



# Comparison of semiclassical transfer to continuum model with Ichimura-Austern-Vincent model in medium energy knockout reactions



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## ABSTRACT

The full quantum mechanical (QM) model of inclusive breakup of Ichimura-Austern-Vincent (IAV) is implemented in this paper to calculate breakup from heavy radioactive nuclei on a  $^9\text{Be}$  target at intermediate energies. So far it had been implemented and applied only to low energy reactions with light projectiles. The IAV model is successful in predicting absolute cross sections among other observables. In order to get insight on the content of the model in the case of the complicated heavy-ion reactions, results are compared with those of the semiclassical transfer to the continuum (TC) model. Because the TC is based on analytical formulae the dynamics of the breakup as it is contained in the rather involved IAV formalism will become more transparent. Heavy-ion reactions at high energies ( $>50$  A.MeV) are demanding from the computational point of view because of the high number of partial waves involved, typically around 100. The TC constitutes a useful alternative to the full QM calculations whenever predictions and/or estimates are necessary. It allows also for a systematic, fast evaluation of breakup observables. In the applications of both methods we use state-of-the-art optical potentials and structure information. Excellent agreement is found between the calculated results of both methods and with available experimental data which shows that the qualitative and quantitative understanding of most aspects of one nucleon breakup is well under control.

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## 1. Introduction

Breakup in a nucleus-nucleus collision often represents a large part of the total reaction cross section if one of the two nuclei is weakly bound. It has been studied for as long as nuclear reactions have been made, starting with the deuteron projectile case, and its modelling can be more or less complicated depending on whether the experiment is inclusive or exclusive and on whether one considers the breakup of a nucleon or a cluster. Exclusive in our case refers to the identification of the final state of the residue by  $\gamma$ -ray detection. From the point of view of the theoretical reaction model the challenge lies in the description of all final state interactions between the various nucleons/nuclei present after the breakup. In principle both nuclear and Coulomb interactions should be considered but the theoretical description can be simplified in some specific experimental conditions, for example when a light target ion is used the Coulomb potential is often neglected.

To study properties of the valence nucleons in short-lived, exotic nuclei, one-nucleon knockout at intermediate energies has been used in the last fifteen years and has largely contributed to

establishing the picture of shell structure away from stability by extracting the spectroscopic factors [1] for the initial state wave function from the comparison of experimental data to the reaction theory predictions. A recent compilation of experimental knockout cross sections at intermediate energies showed a systematic trend when compared to theoretical calculations based on shell-model predictions for shell occupancy and eikonal approximation for the nucleon removal reactions [2]. However, this marked dependence does not seem to be supported by the results obtained with transfer reactions [3–5] and quasifree scattering with  $(p, 2p)$ ,  $(p, pn)$ , and  $(e, e'p)$  reactions [6]. Thus there are two possibilities: i) The eikonal model is not accurate enough to describe knockout from a deeply bound state and/or ii) there are other dynamical effects to be taken into account beyond those included in a peripheral model.

Here we aim at providing two alternatives to the eikonal model to help disentangling the above problems. One is fully quantum mechanical, the so called Ichimura, Austern, and Vincent (IAV) model [7,8]. The IAV model has been successfully applied to study the inclusive breakup reaction induced by weakly bound nuclei, such as deuteron [9] and  $^6\text{Li}$  [10,11], at energies around the Coulomb barrier. The other is semiclassical, the Transfer to the Continuum model (TC) [12–15]. Semiclassical refers to the fact that

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the relative motion of the reaction partners is treated as a classical trajectory, which allows several simplifications in the use of coordinates and energy conservation conditions. The TC method has also been applied to the description of a large number of inclusive breakup reactions, from normal to exotic projectiles both from deeply bound states as well as from weakly bound states [14–23]. In this paper, we will compare the numerical results of these two models and with experimental data. Both methods contain the correct kinematics and QM effects. The TC treats the relative motion in the semiclassical approximation but the fully QM IAV model does not and at the beginning of the paper we show that indeed the semiclassical approximation for the relative motion is justified. On the other hand the IAV method is fully QM and in this sense it is as good as the Continuum Discretized Coupled Channel method (CDCC) which is commonly used to calculate some observables related to breakup. At the moment the IAV model has a larger range of applicability as it can deal with scattering of heavy-ions at high energy, as we argue in the following. Furthermore IAV (and TC) models can both calculate the so called stripping term (NEB) while the widely used CDCC method [8] can only deal with the elastic breakup (EBU)(diffraction). A CDCC wave function can be applied to the IAV model as described in Refs. [8,24]. However at present this is only possible at low energy and for small nuclei.

It is worth noting that the theoretical cross section depends on the description of the reaction mechanism but also on the choice of the initial state wave function. In this paper we discuss and test for the first time in the literature the validity of semi-classical approximations with a full quantum model for the inclusive breakup reaction, thus including both elastic and non-elastic breakup. This is very important seen the present-state-of-the-art of both experimental and theoretical knockout studies [6] and our findings can open up new avenues to the understanding of nuclei with very unbalanced N/Z ratios.

We restrict ourselves to one nucleon breakup in a heavy-ion reaction. We compare theoretical calculations to inclusive data from reactions in which the projectile-core ( $A_C=A_P-1$ ) nucleus is measured and no information is available on the target final state. Because experiments are made at relatively high energies ( $>50$  A.MeV) and the core is measured intact in the forward direction, the hypothesis is that the target can be excited only by the neutron-target interaction. Thus in the theoretical models the core-target scattering is considered elastic. This is called the core-spectator model. This hypothesis is satisfied for heavy targets [25] but it has never been proven true for light targets.

Because the two models have already been used in a large number of cases they are just briefly described in the supplementary material. A detailed discussion of these two models can be found in Refs. [7–10,12–15,23,26–44]. Section 2 contains comparisons and discussion of calculated results with available experimental data and finally Sec. 3 contains our conclusions and outlook.

## 2. Theoretical results and comparison with experimental data

The calculations and comparisons to data presented in this paper refer to experiments made on a  $^9\text{Be}$  target. Both the IAV model and the TC use the  $n$ -target energy-dependent (AB) optical potential of Bonaccorso and Charity [45] and the single folding method of Ref. [46,47] for the core(projectile)-target potentials. The TC in general can deal with spin in both the initial bound state and the final continuum state. The AB and DOM potentials of [45] both contain a spin-orbit term. The IAV model can also deal with spin but so far it has been implemented numerically only for NEB. Therefore we will make comparisons between the TC and IAV results without using the spin-orbit potential. Note that also the

eikonal model does neglect the spin-orbit part of the optical potentials.

Being analytical the TC model can deal with large number of partial waves and final energies without numerical problems. Usually convergence is easily attained by taking the neutron maximum energy as twice the incident energy per nucleon, twenty  $n$ -target partial waves and the core-target impact parameter up to twice the sum of the projectile and target radii. On the other hand for the IAV model, the numerical calculations are much more complicated compared to the TC model. Here, we use the partial wave formalism mentioned in Ref. [27], in which one needs to perform the Jacobi coordinates transformation between the incoming Jacobi coordinate,  $(C+n)+T$ , and outgoing Jacobi coordinate,  $(n+T)+C$  (cf. Fig. 1 of the supplementary material). The computing time of this kind of transformation is proportional to the cube of the maximum number of the partial waves used in the calculations. For the current study cases on the  $^9\text{Be}$  target, the maximum number of partial waves has been set to be around 100 comparing to the low energy cases which only require at most 30 partial waves. In the calculations, the neutron-target energy range is chosen by stopping the calculation when the  $d\sigma/dE$  is less than 1% of the maximum. Note that the IAV method has been applied so far to light projectiles and low incident energies. This is the first attempt to extend the challenging numerical calculations to higher energies and heavier projectiles.

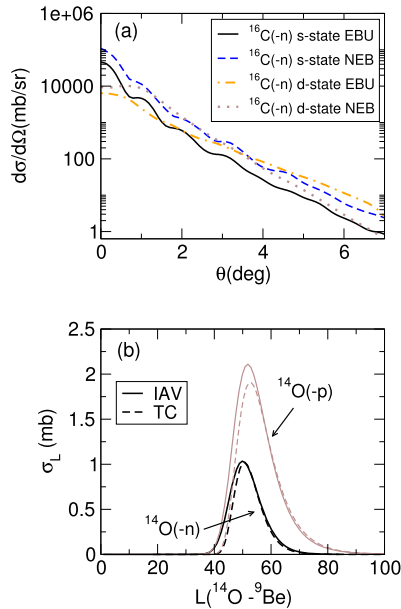
The results of four reactions first presented in [22] are discussed in this paper: both proton and neutron knockout from  $^{14}\text{O}$  and  $^{16}\text{C}$  on a  $^9\text{Be}$  target. In the case of the TC method the proton is treated by fitting its wave-function to a neutron wave function of appropriate effective separation energy as discussed in [42,48]. Details of separation energies, angular momentum states, asymptotic normalization constants and other important parameters are given in Table 1. Only the states for which experimental data and spectroscopic factors were given in Ref. [22] are considered here. Namely the breakup from the valence neutron and proton states of the two projectiles and the one-neutron removal from  $^{16}\text{C}$  leading to the  $^{15}\text{C}$   $(5/2)^+$  bound excited state at 740 keV above its  $(1/2)^+$  ground state. In Table 1 one can already see that the cross sections of both EBU and NEB computed by the TC method are very close to the ones obtained by the IAV model. However to better investigate the relation between IAV and TC, the first thing we are interested in checking is the core-residual nucleus angular distribution which can confirm whether the relative motion follows a classical path or not. This condition is necessary to make the comparison of the two methods meaningful. In Fig. 1 (a), top part, we show the angular distribution of the core in the lab frame calculated by the IAV model. The solid, dashed, dash-dotted and dotted lines are the angular distribution of  $s$ -state EBU,  $s$ -state NEB,  $d$ -state EBU, and  $d$ -state NEB, respectively. We find that the cross section decreases smoothly, it is forward peaked and basically single valued for the case of the weakly bound  $d$ -neutron state in  $^{16}\text{C}$ . The case of the  $s$ -state is interesting because it shows some small oscillations which remind of those seen in Ref. [49] for the breakup of the halo  $s$ -state in  $^{11}\text{Be}$ . According to Ref. [49] they are due to small diffraction effects.

On the other hand in Fig. 1 (b), bottom part, the projectile-target angular momentum distribution of the breakup cross section for  $^{14}\text{O}+^9\text{Be}$  is presented. This figure contains the TC (dashed lines) and IAV (solid lines) results for both neutron (thick lines) and proton (thin lines) breakup. In order to make the comparison with the TC method that is based on an integral over the core-target impact parameter rather than on the sum over partial waves of the IAV model we have used the relationship  $L+1/2=b_c K$  where  $K$  is the momentum of relative motion at infinity.  $K=8.7\text{ fm}^{-1}$  for the presented case. First we notice that the distributions are similar even if the neutron is strongly bound while the proton

**Table 1**

Nucleon breakup single particle cross sections in mb for the one nucleon breakup reactions  $^{14}\text{O}$  at 53 A.MeV and  $^{16}\text{C}$  at 75 A.MeV on a  $^9\text{Be}$  target [22]. Separation energies in MeV, asymptotic normalization constants  $C_i$  in  $\text{fm}^{-1/2}$  and cross section in mb.  $R_f$  is the ratio between the experimental and IAV cross section including the shell model spectroscopic factor. Experimental and eikonal cross sections (including already the spectroscopic factors) and spectroscopic factors from Ref. [22]. See text for details.

	$S_{n(p)}$	$nlj$	$C_i$		$\sigma_{IAV}$	$\sigma_{TC}$	$C^2S$	$\sigma_{eik}$	$\sigma_{exp}$	$R_f$
$^{14}\text{O}(-n)$	23.12	$1p_{3/2}$	17.74	TOT	13.72 (6.86)	12.65	3.15	54(0.26)	14	0.3 (0.65)
				EBU	3.55	2.37				
				NEB	10.17	10.28				
$^{14}\text{O}(-p)$	4.63	$1p_{1/2}$	4.20	TOT	33.91	30.5	1.55	55(1.05)	58	1.10
				EBU	12.50	10.3				
				NEB	21.41	20.2				
$^{16}\text{C}(-n)$	4.25	$2s_{1/2}$	3.83	TOT	58.42	47.7	0.89	60(0.6)	36	0.7
				EBU	16.09	14.3				
				NEB	42.33	32.4				
	4.99	$1d_{5/2}$	0.90	TOT	36.29	26.9	0.90	30(1.54)	46	1.4
				EBU	10.99	7.1				
				NEB	25.30	19.8				
$^{16}\text{C}(-p)$	22.56	$1p_{3/2}$	19.26	TOT	7.45	7.48	2.95	50(0.36)	18	0.82
				EBU	1.21	1.10				
				NEB	6.24	6.38				

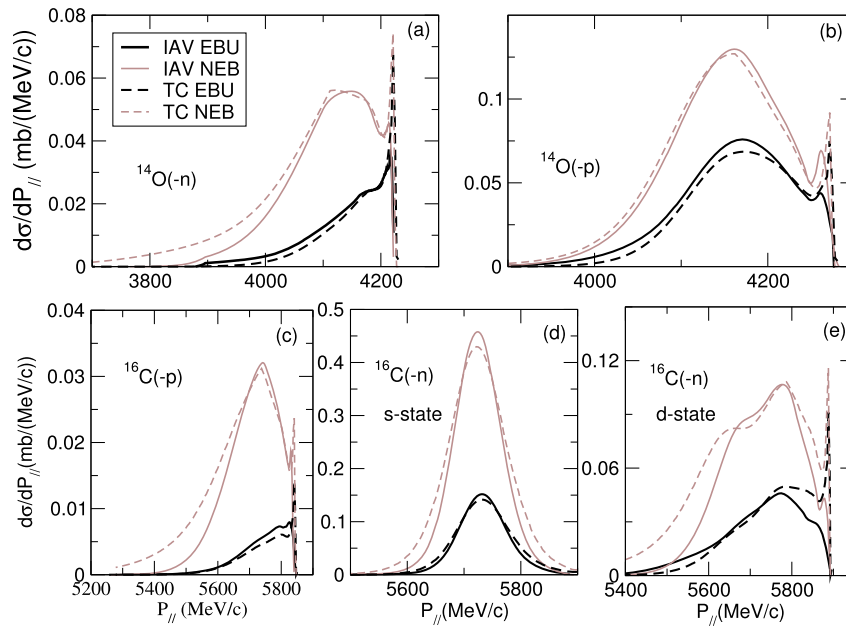


**Fig. 1.** Top (a): Differential cross section angular distribution of the core in lab frame for one neutron removal reaction of  $^{16}\text{C}$  calculated with IAV model. Bottom (b): Projectile-target angular momentum distribution of the summed breakup reaction cross section (EBU+NEB) for the  $^{14}\text{O} + ^9\text{Be}$  system (see text for more details).

is not. The peaks are at about the same  $L$  but of course the absolute values are larger for the weakly bound proton. This result suggests that in presence of such a strongly competing channel the knockout of the strongly bound particle will be suppressed in the experimental data. However the two processes are considered independent in the present existing models and this might partially explain the large difference between the cross section experimental values and those calculated. Both models predict the same bell-shaped distribution, because at small impact parameters breakup is suppressed by the strong core-target absorption while at large  $L$  (or  $b_c$ ) the breakup probability decreases. This kind of distribution was first predicted by Hussein-McVoy [50] on the basis of an eikonal approximation to the IAV model. Again we see a

very classical behaviour with the peak close to the strong absorption radius corresponding to a grazing collision between projectile and target. Because the wave function of the weakly bound proton has a long tail breakup extends to larger impact parameters than in the case of the strongly bound neutron. The IAV model calculates the  $L$ -distributions in the system of the incoming Jacobi coordinates and of the outgoing Jacobi coordinates, as mentioned above. The two distributions are the same but shifted by a few partial waves. The TC considers only the core-target partial waves (impact parameters). Because of this there is a small difference with the IAV results for the low  $L$ . The overall agreement of the two models is excellent which demonstrates that both models describe the same physical content. In particular it shows that the use of semiclassical approximations for heavy-ions at medium to high incident energies is justified and therefore in the future QM numerical method implementations could be simplified in as far as the relative motion wave function is concerned. It shows also that the approximations inherent to the TC model do not change the rate of convergence. Furthermore the absolute values of the cross sections are very close in the two models at each partial wave and in total (cf. Table 1).

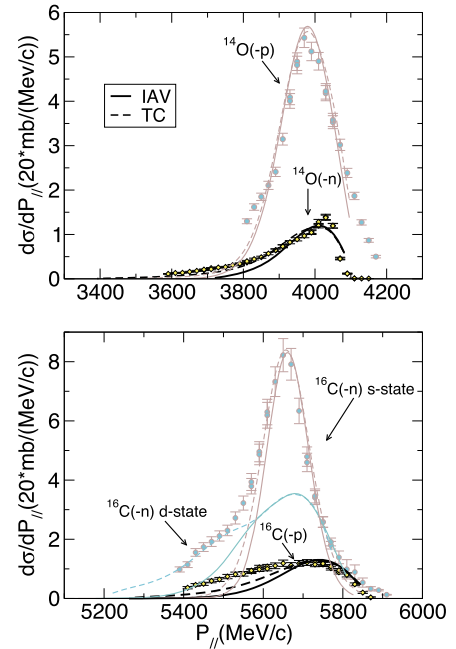
Table 1 contains the total cross section values and the information on the initial single particle states. The nucleon wave functions were calculated by fitting the depth of a Woods-Saxon potential to the nucleon experimental separation energy. Radius parameter and diffuseness were 1.25 fm and 0.7 fm respectively in all but the  $^{14}\text{O}$  neutron case in which we had  $r_0=1.4$  fm. The latter value was chosen following Ref. [22] and also Ref. [4] where the  $^{14}\text{O}(d,p)^{13}\text{O}$  reaction was studied. The  $R_f$  value given in parentheses in the last column corresponds to the standard  $r_0=1.25$  fm as explained in the following. The first column indicates the reactions studied in this paper, the separation energies and quantum numbers of the initial single particle states and the asymptotic normalization constants used in the TC calculations second to fourth column respectively. Sixth and seventh column contain the single particle total cross sections from the IAV and TC calculations as indicated. The eighth and ninth columns provide the shell model spectroscopic factors and eikonal calculation results presented in Ref. [22]. In the same table, last column we show the so called “reduction factor” [6] that we indicate here as  $R_f = \sigma_{exp}/(C^2S\sigma_{IAV})$



**Fig. 2.** Core momentum distribution calculated by IAV (solid lines) and TC (dashed lines) for the reactions of (a)  ${}^9\text{Be}({}^{14}\text{O}, {}^{13}\text{O})\text{X}$ , (b)  ${}^9\text{Be}({}^{14}\text{O}, {}^{13}\text{N})\text{X}$ , (c)  ${}^9\text{Be}({}^{16}\text{C}, {}^{15}\text{B})\text{X}$ , (d)  ${}^9\text{Be}({}^{16}\text{C}, {}^{15}\text{C})\text{X}$  where  $n$  is in  $s$ -state, and (e)  ${}^9\text{Be}({}^{16}\text{C}, {}^{15}\text{C})\text{X}$  where  $n$  is in  $d$ -state. The distributions are renormalized to fit each other near the peak value.

in order to distinguish it from the strong absorption radius  $R_s$ . Its deviation from one is a measure of possible inaccuracies in both the reaction model and the shell model [1]. The numbers in parentheses next to the eikonal cross section values are the  $R_f = \sigma_{\text{exp}}/\sigma_{\text{eik}}$ . These eikonal values are given just as typical examples of results from other theoretical methods but one has to keep in mind that while we have used in this paper the same initial bound state wave functions as in Ref. [22] the neutron-target and core-target potentials used for the eikonal calculations in Ref. [22] were different. Just for information of the potential readers we provide here the reduction factors obtained from the analysis of other reactions, namely  $R_f=0.5$  from  ${}^{14}\text{O}(p,2p)$  [51],  $R_f=0.6$  from  ${}^{14}\text{O}(p,2p)$  [52],  $R_f=0.73$  from  ${}^{14}\text{O}(d,{}^3\text{He})$  [4] and  $R_f=0.54$  from  ${}^{14}\text{O}(d,t)$  [4].

Finally in Fig. 2 we compare the momentum distributions obtained with the two methods for both EBU and NEB and in Fig. 3 we show the total distributions sum of EBU and NEB, folded with the experimental resolution and compared to the data. The solid lines are the results obtained by IAV model and the dashed lines are calculated by TC method. We notice that the overall agreement is very good, in particular for the EBU part of the calculations the two methods give almost identical results. For the NEB the TC models provide more extended tails. These are present in the data not only those discussed in this paper, but also those in the literature relative to exotic nuclei [23,48] and ordinary nuclei breakup [17]. They appear to be due the higher  $n$ -target partial waves whose contribution is centered at higher energies in the TC results than in the IAV calculations. In Ref. [53] the tails were studied in detail and attributed to kinematical effects in the case of EBU from a proton target. This difference is intriguing because the total cross sections (c.f. Table 1) are very close in the two methods and also the total number of partial waves necessary for convergence is basically the same in the two methods (around twelve partial waves for the present data analysis). One possible explanation is that the TC method is based on the calculation of scattering  $S$ -matrices for both the  $n$ -target interaction and the core-target interaction. In the IAV method, on the other hand, the EBU is calculated also via  $S$ -matrix formalism, while NEB is calculated via a source function which provides a stronger localization of the interaction in the volume region of the target. Thus it might be possible



**Fig. 3.** Experimental and calculated cross section momentum distribution for the breakup reaction of (a)  ${}^{14}\text{O}$  and (b)  ${}^{16}\text{C}$  induced reactions.

that the IAV is more sensitive to volume aspects of the  $n$ -target final state interaction, while the TC is more sensitive to surface properties. For both methods the overall agreement with the data for the total knockout spectrum is excellent in the case of knockout of a weakly bound nucleon ( $-p$  for  ${}^{14}\text{O}$  and  $-n$  for  ${}^{16}\text{C}$ ), while some differences can be seen in the tails of the momentum spectra in the case of the strongly bound nucleons.

### 3. Conclusions and outlook

In this paper we have calculated and compared to data, total knockout cross sections, core parallel momentum distributions and angular momentum distributions due to elastic and non elas-



tic breakup from two theoretical methods: the IAV and the TC, which have been used to enlighten some semiclassical aspects of the dynamics of the knockout reaction at intermediate energies. Also the angular distributions of the cores have been calculated with the IAV method. Both the angular distributions of the core and the relative motion angular momentum distributions of the core vs. residual nucleus show typical semiclassical patterns. These characteristics are intrinsic of the TC model and have been confirmed by the fully quantum mechanical IAV model. Because there is also a quantitative agreement between the two methods and the data, one possible way to proceed in the future would be to implement semiclassical forms of the core-target wave functions in the IAV model. Without going to the extreme eikonal approximation suggested in Ref. [50], an intermediate approach could be to use the WBK approximation for the relative motion distorted waves which would simplify the part relative to the projectile target sum over partial waves, similar to what was done in Ref. [41], but retaining the full QM evaluation of the breakup form factor. This would result in a simplification of the numerical calculations, an increased speed of them and an enlargement in the applicability of the method. In the meantime the TC has been demonstrated to be a valid and accurate alternative.

The interactions used in this work, namely the  $n$ - $^9\text{Be}$  optical potential [45] and the single folding model [46,47] for the core-target optical potential, were tested on the  $n$ - $^9\text{Be}$  and projectile- $^9\text{Be}$  free particle experimental cross sections. The present results confirm that they are accurate also in breakup calculations leading to a good agreement with the experimental momentum distribution asymmetries and absolute cross sections. Finally with the use of standard parameters for the single particle initial state potentials, we have obtained with both methods that the extracted spectroscopic factors have at most a quenching of 35–40% with respect to shell-model spectroscopic factors for the reactions studied in this paper.

Only four reactions from Ref. [22] have been analyzed but as they include neutron and proton breakup from weakly bound and strongly bound states and a smaller ( $^{14}\text{O}$  at 53 A.MeV) and a larger ( $^{16}\text{C}$  at 75 A.MeV) incident energy our conclusions can be considered rather general. Encouraged by this rather satisfactory results we hope to extend our calculations to a number of other knockout reactions such as those of Ref. [2] in order to help understanding the strong differences between experimental and theoretical cross sections discussed there.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.physletb.2020.136032>.

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