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Transferring four-dimensional atomic states at one step between separated cavities

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Abstract. This paper is concentrated on one-step transferring four-dimensional atomic states between atoms trapped in distant cavities connected by an optical fiber. Compared with the conventional schemes, one-step transferring four-dimensional atomic states means higher efficiency in preparing quantum network. In addition, we also calculate the effects of photon leakage out of cavity and fiber in this quantum process. The result shows the high fidelity can be reached based on the current experimental conditions.

1. Introduction

Quantum information processing (QIP), with its study work covering so many science fields, is now in the groping and developing phase and has been one of the hottest research areas in the past two decades. Quantum network, as a carrier of transferring quantum information, its fundamental conception is a system, which consists of spatially separated nodes and transfers quantum information between two nodes through a quantum communication channel. Quantum network plays an important role in the new age of QIP. Because the property of diverse QIP systems ranging over distributed quantum computation [1, 2], quantum secret sharing [3], quantum cryptography [4], quantum dense coding [5] and even quantum teleportation [6] depends on the quality and efficiency of transferring quantum information in quantum network to a great extent.

In recent years, many schemes have been presented for transferring quantum states and preparing quantum network based on cavity QED [7-9], trapped ions [10, 11], superconducting circuits [12, 13], nitrogen-vacancy centers [14], linear optical [15, 16] and impurity spins in solids [17] systems etc. Among these ones, the cavity QED is a promising tool for transferring quantum information as it has strong coupling rate between atomic and optical qubits [18, 19]. Based on the assumption of Kimble [20], atom, as a well storing qubit for its long-lived internal states, is trapped in cavity in order to compose a quantum node. And photon, as a good flying qubit, is perfect medium for the transmission of quantum information. And fiber is used as quantum channels linking the neighboring quantum nodes. This quantum network consisting of cavity-fiber-cavity (CFC) system was conceived by Cirac et al. [21] and Pellizzari [22] successively in 1997. Then many important schemes have been presented in recent years for transferring or exchanging quantum information based on the CFC model [23–35]. For example, Yin and Li [23] proposed a proposal for realizing quantum state transfer (QST) and logical gates by trapping multiple two-level atoms in each of the cavities and encoding a qubit in zero- and single-excitation Dicke states of the atoms. Zhou et al. [24] presented a scheme for transferring three-dimensional(3D) quantum states between two distant cavities via adiabatic passage. The advantage of the scheme is the decay of cavities, optical fiber and the atomic spontaneous emission can be ignored for its adiabatic passage along dark states. But it needs to change the laser intensities driving the atom slowly to accomplish the process of transferring information, its efficiency is not good in practical situations. In 2016, Bin Zheng et al.'s [31] proposal is a one-step method to create entanglement between two atomic qubits that are trapped in separate cavities without a direct connection. Additionally, this method allows for the transfer of a quantum state between the cavities. The process involves engineering an effective, asymmetric interaction between the two atomic qubits and an auxiliary atom that is trapped in an intermediate cavity. This interaction is created by virtually manipulating the excited states of the atoms and photons. In 2021 year, He et al.'s [35] study explores the dynamics of non-classical correlations and quantum state transfer in an atom-cavity-fiber system. This system consists of two identical subsystems, each comprising multiple two-level atoms that are confined to two remote optical cavities connected by an optical fiber. Their research reveals that applying parity kicks pulses can significantly enhance both non-classical correlations and the fidelity of quantum state transfer between the atoms.

In this paper, we propose an alternative scheme for generating quantum states transfer (QST) based on cavity-fiber-cavity system. Unlike the conventional QST schemes, our proposal realize one-step transferring four-dimensional atomic information between two quantum nodes. Therefore, our proposal is obviously more efficient for transferring more abundant information compared with the schemes that transfer quantum information of an atom.

The paper is organized as follows. In Sec. 2, we present the model and its theoretical description. In Sec. 3, we show how four-dimensional atomic quantum information can be transferred with this model. In addition, we will add the discussion of effects of photon leakage out of the cavities and fiber. In Sec 4, we give the conclusion.

2. System model

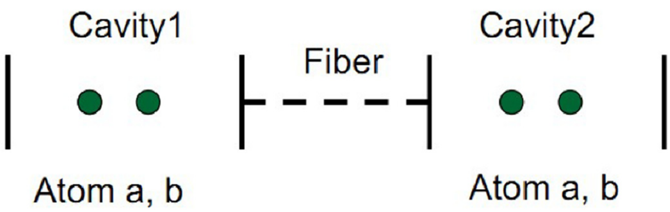

Figure 1. (Color online) Atom-field coupling scheme.

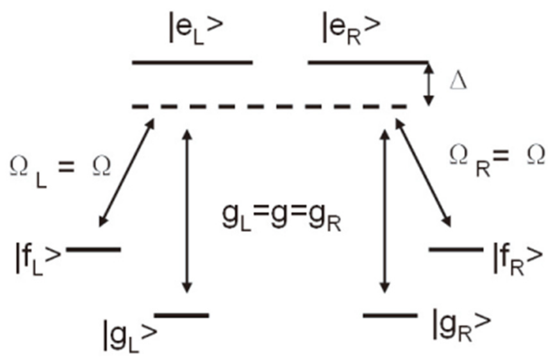

Figure 2. Atomic level configuration for atom m in cavity n.

The framework of our proposal is shown in Figure 1. We trap two atoms in each of the cavities respectively. The atoms are modelled by six-level systems with four ground states $|g_L\rangle_{nm}$, $|f_L\rangle_{nm}$, $|g_R\rangle_{nm}$, $|f_R\rangle_{nm}$ in the same energy level and two excited states $|e_L\rangle_{nm}$, $|e_R\rangle_{nm}$ in the same energy level as shown in Figure 2. The index nm (n = 1, 2 and m = a, b) denotes atom m in cavity n. In the model, the transition $|e_{L(R)}\rangle_{nm} \leftarrow\rightarrow |f_{L(R)}\rangle_{nm}$ is coupled to two classical fields with Rabi frequency $\Omega_{nL(R)}$ respectively. Similar to Ref. [29], the transition $|e_{L(R)}\rangle_{nm} \leftarrow\rightarrow |g_{L(R)}\rangle_{nm}$ is coupled with cavity model with left-circular (right-circular) polarization. The coupling strength for the nth cavity is $g_{nL(R)}$. And $\Delta_{L(R)}$ is defined as energy difference between the atomic transition $|e_{L(R)}\rangle_{nm} \leftarrow\rightarrow |g_{L(R)}\rangle_{nm}$ and the relevant cavity mode. For simplicity, we assume that $\Omega_{nL} = \Omega_{nR} = \Omega$, $g_L = g_R = g$, $\Delta_L = \Delta_R = \Delta$. Considering the large detuning condition, the excited state can be adiabatically eliminated from evolution state space (An electron cannot remain populated in the excited state by absorbing a photon of much lower energy, i.e., law of conservation of energy). Then, under the dipole and rotating wave approximation, the interaction Hamiltonian of the atom−cavity system in the interaction picture can be written as (i = 1 is used throughout this paper) [22, 24, 25]:

$$H_{ac} = \sum_{n=1,2}\sum_{m=a,b}[-\frac{\Omega^2}{\Delta}(|f_L\rangle_{nm}\langle f_L| + |f_R\rangle_{nm}\langle f_R|) - \frac{g^2}{\Delta}(|g_L\rangle_{nm}\langle g_L|a_{nL}^{+}a_{nL} + |g_R\rangle_{nm}\langle g_R|a_{nR}^{+}a_{nR})]$$
$$-\frac{\Omega g}{\Delta}\sum_{n=1,2}\sum_{m=a,b}(|f_L\rangle_{nm}\langle g_L|a_{nL} + |f_R\rangle_{nm}\langle g_R|a_{nR} + H.C.). \tag{1}$$

where $a_{n\,L(R)}^{+}$ and $a_{nL(R)}$ are the creation and annihilation operators for the cavity n with left-circular (right-circular) polarization. For simplicity, we assume the parameters $g = \Omega = 1$, $\Delta = 10$ in this paper.

Then we will consider the interaction Hamiltonian describing the coupling rate between the cavity fields and the fiber modes. It can be broadly described as [22]:

$$H_{cf} = \sum_{n=1}^{\infty}\sum_{k=L,R}\Delta'_{nk}b_{nk}^{+}b_{nk} + v_{nk}\{b_{nk}[a_{1k}^{+} + (-1)^n e^{i\varphi}a_{2k}^{+})] + H.C.\}. \tag{2}$$

where Δ_{nk} is the frequency difference of the nth polarized fiber mode and the cavity mode with the corresponding polarization; b_{nk}^{+} and $a_{1k}^{+}(a_{2k}^{+})$ are the creation operators for the polarized modes of the fiber cavity 1(2); v_{nk} is the corresponding coupling strength, and the phase φ is due to the propagation of the field through the fiber of length L [36]. But in the condition of short fiber limit $2Lv/(2\pi c) \ll 1$, where v is the decay rate of the cavity fields into a continuum of the fiber modes, only the resonant modes b_L^{+} and b_R^{+} of the fiber are excited and coupled to the cavity modes. So the Hamiltonian can be rewritten as: [2]

$$H_{cf} = \sum_{k=L,R} v_k[b_k(a_{1k}^{+} + a_{2k}^{+}) + H.C]. \tag{3}$$

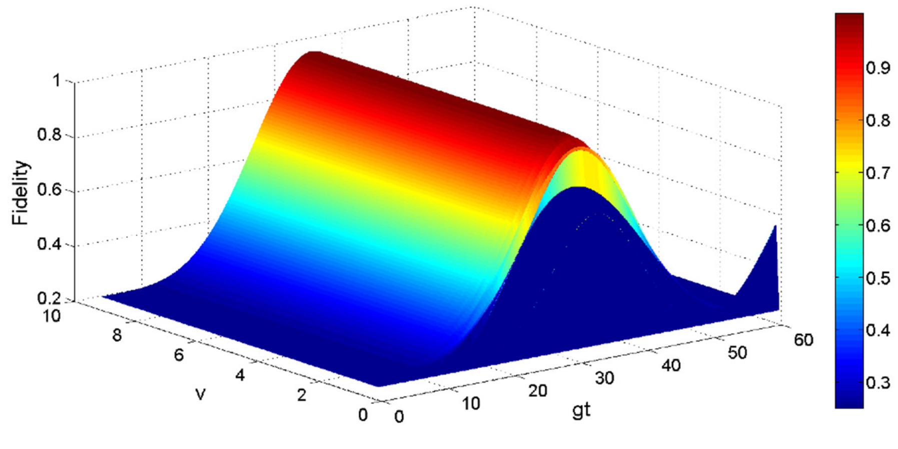

Figure 3. (Color online) Fidelity of the quantum state transfer versus parameter gt and v.

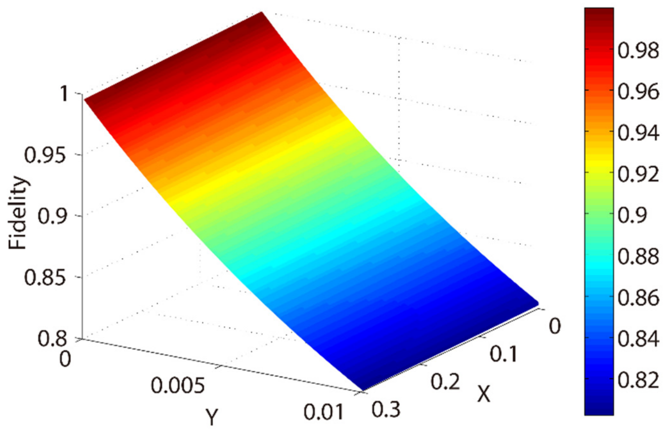

Figure 4. (Color online) Fidelity of the quantum state transfer versus X and Y axes, denoting decay rate τ and κ respectively. The system parameter α=β =V=λ=1/^2, v = 5.8.

where the phase φ has been absorbed into the annihilation and creation operators of the mode of the second cavity field. In this paper, we need $v = v_L = v_R$. The total Hamiltonian for the total system of atom-cavity fiber is

$$H_{tot} = H_{cf} + H_{ac} \tag{4}$$

3. Quantum state transfer

Then we will show how to transfer quantum states by the Hamiltonian Htot. We will describe the physical process qualitatively at first and give the numerical simulation afterwards in ideal and dissipation condition respectively. We assume the quantum information of atom 1a is stored in energy level $|g_L\rangle_{1a}$ and $|f_L\rangle_{1a}$, while the information of the atom 1b is stored in energy level $|g_R\rangle_{1b}$ and $|f_R\rangle_{1b}$. If the atoms 1a, 1b are independent, the atom states in cavity 1 can be set in state $(\alpha|f_L\rangle_{1a}+\beta|g_L\rangle_{1a}) \otimes (\gamma|f_R\rangle_{1b}+\lambda|g_R\rangle_{1b})$ and the atoms in cavity 2 is in the quantum state $|g_L\rangle_{2a}|g_R\rangle_{2b}$ and the photon system is in vacuum state $|0\rangle_{1L}|0\rangle_{1R}|0\rangle_{fL}|0\rangle_{fR}|0\rangle_{2L}|0\rangle_{2R}$ in initial. Here $|0\rangle_{1(2)L(R)}$ means the number of left-circular (right-circular) polarization photons in cavity1(2) is zero, and $|0\rangle_{fL(R)}$ means the number of left-circular (right-circular) polarization photons in fiber is zero. We can find that the atom 1a only has interaction with atom 2a and the atom 1b also only has interaction with atom 2b from the initial states of the whole system. So the whole physical process can be divided into two non-interfering part- s: $|f_L\rangle_{1a}|g_L\rangle_{2a}|000\rangle_L \longrightarrow |g_L\rangle_{1a}|g_L\rangle_{2a}|100\rangle_L \longrightarrow |g_L\rangle_{1a}|g_L\rangle_{2a}|010\rangle_L \longrightarrow |g_L\rangle_{1a}|g_L\rangle_{2a}|001\rangle_L \longrightarrow |g_L\rangle_{1a}|f_L\rangle_{2a}|000\rangle_L$ and $|f_R\rangle_{1b}|g_R\rangle_{2b}|000\rangle_R \longrightarrow |g_R\rangle_{1b}|g_R\rangle_{2b}|100\rangle_R \longrightarrow |g_R\rangle_{1b}|g_R\rangle_{2b}|010\rangle_R \longrightarrow |g_R\rangle_{1b}|g_R\rangle_{2b}|001\rangle_R \longrightarrow |g_R\rangle_{1b}|f_R\rangle_{2b}|000\rangle_R$. Here the |number1, number2, number3$\rangle_{L(R)}$ means the number of left-circular (right-circular) polarization photons in cavity, fiber and cavity2 in turn. In addition, we have assumed that all the parameters in the two parts are equal. And thus the two evolutionary processes are equivalent. Therefore, the whole physical process can be simulated numerically only by calculating one of the two evolutionary processes. We can find the information stored in cavity 1 is transferred into cavity 2 and the system is left with vacuum state finally.

Fidelity of the QST is defined as $F_s = |\langle\psi(t)|\psi_s\rangle|^2$. Where the state $|\psi_s\rangle$ is defined as the final state of the system after ideal QST. In Figure 3, the fidelity is shown for the state transfer $\frac{1}{\sqrt{2}}(|f_L\rangle_{1a}+|g_L\rangle_{1a}) \otimes \frac{1}{\sqrt{2}}(|f_R\rangle_{1b}+|g_R\rangle_{1b}) \otimes |g_L\rangle_{2a}|g_R\rangle_{2b} \longrightarrow |g_L\rangle_{1a}|g_R\rangle_{1b} \otimes \frac{1}{\sqrt{2}}(|f_L\rangle_{2a}+|g_L\rangle_{2a}) \otimes \frac{1}{\sqrt{2}}(|f_R\rangle_{2b}+|g_R\rangle_{2b})$ versus evolution time gt and the coupling rate v. It's shown that the fidelity increases rapidly with the increase of coupling rate v and reaches a maximum at gt = 10π. When v > 1, the fidelity changes slightly, and its maximum is up to 99 .99% in condition of v > 5.8. The result is similar to the paper [29] which transferring 3D quantum state between two spatially atoms based on large detuning condition.

Then we will discuss influence of the decay of the system on the QST. Because of the excited states are adiabatically eliminated, we only take the effect of photon leakage out of the cavities and fiber into account. In Markovian environment, the master equation of motion for the density matrix of the entire system can be written as:

$$\begin{aligned}\dot{\rho} &= -i[H,\rho] \\ &- \sum_{n=1,2}\frac{\kappa_{n,L}}{2}(a_{n,L}^{+}a_{n,L}\rho - 2a_{n,L}\rho a_{n,L}^{+} + \rho a_{n,L}^{+}a_{n,L}) \\ &- \sum_{n=1,2}\frac{\kappa_{n,R}}{2}(a_{n,R}^{+}a_{n,R}\rho - 2a_{n,R}\rho a_{n,R}^{+} + \rho a_{n,R}^{+}a_{n,R}) \\ &- \sum_{k=L,R}\frac{\tau_{k}}{2}(b_{k}^{+}b_{k}\rho - 2b_{k}\rho b_{k}^{+} + \rho b_{k}^{+}b_{k}).\end{aligned} \tag{5}$$

Where $\kappa_{nL(R)}$ and $\tau_{L(R)}$ are decay rate of left-circular (right-circular) polarization photons in the nth cavity and decay rate of left-circular (right-circular) polarization photons in the fiber respectively. For simplicity, we assume $\kappa_{nL(R)} = \kappa$, $\tau_{L(R)} = \tau$ (n= 1, 2). Figure 4 shows fidelity of the QST versus photon leakages out of cavity and fiber. It's obvious that the fidelity is almost unaffected by the variation of decay rate τ but decreasing with the increase of decay rate κ evidently. The fidelity only falls to 99 .93%, when $\tau = 0.3$, $\kappa = 0$. Hence, the demand of the quality of coupling cavity and fiber can be reduced in our scheme. Moreover, the fidelity still can be larger than 96% when $\tau = 0.3$, $\kappa = 0.005$. These numerical results can be qualitatively explained as that the parameters v > g, Ω, photons coupled from one cavity into the fiber would be coupled into the other quickly, so the average population of photons in the fiber are very low in the QST process. Compared to the recent scheme[31], our fidelity has better robustness against effects of spontaneous emission and photon leakage. And furthermore, in the case of large detuning, the longer interaction time makes the fidelity more sensitive to quality factor of the cavity. Considering the actual situation, we can choose the recent experimental parameters about realizing high−Q cavity and strong atom−cavity coupling as the system parameters of our scheme [18] Ω = g = 750MHz, $\tau = \kappa$ = 3.12MHz. Then we have $\tau = \kappa$ = 0.0042 for our scheme (setting g = 1) and the corresponding fidelity is 96 .94%. According to the previous paper [29], our result can meet the demand of realizing QST with high fidelity (F > 95%).

4. Conclusion

In summary, we propose a scheme for the quantum transmission of four-dimensional atomic states between two remote nodes connected by optical fiber firstly, which is almost impossible to be realized in conventional schemes that trapped one atom in each cavity. In condition of large detuning, the atomic excited states are adiabatically eliminated and thus the atomic spontaneous emission can be ignored in this scheme. In addition, analysis shows the fidelity of QST is in a high level with current experiment parameters. Thus, the new scheme may be an effective way to improve the secure and efficiency in preparing distributed and scalable quantum network and other QIP compared with other similar schemes. Furthermore, as an additional function, this system can implement a two-qubit controlled phase gate in particular case. So it has more flexibilities in application.

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