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A Relativistic Symmetry in Nuclei

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Abstract. We review some of the empirical and theoretical evidence supporting pseudospin symmetry in nuclei as a relativistic symmetry. We review the case that the eigenfunctions of realistic relativistic nuclear mean fields approximately conserve pseudospin symmetry in nuclei. We discuss the implications of pseudospin symmetry for magnetic dipole transitions and Gamow-Teller transitions between states in pseudospin doublets. We explore a more fundamental rationale for pseudospin symmetry in terms of quantum chromodynamics (QCD), the basic theory of the strong interactions. We show that pseudospin symmetry in nuclei implies spin symmetry for an anti-nucleon in a nuclear environment. We also discuss the future and what role pseudospin symmetry may be expected to play in an effective field theory of nucleons.

1. Introduction

George Sudarshan was a young professor at the University of Rochester at the time that I was graduate student. He himself had just received his PhD there a few years earlier. Therefore he would mix easily with us graduate students, discussing physics as well as other subjects, and playing volleyball at picnics. He always challenged us intellectually. I believe that the subject I shall discuss in this paper will pique his interest.

Almost fourty years ago it was observed that certain energy levels of atomic nuclei were almost degenerate in energy [1, 2]. The spherical shell model orbitals that were observed to be quasi-degenerate have non-relativistic quantum numbers (n_r, ℓ, $j = \ell + 1/2$) and ($n_r - 1$, $\ell + 2$, $j' = j + 1 = \ell + 3/2$) where n_r, ℓ, and j are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively. This doublet structure is expressed in terms of a "pseudo" orbital angular momentum $\tilde{\ell} = \ell + 1$, the average of the orbital angular momentum of the two states in doublet, and "pseudo" spin, $\tilde{s} = 1/2$. For example, ($n_r s_{1/2}$, $(n_r - 1)d_{3/2}$) will have $\tilde{\ell} = 1$, ($n_r p_{3/2}$, $(n_r - 1)f_{5/2}$) will have $\tilde{\ell} = 2$, etc. Since $j = \tilde{\ell} - \frac{1}{2}$ and $j' = \tilde{\ell} + \frac{1}{2}$, the energy of the two states in the doublet are then appproximately independent of the orientation of the pseudospin.

These doublets persist for deformed nuclei as well [3]. The axially-symmetric deformed single-particle orbits with non-relativistic asymptotic quantum numbers $[N, n_3, \Lambda]\Omega = \Lambda + 1/2$ and $[N, n_3, \Lambda' = \Lambda + 2]\Omega' = \Lambda + 3/2$ are quasi-degenerate independent of deformation. Here N is the total harmonic oscillator quantum number, n_3 is the number of quanta for oscillations along the symmetry axis, taken to be in the z-direction, Λ and Ω are respectively the components of the orbital and total angular momentum projected along the symmetry axis. In this case, the doublet structure is expressed in terms of a "pseudo" orbital angular momentum projection, $\tilde{\Lambda} = \Lambda + 1$, which is added to a "pseudo" spin projection, $\tilde{\mu} = \pm 1/2$ to yield the above mentioned doublet of states with $\Omega = \tilde{\Lambda} - 1/2$ and $\Omega' = \tilde{\Lambda} + 1/2$. This approximate pseudospin "symmetry" has been used to explain features of deformed nuclei, including superdeformation[4] and identical bands[5, 6, 7, 8] as well.

In Fig 1 the spectrum of the ground state band and the neighboring band in ^{187}Os are plotted. The states of the two bands are almost degenerate in energy differing only by about 9 KeV. The two bands are built on the $[5, 1, 0]\frac{1}{2}$ and $[5, 1, 2]\frac{3}{2}$ deformed orbitals respectively which are pseudo-spin doublets with $\tilde{\Lambda} = 1$ and $\Omega = \tilde{\Lambda} \pm \frac{1}{2}$.

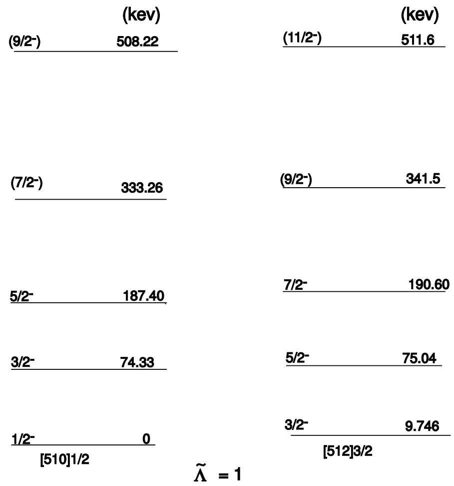

Figure 1. The spectrum of the ground state band (pseudospin unaligned) and the neighboring band (pseudospin aligned) in ^{187}Os which are based on a deformed pseudospin doublet with $\tilde{\Lambda} = 1$[8].

Although there have been attempts to understand the origin of this "symmetry"[9, 10], only recently has it been shown to arise from a relativistic symmetry of the Dirac Hamiltonian[11, 12, 13] which we review in Sec. 2. This relativistic symmetry implies conditions on the Dirac eigenfunctions[14] which we discuss in Sec. 3. These relationships have been studied extensively for spherical nuclei[14, 15, 16, 17, 18, 19] and for deformed nuclei[20, 21, 22, 23] and we shall review them in Sec. 4.

2. The Dirac Hamiltonian and Pseudospin Symmetry

The Dirac Hamiltonian, H, with an external scalar, $V_S(\vec{r})$, and vector, $V_V(\vec{r})$, potentials is given by:

$$H = \vec{\alpha} \cdot \vec{p} + \beta(M + V_S(\vec{r})) + V_V(\vec{r}) \,, \tag{1}$$

where $\vec{\alpha}$, β are the usual Dirac matrices, M is the nucleon mass, and we have set $\hbar = c = 1$. The Dirac Hamiltonian is invariant under a SU(2) algebra for two limits: $V_S(\vec{r}) = V_V(\vec{r}) + C_s$ and $V_S(\vec{r}) = -V_V(\vec{r}) + C_{ps}$ where C_s, C_{ps} are constants [24]. The former limit has application to the spectrum of mesons for which the spin-orbit splitting is small [25] and for the spectrum of an antinucleon in the mean-field of nucleons [11, 26, 27]. The latter limit leads to pseudospin symmetry in nuclei [12]. This symmetry occurs independent of the shape of the nucleus: spherical, axial deformed, or triaxial.

3. Pseudospin Symmetry Generators

The generators for the pseudospin SU(2) algebra, $\tilde{S}_i (i = x, y, z)$, which commute with the Dirac Hamiltonian, $[H_{ps}, \tilde{S}_i] = 0$, for the pseudospin symmetry limit $V_S(\vec{r}) = -V_V(\vec{r}) + C_{ps}$, are given by[13]

$$\tilde{S}_i = \begin{pmatrix} \tilde{s}_i & 0 \\ 0 & s_i \end{pmatrix} = \begin{pmatrix} U_p\, s_i U_p & 0 \\ 0 & s_i \end{pmatrix}, \tag{2}$$

where $s_i = \sigma_i/2$ are the usual spin generators, σ_i the Pauli matrices, and $U_p = \frac{\vec{\sigma}\cdot\vec{p}}{p}$ is the momentum-helicity unitary operator [10]. Thus the operators $\tilde{S}_i$ generate an SU(2) invariant symmetry of H_{ps}. Therefore, each eigenstate of the Dirac Hamiltonian has a partner with the same energy,

$$H_{ps}\Phi^{ps}_{\tilde{k},\tilde{\mu}}(\vec{r}) = E_{\tilde{k}}\Phi^{ps}_{\tilde{k},\tilde{\mu}}(\vec{r}), \tag{3}$$

where $\tilde{k}$ are the other quantum numbers and $\tilde{\mu} = \pm\frac{1}{2}$ is the eigenvalue of $\tilde{S}_z$,

$$\tilde{S}_z\Phi^{ps}_{\tilde{k},\tilde{\mu}}(\vec{r}) = \tilde{\mu}\Phi^{ps}_{\tilde{k},\tilde{\mu}}(\vec{r}). \tag{4}$$

The eigenstates in the doublet will be connected by the generators $\tilde{S}_\pm = \tilde{S}_x \pm i\tilde{S}_y$,

$$\tilde{S}_\pm\Phi^{ps}_{\tilde{k},\tilde{\mu}}(\vec{r}) = \sqrt{\left(\frac{1}{2} \mp \tilde{\mu}\right)\left(\frac{3}{2} \pm \tilde{\mu}\right)}\,\Phi^{ps}_{\tilde{k},\tilde{\mu}\pm 1}(\vec{r}). \tag{5}$$

The fact that Dirac eigenfunctions belong to the spinor representation of the pseudospin SU(2), as given in Eqs. (4)–(5), leads to the conditions on the corresponding Dirac amplitudes that are reviewed in the next subsection.

4. Pseudospin Symmetry in Nuclear Eigenfunctions

From the fact that the lower entry for the pseudospin generators given in Eq. (2) is given by the spin generators only leads to the prediction that the lower amplitudes of the Dirac eigenfunctions for the two states in the doublet are equal[12].

On the other hand from Eqs. (4-5) the upper amplitudes are related by a first order differential equation[14, 19]

$$\left(\frac{\partial}{\partial r} + \frac{\tilde{\ell}+2}{r}\right) g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}+\frac{1}{2}}(r) = \left(\frac{\partial}{\partial r} - \frac{\tilde{\ell}-1}{r}\right) g_{\tilde{n}_r,\tilde{\ell},\tilde{\ell}-\frac{1}{2}}(r). \tag{6}$$

Indeed the lower amplitudes have been shown to be very similar for spherical and deformed nuclei and also the differential relations for the upper amplitudes have been shown to be approximately valid for spherical and deformed nuclei [11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 28]. In both spherical and deformed nuclei investigations show that pseudospin symmetry conservation increases as the binding energy decreases and as the pseudo-orbital angular momentum decreases.

5. Magnetic Transitions and Pseudospin Symmetry

Magnetic dipole transitions between pseudospin doublets are forbidden non-relativistically because the states differ by two units of angular momentum. However, they are not forbidden relativistically which means that they are proportional to the lower component of the Dirac eigenfunction. In addition, pseudospin symmetry imposes a condition between the magnetic moments of the states and the magnetic dipole transtion between them which follows from the fact that the radial amplitudes of the lower components of the two states in a pseudospin doublet are equal. Therefore the magnetic dipole transition between the two states in the doublet can be predicted if the magnetic moments of the states are known [11, 29].

The nuclei ${}^{39}_{19}\mathrm{K}_{20}$ and ${}^{39}_{20}\mathrm{Ca}_{19}$ are mirror nuclei. The ground state and first excited state of ${}^{39}_{19}\mathrm{K}_{20}$ are interpreted as a $0d_{3/2}$ and $1s_{1/2}$ proton hole respectively, while the ground state and first excited state of ${}^{39}_{20}\mathrm{Ca}_{19}$ are interpreted as a $0d_{3/2}$ and $1s_{1/2}$ neutron hole respectively. These states are members of the $\tilde{n}_r = 1, \tilde{\ell} = 1$ pseudospin doublet. The M1 transitions between these two states in both of these nuclei have been measured, although they are forbidden in a non-relativistic single-nucleon model, and are indeed small [30, 31]. Using the magnetic moment of ^{39}Ca, $\mu = 1.02168(12)\mu_0$,[32] a transition rate is calculated which is only about 37% larger than the measured transition as seen in Table 1. However, the two states in the doublet are not pure single-particle states. A modification of these relations has been derived which take into account the fact that these states are not pure single particle states [11, 33]. The modified relations give a transtion rate that agrees with the measured value to within experimental error as seen in Table 1.

Table 1. B(M1) rates for ${}^{39}Ca$

	$B(M1)$
Pseudospin symmetry prediction	0.0166
Pseudospin symmetry and s.p. corrections prediction	0.0121
EXP Ref. 33	0.0121 (14)

Pseudospin symmetry also predicts Gamow-Teller transitions between doublets which are also forbidden non-relativistically. Using reasonable effective coupling constants the Gamow-Teller transition ${}^{39}Ca \rightarrow {}^{39}K$ can be reproduced[11, 29] within the experimental error.

In the non-relativistic shell model an effective tensor term $g_{\text{eff}}[Y_2\sigma]^{(1)}$ is added to the magnetic dipole operator and the Gamow-Teller operator to produce a transition, where g_{eff} is a calculated effective coupling constant. However, the magnetic dipole transition calculated between the same states is an order of magnitude lower than the experimental transition[31] although the calculated Gamow-Teller agrees with the experimental value within the limits of experimental and theoretical uncertainty. This inconsistency has been a puzzle for the non-relativistic shell model. On the other hand, as we just have seen, the relativistic single-nucleon model gives a consistent description of both of these transitions.

A global prediction of magnetic dipole transitions throughout the periodic table has had reasonable success as well [11, 33]. Pseudospin symmetry can also be used to relate quadrupole transitions between multiplets [11].

6. QCD Sum Rules

QCD sum rules have been used to show that $V_S \approx -V_V$ in nuclear matter [34]. Since the derivation is too complex to repeat here, we shall only attempt to justify the result [11].

The vector potential will be proportional to the nuclear matter density, which in nuclear matter is uniform, $V_V \approx \rho_N$. The value of ρ_N is taken to be the central matter density of nuclei which has been measured in electron scattering from nuclei.

In order to determine the scalar potential we need to know the nuclear scalar density. There is no direct measurement of the nuclear scalar density, but the scalar density of quarks in a nucleon can be measured experimentally. This scalar density of quarks is given in terms of what is called the sigma term

$$\sigma_N = 2m_q(\langle N|\bar{q}q|N\rangle - \langle 0|\bar{q}q|0\rangle)\,, \tag{7}$$

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where q is quark field operator, $\bar{q} = q^\dagger\gamma_0$, m_q is the quark mass, $|N\rangle$ is the nucleon state, and $|0\rangle$ is the vacuum state. This sigma term can be measured in pion-nucleon scattering [35, 36] and, after a sophisticated analysis, is determined to be $\sigma_N \approx 45 \pm 8$ MeV. Averaging over all the nucleons in nuclear matter and ignoring nuclear interactions,

$$\sigma_N \rho_N = 2m_q(\langle\bar{q}q\rangle_{\rho_N} - \langle\bar{q}q\rangle_{\rm vac}) \,. \tag{8}$$

The scalar density of quarks in nuclear matter relative to the vacuum is then

$$\frac{\langle\bar{q}q\rangle_{\rho_N}}{\langle\bar{q}q\rangle_{\rm vac}} = 1 + \frac{\sigma_N\rho_N}{2\; m_q\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{\sigma_N\rho_N}{m_\pi^2 f_\pi^2} \tag{9}$$

where the last term follows from the Gell-Mann-Oakes-Renner relation,[37]

$$2m_q\langle\bar{q}q\rangle_{\rm vac} = -m_\pi^2 f_\pi^2 \,. \tag{10}$$

Both the nucleon mass and the scalar potential are Lorentz scalars. The effective mass will be proportional to the left-hand side of Eq. (9) with the first term on the right-hand side proportional to the mass of the nucleon and the second proportional to the scalar potential.

The detailed QCD sum rule gives

$$V_S = -\frac{4\pi^2\sigma_N\rho_N}{M^2 m_q} \tag{11}$$

$$V_V = \frac{32\pi^2\rho_N}{M^2} \tag{12}$$

Since all the quantities on the right-hand side of Eq. (12) are positive, the scalar potential is attractive and the vector potential is repulsive, just as one finds in relativistic mean field models. Furthermore the ratio becomes

$$\frac{V_S}{V_V} = -\frac{\sigma_N}{8\; m_q} \approx -1.1 \,, \tag{17}$$

using accepted values of the average quark mass in the proton (≈ 5 MeV) and the value of σ_N (≈ 45 MeV), which is uncannily close to the ratio of the potentials at the center of the nucleus as determined in relativistic mean field models [11] and indicative of pseudospin symmetry. Also the negative sign originates in the vacuum expectation of the quark scalar density given in Eq. (10). These features suggest that perhaps pseudospin has a more fundamental foundation in terms of QCD.

7. Anti-nucleon Spectrum

The potential of the anti-nucleon in the nuclear environment is the charge conjugate of the nucleon potential. Under charge conjugation the scalar potential remains invariant, $\bar{V}_S(\vec{r}\,) = C^\dagger V_S(\vec{r}\,)C = V_S(\vec{r}\,)$, but the vector potential changes sign, $\bar{V}_V(\vec{r}\,) = C^\dagger V_V(\vec{r}\,)C = -V_V(\vec{r}\,)$. Therefore for an anti-nucleon in a nuclear environment $\bar{V}_S(\vec{r}\,) \approx \bar{V}_V(\vec{r}\,)$, and we have approximate spin symmetry [11]. In fact the negative energy solutions to the nucleon mean field do show a strong spin symmetry [38]. However, there are self-consistent effects which mitigate this conclusion [39]. Also the annihilation potential needs to be taken into account to give a reliable prediction of the anti-nucleon spectrum. But, since the annihilation potential exists only for the anti-nucleon mean field potential and not the nucleon mean field potential, the annihilation potential must be equally scalar and vector so that it will vanish under charge conjugation. This means that approximate spin symmetry will remain intact. Indeed, the limited polarized antinucleon scattering data available shows a vanishing small polarization which implies approximate spin symmetry [40].

8. Summary of the Mean Field Results and Future

We have reviewed some of the theoretical and empirical evidence for relativistic pseudospin symmetry in nuclei. Small energy splittings between pseudospin doublets are measured in nuclei and calculated in the relativistic mean field approximation. These same relativistic mean field calculations show that the scalar and vector potentials approximately satisfy the conditions for pseudospin symmetry and their eigenfunctions approximately satisfy the conditions imposed by pseudospin symmetry. Pseudospin symmetry conservation increases as the binding energy decreases and as the pseudo-orbital angular momentum decreases.

Pseudospin symmetry predicts the small magnetic dipole transitions measured between the states in the doublets very well for ^{39}Ca and quite well throughout the nuclear table. The Gamow-Teller transition $^{39}Ca \rightarrow ^{39}K$ is predicted well but empirical tests throughout the nuclei chart have yet to be carried out.

Although not covered in this brief reveiw, expectations are that the pseudospin symmetry will improve for the neutron orbitals in neutron rich nuclei and for the proton orbitals in proton rich nuclei [11]. Such nuclei shall be produced in rare isotope accelerators.

Also not covered is the scattering of even-even nuclei by nucleons. If the spin polorization and spin rotation functions are measured with respect to scattering angle, the amount of pseudospin violation can measured versus the scattering angle. In fact, as the scattering energy increases the pseudospin conservation increases [11, 41, 42, 43].

Pseudospin symmetry and charge conjugation together predict that an anti-nucleon in nuclear matter has spin symmetry. Anti-nucleon scattering from nuclei produces zero polarization and thus spin symmetry is confirmed for the limited data available. Perhaps more data will become available as more intense polarized anti-nucleon beams are produced.

9. Beyond the Mean Field and Future

The fact that nuclei, which are dominantly non-relativistic, have a relativistic symmetry is reminiscent of Alice in Wonderland. In her journey through Wonderland, Alice encounters a cat with a grin which she finds curious. While conversing with the cat, the cat slowly disappears except for its smile. "Well! I've often seen a cat without a grin," thought Alice, "but a grin without a cat! It's the most curious thing I ever saw in all my life!" [44]. Relativistic dynamics in nuclei is like the cat whereas pseudospin symmetry is like the grin and to have one without the other is indeed curious. Even more curiouser is the fact that the conditions for pseudospin symmetry have origins in QCD.

This situation implies that there is a fundamental rationale for pseudopsin symmetry. For that reason we have investigated pseudospin conservation in nucleon-nucleon scattering [11, 45]. We found that pseudospin symmetry is an approximate dynamic symmetry in the isospin one channel and less so in the isospin zero channel. How this translates into pseudospin symmetry conservation in heavy nuclei needs to be explored further.

Effective field theory has been making inroads in deriving the nucleon-nucleon interaction from QCD [46]. This theory gives an expansion in terms of the momentum divided by a QCD scale factor. Different terms will have coefficients which are determined by comparison to experiment. In this expansion the non-relativistic spin generators s_i appear. But we have seen that pseudospin plays a role in nuclei so we would expect that the non-relativistic pseudospin generators $\tilde{s}_i = U_p s_i U_p$ appear as well, where p is the relative momentum between the two particles. To date, the spin dependence between the two particles is expressed in terms of the spin-spin interaction, $s(1) \cdot s(2)$ and the tensor interaction $s(1) \cdot p \; s(2) \cdot p$. In fact we can show that

$$s(1) \cdot s(2) = \tilde{s}(1) \cdot \tilde{s}(2), \qquad (18)$$

which means that the spin and pseudospin interaction are the equivalent, and furthermore,

$$s(1) \cdot p \; s(2) \cdot p = p^2[\tilde{s}(1) \cdot s(2) + s(1) \cdot \tilde{s}(2) + s(1) \cdot s(2) + \tilde{s}(1) \cdot \tilde{s}(2)], \tag{19}$$

which means that the tensor interaction is an interaction between the spin and pseudospin. Thus effective field theory to date is in fact symmetrical between spin and pseudospin, a fact which has not been recognized previously. Higher order terms involving spin and angular momentum should be symmetrical in form between pseudospin and pseudo-orbital angular momentum. That remains to be seen. Of course, this does not mean that the realistic interaction will be symmetrical in the operators and their pseudo operators counterparts. That distinction will be made by the fact that the numerical coefficients in the realistic interaction, which are determined by experimental data, will have different numerical values for the orbital angular momentum terms and the pseudo-orbital angular momentum terms.

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