Comparative study of energy for DNM with perturbative to mean field results within Fermi liquid theory approach

Kausik Pal*
Department of Physics, Serampore College, Serampore 712201, India.

I. Introduction

In nuclear physics Fermi liquid theory [1, 2] was first extended and used by Migdal [3] to study the properties of both unbound nuclear matter and Finite nuclei. Most of the earlier nuclear matter calculations which involved Landau theory were done in a non-relativistic framework. The relativistic extension of the FLT was first developed by Baym and Chin [1] in the context of studying the properties of dense nuclear matter (DNM). In [1] the authors invoked Walecka model (WM) to calculate various interaction parameters ($f_{pp'}$) but did not consider mean fields (MF) for the σ and ω meson *i.e.* there the FLPs are calculated perturbatively. Later Matsui revisited the problem in [4] where one starts from the expression of energy density in presence of scalar and vector meson MF and takes functional derivatives to determine the FLPs. The results are qualitatively different than the perturbative results as may be seen from [1, 4]. In this work we present the comparative study of LPs, chemical potential and energy density obtained perturbatively with mean field results.

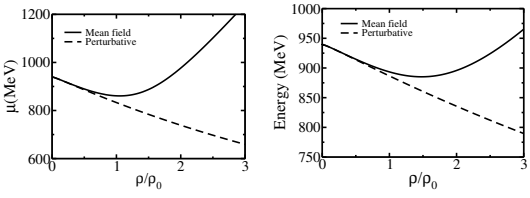

FIG. 1: Chemical potential and energy density with σ and ω meson exchange in symmetric nuclear matter.

*Electronic address: kausik.sinp@gmail.com

II. Formalism and LPs

In FLT total energy E of an interacting system is the functional of occupation number n_p of the quasi-particle states of momentum p [1],

$$E = E^0 + \sum_s \int \frac{d^3p}{(2\pi)^3}\varepsilon^0_{ps}\delta n_{ps} + \frac{1}{2}\sum_{ss'} \int \frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3} f_{ps,p's'}\delta n_{ps}\delta n_{p's'}, \quad (1)$$

where E^0 is the ground state energy and s is the spin index, and the quasi-particle energy can be written as,

$$\varepsilon_{ps} = \varepsilon^0_{ps} + \sum_{s'} \int \frac{d^3p'}{(2\pi)^3} f_{ps,p's'}\delta n_{p's'}, \quad (2)$$

where ε^0_{ps} is the non-interacting single particle energy.

Since quasi-particles are well defined only near the Fermi surface, one assumes

$$\left.\begin{array}{ll} & \varepsilon_p = \mu + v_f(p - p_f) \\ \text{and} & p \simeq p' \simeq p_f. \end{array}\right\} \quad (3)$$

Then LPs f_ls are defined by the Legendre expansion of $f_{ps,p's'}$ as [1],

$$f_l = \frac{2l+1}{4}\sum_{ss'}\int \frac{d\Omega}{4\pi} P_l(\cos\theta) f_{ps,p's'} \quad (4)$$

where θ is the angle between p and p', both taken to be on the Fermi surface, and the integration is over all directions of p. We restrict ourselves for $l \leq 1$ *i.e.* f_0 and f_1, as higher l contribution decreases rapidly [1, 4].

The spin averaged scattering amplitude ($f_{pp'}$) is given by [1],

$$f_{pp'} = \frac{1}{4}\sum_{ss'}\frac{M}{\varepsilon^0_p}\frac{M}{\varepsilon^0_{p'}}\mathcal{M}_{ps,p's'}. \quad (5)$$

where the Lorentz invariant matrix $\mathcal{M}_{ps,p's'}$ consists of the usual direct and exchange amplitude, which may be evaluated directly from the relevant Feynman diagrams.

a. Perturbative calculation

Let us calculate the LPs perturbatively due to the exchange of scalar and vector mesons between the nucleons [1]. The direct contribution is given by [1]

$$\left.\begin{array}{rl} f_{pp'}^{dir,\sigma} & = -\frac{g_\sigma^2}{m_\sigma^2}\frac{M^2}{\varepsilon_p^0\varepsilon_{p'}^0} \\ f_{pp'}^{dir,\omega} & = \frac{g_\omega^2}{m_\omega^2}\frac{P.P'}{\varepsilon_p^0\varepsilon_{p'}^0}, \end{array}\right\} \quad (6)$$

With the help of Eq.(4) and Eq.(6), the LPs become

$$\left.\begin{array}{rl} f_0^{dir,\sigma} & = -\frac{g_\sigma^2}{m_\sigma^2}\frac{M^2}{\varepsilon_f^2} \\ f_0^{dir,\omega} & = \frac{g_\omega^2}{m_\omega^2}, \end{array}\right\} \quad (7)$$

$$\left.\begin{array}{rl} f_1^{dir,\sigma} & = 0 \\ f_1^{dir,\omega} & = -\frac{g_\omega^2}{m_\omega^2}\frac{p_f^2}{\varepsilon_f^2}. \end{array}\right\} \quad (8)$$

One may now, for the direct contribution plug in $f_{pp'}^{dir,\sigma}$ and $f_{pp'}^{dir,\omega}$ in Eq.(1) and Eq.(2) to obtain the energy density and the SPE spectrum, respectively. The SPE spectrum is given by [1]

$$\varepsilon_p^{dir} = \varepsilon_p^0 + \frac{g_\omega^2}{m_\omega^2}\rho - \frac{g_\sigma^2}{m_\sigma^2}\frac{M}{\varepsilon_p^0}n_s. \quad (9)$$

Here ρ and n_s are the baryon and scalar density. The energy density for direct contribution is [1]

$$E^{dir} = E^0 + \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho^2 - \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}n_s^2. \quad (10)$$

One can derive the chemical potential directly from Eq.(9) as $\mu = \varepsilon_p\Big|_{p=p_f}$.

b. Mean field results

It is well known that in the MF approximation, one replaces the mesonic fields by their vacuum expectation values viz. $\sigma \rightarrow <\sigma> = \sigma_0$, $\omega \rightarrow <\omega> = \delta_{\mu 0}\omega^\mu$. In the MF approximation the energy density can be written as

$$E^{MFT} = \frac{1}{2}\frac{g_\omega^2}{m_\omega^2}\rho^2 + \frac{1}{2}\frac{g_\sigma^2}{m_\sigma^2}n_s^2 + \sum_i n_i\sqrt{p_i^2 + M^{*2}}. \quad (11)$$

In the above equation M^* denotes the effective nucleon mass to be determined self consistently [4]. The interaction parameter is given by [4]

$$f_{pp'}^{MFT} = \frac{g_\omega^2}{m_\omega^2} - \frac{g_\sigma^2}{m_\sigma^2}\frac{M^{*2}}{\varepsilon_p^0\varepsilon_{p'}^0}\left[1 + \frac{g_\sigma^2}{m_\sigma^2}\zeta(M^*)\right]^{-1} \quad (12)$$

$$\zeta(M^*) = \sum_i n_i\frac{p_i^2}{(p_i^2 + M^{*2})^{3/2}} \quad (13)$$

The inverse part of Eq.(12) reduces the magnitude of interaction parameter compared to what is obtained in absence of the MF Eq.(6). Using the interaction parameter we can easily derive the LPs and other quantities, similarly as obtained for perturbative calculation.

III. Results and Discussions

In Fig(1) we present the comparative study of the chemical potential and energy density obtained perturbatively and with MF calculation. At low density they tend to merge, while at higher density MF results become larger than the perturbative results.

Acknowledgments

I am indebted to late A.K.Dutt-Mazumder who introduced me to this topic.

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