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SPECTRAL AND STATISTICAL PROPERTIES OF COLLECTIVE RESONANCE FLUORESCENCE IN A SQUEEZED VACUUM

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I. Introduction

The fundamental property of squeezed light is the reduced quantum fluctuations in the one-quadrature phase.

After the early works by Caves [1], in which the potential application of squeezed light for detection of gravitational waves was shown, a large amount of theoretical [2-16] and experimental works [17-18] is concentrated on the generation of squeezed light and its application to overcome the short--noise limit [19]. In recent works the radiative decay [21] and spectroscopic properties [22] of an atom interacting with a broad-band squeezed vacuum have been considered.

In this work we discuss the collective resonance fluorescence from N driven atoms which are damped by white squeezed noise. In the case of intense external field the stationary solution for the density operator of the atomic system is given. In the general case (without the case of exact resonance) the density matrix of the atomic system is dependent on a phase difference of the driving field and squeezed vacuum. The dependence of the spectrum of fluorescence and photon statistics of spectrum components on the parameters of the squeezed vacuum are analyzed.

II. Basic equations

Let N two-level atoms be concentrated in a region small compared to the wave-length of all the relevant radiation modes (Dicke model). The atoms are interacting with a classical driving field of frequency ω_L and with the quantized multimode rediation field. The Hamiltonian of the system in the electric dipole, rotating-wave approximations and in the interaction picture can be written in following form:

$$H = \frac{1}{2}\delta(J_{22} - J_{11}) + G e^{i\phi_L} J_{21} + G e^{-i\phi_L} J_{12}$$
$$+ H_{rad} + (J_{21}\Gamma + J_{12}\Gamma^{+}) \qquad , \qquad (1)$$

where $\delta = \omega_{21} - \omega_L$ is the detuning of the laser frequency from the atomic resonance frequency ω_{21}; $G e^{-i\phi_L} = \mu E$, where μ is the atomic dipole moment and E is the amplitude of the driving field; J_{ij} $(i,j = 1,2)$ are the collective (angular) atomic operators which satisfy the commutation relations

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{i'j} - J_{i'j}\delta_{ij'} \qquad , \qquad (2)$$

H_{rad} is the free Hamiltonian for the quantized radiation field; Γ and Γ^{+} are operators defined in terms of the positive and negative frequency components of this field, respectively.

The normal treatment of the resonance fluorescence is considered in many works [23-25] in which the quantized radiation field is initially taken in the usual vacuum state.

In this work we assume that all the quantized radiation modes coupling to the atoms initially are squeezed [20-22]. The band-width of the squeezing is assumed to be sufficiently broad so that the squeezed vacuum appears as δ-correlated squeezed white noise to the atoms. Then, the correlation functions for free parts Γ_{free} and Γ^{+}_{free} (the noise operators) of the operators Γ and Γ^{+} can be written as [20-22]

$$\langle \Gamma^{+}_{free}(t)\Gamma_{free}(t')\rangle = \gamma P \delta(t-t') \quad ,$$
$$\langle \Gamma_{free}(t)\Gamma^{+}_{free}(t')\rangle = \gamma (P+1) \delta(t-t') \quad ,$$
$$\langle \Gamma_{free}(t)\Gamma_{free}(t')\rangle = \gamma |Q| e^{i\phi_v} \delta(t-t') \quad , \qquad (3)$$
$$\langle \Gamma^{+}_{free}(t)\Gamma^{+}_{free}(t')\rangle = \gamma |Q| e^{-i\phi_v} \delta(t-t') \quad ,$$

where γ is the spontaneous emission rate of an atom in the usual unsqueezed vacuum; P and $Q = |Q| e^{i\phi_v}$ are the parameters characterizing the squeezing with $|Q|^2 \leq P(P+1)$, where the equality holds for a minimum-uncertainty squeezed state.

Using the relations (1) and (3) and after making the unitary transformation $U = e^{-i\frac{\phi_L}{2}(J_{22} - J_{11})}$, one finds the master equation for the reduced density operator ρ of the atoms in the following form [20]:

$$\frac{\partial \rho}{\partial t} = -i\left[\frac{1}{2}\delta.(J_{22} - J_{11}) + G(J_{12} + J_{21}), \rho\right]$$
$$+ \frac{\gamma}{2}(P+1)(J_{12}\rho J_{21} - J_{21}J_{12}\rho + H.C.)$$
$$+ \frac{\gamma}{2}P.(J_{21}\rho J_{12} - J_{12}J_{21}\rho + H.C.)$$
$$- \frac{\gamma}{2}|Q| e^{-i\phi}(J_{21}\rho J_{21} - J_{21}^{2}\rho + H.C.)$$
$$- \frac{\gamma}{2}|Q| e^{i\phi}(J_{12}\rho J_{12} - J_{12}^{2}\rho + H.C.) \equiv L\rho \quad , \qquad (4)$$

where $\phi = 2\phi_L - \phi_v$ is the phase difference of the driving field and squeezed vacuum.

Following previous works [23,26] we introduce the Schwinger representation for the collective atomic operators J_{ij} .

$$J_{ij} = a_i^{+} a_j \qquad (i,j = 1,2) \qquad , \qquad (5)$$

where the operators a_i and a_i^{+} obey the boson commutation relations

$$[a_i, a_j^{+}] = \delta_{ij}$$

and can be treated as the annihilation and creation operators for the atoms being populated in the level $|i\rangle$.

After performing the canonical (dressing) transformation

$$a_1 = c_1 \cos\varphi + c_2 \sin\varphi \quad ,$$
$$a_2 = -c_1 \sin\varphi + c_2 \cos\varphi \quad , \qquad (6)$$

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where $\quad tg(2\varsigma) = 2G/\delta \quad ,$

one can split the Liouville operator L appearing in equation (4) into the slowly varying part and the terms oscillating at frequencies 2Ω and 4Ω, with Ω denoting one half of the Rabi frequency. We assume here that the Rabi frequency is sufficiently large and satisfies the relation

$$\Omega = \left(\tfrac{1}{4}\delta^2 + G^2 \right)^{1/2} \gg N\gamma \qquad (7)$$

In this case the secular approximation [23-24] is justified and we retain only the slowly varying part of the Liouville operator. We have then the master equation

$$\begin{aligned} \frac{\partial \tilde{\varrho}}{\partial t} = & -i\left[\Omega (R_{22} - R_{11}) , \tilde{\varrho} \right] + \\ & + B \left(R_3 \tilde{\varrho} R_3 - R_3^2 \tilde{\varrho} + H.C. \right) \\ & + \left\{ X_1 \left(R_{12} \tilde{\varrho} R_{21} - R_{21} R_{12} \tilde{\varrho} \right) + \right. \\ & \left. + X_2 \left(R_{21} \tilde{\varrho} R_{12} - R_{12} R_{21} \tilde{\varrho} \right) + H.C. \right\} \quad , \end{aligned} \qquad (8)$$

where $R_{ij} = C_i^{+} C_j \; (i,j = 1,2)$ are the collective operators of "dressed" atoms satisfying the commutation relation

$$\left[R_{ij} , R_{i'j'} \right] = R_{ij'} \delta_{i'j} - R_{i'j} \delta_{ij'} \quad , \qquad (9)$$

$$B = \gamma \left(P + \tfrac{1}{2} - |Q| \cos\phi \right) \sin^2\varsigma \cdot \cos^2\varsigma \quad , \qquad (10)$$

$$X_1 = \frac{\gamma}{2} \left[P(\cos^4\varsigma + \sin^4\varsigma) + \cos^4\varsigma + 2|Q|\cos\phi \cdot \sin^2\varsigma \cos^2\varsigma \right] \quad , \qquad (11)$$

$$X_2 = \frac{\gamma}{2} \left[P(\cos^4\varsigma + \sin^4\varsigma) + \sin^4\varsigma + 2|Q|\cos\phi \cdot \sin^2\varsigma \cos^2\varsigma \right] \quad , \qquad (12)$$

$$R_3 = R_{22} - R_{11} \quad , \qquad (13)$$

$\tilde{\varrho} = T \varrho T^{+}$, where T is the unitary operator representing the canonical transformation (6).

The master equation (8) gives the exact steady-state solution

$$\tilde{\varrho} = Z^{-1} \sum_{n_1=0}^{N} X^{n_1} \, | n_1 \rangle \langle n_1 | \quad , \qquad (14)$$

where

$$X = X_1 / X_2 \quad , \qquad (15)$$

$$Z = \left(X^{N+1} - 1 \right) / (X - 1) \quad . \qquad (16)$$

The state $|n_1\rangle$ is the eigenstate of the operators R_{11} and $R_{11} + R_{22}$. The solution (14) allows one to calculate all the stationary expectation values of the atomic observables. Some of the results that will be needed for our further considerations are given in Appendix. In the case of exact resonance $ctg^2\varsigma = 1$ one shows from (11)-(12) and (15) that $X = 1$ and the solution (14) reduces to

$$\tilde{\varrho} = (N+1)^{-1} \sum_{n_1=0}^{N} | n_1 \rangle \langle n_1 | \quad . \qquad (17)$$

The solution (17) is independent of the parameters P, Q of the squeezed vacuum, consequently all the one-time expectation values of the atomic observables are independent of that the vacuum is squeezed or not. In the general off-resonance case the density operator $\tilde{\varrho}$ in (14) is phase-sensitive and all the steady-state expectation values of the atomic and field observables are dependent on the parameters of squeezed vacuum. In sections III and IV we investigate the influence of the squeezed vacuum on the fluorescent spectrum and photon statistics of spectrum components.

III. <u>Steady-state fluorescence spectrum</u>

Following the work [22] the steady-state spectrum of the fluorescent light has been calculated as the Fourier transform of the atomic correlation function

$$\langle J_{21}(\tau) J_{12} \rangle_s = \lim_{t \to \infty} \langle J_{21}(t+\tau) J_{12}(t) \rangle \quad . \qquad (18)$$

This scheme for calculating the spectrum assumes that there exists a small "window" of unsqueezed vacuum modes through which we can view the fluorescence. For the case $N \gg P$ the intensity of the fluorescent field dominates over the squeezed noise thus a "window" of unsqueezed vacuum modes is unnecessary, where $\langle \cdots \rangle_s$ denotes an expectation value over the steady-state (14). According to the transformation (16) we have

$$J_{21}(t) = \sin\varphi \cdot \cos\varphi \, R_3(t) + \cos^2\varphi \, R_{21}(t) - \sin^2\varphi \, R_{12}(t) . \tag{19}$$

The equations of motion for $\langle R_{ij}(t) \rangle$ can be derived by using the master equation (8), and have the following form

$$\frac{d}{dt} \langle R_3(t) \rangle = -\gamma_0 \langle R_3(t) \rangle - \gamma_c \langle R_3^2(t) \rangle + \gamma_c (N^2 + 2N) , \tag{20}$$

$$\frac{d}{dt} \langle R_{12}(t) \rangle = -2i\Omega \langle R_{12}(t) \rangle - \gamma_+ \langle R_{12}(t) \rangle - \\ - \frac{\gamma_c}{2} \langle \{ R_3(t), R_{12}(t) \} \rangle , \tag{21}$$

$$\frac{d}{dt} \langle R_{21}(t) \rangle = \frac{d}{dt} \langle R_{12}(t) \rangle^* , \tag{22}$$

where

$$\{ R_3(t), R_{ij}(t) \} \equiv R_3(t) R_{ij}(t) + R_{ij}(t) R_3(t) ,$$

$$\gamma_0 = 2\gamma \left[(P + \tfrac{1}{2})(\sin^4\varphi + \cos^4\varphi) + 2|Q| \cos\phi \cdot \cos^2\varphi \cdot \sin^2\varphi \right] , \tag{23}$$

$$\gamma_+ = \gamma \left[(P + \tfrac{1}{2})(1 + 2 \sin^2\varphi \cdot \cos^2\varphi) - 2|Q| \cos\phi \cos^2\varphi \sin^2\varphi \right] , \tag{24}$$

$$\gamma_c = \frac{\gamma}{2} (\sin^2\varphi - \cos^2\varphi) . \tag{25}$$

Equations (20)-(22) are so far exact. They contain, however, terms with the products of operators which make them unsolvable in the general case.

For the one-atom case one can use the well-known operator relation

$$R_{ij} R_{i'j'} = R_{ij'} \delta_{i'j} \quad (i, j, i', j' = 1, 2) \tag{26}$$

and equations (20)-(22) reduce to the linear differential equations

$$\frac{d}{dt} \langle R_3(t) \rangle = -\gamma_0 \langle R_3(t) \rangle + 2\gamma_c , \tag{27}$$

$$\frac{d}{dt} \langle R_{12}(t) \rangle = -2i\Omega \langle R_{12}(t) \rangle - \gamma_+ \langle R_{12}(t) \rangle , \tag{28}$$

$$\frac{d}{dt} \langle R_{21}(t) \rangle = \frac{d}{dt} \langle R_{12}(t) \rangle^* . \tag{29}$$

Linear differential equations (27)-(29) are exact solvable and their solutions are in agreement with the previous work on the resonance fluorescence of an atom in a squeezed vacuum [22].

For the case of exact resonance, i.e., $\mathrm{ctg}^2\varphi = 1$ we have $\gamma_c = 0$ and all equations (20)-(22) reduce to exact solvable linear differential equations.

In the general case, to deal with the product terms we apply a decorrelation scheme similar to that used by Compagno and Persico [25]. The only difference consists in the fact that we decorrelate symmetrized products of operators (anticommutator). This allows us to preserve one-atom terms unchanged and clearly separate them from the collective terms. The decorrelated operators that do not enter into the equations as "proper" variables are replaced by their steady-state averages calculated with the density matrix (14).

$$\langle \{ R_3 , R_{ij} \} \rangle = 2 \langle R_3 \, R_{ij} \rangle = 2 \langle R_3 \rangle_s R_{ij} . \tag{30}$$

With such approximations equations (20)-(22) have simple exponential solutions with the one-atom (i.e. γ_0, γ_+) and collective (i,e. $\gamma_c \langle R_3 \rangle_s$) damping constant clearly separated. Upon neglecting the collective part one immediately obtains the one-atom results. By using the density matrix (14), one can show that [23,25] in the case of large N the decorrelation (30) yields a small error (with an order of $N^{-1/2}$) in the calculation of the steady-state fluorescent spectrum. The explicit expressions for the collective terms of the damping constants can be obtained with the use of the steady-state averages given in Appendix.

Using the relation (19), the solutions of equations (20)-(22) and applying the quantum regression theorem [21], one obtains the following expressions for the correlation function (18)

$$\begin{aligned}\langle J_{21}(\tau)J_{12}\rangle_s &= \sin^4\varsigma\cdot\langle R_{12}R_{21}\rangle_s\, e^{-\tilde{\gamma}_+\tau-2i\Omega\tau}\\ &+\cos^4\varsigma\cdot\langle R_{21}R_{12}\rangle_s\, e^{-\tilde{\gamma}_+\tau+2i\Omega\tau}\\ &+\sin^2\varsigma\cdot\cos^2\varsigma\,(\langle R_3^2\rangle_s-I_e)\,e^{-\tilde{\gamma}_0\tau}+\sin^2\varsigma\cdot\cos^2\varsigma\, I_e\ ,\end{aligned}$$

where (31)

$$I_e = \gamma_c\,(N^2+2N)\langle R_3\rangle_s\,/\,\tilde{\gamma}_0\ , \qquad (32)$$

$$\tilde{\gamma}_0 = \gamma_0 + \gamma_c\,\langle R_3\rangle_s\ , \qquad (33)$$

$$\tilde{\gamma}_+ = \gamma_+ + \gamma_c\,\langle R_3\rangle_s\ . \qquad (34)$$

The expressions for the weighting factors of the particular exponents are given in Appendix.

The steady-state spectrum of the fluorescent light is proportional to the Fourier transform of the correlation function (31) and has the following form:

$$\begin{aligned}S(\omega) &\sim \frac{1}{2}\,\mathrm{Re}\left[\int_0^\infty e^{-i(\omega-\omega_L)\tau}\cdot\langle J_{21}(\tau)J_{12}\rangle_s\, d\tau\right]=\\ &= \sin^2\varsigma\cdot\cos^2\varsigma\,(\langle R_3^2\rangle_s-I_e)\cdot\frac{\tilde{\gamma}_0}{(\omega-\omega_L)^2+\tilde{\gamma}_0^2}\ +\\ &+\cos^4\varsigma\cdot\langle R_{21}R_{12}\rangle_s\cdot\frac{\tilde{\gamma}_+}{(\omega-\omega_L-2\Omega)^2+\tilde{\gamma}_+^2}\\ &+\sin^4\varsigma\cdot\langle R_{12}R_{21}\rangle_s\cdot\frac{\tilde{\gamma}_+}{(\omega-\omega_L+2\Omega)^2+\tilde{\gamma}_+^2}\\ &+\frac{1}{2}\sin^2\varsigma\cos^2\varsigma\ I_e\ \delta(\omega-\omega_L)\ .\end{aligned}\qquad (35)$$

The fluorescence spectrum (35) contains three spectral lines centered at frequencies $\omega=\omega_L;\ \omega_L\pm 2\Omega$. In the off-resonance case, i.e., when $\mathrm{ctg}^2\varsigma\neq 1$ the central line at $\omega=\omega_L$ contains the elastic component with the intensity proportional to N^2 and the Lorentzian shaped component with the linewidth $\tilde{\gamma}_0$ and intensity $\sin^2\varsigma\cos^2\varsigma\,(\langle R_3^2\rangle_s-I_e)$. The two-sidebands are Lorentzians of linewidth $\tilde{\gamma}_+$ centered at frequencies $\omega=\omega_L-2\Omega$ and $\omega=\omega_L+2\Omega$, and having the intensities which are proportional to $\sin^4\varsigma\,\langle R_{12}R_{21}\rangle_s$ and $\cos^4\varsigma\,\langle R_{21}R_{12}\rangle_s$, respectively. In the case of exact resonance we have $\gamma_c=0$ and the spectrum (34) reduces to

$$S(\omega)\sim\frac{1}{12}N(N+2)\left\{\frac{\bar{\gamma}_0}{(\omega-\omega_L)^2+\bar{\gamma}_0^2}+\frac{3}{4}\,\frac{\bar{\gamma}_+}{(\omega-\omega_L-2\Omega)^2+\bar{\gamma}_+^2}+\frac{3}{4}\,\frac{\bar{\gamma}_+}{(\omega-\omega_L+2\Omega)^2+\bar{\gamma}_+^2}\right\}, \qquad (36)$$

where $\bar{\gamma}_0$ and $\bar{\gamma}_+$ are the one-atom linewidths (see eqs. (23)-(24)) being taken for the case of exact resonance

$$\bar{\gamma}_0 = \gamma\left[\,P+\frac{1}{2}+|Q|\cos\phi\,\right]\ , \qquad (37)$$

$$\bar{\gamma}_+ = \frac{\gamma}{2}\left[\,3P+\frac{3}{2}-|Q|\cos\phi\,\right]\ . \qquad (38)$$

It is clear from the relations (36)-(38) that in the case of exact resonance the elastic component vanishes, the intensities of all three inelastic components are proportional to N^2 and are independent of the parameters of squeezed vacuum while the linewidths, and consequently the peak intensities, are dependent on the parameters of the squeezed vacuum and become the phase-sensitive quantities. For an illustration we assume the squeezed vacuum being in a highly squeezed minimum-uncertainty state, i.e., $P\gg 1$ and $Q=(P+P^2)^{1/2}$ we have:

$$\begin{cases}\bar{\gamma}_0\approx 2P\gamma & \text{if}\quad \phi=0\ ,\\ \bar{\gamma}_0\approx \dfrac{\gamma}{8P} & \text{if}\quad \phi=\pi\ ,\end{cases}\qquad (39)$$

$$\begin{cases}\bar{\gamma}_+\approx P\gamma & \text{if}\quad \phi=0\ ,\\ \bar{\gamma}_+\approx 2P\gamma & \text{if}\quad \phi=\pi\ ,\end{cases}\qquad (40)$$

thus, the sidebands are broadened while the central peak has a supernatural or subnatural linewidth by changing the phase difference ϕ by π.

In another case when $\rho \ll 1$ and $Q = (\rho + \rho^2)^{1/2}$ we have

$$\left\{ \begin{array}{lll} \bar{\gamma}_0 \approx \gamma\left(\frac{1}{2} + \sqrt{\rho}\right) > \frac{\gamma}{2} & \text{if} & \phi = 0 , \\ \bar{\gamma}_0 \approx \gamma\left(\frac{1}{2} - \sqrt{\rho}\right) < \frac{\gamma}{2} & \text{if} & \phi = \pi , \end{array} \right. \tag{41}$$

$$\left\{ \begin{array}{lll} \bar{\gamma}_+ \approx \frac{\gamma}{2}\left(\frac{3}{2} - \sqrt{\rho}\right) < \frac{3}{4}\gamma & \text{if} & \phi = 0 , \\ \bar{\gamma}_+ \approx \frac{\gamma}{2}\left(\frac{3}{2} + \sqrt{\rho}\right) > \frac{3}{4}\gamma & \text{if} & \phi = \pi , \end{array} \right. \tag{42}$$

thus the narrowing or expansion of the linewidths of the central component and sidebands take place in the dependence on the phase difference ϕ. We note that the narrowing or expansion of the linewidths are sufficiently large when squeezed vacuum is in highly squeezed $(\rho \gg 1)$ minimum-uncertainty state (see relations (39)-(40)).

In the off-resonance case, as is clear from eqs. (32)-(33) the linewidths contain the one-atom $(\gamma_0 ; \gamma_+)$ and collective parts $(\gamma_c \langle R_3 \rangle_s)$ and they are, in the general case, dependent on the parameters of squeezed vacuum. For large numbers of atoms $N \gg 1$ from relation for $\langle R_3 \rangle_s$ given in Appendix one finds

$$\gamma_c \langle R_3 \rangle_s \approx \frac{\gamma}{2} N \left| \sin^2\varphi - \cos^2\varphi \right| \qquad \text{if} \quad ctg^2\varphi \neq 1 , \tag{43}$$

thus, for the case of $N \gg 1$ the collective part of the spectrum linewidths is independent of the parameters of squeezed vacuum and dominates over the one-atom parts γ_0, γ_+. Consequently the spectrum linewidths are approximately independent of that the vacuum is squeezed or not.

Contrary to the exact-resonance case the intensities of the spectrum components are strongly depend on the parameters of the squeezed vacuum. In fig. 1 a-d the relative intensities

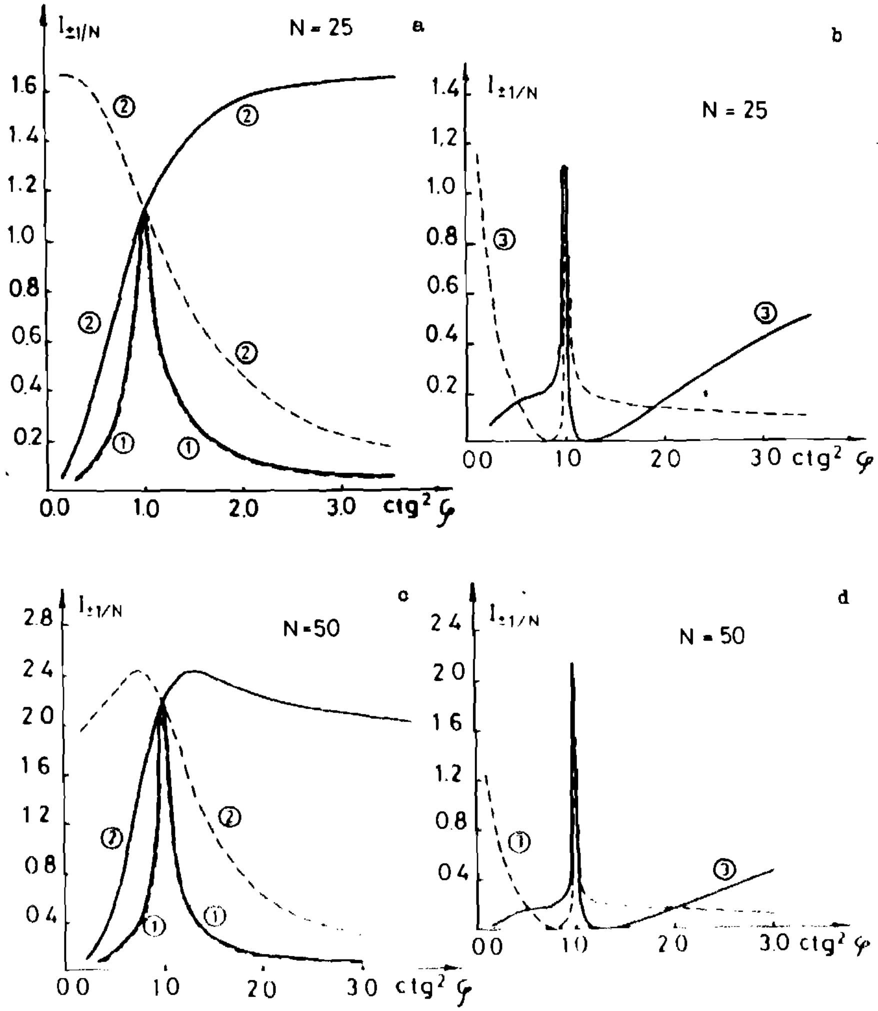


Fig. 1.

The relative intensities I_{+1}/N (solid curves) and I_{-1}/N (dashed curves) as functions of the parameter $ctg^2\varphi$ for $N = 25$ (a-b) and $N = 50$ (c-d). The curves (1)-(3) correspond to: $\rho = |Q| = 0$; $\rho = 2, |Q| = (\rho^2 + \rho)^{1/2}$, $\phi = 0$ and $\rho = 2, |Q| = (\rho^2 + \rho)^{1/2}$, $\phi = \pi$, respectively.

of the two sidebands, i.e., the quantities $I_{-1}/_N = \sin^4\varphi \frac{\langle R_{12}R_{21}\rangle_s}{N}$ (dashed curves) and $I_{+1}/N = \cos^4\varphi \cdot \langle R_{21}R_{12}\rangle_s / N$ (solid curves) are plotted as functions of the parameter $ctg^2\varphi$ for various values of P , $|Q|$ and ϕ . In fig. 2a-b the relative intensity of the central line (the sum of elastic and inelastic components), i.e., the quantity $I_o/_{N^2} = \sin^2\varphi \cos^2\varphi \langle R_3^2\rangle_s / N^2$ is plotted as a function of the parameter $ctg^2\varphi$ for the same values of P, $|Q|$ and ϕ . It is clear from figs. (1)-(2), except for the point of exact resonance, that the intensities of spectrum components are strongly dependent on the parameters of squeezed vacuum and become phase-sensitive values. In unsqueezed vacuum (see curves 1 in fig. 1, 2 the intensities of the two sidebands are equal and spectrum is symmetric. In the squeezed vacuum (see curves 2 and 3 in figs.1 a-d) the intensities of two sidebands are quite different for the off-resonance case $ctg^2\varphi \neq 1$ and spectrum becomes asymmetric. For a large number of atoms $N \gg 1$ from the relations for $\langle R_{21}R_{12}\rangle_s$ $\langle R_{12}R_{21}\rangle_s$ and $\langle R_3^2\rangle_s$ given in Appendix, one finds

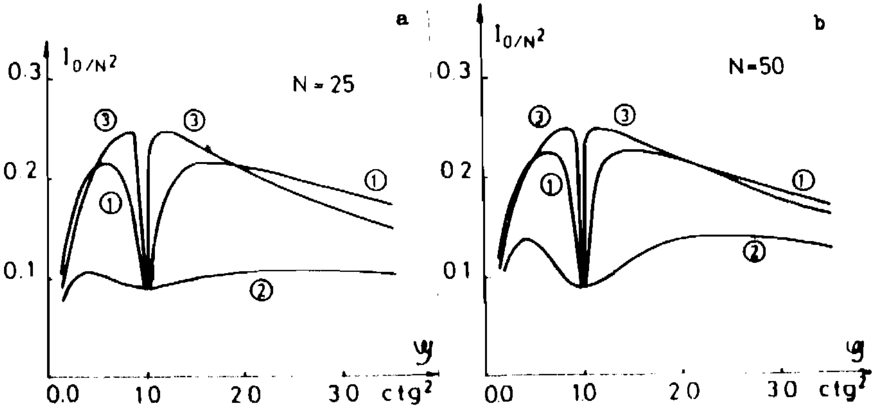


Fig. 2.

The relative intensity I_o/N^2 as a function of the parameter $ctg^2\varphi$ for $N = 25$ (a) and $N = 50$ (b). The curves (1)-(3) correspond to: $P = |Q| = 0$; $P = 2, |Q| = (P^2 + P)^{1/2}, \phi = 0$ and $P = 2, |Q| = (P^2+P)^{1/2}, \phi = \pi$, respectively.

$$I_1 \approx \cos^4\varphi \cdot N/|X-1| \qquad \text{if} \quad ctg^2\varphi \neq 1 ,$$

$$I_{-1} \approx \sin^4\varphi \cdot NX/|X-1| \qquad \text{if} \quad ctg^2\varphi \neq 1 ,$$

$$I_o \approx \begin{cases} \sin^2\varphi \cdot \cos^2\varphi \left(N^2 - \frac{4N}{X-1}\right) & \text{if} \quad ctg^2\varphi > 1 , \\ \sin^2\varphi \cdot \cos^2\varphi \left(N^2 - \frac{4NX}{1-X}\right) & \text{if} \quad ctg^2\varphi < 1 , \end{cases}$$

thus in the off-resonance case the intensities of spectrum components are strongly dependent on squeezed vacuum even for a large number of atoms $N \gg 1$ when the spectrum linewidths are independent of one.

IV. Photon statistics of the spectrum components

In this section we discuss the influence of the squeezed vacuum on the photon statistics of the spectrum components.

As is clear from the previous section and eq.(19), the operators $\cos^2\varphi R_{21}$, $\sin\varphi\cos\varphi R_3$ and $-\sin\varphi R_{12}$ can be considered as operator-sources of the spectral lines centered at frequencies $\omega = \omega_L + 2\Omega$; ω_L and $\omega_L - 2\Omega$ and for later use these operators will be denoted by S_{+1}^+ , $S_o^+ = S_o$ and S_{-1}^+ , respectively. Following refs. [23,28] we introduce the degree of second-order coherence for the spectrum component S_ℓ ($\ell = 0, \pm 1$) in the following form

$$G_{\ell,\ell}^{(2)} = \frac{\langle S_\ell^+ S_\ell^+ S_\ell S_\ell \rangle}{(\langle S_\ell^+ S_\ell \rangle)^2} \qquad (\ell = 0, \pm 1) . \quad (44)$$

The quantity $G_{\ell,\ell}^{(2)}$ describes the photon statistics of the spectrum component S_ℓ .

Using eqs. (14) and (19) one finds

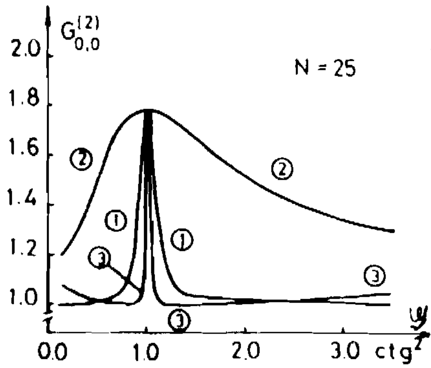
$$G_{0,0}^{(2)} = \langle R_3^4 \rangle_s / (\langle R_3^2 \rangle)^2 , \qquad (45)$$

$$G^{(2)}_{-1,-1} = \langle R_{12}R_{12}R_{21}R_{21}\rangle_s / (\langle R_{12}R_{21}\rangle_s)^2 = G^{(2)}_{1,1} \quad , \qquad (46)$$

where the expectation values $\langle R_3^4\rangle_s$, $\langle R_3^2\rangle_s$, $\langle R_{12}R_{21}\rangle_s$, $\langle R_{12}R_{12}R_{21}R_{21}\rangle_s$ are given in the Appendix.

Fig. 3.

The degree of second-order coherence $G^{(2)}_{0,0}$ as a function of the parameter $ctg^2\varphi$ for fixed N= 25. The curves (1)-(3) correspond to: $P=|Q|=0$; $P=2$, $|Q|=(P^2+P)^{1/2}$, $\phi=0$ and $P=2$, $|Q|=(P^2+P)^{1/2}$, $\phi=\pi$, respectively.



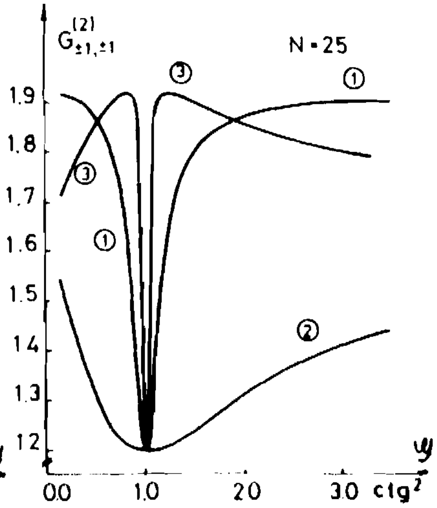


Fig. 4.

The degree of second-order coherence $G^{(2)}_{\pm1,\pm1}$ as a function of the parameter $ctg^2\varphi$ for fixed N= 25. The curves (1)-(3) correspond to: $P=|Q|=0$; $P=2$, $|Q|=(P^2+P)^{1/2}$, $\phi=0$ and $P=2$, $|Q|=(P^2+P)^{1/2}$, $\phi=\pi$, respectively.

For the one-atom case, by using the operator relation (26) one finds

$$G^{(2)}_{0,0} = 1 \quad , \quad G^{(2)}_{1,1} = G^{(2)}_{-1,-1} = 0 \quad ,$$

thus the photon statistics of the central component remains Poissonian and the sidebands have subpoissonian statistics as for the case in the unsqueezed vacuum.

For the collective case, the degrees of second-order coherence for the spectral lines given in (45)-(46) are dependent on the parameters of squeezed vacuum and become phase-sensitive quantitites. The behaviour of the degrees of coherence $G^{(2)}_{0,0}$ and $G^{(2)}_{\pm1,\pm1}$ as a function of the parameter $ctg^2\varphi$ for fixed N=25 and for various values of P, $|Q|$ and ϕ is plotted in figs. 3 and 4, respectively. As is clear from figs. 3-4, except for the point of the exact resonance $ctg^2\varphi = 1$, the parameters of the squeezed vacuum including the phase difference ϕ play an important role in determining photon statistics of the Mollow triplet.

v. Conclusions

We have considered the problem of collective resonance fluorescence in the squeezed vacuum. For the intense external field the analytical solution for the steady-state density operator for the atomic system is found. Analytical formulas have been derived for the spectrum of the resonance fluorescence and for the degrees of the second-ordered coherence for the spectrum components.

It has been shown that in the case of exact resonance the intensities of the spectrum component are independent of squeezed vacuum while the linewidths are subnatural or supernatural in the dependence on the parameters P, $|Q|$ and phase difference

ϕ of the driving field and squeezed vacuum. Contrary to the exact resonance case, in the off-resonance case the intensities of the spectrum components are dependent on the squeezed vacuum and the spectrum become , in the general case, asymmetric while the linewidths are approximately independent of the squeezed vacuum for a large number of atoms $N \gg 1$.

We have also shown that in the collective and off-resonance case the photon statistics of spectrum components are dependent on the parameters of the squeezed vacuum and become phase-sensitive quantities.

Appendix

In this Appendix we give the explicit expressions for the steady-state averages of the atomic operators that can be calculated with the use of the density matrix (14) :

$$\langle R_{11} \rangle_s = Z^{-1}\left[N X^{N+2} - (N+1) X^{N+1} + X \right] / (X-1)^2 , \tag{A.1}$$

$$\langle R_{11}^2 \rangle_s = Z^{-1}\left[N^2 X^{N+3} - (2N^2+2N-1) X^{N+2} + (N+1)^2 X^{N+1} - X^2 - X \right] / (X-1)^3 , \tag{A.2}$$

$$\begin{aligned}\langle R_{11}^3 \rangle_s = Z^{-1}\big[& N^3 X^{N+4} - (3N^3+3N^2-3N+1) X^{N+3} + \\ & + (3N^2+6N-4) X^{N+2} - (N^3+3N^2+3N+1) X^{N+1} + \\ & + X^3 + 4X^2 + X \big] / (X-1)^4 ,\end{aligned} \tag{A.3}$$

$$\begin{aligned}\langle R_{11}^4 \rangle_s = Z^{-1}\big[& N^4 X^{N+5} - (4N^4+4N^3-6N^2+6N-1) X^{N+4} + \\ & + (6N^4+12N^3-6N^2-12N+1) X^{N+3} \\ & - (4N^4+12N^3+6N^2-12N-11) X^{N+2} + (N^4+4N^3+ \\ & + 4N+1) X^{N+1} - X^4 - 11X^3 - 11X^2 - X \big] / (X-1)^5 ,\end{aligned} \tag{A.4}$$

$$\langle R_3 \rangle_s = N - 2\langle R_{11} \rangle_s , \tag{A.5}$$

$$\langle R_3^2 \rangle_s = 4\langle R_{11}^2 \rangle_s - 4N\langle R_{11} \rangle_s + N^2 , \tag{A.6}$$

$$\langle R_{12} R_{21} \rangle_s = -\langle R_{11}^2 \rangle_s + (N+1)\langle R_{11} \rangle_s , \tag{A.7}$$

$$\langle R_{21} R_{12} \rangle_s = -\langle R_{11}^2 \rangle_s + (N-1)\langle R_{11} \rangle_s + N , \tag{A.8}$$

$$\begin{aligned}\langle R_3^4 \rangle_s = {} & 16\langle R_{11}^4 \rangle_s - 32N\langle R_{11}^3 \rangle_s + 24N^2\langle R_{11}^2 \rangle_s \\ & - 8N^3\langle R_{11} \rangle_s + N^4 ,\end{aligned} \tag{A.9}$$

$$\begin{aligned}\langle R_{12} R_{12} R_{21} R_{21} \rangle_s = {} & \langle R_{11}^4 \rangle_s - 2(N+2)\langle R_{11}^3 \rangle_s + \\ & + (N^2+5N+5)\langle R_{11}^2 \rangle_s - (N^2+3N+2)\langle R_{11} \rangle_s .\end{aligned} \tag{A.10}$$

References

[1] Caves,C.M., Phys.Rev. D23 (1983) 1693

[2] Walls, D.F., Nature (London) 306 (1983) 141

[3] Collett, M.J., Loudon, R., JOSA B4 (1987) 1525

[4] Yurke, B., Phys.Rev. A29 (1984) 408

[5] Bondurant, R.S., Kumar, P., Shapiso, J.H., Maeda, M., Phys.Rev. A30 (1984) 343

[6] Mandel, L., Phys.Rev.Lett. 49 (1982) 136

[7] Walls, D.F., Zoller, P., Phys.Rev.Lett. 47 (1981) 709

[8] Lakshmi P.A., Agarwal, G.S., Phys.Rev. A29 (1984) 2260

[9] Bogolubov, N.N. (Jr.), Shumovsky, A.S., Tran Quang. Phys.Lett. A118 (1986) 315 and A116 (1986) 175

[10] Bogolubov, N.N. (Jr.), Shumovsky, A.S., Tran Quang. Opt.Commun. 62 (1987) 49 and 64 (1987) 351

[11] Ficek, Z., Tanas,R., Kielich, S. J. de Phys. 48 (1987) 1697

[12] Kozierowski, M., Kielich, S. Phys.Lett. A94 (1983) 213

[13] Golubyev, Yu.M., Sov. J.exper.theor.phys. 93 (1987) 463

[14] Dodonov, V.V., Kunmyshev, E.V., Man'ko, V.I., Phys.Lett. 79A (1980) 150

[15] Heidman, A., Reynaud, S., J. de Phys. 46 (1985) 1937

[16] Wodkiewicz, K., Knight, P.L., Buckle, S.J., Barnett, S.M., Phys.Rev. A35 (1987) 2567

[17] Slusher, R.C., Hollberg, L.W., Yurke, B., Mertz, J.C., Valley, J.F., Phys.Rev.Lett. 55 (1985) 2409

[18] Ling-An Wu, Kimble, H.J., Hall, J.L., Huija Wu. Phys.Rev. Lett. 57 (1986) 2520

[19] Min Xiao, Ling-An Wu, Kimble, H.J., Phys.Rev.Lett. 59 (1987) 278

[20] Collett, M.J., Gardiner, C.W., Phys.Rev. A31 (1985) 3761

[21] Gardiner, C.W., Phys.Rev.Lett. 56 (1986) 1917

[22] Carmichael, H.J., Lane, A.S., Walls, D.F., Phys.Rev.Lett. 58 (1987) 2539

[23] Bogolubov, N.N. (Jr.), Aliskenderov, E.A., Shumovsky, A.S., Tran Quang. J.Phys. B20 (1987) 1885

[24] Agarwal, G.S., Narducci, L.M., Feng, P.H., Gilmore, R., Phys.Rev.Lett. 42 (1979) 1260

[25] Compagno, G., Persico, F., Phys.Rev. A25 (1982) 3138

[26] Schwinger, J.V. In "Quantum Theory of Angular Momentum" ed. by L.C.Biedenharn and H.Van Dam (Academic Press, New York, 1965)

[27] Lax, M., Phys.Rev. 172 (1968) 350

[28] Loudon, R., Rep.Prog.Phys. 43 (1980) 38

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Спектральные и статистические свойства
коллективной резонансной флуоресценции
в сжатом вакууме

Обсуждается проблема коллективной резонансной флуоресценции атомов в белом сжатом шуме. Получены аналитические формулы для спектра резонансной флуоресценции и для степени когерентности второго порядка спектральных компонент. Показаны сужение, уширение спектральных линий и асимметрия спектра, которые возникли из-за сжатого шума.

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Spectral and Statistical Properties of
Collective Resonance Fluorescence in
a Squeezed Vacuum

The problem of collective resonance fluorescence of N driven atoms which are damped by a white squeezed noise is discussed. Analytical formulas for the spectrum of resonance fluorescence and for the degree of second-order coherence for spectral lines are calculated. The narrowing, expansion of linewidths and asymmetry of spectrum caused by squeezed vacuum are shown.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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