clouds, predict between
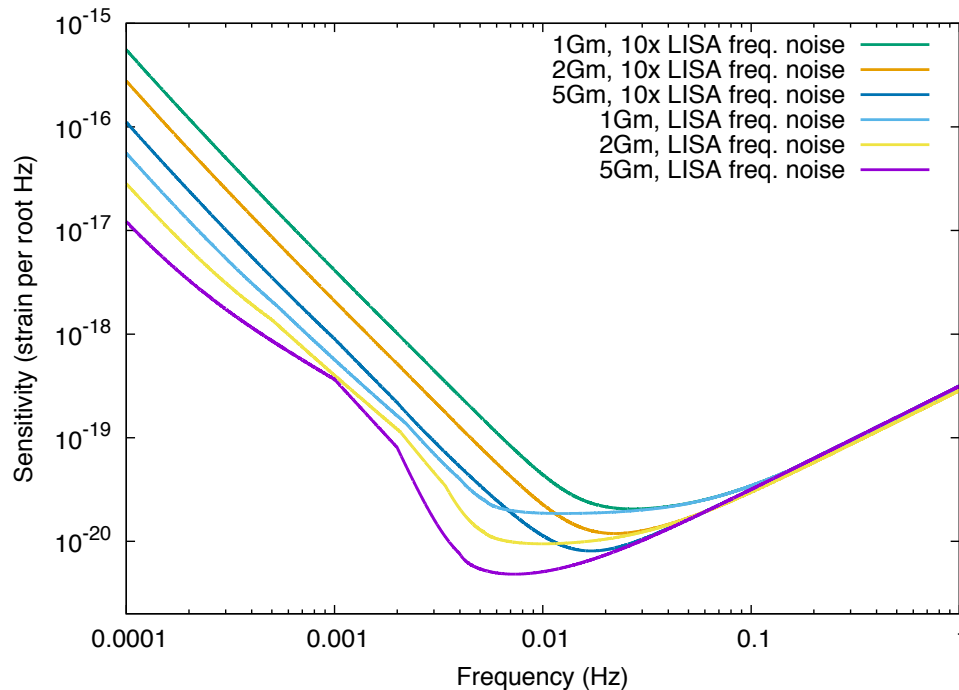
Figure 1. Sensitivity as a function of frequency for six different possible configurations of the eLISA mission. The configurations differ in arm length, which range from 1Gm to 5Gm, and in the acceleration noise performance at low frequencies, which is taken either equal to the original design specification for LISA, or ten times worse. The bump in the sensitivity curves at low frequencies is caused by the astrophysical confusion foreground from compact white dwarf binaries in the Milky Way.

a few tens of events and a few hundreds of events, depending whether it is assumed that there is a delay between the merger of the host galaxies and the formation of the black hole binary or not. Table 1 tabulates the relative gain in the expected number of events for the various alternative eLISA configurations described above. We see that increasing the arm lengths can significantly increase the number of events observed in light seed models. These extra events are primarily low-mass/high-redshift events for which the signal-to-noise ratio (S/N) is too small for them to be seen by the baseline eLISA mission [9, 10]. The arm length has little effect on event numbers for heavy seed models, in which the events are all higher mass and lower redshift and will have high S/N even in the eLISA baseline mission. Event rates for all models are significantly impacted if the acceleration noise performance is much worse than the LISA target, although these events can be partially recovered with longer arm lengths or 6-links.

Many of these SMBH binaries will be observed with high S/N, which will allow very precise determination of the parameters of the systems. A mission with the baseline eLISA design would be expected, for typical systems, to determine the masses of both black holes to between a tenth of a percent and one percent, the spins of the black holes to better than ten per cent, the sky location of the source to ~ 100 square degrees and the luminosity distance to the source to about 10% [9]. In particular, we would expect to determine both (redshifted) masses for ~ 5, ~ 15 or ~ 40 systems (for the light seed/heavy seed no delay/heavy seed with delay models respectively) and the spin of the primary for ~ 15, ~ 30 or ~ 1 systems. We would only expect, however, to measure *both* the sky position to 10 square degrees *and* the distance to 10% for one

Model	eLISA configuration					
	LISA acc. noise			10×LISA acc. noise		
	1Gm	2Gm	5Gm	1Gm	2Gm	5Gm
LS	1.00 (1.62)	2.40 (3.67)	5.25 (6.76)	0.29 (0.50)	0.73 (1.22)	1.58 (2.54)
HS with delay	1.00 (1.07)	1.06 (1.10)	1.09 (1.10)	0.32 (0.45)	0.57 (0.72)	0.89 (0.99)
HS no delay	1.00 (1.10)	1.12 (1.24)	1.26 (1.33)	0.21 (0.30)	0.41 (0.55)	0.75 (0.92)

Table 1. Change in expected number of observed SMBH merger events relative to eLISA baseline design for different mission configurations. In each cell the first number is for a 4-link configuration while the number in brackets is for a 6-link design. Each row corresponds to one of the three astrophysical models described in the text (LS = "light seed", HS = "heavy seed"). The first column (LISA acceleration noise, 1Gm arm length and 4-links) is the eLISA baseline, which would be expected to observe ~ 100, ~ 35 and ~ 450 events in the three models.

system. This increases dramatically, to about ten systems, if the configuration of the mission is changed from 4 to 6 links. Other changes to the mission design lead to more modest changes (by factors of a few up or down, following the same trend as the event rates) in this number and the number of systems with good mass and spin determinations. If systems can be well localised on the sky, there is a much better chance that the host galaxy of the source can be identified. Performing combined electromagnetic and gravitational wave observations on such sources would dramatically increase their scientific value. It is therefore clear that the single most important change to the mission design from the perspective of doing science with SMBH mergers is to upscope from 4 to 6 links, if this is possible within the budgetary constraints.

2.2. Extreme-mass-ratio inspirals
The galacto-centric massive black holes described in the previous section are typically surrounded by clusters of stars. Stars in these clusters evolve as normal and end their lives as compact stellar remnants (black holes, neutron stars or white dwarfs). Encounters with other stars can put these compact objects onto orbits that pass very close to the central black hole so that they become captured and then gradually inspiral into the SMBH via gravitational wave emission [11]. The mass ratio of these systems is typically $10^{-6} - 10^{-4}$ and so these are called extreme-mass-ratio inspirals (EMRIs). EMRIs are particularly interesting because the large mass ratio means that the small object completes a large number of orbits in the strong field region of the spacetime close to the central SMBH. The gravitational waves emitted trace out the orbit the object follows, which allows the reconstruction of a map of the spacetime structure that can be used for astrophysics and to test general relativity [12].

The intrinsic rate at which EMRIs occur in individual galaxies is very uncertain, so there is also significant uncertainty in the number of EMRI events that eLISA might observe. Inspirals of black holes are likely to completely dominate the set of observed events, since these are preferentially captured due to mass segregation and can be seen to much greater distances due to their higher intrinsic S/N. Simulations suggest that the rate should scale with black holes mass as a power law with exponent of -0.19 [13], and a Milky Way-like black hole of mass $3 \times 10^6 M_\odot$ should host ~ 400 black hole inspirals per Gyr [14]. With these assumptions and additionally assuming that the galaxies that might host EMRIs have a flat number density distribution in log mass, $dn/d\ln M = 0.002\text{Mpc}^{-3}$, we predict the baseline eLISA mission would see ~ 650, ~ 350 or ~ 200 events depending on whether the assumed S/N threshold for detection is 15, 20 or 25 [9]. It was initially thought that EMRI detection would require a high threshold S/N due to computational limitations associated with the large parameter space that must be explored in such a search [15]. However, successful parameter estimation for EMRIs with

S/N threshold	eLISA configuration					
	LISA acc. noise			10×LISA acc. noise		
	1Gm	2Gm	5Gm	1Gm	2Gm	5Gm
15	1.00 (2.08)	3.27 (5.11)	6.55 (7.83)	0.14 (0.33)	0.60 (1.30)	1.88 (3.44)
20	1.00 (2.26)	3.93 (6.82)	9.88 (12.94)	0.12 (0.29)	0.54 (1.32)	2.04 (4.00)
25	1.00 (2.45)	4.63 (8.89)	14.29 (20.00)	0.13 (0.32)	0.55 (1.34)	2.32 (4.84)

Table 2. As Table 1 but now showing relative numbers of EMRI events expected to be observed, for three different specifications of the S/N required for detection. The eLISA baseline configuration is predicted to observe 650/350/200 events respectively for these three thresholds.

S/N less than 20 has subsequently been demonstrated in the mock LISA data challenges [16]. The latter demonstration was for a highly simplified data set, so the threshold that will be required in practice is still not known exactly. Table 2 shows how the event rate varies as the mission configuration is adjusted. The trends are similar to those seen in Table 1 for the black hole mergers. However, the EMRI rate can be much more significantly enhanced by improving the detector sensitivity, since EMRIs are sensitivity rather than astrophysical-rate limited — doubling the distance reach of the detector increases the number of observed events by a factor of 8 for EMRIs, but not for SMBH mergers, since there are no more sources in the added volume.

Despite their lower S/N, the parameters of EMRIs can be determined even more precisely than those of SMBH mergers, because any given EMRI generates so many cycles of gravitational radiation in the strong field region of the spacetime. We expect to measure the masses of both objects and the spin of the central massive black hole in any observed EMRI to a few hundredths of a percent, the sky location to a few square degrees and the luminosity distance to about ten percent [9, 16]. This is for an EMRI observed with S/N $\approx$ 30 and does not depend very strongly on which mission configuration is used. More sensitive configurations will observe any given EMRI with higher S/N and hence improved parameter estimation (errors scale as 1/(S/N)), but at fixed S/N all configurations perform equally well.

2.3. Other sources

eLISA will also observe thousands of compact binaries (primarily white dwarf-white dwarf binaries) with $\sim$ 1 hour periods in the Milky Way. Some such systems are known to exist as they have been observed electromagnetically and these "verification binaries" can be used to check that the eLISA instrument is performing as expected. The baseline eLISA mission would be expected to observe an additional $\sim$ 5250 individually resolved binaries with S/N > 7, of which $\sim$ 1350 would be accurately resolved on the sky and of those $\sim$ 450 would also have well determined distance [17]. Although these signals are almost monochromatic, over a two year mission with the baseline eLISA configuration, we would also expect to measure frequency derivatives for $\sim$ 1500 systems and a second-derivative of frequency for $O(1)$ system. These numbers change in a similar way to the EMRI events (see Table 2) as the mission configuration is varied, although the number of individually resolvable events does not get above $\sim$ 20000, since sensitivity is ultimately limited by confusion noise from unresolved compact binaries at low frequency, as indicated by the low-frequency bump in the sensitivity curves shown in Figure 1.

eLISA could also observe more exotic sources of cosmological origin, such as bursts from kinks or cusps on cosmic strings, or a stochastic background of millihertz gravitational radiation generated in the early Universe. Millihertz gravitational waves are generated at the TeV energy scale, which is the scale of the electroweak phase transition. The detection of a background at these frequencies would therefore have a profound impact on our understanding of the physics of that energy scale. We refer the reader to [18] for more details.

3. Science with space-based gravitational wave detectors

eLISA observations of the sources described above have the potential to transform our scientific understanding in several areas. We cannot go into any detail here, but in the following we will very briefly describe some of the potential scientific outcomes of eLISA observations. We refer the interested reader to [19] for a more complete description and additional references.

3.1. Astrophysics

The black holes that eLISA will observe play a vital role in the assembly of structure in the Universe. SMBH mergers trace out galactic mergers and so eLISA observations of these sources will probe the hierarchical assembly of structure. Several models exist that are consistent with current observations of higher mass systems, but these make different predictions in the low mass range that eLISA will probe. eLISA will be able to distinguish between a wide variety of different models for the growth of structure [8] and will also be able to identify when observed sources come from a mixture of different populations, and determine the mixture fraction to a precision of $\sim \pm 0.2$ [8, 9]. SMBH merger observations will also probe the accretion history of black holes, through measurements of black hole spins.

eLISA observations of EMRIs can be used to probe the number density and characteristics of massive black holes in the eLISA range in the local Universe. Observations of 10 EMRIs will be sufficient to constrain the slope of the mass function to ± 0.3, the precision with which it is currently known, and this improves as the square root of the number of events [20]. EMRI observations alone will not put constraints on the evolution of the mass function with redshift, but this might be possible by combining EMRI observations and observations of SMBH mergers. EMRIs also probe the physics of dense stellar clusters — the masses of the inspiring compact objects provide a probe of the stellar initial mass function and the efficiency of mass segregation, while the inclination and eccentricity probe the channel by which the EMRI formed.

Observations of galactic compact binaries trace stellar populations in the Milky Way, which will place indirect constraints on compact binary merger rates. Measurements of frequency derivatives in such systems also offer clues to the physics of mass transfer and of tidal interactions in stellar binaries [5, 9, 17].

3.2. Fundamental physics

eLISA observations will probe the predictions of general relativity in a regime of strong field, dynamical gravity in which it has never been tested. All future gravitational wave observations can be used for testing fundamental physics, but eLISA will provide particularly strong constraints since eLISA sources will be detected with high S/N (SMBH mergers could have S/N of thousands), they can be observed for many cycles (in particular, EMRIs can be tracked for hundreds of thousands of orbits in the strong field), the source dynamics are rich (both components will have significant spins for SMBH mergers, while EMRIs are expected to be on eccentric and inclined orbits about the central SMBH) and the systems are expected to be astrophysically "clean", i.e., the sources are mostly black holes and the influence of other material is expected to be small. There is an extensive literature describing tests of fundamental physics that will be possible with eLISA observations. We cannot describe everything here, but will give a few highlights and refer the reader to [21] for a comprehensive review.

eLISA observations can test aspects of gravitational physics, including detecting non-GR polarisations of gravitational waves [22], measuring the speed of propagation of gravitational waves through the dispersion of chirp signals [23] and detect differences in the rate at which energy is being carried away from a system which might reveal other emission channels, for instance dipole radiation in scalar-tensor theories of gravity [24]. It will also be possible to place constraints on generic deviations from the predictions of general relativity, by comparing measured values of the post-Newtonian coefficients of an expansion of the phase evolution as a

function of frequency with the values predicted by GR [25]. Deviations of a tenth of a percent in low order coefficients should be identifiable. Alternatively, generic additional terms can be included in the phase expansion and constrained by observations (the parameterised post Einsteinian formalism [26]). eLISA will be able to exclude a large part of the ppE parameter space, mostly at higher orders in frequency that are not probed by binary systems where velocities are typically much smaller [27].

eLISA observations of EMRIs will probe the nature of black hole spacetimes. General relativity predicts that a black hole is completely characterised by two parameters — a mass and a spin. All higher terms in an expansion of the gravitational field in powers of distance are determined by the mass and spin. The waveforms generated by EMRIs encode a map of the spacetime structure from which those higher terms can be measured and compared to the predictions [12]. Spacetime mapping can also be done using eLISA measurements of the modes of the ringdown radiation from SMBH mergers [28]. Spacetime mapping with EMRIs has been explored extensively (see [21] for a full review). eLISA's ability to measure "bumpiness" of black holes was characterised using various different models for the bumps, e.g., [29, 30, 31, 32]. The conclusion was that typical EMRI observations would simultaneously measure the central black hole mass and spin to a hundredth of a percent and constrain any deviations in the quadrupole moment at the level of a tenth of a percent [33]. EMRI observations could also indicate the presence of a horizon in the spacetime [34], reveal the physics of the tidal coupling interaction between the two bodies [35] and detect the influence of perturbing matter [36] or stellar objects [37]. These astrophysical imprints are distinct from those of the central object.

3.3. Cosmology

The direct detection of a millihertz gravitational wave background would provide constraints on the physics of the early Universe at the TeV scale, as mentioned earlier. However, eLISA sources can also be used indirectly to probe the expansion history of the Universe. SMBH mergers and EMRIs will both provide reasonably accurate measurements of luminosity distance, but to probe the expansion history we also need an estimate of redshift. Redshift only enters the gravitational wave signal in the form of a redshifted mass, but the intrinsic mass is unknown. The redshift could be determined if an electromagnetic counterpart to a source was observed, but the large error boxes make this unlikely. A more promising approach is to use a statistical technique, averaging over many events. If the EMRI event rate is as high as predicted, EMRIs could provide a measurement of the Hubble constant. A $1\% - 2\%$ measurement would be possible using 20 EMRI events at redshift $z < 5$ [38]. Applying similar statistical techniques to SMBH mergers could provide a probe of the equation of state of dark energy [39], if sufficient systems are observed. Any cosmological constraints that eLISA provides are unlikely to be better than those which will have been derived electromagnetically by that time. However, the eLISA measurements are completely independent of all other techniques and so will provide a completely independent verification of previous results.

4. Summary

The millihertz gravitational wave band is rich in sources and will be finally opened up by the ESA L3 mission in 2034. This mission is expected to observe gravitational waves from the mergers of binaries of massive black holes with mass in the range $10^4 - 10^7 M_\odot$, from the extreme-mass-ratio inspirals of stellar compact remnants into such black holes, from hour-period compact object binaries in the Milky Way and possibly a cosmological background or bursts from cusps on cosmic strings. The design of the eLISA satellite that will constitute this mission has not yet been finalised and so work is underway to explore the impact of design choices on the potential scientific impact of the mission. Increasing the arm length increases the expected number of observed events for all source types, as does increasing the number of laser

links in the constellation from 4 to 6. However, if the acceleration noise at low frequencies is significantly worse than the design target it will reduce the number of sources. The precision of parameter estimation for most sources responds in a similar way to the number of events under these design modifications, with the exception of the precision of localisation (on the sky and in distance) of SMBH merger events. For those events, there is a dramatic improvement when the number of links is increased. Therefore, of all the modifications that might be achievable within the budgetary constraints, increasing the number of links is likely to have the biggest impact on the science. Whatever the final design, eLISA observations have the potential to revolutionise our understanding of astrophysics, fundamental physics and cosmology. They will provide unique constraints on the hierarchical assembly of large scale structure, they will provide precise tests of the predictions of general relativity in a previously unexplored regime, they will provide verification of the nature of the spacetime exterior to black holes and will place new independent constraints on the expansion history of the Universe. This science will only be realised with appropriate design of the mission and careful development of techniques for the analysis of the data. There is still much work to be done between now and the L3 launch to prepare the way for the advent of millihertz gravitational wave astronomy!

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