

# Applications Of The Extended Tolman-Oppenheimer-Volkoff Equation In $f(R)$ Gravity

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**Abstract.** The Tolman-Oppenheimer-Volkoff (TOV) equation determines the structure of a spherically symmetric body of isotropic material in equilibrium, in general relativity (GR). It is widely used in the study of properties of compact stars. In case of charged compact stars, an extended TOV equation was found first by Bekenstein in 1971.

We aim to use, the  $f(R)$  theory of gravity, one among several proposed extended theories of gravity to find an equivalent equation to TOV equation, and use it to study electrically charged compact stars. We assume that the charge density is proportional to the energy density of the star and we choose the polytropic equation of state to describe the state of the charged star. We perform a detailed numerical study using Maple 2016, and our results are compared to those found in the literature in the framework of GR.

## 1. Introduction

Recent observations show that the universe is expanding at an accelerating rate. GR doesn't predict this to happen. Moreover other shortcomings of this theory emerged in cosmology at infrared and ultraviolet scales. To cure these problems, two general ways have been taken: introducing a new type of energy or modifying the theory of gravity. Here, we use  $f(R)$  gravity, a modification of GR to study electrically charged compact stars [1]. To test alternative theories of gravity like  $f(R)$  gravity, we generally study compact stars. These objects are well studied in the literature. For charged compact stars, an extended TOV equation was found first by Bekenstein, in 1971 [2], then followed by other authors [3], [4] and [5]. In this paper, we review the study of compact charged spheres made of a charged perfect fluid with a polytropic equation of state as studied in the literature [5]. The charge density is chosen to be proportional to the energy density. The study is performed by solving the extended Tolman-Oppenheimer-Volkoff equation (TOV), which describes the hydrostatic equilibrium, in GR. The goal that we want to reach is to apply  $f(R)$  gravity to the results found for the polytropic charged stars and then analyses the effect that would appear in adding that correction. We choose the quadratic correction

$$f(R) = R + \frac{1}{2}\beta R^2 \quad (1)$$

to the Hilbert-Einstein action, because it is the simplest case and it has shown consistence results for some cosmological phenomena.  $\beta$  is called the parameter of quadratic corrections.

This paper is organized as follows: first we briefly review the ETG in section 2 and then in section 3 we describe the equations governing the charged compact stars, an equation of state of polytropic form and a charge density are defined. We plotted new graphs for the mass- radius characteristic for a fixed polytropic exponent and different charge fractions in [5]. In section 4, we derive the modified extended TOV equation in  $f(R)$  gravity. Finally we conclude.

## 2. The Extended Theory of Gravity (ETG)

The  $f(R)$  theory of gravity is one type of ETG which consists in modifying the Einstein-Hilbert action by replacing the Ricci scalar  $R$  by a generic function of it, taking it to higher orders in  $R$ :

$$L = \sqrt{-g} f(R) \quad (2)$$

In this formalism we write the action as:

$$A = \int d^4x \sqrt{-g} [f(R) + XL_m] \quad (3)$$

where  $L_m$  is the minimally coupled ordinary matter Lagrangian density and  $X = 8\pi G$  is a constant with  $G$  being the Newtonian constant. Variation with respect to the metric tensor yields the field equations [1]:

$$F(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) + g_{\mu\nu}\square F(R) - \nabla_\mu \nabla_\nu F(R) = XT_{\mu\nu} \quad (4)$$

The trace of the field equations is:

$$F(R)R - 2f(R) + 3\square F(R) = XT \quad (5)$$

Where  $F(R) = \frac{df(R)}{dR}$ . Here,  $T_{\mu\nu} = \frac{-1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}$  is the the energy-momentum tensor of matter,

$T$  is the trace,  $R_{\mu\nu}$  is the Ricci tensor,  $g_{\mu\nu}$  the metric tensors and  $\square$  is the d'Alembert operator.

## 3. Polytropic Charged Spheres

Compact stars are well suited to test Alternative Theories of Gravity (like  $f(R)$  gravity) by studying their formation and evolution across time. In fact these objects are well studied in the literature: Chandrasekhar found a mass limit for white dwarfs in Newtonian gravitation, whereas Tolman, Oppenheimer and Volkoff found a mass limit for neutron stars, a more compact objects in relativistic gravitation. Regarding charged compact stars, it was Bekenstein who, first, studied their stability. By making an ansatz, that the interior of the stars are composed of radiation and perfect fluid, he found the extended TOV equation which tends, in Newtonian limit, to the well known hydrostatic equation of equilibrium in the case of uncharged star. In this work we consider charged compact stars and study their properties in the framework of  $f(R)$  gravity. To do so we must compare our found results with those found in the literature in case of GR. We will follow the work given in reference [5]. In fact, before proceeding to use  $f(R)$  gravity, we have first derived all the results found by Arbanil et al., in reference [5] and studied the mass-radius characteristic for other values of parameters involved in the same reference.

### 3.1. The equations of equilibrium

The Einstein-Maxwell equations in the presence of charged matter are, with ( $\chi = 8\pi$ )

$$G_{\mu\nu} = -8\pi T_{\mu\nu} \quad (6)$$

$$\nabla_\nu F^{\mu\nu} = 4\pi J^\mu \quad (7)$$

where  $\mu, \nu = 0..3$  are the Greek indices,  $J^\mu$  is the electric current density and  $T_{\mu\nu}$  stands for the energy momentum tensor, which, in the present study is written as [6]:

$$T_{\mu\nu} = E_{\mu\nu} + M_{\mu\nu} \quad (8)$$

where  $M_{\mu\nu}$  is the energy-momentum tensor of a perfect fluid given by:

$$M_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \quad (9)$$

$\rho$  is the energy density,  $p$  is the pressure and  $u_\mu$  is the fluid's four velocity.

$E_{\mu\nu}$  is the electromagnetic energy-momentum tensor given by:

$$E_{\mu\nu} = \frac{1}{4\pi} \left( F_\mu^\gamma F_{\nu\gamma} - \frac{1}{4} g_{\mu\nu} F_{\gamma\beta} F^{\gamma\beta} \right) \quad (10)$$

where the Faraday-Maxwell strength tensor is:

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu \quad (11)$$

with  $\nabla_\mu$  representing the covariant derivative, and  $A_\mu$  the electromagnetic gauge field. In addition, the electric current density is written as:

$$J^\mu = \rho_e u^\mu \quad (12)$$

where  $\rho_e$  is the electric charge density.

To describe a static fluid distribution with spherically symmetric, we have to write the metric as:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (13)$$

so that the metric functions  $A(r)$  and  $B(r)$  depend only on  $r$ . ( $t, r, \theta, \varphi$ ) are the Schwarzschild-like coordinates. The electric field being static spherically symmetric, so the only nonzero components of the Maxwell strength tensor are  $F^{tr}(r) = -F^{rt}(r)$ , with  $F^{tr}(r)$  a function of the radial coordinate  $r$  alone. Thus, the only non-vanishing component of the Maxwell equations is:

$$\frac{dq(r)}{dr} = 4\pi r^2 \rho_e(r) \sqrt{A(r)} \quad (14)$$

where  $q(r) = r^2 \sqrt{A(r)} F^{tr}(r)$  is the total electric charge inside a sphere of radial coordinate  $r$ .

### 3.2. The extended Tolman-Oppenheimer-Volkoff (TOV) equation

Following reference [5], the non-zero Einstein equations derived there, are

$$\frac{d}{dr}(rA^{-1}) = 1 - 8\pi r^2 \left( \rho + \frac{1}{8\pi} \frac{q(r)^2}{r^4} \right) \quad (15)$$

$$\frac{r}{AB} \frac{dB}{dr} + \frac{1}{A} = 1 + 8\pi r^2 \left( p - \frac{1}{8\pi} \frac{q(r)^2}{r^4} \right) \quad (16)$$

By using the Bianchi identities,  $\nabla_\nu T^{\nu\mu} = 0$ , we have the conservation equation:

$$\frac{dB(r)}{dr} = \frac{B(r)}{\rho(r) + p(r)} \left( \frac{q(r)}{2\pi r^4} \frac{dq(r)}{dr} - 2 \frac{dp(r)}{dr} \right) \quad (17)$$

A new quantity  $m(r)$  representing the mass inside the shell of radial coordinate  $r$  is defined [6] such that:

$$a(r) = A(r)^{-1} = 1 - \frac{2m(r)}{r} + \frac{q^2(r)}{r^2} \quad (18)$$

Replacing equations (14) and (19) into the Einstein equation (16), we get the extended TOV equation initially found by Bekenstein [1] and was widely used in the literature since then:

$$\frac{dp}{dr} = -(\rho + p)a^{-1} \left( 4\pi pr + \frac{m}{r^2} - \frac{q^2}{r^3} \right) + \rho_e \left( \frac{q}{r^2} a^{-1/2} \right) \quad (19)$$

### 3.2.1. Equation of state:

The equations above are not sufficient to determine the parameters needed to completely describe the compact charged star, namely  $q(r)$ ,  $A(r)$ ,  $m(r)$ ,  $\rho_e(r)$ ,  $p(r)$  and  $\rho(r)$ . We need two more equations;

- one is the equation of state providing a relationship between energy density and pressure. Namely we chose a polytropic equation of state of the form:

$$p = \omega \rho^\gamma \quad (20)$$

where  $\omega$  is a constant and  $\gamma$  is the polytropic exponent.

- The second equation is given by assuming a proportionality relation between the energy density and the charge density:

$$\rho_e = \alpha \rho \quad (21)$$

where  $\alpha$  is the charge fraction which is dimensionless in geometric units.

### 3.2.2. Numerical analysis:

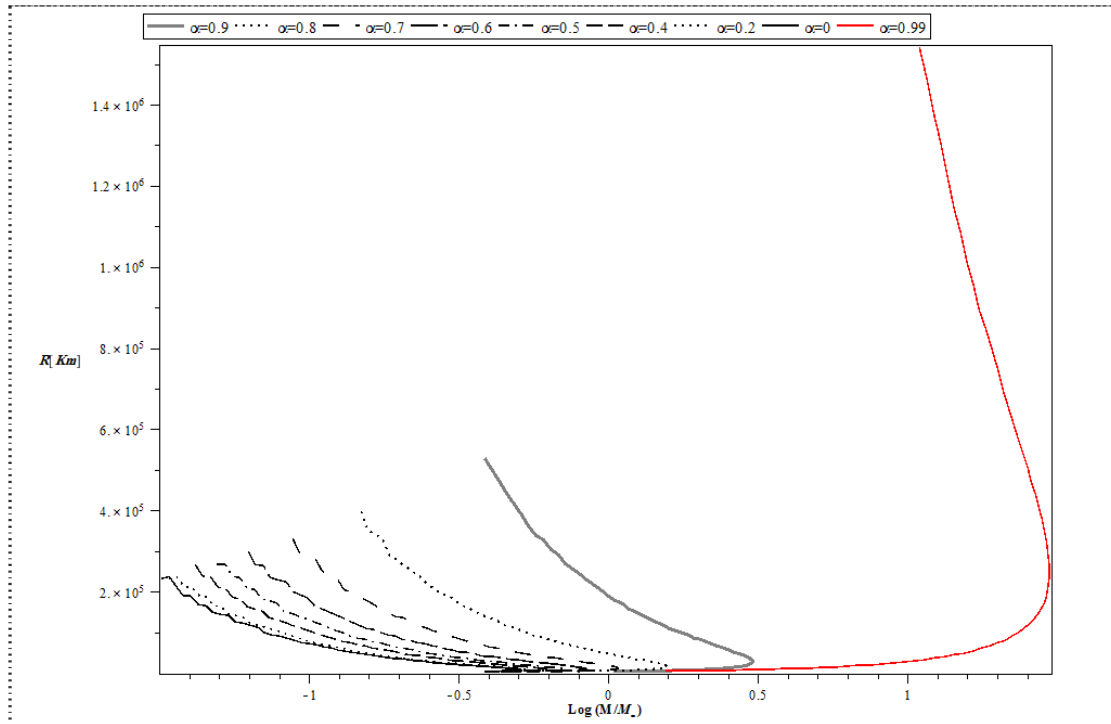
By using equations (20) and (21), together with the three Einstein equations (15-17) and the Maxwell equation (14), we get a complete system of equations that we solve numerically, using Maple 2016 software. As the system contains differential equations, we need to define boundary conditions. They are chosen at the centre of the compact charged star as:

$$m(r=0) = 0, \quad q(r=0) = 0, \quad p(r=0) = p_{cr}, \quad \rho(r=0) = \rho_{cr}, \quad \rho_e(r=0) = \rho_{ecr} \quad (22)$$

At the surface of the star, the pressure is zero, so the radius of the star, denoted  $R_{st}$  (to avoid any confusion with the Ricci scalar  $R$ ), is found when  $p(R_{st}) = 0$  holds.

We used to check all the graphs given in [5] and plotted new graphs for a given polytropic exponent  $\gamma = 4.5/3$ . In figure 1 we plot the radius of the resulting spheres as a function of the mass, normalized to the Sun's mass  $M_\odot$ , for  $\gamma = 4.5/3$ , and for few values of the charge fraction. The considered central energy densities lie in the interval  $[10^{13}, 10^{20}] \text{ kg/m}^3$ . We get the following graphs shown in figure (1) below.

We can see clearly the Oppenheimer-Volkoff mass limit in these figures. Notice that the Oppenheimer-Volkoff mass limit appears in relativistic gravitation as for neutron stars.



**Figure 1.** The radius of the resulting spheres as a function of the mass for  $\gamma = 4.5 / 3$ , for few values of the charge fraction  $\alpha$ . We consider the central energy densities in the range  $[10^{13}, 10^{20}] \text{ kg} / \text{m}^3$ .

#### 4. The Extended TOV Equation in $f(R)$ gravity

Now, by using the gravitational field equations (4), we find the following equations that extend those found in GR in reference [5].

$$\begin{aligned}
 & \frac{1}{16A^3 r^4 B^4} \left\{ 4A^2 \left( \frac{d^2 B}{dr^2} \right)^2 B^2 \beta r^4 - 4r^3 \beta A B \left[ \left( \frac{dB}{dr} \right)^2 A r + \frac{dB}{dr} \frac{dA}{dr} r B + 4 \frac{dA}{dr} B^2 \right] \frac{d^2 B}{dr^2} \right. \\
 & + A^2 \left( \frac{dB}{dr} \right)^4 \beta r^4 + 2A \left( \frac{dB}{dr} \right)^3 \frac{dA}{dr} B \beta r^4 \left[ r^2 \left( \frac{dA}{dr} \right)^2 + 8rA \frac{dA}{dr} - 16A^2 \right] \left( \frac{dB}{dr} \right)^2 + 8B^2 \left[ r^2 \right. \\
 & \left. \left( \frac{dA}{dr} \right)^2 + 8rA \frac{dA}{dr} - 16A^2 \right] \left( \frac{dB}{dr} \right)^2 + 8B^3 \left[ r^2 \beta \left( \frac{dA}{dr} \right)^2 + 2 \left( (r^2 + 2\beta)A - 2\beta \right) A^2 \right] r \frac{dB}{dr} \\
 & \left. + 16B^4 \left[ r^2 \beta \left( \frac{dA}{dr} \right)^2 - (A-1)(A-1)A^2 \left( (r^2 + \beta)A - \beta \right) \right] \right\} = \frac{8Ar^2 \pi p - q^2 A}{r^4}
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 & \frac{1}{16r^2A^5B^4} \left\{ -16A^3 \frac{d^4B}{dr^4} B^3 \beta r^4 + 48B^2 \beta \left[ \frac{dB}{dr} Ar + \frac{5}{6} \left( r \frac{dA}{dr} - \frac{4A}{5} \right) B \right] A^2 r^3 \frac{d^3B}{dr^3} \right. \\
 & + 8r^3 \beta A^2 B^3 \left( \frac{dB}{dr} r + 4B \right) \frac{d^3A}{dr^3} + 28A^3 \left( \frac{d^2B}{dr^2} \right)^2 B^2 \beta r^4 - 108 \left[ \frac{8r^2 \beta A B^2 d^2A}{27 dr^2} + r^2 \right. \\
 & \beta A^2 \left( \frac{dB}{dr} \right)^2 + \frac{7}{9} B \beta \left( r \frac{dA}{dr} - \frac{22A}{21} \right) Ar \frac{dB}{dr} + \frac{16}{27} B^2 \left( r^2 \beta \left( \frac{dA}{dr} \right)^2 - \frac{A^3 r^2}{8} - \frac{9r \beta A}{8} \right. \\
 & \left. \left. - \frac{dA}{dr} - A^2 \beta \right) \right] BA r^2 \frac{d^2B}{dr^2} - 24B^2 \left[ \left( \frac{dB}{dr} \right)^2 Ar^2 + 2B \left( r \frac{dA}{dr} - \frac{2A}{3} \right) r \frac{dB}{dr} \right] + 8 \left( r \frac{dA}{dr} \right. \\
 & \left. + \frac{A}{6} \right) B^2 \beta A r^2 \frac{d^2A}{dr^2} + 47A^3 \left( \frac{dB}{dr} \right)^4 \beta r^4 + 46 \left( r \frac{dA}{dr} - \frac{30A}{23} \right) B \beta A^2 r^3 \left( \frac{dB}{dr} \right)^3 + 47B^2 \\
 & A \left[ r^2 \beta \left( \frac{dA}{dr} \right)^2 - \frac{64}{47} r \beta A \frac{dA}{dr} - \frac{4A^2 (Ar^2 + 16\beta)}{47} \right] r^2 \left( \frac{dB}{dr} \right)^2 + 48 \left[ r^3 \beta \left( \frac{dA}{dr} \right)^3 - \frac{17}{12} \right. \\
 & \left. r^2 \beta A \left( \frac{dA}{dr} \right)^2 - \frac{dA}{dr} \frac{A^2 r (Ar^2 + 16\beta)}{12} + \frac{((r^2 - 2\beta)A - 6\beta)A^3}{6} \right] B^3 r \frac{dB}{dr} + 192 \left[ r^3 \beta \right. \\
 & \left. \left( \frac{dA}{dr} \right)^3 + \frac{r^2 \beta A}{3} \left( \frac{dA}{dr} \right)^2 - \frac{((r^2 - 2\beta)A + 10\beta)A^2 r dA}{24 dr} + \frac{\beta A^3 (A + 11)(A - 1)}{12} \right] B^4 \left. \right\} \tag{24} \\
 & = \frac{8\pi pr^4 + q^2}{r^2}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{16B^3A^5r^4} \left\{ 16A^3 \frac{d^4B}{dr^4} B^3 \beta r^4 - 48B^2 \beta \left[ \frac{dB}{dr} Ar + \frac{5}{6} \left( r \frac{dA}{dr} - \frac{4A}{5} \right) B \right] A^2 r^3 \right. \\
 & \frac{d^3B}{dr^3} - 8r^3 \beta A^2 B^3 \left( \frac{dB}{dr} r + 4B \right) \frac{d^3A}{dr^3} - 36A^3 \left( \frac{d^2B}{dr^2} \right)^2 B^2 \beta r^4 + 116 \left[ -\frac{8r^2A}{29} \right. \\
 & B^2 \frac{d^2A}{dr^2} r^2 A^2 \left( \frac{dB}{dr} \right)^2 + \frac{23}{29} \left( r \frac{dA}{dr} - \frac{22A}{21} \right) BA r \frac{dB}{dr} + \frac{16B^2}{29} \left( r^2 \left( \frac{dA}{dr} \right)^2 - \frac{rA}{8} \right. \\
 & \left. \left. \frac{dA}{dr} - A^2 \right) \right] B \beta A r^2 \frac{d^2B}{dr^2} + 24B^2 \left[ \left( \frac{dB}{dr} \right)^2 Ar^2 + 2B \left( r \frac{dA}{dr} - \frac{2A}{3} \right) r \frac{dB}{dr} + 8 \right. \\
 & \left( r \frac{dA}{dr} + \frac{A}{6} \right) B^2 \left. \right] \beta A r^2 \frac{d^2A}{dr^2} - 49A^3 \left( \frac{dB}{dr} \right)^4 \beta r^4 - 50 \left( r \frac{dA}{dr} - \frac{36A}{25} \right) B \beta A^2 \\
 & r^3 \left( \frac{dB}{dr} \right)^3 - 49B^2 \beta \left[ r^2 \left( \frac{dA}{dr} \right)^2 - \frac{72rA}{49} \frac{dA}{dr} - \frac{48A^2}{49} \right] Ar^2 \left( \frac{dB}{dr} \right)^2 - 48B^3 \beta \\
 & \left[ r^3 \left( \frac{dA}{dr} \right)^3 - \frac{4r^2A}{3} \left( \frac{dA}{dr} \right)^2 - \frac{4rA^2}{3} \frac{dA}{dr} - \frac{4A^3}{3} \right] r \frac{dB}{dr} - 192B^4 \left[ r^3 \beta \left( \frac{dA}{dr} \right)^3 + \right. \\
 & \left. \frac{r^2 \beta A}{4} \left( \frac{dA}{dr} \right)^2 - \frac{A^2 ((r^2 + 2\beta)A + 2\beta)r dA}{12 dr} - \frac{(A - 1)A^3 ((r^2 + \beta)A - 13\beta)}{12} \right] \left. \right\} \tag{25} \\
 & = \frac{B(8\pi pmr^4 + q^2)}{r^4}
 \end{aligned}$$

where, to shorten equations and simplify notation, we dropped the function dependence:  $B(r) = B$ ,  $q(r) = q$ , and so on.

To obtain the extended TOV equation in  $f(R)$  gravity, we should replace the conservation equation (17) and its derivative with respect to  $r$ , into the 00-component of the gravitational field equations (4), namely the equation (23) and we get:

$$\begin{aligned}
& \frac{1}{64r^{12}\pi^4(p+\rho m)^4} \left\{ A^2 \left[ -8r^8\pi^2(p+\rho m)\frac{d^2p}{dr^2} + 2qr^4\pi(p+\rho m)\frac{d^2q}{dr^2} + \left(\frac{dq}{dr}\right)^2 q^2 \right. \right. \\
& \left. \left. -10q \left( r\frac{dp}{dr} + \frac{d\rho m}{dr} \frac{mr}{5} + \frac{4p}{5} + \frac{4\rho m}{5} \right) r^3\pi\frac{dq}{dr} \right] + 24r^8 \left( \frac{dp}{dr} + \frac{d\rho m}{dr} \frac{m}{3} \right) \frac{dp}{dr} \pi^2 \right\} \beta B^4 \Bigg\} \\
& - \frac{1}{64r^{12}\pi^4(p+\rho m)^4} \left\{ \left[ A \left( \frac{dq}{dr} \right)^2 q^2 - 8q \left( A \frac{dP}{dr} - \frac{1}{4} \left( p + \rho m \frac{dA}{dr} \right) \right) r^4 \pi \frac{dq}{dr} + 16r^7 \right. \right. \\
& \left. \left( rA \left( \frac{dp}{dr} \right)^2 - \frac{r}{2} \frac{dA}{dr} (p + \rho m) \frac{dp}{dr} + \frac{dA}{dr} (p + \rho m)^2 \right) \pi^2 \right] A \left[ -8r^8\pi^2(p+\rho m)\frac{d^2p}{dr^2} \right. \right. \\
& \left. \left. + 2qr^4\pi(p+\rho m)\frac{d^2q}{dr^2} + \left(\frac{dq}{dr}\right)^2 q^2 - 10q \left( r\frac{dp}{dr} + \frac{d\rho m}{dr} \frac{mr}{5} + \frac{4p}{5} + \frac{4\rho m}{5} \right) r^3\pi\frac{dq}{dr} \right. \right. \\
& \left. \left. + 24r^8 \left( \frac{dp}{dr} + \frac{d\rho m}{dr} \frac{m}{3} \right) \frac{dp}{dr} \pi^2 \right] \beta B^4 \right\} + \frac{A^2 B^4}{256r^{12}\pi^4(p+\rho m)^4} \left( -4\frac{dp}{dr} r^4\pi + q\frac{dq}{dr} \right)^4 \beta \\
& + \frac{A^2 B^4 \left( -4\frac{dp}{dr} r^4\pi + q\frac{dq}{dr} \right)^3 \frac{dA}{dr} \beta}{64r^8\pi^3(p+\rho m)^3} + \frac{1}{64r^6\pi^2(p+\rho m)^2} \left[ \beta B^4 \left( r^2 \left( \frac{dA}{dr} \right)^2 + 8rA \right. \right. \\
& \left. \left. \frac{dA}{dr} - 16A^2 \left( -4\frac{dp}{dr} r^4\pi + q\frac{dq}{dr} \right)^2 \right] + \frac{1}{4r^3\pi(p+\rho m)} \left[ \left( r^2\beta \left( \frac{dA}{dr} \right)^2 + 2((r^2+2\beta)A) \right. \right. \\
& \left. \left. - 2\beta)A^2 \right) \left( -4\frac{dp}{dr} r^4\pi + q\frac{dq}{dr} \right) B^4 \right] + B^4 \left[ r^2\beta \left( \frac{dA}{dr} \right)^2 - (A-1)A^2((r^2+\beta)A-\beta) \right] \\
& = A^4 B^4 (8\pi p r^4 - q^2)
\end{aligned} \tag{26}$$

Notice that this extended TOV equation in  $f(R)$  gravity should be used to study charged compact stars. We postpone the discussion of the results to future work.

## 5. Conclusion

We have studied the effect of electric charge in compact stars assuming that the charge distribution is proportional to the mass density as is done in reference [2]. We have checked our numerical results (programs written using Maple 2016) with that found in the literature in case of GR and we further that work using extended theory of gravity. Next we will use that extended TOV equation to study charged compact stars and compare the results to those found in case of GR.

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