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Photon Emission by an Electron in a Constant Background Field Modeling a Lorentz-Noninvariant Vacuum

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We considered the radiative transition of an electron due to its spin-flip in a constant background field of the quasielectric type, which models the Lorentz-noninvariant vacuum. The power and probability of radiation and the degree of its linear polarization are calculated. It is shown that this radiative transition leads to the complete polarization of the initially unpolarized electron beam. The investigated effects increase with increasing electron energy.

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1. Introduction

The Standard Model (SM) received full experimental confirmation after the discovery of the Higgs boson at the Large Hadron Collider in 2012 [1]. However, this theory cannot be considered as a final theory of fundamental interactions of elementary particles because it does not provide solutions to a number of fundamental problems (see, e.g. Ref. [2]), which include, inter alia, the enormous hierarchy of particle masses, baryon asymmetry, dark matter, and dark energy in the Universe. In order to address them, various generalizations of the SM are currently being actively developed [3]. It is supposed, in particular, that the effects of quantum gravitation, which are not described by the SM, become especially significant at ultrahigh energies of the order of the Planck energy ($\sim 10^{19}$ GeV). In the region of comparatively low energies these effects lead to the Lorentz invariance violation (LIV), which can be described in the framework of the so-called Standard Model Extension (SME) [4–6], which is one variant of the effective field theory [7]. The LIV effects are described in the SME framework by adding to the Lagrangian of the SM various combinations of standard fields with free tensor indices, which are convolved with corresponding constant tensors. These tensors are interpreted as constant background fields describing a Lorentz-noninvariant structure of the vacuum.

Some effects in the presence of the background axial-vector field were investigated in a number of works: birefringence of light [5,8]; production of an electron–positron pair by a photon and emission of a photon by an electron and a positron [9,10]; synchrotron radiation of an electron taking into account its anomalous magnetic moment [11]; radiation of a hydrogen-like atom [12]; generation of a vacuum current [13].

In our works we considered the following radiative effects in a background tensor field of the quasimagnetic type: the one-loop mass and vertex (at zero momentum transfer) operators of the electron [14] and emission of a photon by an electron [15–17].

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In the present paper, we study the photon emission of an electron moving in a constant tensor background field of the quasielectric type using the Lagrangian of the form [15–17]:[1]

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{T}}, \tag{1}$$

where $\mathcal{L}_{\text{QED}}$ is the standard Lagrangian of quantum electrodynamics, and

$$\mathcal{L}_{\text{T}} = -\frac{1}{2}\bar{\psi}\sigma^{\mu\nu}H_{\mu\nu}\psi \tag{2}$$

is the Lagrangian of the interaction of the electron-positron field ψ with the constant tensor background field $H^{\mu\nu} = -H^{\nu\mu}$.

For the case of a quasielectric background field, we have

$$H^{\mu\nu}H_{\mu\nu} < 0, \quad H^{\mu\nu}\widetilde{H}_{\mu\nu} = 0,$$

where the dual tensor $\widetilde{H}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}H^{\alpha\beta}/2$ (note that for the quasimagnetic field $H^{\mu\nu}H_{\mu\nu} > 0$).

Then there exists a special reference frame in which only the following components are different from zero (at the corresponding orientation of the coordinate axes):

$$H_{03} = -H_{30} = \widetilde{H}_{12} = -\widetilde{H}_{21} = b, \tag{3}$$

so that the tensor background field in the considered case is equivalent to a 3-vector (we assume $b > 0$)

$$\mathbf{b} = b\mathbf{e}_z,\ b = \left(-H^{\mu\nu}H_{\mu\nu}/2\right)^{1/2}. \tag{4}$$

2. Wave functions of an electron in the quasielectric background field

Using Eqs. (1–3), we obtain the Dirac equation for the wave function of an electron of mass m in the Hamiltonian form:

$$i\frac{\partial\psi}{\partial t} = \widehat{H}\psi, \qquad \widehat{H} = \boldsymbol{\alpha}\cdot\widehat{\mathbf{p}} + m\gamma^0 + ib\gamma^3, \tag{5}$$

where $\widehat{\mathbf{p}} = -i\nabla$.

The solution of Eq. (5) has the form

$$\psi_{\mathbf{p}\zeta}(t,\mathbf{r}) = \frac{1}{\sqrt{V}}u(\mathbf{p},\zeta)\exp(-iEt + i\mathbf{p}\cdot\mathbf{r}), \qquad u(\mathbf{p},\zeta) = A\begin{pmatrix} Be^{-i\phi} \\ i\zeta B \\ i\zeta e^{-i\phi} \\ 1 \end{pmatrix}. \tag{6}$$

Here V is the normalization volume;

$$A = \frac{1}{2}\left(1 - \frac{m}{E}\right)^{1/2}, \quad B = \frac{q}{E-m}, \qquad q = p_\perp - \zeta(b - ip_z); \tag{7}$$

the spin quantum number $\zeta = \pm 1$, the electron energy

$$E = \sqrt{m^2 + |q|^2} = \sqrt{m^2 + p_z^2 + (p_\perp - \zeta b)^2} \tag{8}$$

[1] We use a system of units where $\hbar = c = 1$, $\alpha = e^2/4\pi \simeq 1/137$, a pseudo-Euclidean metric with the signature $(+\,-\,-\,-)$; and the Dirac matrices $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$, $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$.

depends on ζ, and the longitudinal p_z and transverse $p_\perp = \sqrt{p_x^2 + p_y^2}$ components of the momentum $\mathbf{p}$; and the angle ϕ in the expression for the wave function (6) defines the direction of the transverse momentum according to $\mathbf{p}_\perp = (p_x, p_y, 0) = p_\perp(\cos\phi, \sin\phi, 0)$.

The wave function (6) is an eigenstate of the Hamiltonian $\widehat{H}$ (5), the momentum operator $\widehat{\mathbf{p}}$, and the spin operator [18]

$$\widehat{\Pi} = \gamma^5 \gamma^\mu \widetilde{H}_{\mu\nu} p^\nu, \tag{9}$$

where $p^\nu = (E, \mathbf{p})$. Taking into account Eqs. (3) and (4), we represent the operator (9) as follows:

$$\widehat{\Pi} = \mathbf{b} \cdot \boldsymbol{\epsilon} = b\epsilon_z = b\gamma^5(\gamma^1 p_y - \gamma^2 p_x). \tag{10}$$

Here the 3-vector of electric polarization is introduced [19]:

$$\boldsymbol{\epsilon} = \gamma^0(\mathbf{p} \times \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \gamma^5 \boldsymbol{\alpha}. \tag{11}$$

The eigenvalues of the operator (10) are related to the spin number ζ by the relation

$$\widehat{\Pi}\psi_{\mathbf{p}\zeta} = \zeta b p_\perp \psi_{\mathbf{p}\zeta}.$$

3. Radiation power

Using the general formulas of radiation theory [18], we find the angular distribution of the radiation power at the radiative transition of an electron $|i\rangle = |\mathbf{p}, \zeta\rangle \to |f\rangle = |\mathbf{p}', \zeta'\rangle$ with emission of a photon with 4-momentum $k^\mu = (\omega, \mathbf{k})$ and polarization vector $\mathbf{e}_\lambda$:

$$\frac{dP^{(\lambda)}}{d\Omega} = \frac{\alpha}{2\pi} \frac{\omega^2}{1 - \mathbf{n}\cdot\mathbf{v}} \left|\mathbf{e}_\lambda^* \cdot \langle\boldsymbol{\alpha}\rangle\right|^2. \tag{12}$$

Here the frequency ω of the photon emitted in the direction $\mathbf{n} = \mathbf{k}/\omega$ ($|\mathbf{n}| = 1$) is determined by the conservation of energy and momentum:

$$\omega = \frac{2bv_\perp}{1 - \mathbf{n}\cdot\mathbf{v}} \delta_{\zeta',1}\delta_{\zeta,-1}, \tag{13}$$

and, consequently, the radiation is due to the spin-flip of the electron: $\zeta = -1 \to \zeta' = 1$.

Equation (13) is the leading term of the expansion in the parameter b/m, the smallness of which is provided by the present upper limit on the tensor background field strength [6]:

$$b \lesssim 10^{-18}\ \text{eV}. \tag{14}$$

In this approximation, which we will restrict below, $\mathbf{v} = \mathbf{p}/\varepsilon$ is the velocity of a free electron with energy $\varepsilon = E(b=0) = \sqrt{m^2 + \mathbf{p}^2}$ (see Eq. (8)).

The matrix elements in Eq. (12)

$$\langle\boldsymbol{\alpha}\rangle = u^+\left(\mathbf{p}', \zeta'\right)\boldsymbol{\alpha} u\left(\mathbf{p}, \zeta\right). \tag{15}$$

Taking into account Eqs. (6–8) and (13), we obtain the matrix elements (15) and $\langle\alpha_0\rangle$ ($\alpha_0 = I$ is the unit matrix) in the first order of expansion in the background field b:

$$\begin{pmatrix}\langle\alpha_1\rangle\\ \langle\alpha_2\rangle\end{pmatrix} = \frac{1}{2\varepsilon p_\perp^2}\left[-i\omega\varepsilon + (p_\perp - ip_z)\left(\frac{i\omega p_\perp}{\varepsilon - m} - k_z\right)\right]\begin{pmatrix}p_y\\ -p_x\end{pmatrix},$$
$$\begin{pmatrix}\langle\alpha_3\rangle\\ \langle\alpha_0\rangle\end{pmatrix} = \frac{k_x p_y - k_y p_x}{2\varepsilon p_\perp^2}\begin{pmatrix}p_\perp - ip_z\\ im + \dfrac{p_z(p_\perp - ip_z)}{\varepsilon - m}\end{pmatrix}. \tag{16}$$

Note that the matrix elements (16) satisfy the relation

$$\omega\langle\alpha_0\rangle - \mathbf{k}\cdot\langle\boldsymbol{\alpha}\rangle = 0, \tag{17}$$

which follows from the conservation of the electromagnetic current [20].

Summation in Eq. (12) by polarizations according to the known formula [18]

$$\sum_{\lambda} e_{\lambda}^{*i} e_{\lambda}^{k} = \delta^{ik} - n^i n^k \tag{18}$$

yields in view of Eq. (17)

$$\frac{dP}{d\Omega} = \frac{\alpha}{2\pi} \frac{\omega^2}{1 - \mathbf{n}\cdot\mathbf{v}} \left(|\langle \boldsymbol{\alpha} \rangle|^2 - |\langle \alpha_0 \rangle|^2 \right). \tag{19}$$

Taking into account Eqs. (13) and (16), as well as the axial symmetry of the background field (4), which allows us to set $\mathbf{p} = (p_x, 0, p_z)$, we represent Eq. (19) in the form

$$\frac{dP}{d\Omega} = P_0 \frac{(1 - v^2) v_x^2}{(1 - n_x v_x - n_z v_z)^5} \left[(1 - n_z v_z)^2 - n_x^2 v_x^2 - (1 - v^2) n_y^2 \right], \tag{20}$$

where

$$P_0 = \frac{2\alpha}{\pi} \frac{b^4}{m^2}. \tag{21}$$

In the spherical coordinate system with the polar axis Oz, we have in Eq. (20)

$$\begin{gathered} d\Omega = \sin\theta \, d\theta \, d\varphi, \\ n_x = \sin\theta\cos\varphi, \quad n_y = \sin\theta\sin\varphi, \quad n_z = \cos\theta. \end{gathered} \tag{22}$$

To calculate the total radiation power, it is convenient to express the angles in Eq. (22) through the angles (marked by the index 0) in a reference frame moving with the velocity v_z along the Oz-axis (as in synchrotron radiation theory [19]), using an appropriate boost that does not change the original configuration of the background field (4):

$$\begin{gathered} \begin{pmatrix} n_x \\ n_y \end{pmatrix} = \frac{\sqrt{1 - v_z^2}}{1 + v_z n_{0z}} \begin{pmatrix} n_{0x} \\ n_{0y} \end{pmatrix}, \quad n_z = \frac{n_{0z} + v_z}{1 + v_z n_{0z}}, \\ v_x = v_{0x}\sqrt{1 - v_z^2}, \quad d\Omega = \frac{1 - v_z^2}{(1 + v_z n_{0z})^2} d\Omega_0. \end{gathered} \tag{23}$$

Using Eq. (23), we represent Eq. (20) as

$$\frac{dP}{d\Omega_0} = P_0 (1 + v_z n_{0z}) \frac{v_0^2 (1 - v_0^2)}{(1 - v_0 n_{0x})^5} \left[1 - v_0^2 n_{0x}^2 - (1 - v_0^2) n_{0y}^2 \right]. \tag{24}$$

Here $v_0 \equiv v_{0x}$ is invariant with respect to boosts along the Oz-axis:

$$v_0 = \frac{v_\perp}{\sqrt{1 - v_z^2}}, \quad v_\perp = \sqrt{v^2 - v_z^2}. \tag{25}$$

The integration of the angular distribution (24) is simplified if we choose Ox as the polar axis. Then $n_{0x} = \cos\alpha$, $n_{0y} = \sin\alpha\cos\beta$, $n_{0z} = \sin\alpha\sin\beta$, which allows us to perform independent integration over the angles α and β, and as a result we obtain the total radiation power

$$P = \frac{8\pi}{3} P_0 \frac{v_0^2 (1 + 3v_0^2)}{(1 - v_0^2)^2} = \frac{16}{3} \frac{\alpha b^4}{m^2} t^2 (1 + 4t^2), \tag{26}$$

where

$$t = \gamma v_\perp = \frac{p_\perp}{m} \tag{27}$$

with the Lorentz factor $\gamma = \varepsilon/m = 1/\sqrt{1 - v^2}$.

For the unpolarized electron it is necessary to introduce an additional factor $1/2$ in the right-hand side of Eq. (26). Note that the power (26) is Lorentz invariant (see, e.g. Ref. [18]), which explains its dependence only on v_0.

4. Polarization of the radiation

To describe the polarization of radiation, we introduce, as in the theory of synchrotron radiation, the σ- and π-components of the linear polarization (see Ref. [18]):

$$\mathbf{e}_\sigma = \frac{\mathbf{e}_z \times \mathbf{n}}{|\mathbf{e}_z \times \mathbf{n}|} = \frac{-n_y \mathbf{e}_x + n_x \mathbf{e}_y}{\sqrt{1-n_z^2}}, \quad \mathbf{e}_\pi = \mathbf{n} \times \mathbf{e}_\sigma = \frac{\mathbf{e}_z - n_z \mathbf{n}}{\sqrt{1-n_z^2}}. \tag{28}$$

From Eqs. (12), (16), (23), and (28), we find angular distributions of radiation powers of linear polarization components:

$$\begin{pmatrix} dP^{(\sigma)}/d\Omega_0 \\ dP^{(\pi)}/d\Omega_0 \end{pmatrix} = P_0(1+v_z n_{0z}) \frac{v_0^2(1-v_0^2)}{(1-v_0 n_{0x})^5} \frac{1}{1-n_{0z}^2} \begin{pmatrix} n_{0x}^2 \left[1 - v_0^2(1-n_{0z}^2)\right] \\ n_{0y}^2 \left[v_0^2 + (1-v_0^2) n_{0z}^2)\right] \end{pmatrix}. \tag{29}$$

Their sum gives Eq. (24), as it should be.

By integrating the distributions (29) over the angles, we obtain the total radiation powers of the polarization components

$$\begin{pmatrix} P^{(\sigma)} \\ P^{(\pi)} \end{pmatrix} = \frac{\pi}{3} P_0 \frac{v_0^2}{(1-v_0^2)^2} \begin{pmatrix} (6 + 23 v_0^2 - v_0^4) \\ (2 + v_0^2 + v_0^4) \end{pmatrix}, \tag{30}$$

the sum of which coincides with Eq. (26).

It follows from Eq. (30) that the radiation is linearly polarized (the σ-component predominates), and the degree of polarization is

$$\frac{P^{(\sigma)} - P^{(\pi)}}{P} = \frac{2 + 15t^2 + 12t^4}{4(1+t^2)(1+4t^2)}. \tag{31}$$

It increases monotonically with velocity, from $1/2$ at $t \to 0 (v_0 \to 0)$ to $3/4$ at $t \to \infty (v_0 \to 1)$.

5. Probability of radiative transition

The angular probability distribution of radiation is obtained by multiplying the right-hand side of Eq. (24) by $1/\omega$ and taking into account Eqs. (13), (21), (23), and (25):

$$\frac{dw}{d\Omega_0} = \frac{\alpha b^3}{\pi m^2} \sqrt{1-v_z^2} \frac{v_0(1-v_0^2)}{(1-v_0 n_{0x})^4} \left[1 - v_0^2 n_{0x}^2 - (1-v_0^2) n_{0y}^2\right]. \tag{32}$$

Integration of the distribution (32) over the angles gives the total probability of radiation (decay rate)

$$\begin{gathered} w = \sqrt{1-v_z^2} w^{(0)} = w_0 v_\perp (1+3t^2), \\ w^{(0)} = w_0 \frac{v_0(1+2v_0^2)}{1-v_0^2}, \quad w_0 = \frac{8}{3} \frac{\alpha b^3}{m^2}. \end{gathered} \tag{33}$$

Here, the characteristic dependence on the longitudinal velocity v_z expresses the law of probability transformation at boost according to special relativity.

Consider the average radiative energy loss of an electron, i.e. the average photon energy:

$$\langle \omega \rangle = \frac{P}{w}. \tag{34}$$

From Eq. (34), taking into account Eqs. (26) and (33), we find

$$\langle \omega \rangle = 2b\gamma t \frac{1+4t^2}{1+3t^2}, \tag{35}$$

and the maximum photon energy (see Eq. (13))

$$\omega_{\max} = 2b\gamma t(1+v). \tag{36}$$

According to Eq. (34), on average after a time interval

$$\tau_R = 1/w \tag{37}$$

the electron undergoes a spin-flip transition, emitting a photon and entering a radiation-sterile state (see Eqs. (8) and (13)). Therefore, the initially unpolarized electron beam becomes fully polarized, with τ_R being the characteristic polarization time. Using Eqs. (33) and (37), we find the radiative polarization length

$$L_R = v\tau_R = \frac{v}{w_0 v_\perp (1+3t^2)}. \tag{38}$$

6. Discussion

Let us compare the results obtained for the background quasielectric field (type E) with the corresponding results for the quasimagnetic field (type M) [15–17].

In the region of small velocities they are very different: for type E the power and probability of radiation go to zero at $v \to 0$ (see Eqs. (26) and (33)), whereas for type M even a resting electron emits a photon. This difference is explained by the fact that in the case of type M the contribution of the magnetic moment of the electron dominates [14], whereas for type E the radiation is due to the interaction of the induced electric moment (at $v > 0$) with the quasielectric field (see Eqs. (10) and (11)).

At $v = v_\perp \to 1$ (the high-energy region, $\gamma \gg 1$), the results for types E and M practically coincide. This is explained by the fact that at the boost to the electron's rest frame both configurations of the background field become practically indistinguishable from the configuration of the so-called crossed field (cf. Refs. [20,21]). In this frame, the quasielectric $\mathbf{b}_0$ and quasimagnetic $\mathbf{h}_0$ fields are orthogonal to each other and are equal in magnitude, as it follows from the explicit form of the corresponding transformations (see, e.g. Ref. [22]):

$$\mathbf{b}_0 = \gamma\mathbf{b},\ \mathbf{h}_0 = -\gamma\mathbf{v}\times\mathbf{b} \quad \text{for } \mathbf{v}\cdot\mathbf{b} = 0,$$

and

$$\mathbf{h}_0 = \gamma\mathbf{h},\ \mathbf{b}_0 = \gamma\mathbf{v}\times\mathbf{h} \quad \text{for } \mathbf{v}\cdot\mathbf{h} = 0,$$

respectively.

As can be seen from from Eqs. (26), (33), (35), and (38), the effects of Lorentz violation become most significant for a high-energy electron when it moves transversely with respect to the direction of the background field.

However, our the results are valid provided that the electron recoil is small when emitting a photon, i.e.

$$\chi = \frac{\langle\omega\rangle}{\varepsilon} \ll 1. \tag{39}$$

Hence, taking into account Eqs. (35) and (27), in case $v_\perp \to 1$ we obtain the constraint

$$\chi \simeq \frac{\gamma b}{m} \ll 1. \tag{40}$$

In the same case for the radiative polarization length (38) we obtain (in usual units)

$$L_R \simeq \frac{\lambda\!\!\!^{-}_e}{8\alpha}\left(\frac{m}{b}\right)^3 \gamma^{-2}, \tag{41}$$

where $\lambda\!\!\!^{-}_e$ is the Compton wavelength of the electron.

For numerical estimations, we set $b = 5 \times 10^{-18}$ eV (see Eq. (14)) and $\varepsilon = 10^{16}$ GeV (the Grand Unification scale [2,3]). Then from Eqs. (39–41) we obtain

$$\chi \simeq 2 \times 10^{-4}, \quad \langle \omega \rangle = 2 \times 10^{12}\ \text{GeV}, \quad L_R \simeq 2 \times 10^{21}\ \text{cm}.$$

For comparison, the maximum recorded energy of particles in cosmic rays $\simeq 10^{11}$ GeV [23], and the distance from the Sun to the center of the Galaxy $\simeq 2.5 \times 10^{22}$ cm [1].

7. Conclusion

Within the framework of the SME, the emission of a photon by an electron moving in a constant quasielectric background field, which models the Lorentz-noninvariant vacuum, is investigated. This process, due to the flip of the electron spin during the transition to a radiation-stable state, leads to a complete polarization of the initially unpolarized electron beam along the direction of the background field. It is shown that the radiation has a significant linear polarization, the degree of which reaches 75% for high-energy electrons. It is established that the considered radiation effects can become significant in astrophysical conditions for ultrahigh-energy electrons.

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