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Trajectory of magnetized particles around Schwarzschild black hole in a uniform magnetic field

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We study the motion of a magnetized particle orbiting around a black hole in asymptotically uniform magnetic field. The influence of magnetic parameter on effective potential of the radial motion of magnetized spinning particle around the Schwarzschild black hole using Hamilton–Jacobi formalism is studied. It is found that circular orbits for photons and slowly moving particles may become stable near $r = 3M$. It was argued that the radii of the innermost stable circular orbits are sensitive on the change of magnetic parameter.

I. INTRODUCTION

Particles and fields play an important role in the study of astrophysical compact objects such as neutron stars, white dwarfs and black holes. For this reason, the phenomena and astrophysical processes that occur around black holes attract great interest of scientists. The interest that arose in the early seventies in the theoretical study of classical and quantum processes that can occur in the vicinity of black holes does not weaken even now, despite the large number of works carried out in recent years. New physical concepts that have emerged in the theory of black holes [1, 2] have already had their impact on the general development of research aimed at including gravity in the unified theory of fundamental interactions [3, 4]. The interaction between the gravitational and electromagnetic fields is important for characterizing the motion of particles in strong gravitational fields. Motivation for the study of these phenomena, from the problems of motion and acceleration of particles in gravitational fields. The study of the interaction between particles and the electromagnetic field in a curved spacetime is also of astrophysical interest, for example, in the case of strong synchrotron radiation coming out of galactic nuclei, which can be explained by the existence in those regions of very strong magnetic fields interacting with ultrarelativistic electrons. Such magnetic fields can penetrate into the inner parts of the accretion disk around the central black hole. Here we study the cross section of the gravitational capture of magnetized particles (with a non-zero magnetic moment) by a Schwarzschild black hole located in an asymptotically homogeneous magnetic field. The expression for the magnetic moment of a particle is characterized through the polarization tensor [5], which appears in the Hamilton-Jacobi equation. It is revealed that particles with a magnetic moment are more stable than classical particles. The above results are obtained within the framework of the general theory of relativity, an expression for the capture cross section of magnetized particles that have an arbitrary velocity at infinity, a sufficiently compact spherically symmetric body. A comparison with the corresponding results for classical particles, which were obtained in [6], is given.

Here we consider motion of magnetized particle around non-rotating Schwarzschild black hole and motion of magnetized particle in external magnetic field. Here, innermost stable circular orbits [7, 8], storage and release of energy of particle have been considered taking account of magnetic coupling parameter β [5]. Following the idea of the paper

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Ref. [5], we assume that the black hole is immersed in the external uniform magnetic field. We plan to study the influence of magnetic parameter and magnetic field on stable circular orbits of magnetized particle around compact object in external magnetic field. We obtain the equation of motion of the particles using Hamilton-Jacobi formalism [2].

II. BACKGROUND

In this article we discuss the simplest solution of the Einstein equation, which is called a Schwarzschild black hole.It is assumed that the black hole is in an external magnetic field. For simplicity, assume that the magnetic field is uniform.

Metric for describing the static spherical-symmetric Schwarzschild space-time in spherical coordinates has the following form:

$$ds^2 = -A^2 dt^2 + H^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2 \varphi, \tag{1}$$

where the metric function

$$A^2 = H^{-2} = \left(1 - \frac{2M}{r}\right)$$

expressed in terms of the M parameter, which is the total mass black hole placed in an asymptotically homogeneous magnetic field B_0. For simplicity, the direction of the polar axis is chosen as as before, along the direction of the magnetic field.

A. Equation of motion

The Hamilton-Cacobi equation for magnetized particles is chosen takim, in progress [5],

$$H = \frac{1}{2}\left(\frac{1}{m+U} g^{\mu\nu} P_\mu P_\nu + U\right), \tag{2}$$

from which one can easily derive the radial equation of motion magnetized test particles, as shown in:

$$r^3 \left(\frac{dr}{d\tau}\right)^2 = f(r) \ , \tag{3}$$

where

$$f(r) = (e^2 + \eta - 1)r^3 + 2M(1-\eta)r^2 - l^2 r + 2Ml^2 \ , \tag{4}$$

e and l are, respectively, the energy and momentum of the particle, divided by the mass m and introduce the notation, as done in the work [5]

$$\eta = \frac{\beta\left(1 - \frac{2M}{r}\right)}{\left(1 - \frac{2M}{r} - \Omega^2 r^2\right)^{\frac{1}{2}}} \ . \tag{5}$$

Using the equation of motion

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}} = \frac{e^2 - 1}{2} \tag{6}$$

it is easy to obtain an expression for an effective potential motion of magnetized particles

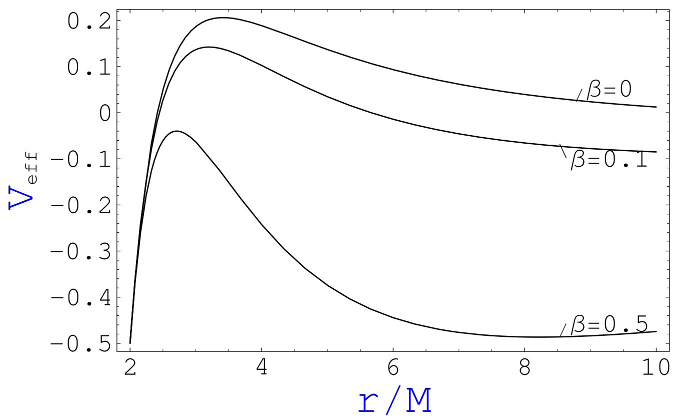


FIG. 1: Dependence of the effective potential on of radial motion of magnetized particles near the Schwarzschild black hole for different values of the β parameter.

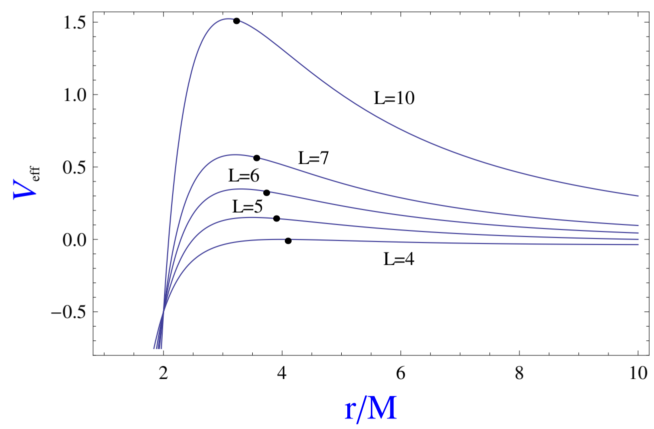


FIG. 2: Dependence of the effective potential on the moment pulse of magnetized particles for a constant value of the parameter $\beta = 10^{-5}$.

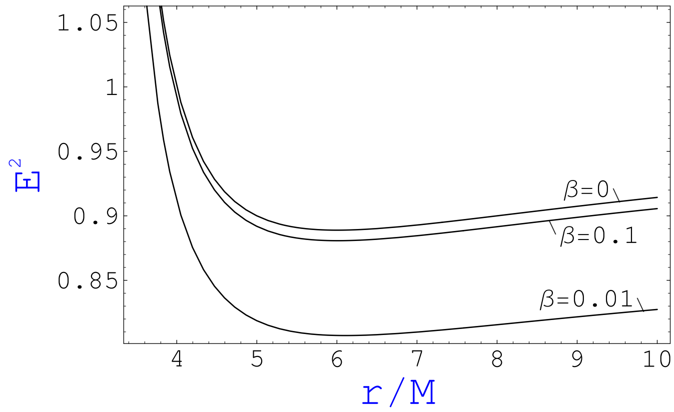
$$V_{\text{eff}} = \frac{1}{2}\left(\frac{l^2}{r^2} - \eta\right)\left(1 - \frac{2M}{r}\right) - \frac{M}{r} \ . \tag{7}$$

Now you can easily find a solution for the radii of the inner stable circular orbit (SKO), that is, the radius r_{VSKO}. Using the equation (3) and conditions for the existence of circular orbit:

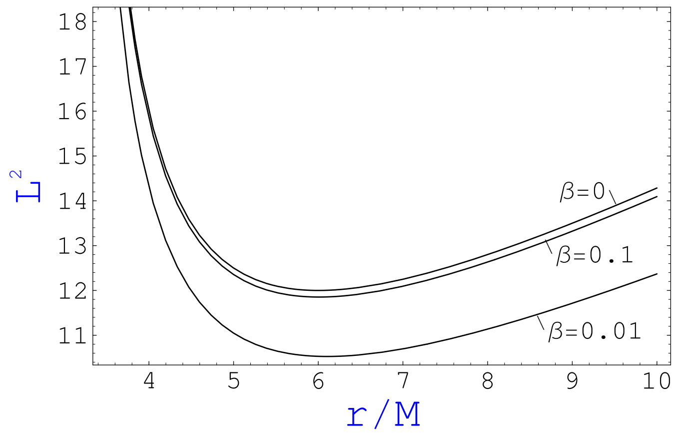
$f(r) = 0\,,\ f'(r) = 0\ ,$

one can find expressions for the energy e and the angular momentum l. A Fig. 3 shows the radial dependence of energy and angular momentum of particles in circular orbits around a black hole shardschild for different values of the β parameter. It can be seen that with by increasing the value of the β parameter, the corresponding circular the orbits move towards the object, i.e., approaching black hole.

As you can see, the equation (7) depends on the angular momentum and radius, and also from the dimensionless value β, which was introduced in the previous chapter. Let us recall that it characterizes interaction of the magnetic moment of the particles with the external magnetic polem. On fig. 1 shows the radial dependence effective potential



a)



b)

FIG. 3: Radial dependence of energy (à) and angular moment (b) in circular orbits around Schwarzschild black hole for different values of the β parameter.

for magnetized particles at different the values of the dimensionless parameter β. It can be seen that the orbits particles become more stable with an increase in the β parameter. This means that the spin particles are stable without spin. How can be seen from the figure the potential has a repulsive character, it is possible say, particles coming from infinity and flying past source may not be captured, that is, they again go into infinity. This means that the shape of the orbit of the particles is only parabolic or hyperbolic, but circular and elliptical orbits exist with an increase in the value of the β parameter. Na Fig. 2 the dependence of the effective potential on angular momentum of magnetized particles for a constant value $\beta = 10^{-5}$.

III. SUMMARY AND CONCLUSION

Here we have obtained analytic expressions for the cross section of the capture magnetized particles of the Schwartzschild black hole. Expressions for capture sections were obtained using the formalism Hamilton-Jacobi.

The form of the Hamilton-Saxobi equation was chosen, as in Work [5, 9]. Such an analysis for particles with zero magnetic moment was carried out for the first time by Zakharov in work [6]. Extensive analysis of the effective potential of the radial motions for magnetized particles have shown that orbits can exist only parabolic or hyperbolic, but round or electric orbits exist with an increase in the dimensionless parameter β, which characterizes the interaction between magnetic field and magnetic moment, i.e. e. captured compact magnetized particles can leave the black hole with an increase in the magnetic moment.

Then we found the dependence of the inner stable circular orbit magnetic particles from magnetic parameters β. This was considered in the works [10–12], where asserts that the inner stable circular orbits r_{ISCO} us It was shown that also in the presence of the magnetic moment of the particles stable circular orbits advancing towards of Schwartzschild black hole.

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