Higgs phase of gravity

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Abstract

Dark energy indicated by the current acceleration of our universe is one of the greatest mysteries in modern cosmology. Although more than 70% of our universe is thought to be dark energy, we do not know what it really is. In this situation, it seems natural to ask whether we can modify gravity at long distances to address the mystery. Ghost condensation is an analog of Higgs mechanism in general relativity, and modifies gravity at long distances without pathologies like ghost instability or strong coupling. In this presentation I discuss gravity and cosmology with ghost condensate.

1 Introduction

Acceleration of the cosmic expansion today is one of the greatest mysteries in both cosmology and fundamental physics. Assuming that Einstein's general relativity is the genuine description of gravity all the way up to cosmological distance and time scales, the so called concordance cosmological model requires that about 70% of our universe should be some sort of energy with negative pressure, called dark energy. However, since the nature of gravity at cosmological scales has never been probed directly, we do not know whether the general relativity is really correct at such infrared (IR) scales. Therefore, it seems natural to consider modification of general relativity in IR as an alternative to dark energy. Dark energy, IR modification of gravity and their combination should be tested and distinguished by future observations and experiments.

2 EFT of ghost condensation

Ghost condensation is an analogue of the Higgs mechanism in general relativity and modifies gravity in IR in a theoretically controllable way [1]. Its basic idea can be pedagogically explained by comparison with the usual Higgs mechanism as in the table shown below. First, the order parameter for ghost condensation is the vacuum expectation value (vev) of the derivative $\partial_\mu\phi$ of a scalar field ϕ, while the order parameter for Higgs mechanism is the vev of a scalar field Φ itself. Second, both have instabilities in their symmetric phases: a tachyonic instability around $\Phi = 0$ for Higgs mechanism and a ghost instability around $\partial_\mu\phi = 0$ for ghost condensation. In both cases, because of the instabilities, the system should deviate from the symmetric phase and the order parameter should obtain a non-vanishing vev. Third, there are stable points where small fluctuations do not contain tachyons nor ghosts. For Higgs mechanism, such a point is characterized by the order parameter satisfying $V' = 0$ and $V'' > 0$. On the other hand, for ghost condensation a stable point is characterized by $P' = 0$ and $P'' > 0$. Fourth, while the usual Higgs mechanism breaks usual gauge symmetry and changes gauge force law, the ghost condensation spontaneously breaks a part of Lorentz symmetry (the time translation symmetry) and changes linearized gravity force law in Minkowski background. Finally, generated corrections to the standard Gauss-law potential is Yukawa-type for the usual Higgs mechanism but oscillating for ghost condensation.

At this point one might wonder if the system really reach a configuration where $P' = 0$ and $P'' > 0$. Actually, it is easy to show that this is the case. For simplicity let us consider a Lagrangian $L_\phi = P(-(\partial\phi)^2)$ in the expanding FRW background with P of the form shown in the upper right part of the table. We assume the shift symmetry, the symmetry under the constant shift $\phi \to \phi + c$ of the scalar field. This symmetry prevents potential terms of ϕ from being generated. The equation of motion for ϕ

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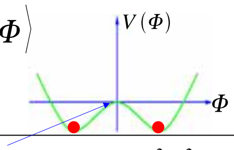
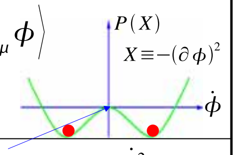
is simply $\partial_t[a^3 P'\dot{\phi}] = 0$, where a is the scale factor of the universe. This means that $a^3 P'\dot{\phi}$ is constant and that

$$P'\dot{\phi} \propto a^{-3} \to 0 \quad (a \to \infty) \tag{1}$$

as the universe expands. We now have two choices: $P' = 0$ or $\dot{\phi} = 0$, namely one of the two bottoms of the function P or the top of the hill between them. Obviously, we cannot take $\dot{\phi} = 0$ since it is a ghosty background and anyway unstable. Thus, we are automatically driven to $P' = 0$ by the expansion of the universe. In this sense the background with $P' = 0$ is an attractor.

	Higgs Mechanism	***Ghost Condensation***
Order Parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-m^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Spontaneous breaking	Gauge symmetry	Lorents symmetry (Time translation)
Modifying	Gauge force	Gravitational force (in flat background)
New potential	Yukawa-type	Oscillating

Having shown that the ghost condensate is an attractor, let us construct a low energy effective field theory around this background. For this purpose the most straightforward approach is to expand a general Lagrangian consistent with the shift symmetry. An alternative, more powerful way is to use the symmetry breaking pattern. In this approach, we actually do not need to specify a concrete way of the spontaneous symmetry breaking.

Here, let us briefly review this approach based on the symmetry breaking pattern. This is more universal and can be applied to any situations as far as the symmetry breaking pattern is the same. We assume that (i) $\langle \partial_\mu \phi \rangle$ is non-vanishing and timelike and that (ii) the background spacetime metric is maximally symmetric, either Minkowski or de Sitter. With the assumption (i), the 4-dimensional Lorentz symmetry is spontaneously broken down and we are left with the 3-dimensional spatial diffeomorphism $\vec{x} \to \vec{x}'(t, \vec{x})$. Our strategy here is to write down the most general action invariant under this residual symmetry. After that, the action for the Nambu-Goldstone (NG) boson π is obtained by undoing the unitary gauge.

For simplicity let us consider the Minkowski background plus perturbation: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The infinitesimal gauge transformation is $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, where ξ^μ is a 4-vector representing the gauge freedom. Under the residual gauge transformation ξ^i $(i = 1, 2, 3)$, the metric perturbation transforms as

$$\delta h_{00} = 0, \quad \delta h_{0i} = \partial_0 \xi_i, \quad \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i. \tag{2}$$

Now let us seek terms invariant under this residual gauge transformation. They must begin at quadratic order since we assumed that the flat spacetime is a solution to the equation of motion. The leading term (without derivatives acted on the metric perturbations) is $\int dx^4 M^4 h_{00}^2$. This is indeed invariant under the residual gauge transformation (2). From this term, we can obtain the corresponding term in the effective action for the NG boson π. Since $h_{00} \to h_{00} + 2\partial_0 \xi_0$ under the broken symmetry transformation ξ^0, by promoting ξ^0 to a physical degree of freedom π, we obtain the term $\int dx^4 M^4 (h_{00} - 2\dot{\pi})^2$. This includes a time kinetic term for π as well as a mixing term. At this point we wonder if we can get the usual space kinetic term $(\vec{\nabla}\pi)^2$ or not. The only possibility would be from $(h_{0i})^2$ since $h_{0i} \to h_{0i} - \partial_i \pi$ under the broken symmetry transformation $\xi^0 = \pi$. However, this term is not invariant under the residual spatial diffeomorphism ξ^i and, thus, cannot enter the effective action. Actually, there are combinations invariant under the spatial diffeomorphism. They are made of the geometrical quantity called extrinsic curvature. The extrinsic curvature K_{ij} in the linear order is $K_{ij} = (\partial_0 h_{ij} - \partial_j h_{0j} - \partial_i h_{0j})/2$ and transforms as a tensor under the spatial diffeomorphism. Thus, $\int dx^4 \tilde{M}^2 K^2$ and $\int dx^4 \bar{M}^2 K^{ij} K_{ij}$ are invariant under spatial diffeomorphism and can be used in the action, where $K = K^i_i$. Since $K_{ij} \to K_{ij} + \partial_i \partial_j \pi$ under the broken symmetry $\xi^0 = \pi$, we obtain $\int dx^4 (\tilde{M}^2 + \bar{M}^2)(\vec{\nabla}^2 \pi)^2$. Combining these terms with the above time kinetic term and properly normalizing the definition of π and M, we obtain

$$L_{eff} = M^4 \left\{ \frac{1}{2}\left(\dot{\pi} - \frac{1}{2}h_{00}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2\pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i\vec{\nabla}^j\pi\right)\left(K_{ij} + \vec{\nabla}_i\vec{\nabla}_j\pi\right) + \cdots \right\}, \quad (3)$$

where α_1 and α_2 are dimensionless constants of order unity. Note that, in deriving the effective action, all we needed was the symmetry breaking pattern. Thus, the low energy EFT of the ghost condensation is universal and should hold as far as the symmetry breaking pattern is the same.

In ghost condensation the linearized gravitational potential is modified at the length scale r_c in the time scale t_c, where r_c and t_c are related to the scale of spontaneous Lorentz breaking M as

$$r_c \simeq \frac{M_{\rm Pl}}{M^2}, \quad t_c \simeq \frac{M_{\rm Pl}^2}{M^3}. \quad (4)$$

Note that r_c and t_c are much longer than $1/M$. The way gravity is modified is peculiar. At the time when a gravitational source is turned on, the potential is exactly the same as that in general relativity. After that, however, the standard form of the potential is modulated with oscillation in space and with exponential growth in time. This is an analogue of Jeans instability, but unlike the usual Jeans instability, it persists in the linearized level even in Minkowski background. The length scale r_c and the time scale t_c above are for the oscillation and the exponential growth, respectively. At the time $\sim t_c$, the modification part of the linear potential will have an appreciable peak only at the distance $\sim r_c$. At larger distances, it will take more time for excitations of the Nambu-Goldstone boson to propagate from the source and to modify the gravitational potential. At shorter distances, the modification is smaller than at the peak position because of the spatial oscillation with the boundary condition at the origin. The behavior explained here applies to Minkowski background, but in ref. [1] the modification of gravity in de Sitter spacetime was also analyzed. It was shown that the growing mode of the linear gravitational potential disappears when the Hubble expansion rate exceeds a critical value $H_c \sim 1/t_c$. Thus, the onset of the IR modification starts at the time when the Hubble expansion rate becomes as low as H_c.

If we take the $M/M_{\rm Pl} \to 0$ limit then the Higgs sector is completely decoupled from the gravity and the matter sectors and, thus, the general relativity is safely recovered. Therefore, cosmological and astrophysical considerations in general do not set a lower bound on the scale M of spontaneous Lorentz breaking, but provide upper bounds on M. If we trusted the linear approximation for all gravitational sources for all times then the requirement $H_c \lesssim H_0$ would give the bound $M \lesssim (M_{\rm Pl}^2 H_0)^{1/3} \simeq 10MeV$, where H_0 is the Hubble parameter today [1]. However, for virtually all interesting gravitational sources the nonlinear dynamics dominates in time scales shorter than the age of the universe. As a result the nonlinear dynamics cuts off the Jeans instability of the linear theory, and allows $M \lesssim 100GeV$ [2].

Note that the ghost condensate provides the second most symmetric class of backgrounds for the system of field theory plus gravity. The most symmetric class is of course maximally symmetric solutions: Minkowski, de Sitter and anti-de Sitter. The ghost condensate minimally breaks the maximal symmetry and introduces only one Nambu-Goldstone boson.

3 Cosmological applications

Now let us discuss some applications to cosmology.

Inflation: We can consider inflation with ghost condensation in the regime of validity of the EFT. In the very early universe where H is higher than the cutoff M, we do not have a good EFT describing the sector of ghost condensation. However, the contribution of this sector to the total energy density ρ_{tot} is naturally expected to be negligible: $\rho_{ghost} \sim M^4 \ll M_p^2 H^2 \simeq \rho_{tot}$. As the Hubble expansion rate decreases, the sector of ghost condensation enters the regime of validity of the EFT and the Hubble friction drives P' to zero. If we take into account quantum fluctuations then P' is not quite zero but is $\sim (H/M)^{5/2} \sim (\delta\rho/\rho)^2 \sim 10^{-10}$ in the end of ghost inflation. In this way, we have a consistent story, starting from the outside the regime of validity of the EFT and dynamically entering the regime of validity. All predictions of the ghost inflation are derived within the validity of the EFT, including the relatively low-H de Sitter phase, the scale invariant spectrum and the large non-Gaussianity [3].

Dark energy: In the usual Higgs mechanism, the cosmological constant (cc) would be negative in the broken phase if it is zero in the symmetric phase. Therefore, it seems difficult to imagine how the Higgs mechanism provides a source of dark energy. On the other hand, the situation is opposite with the ghost condensation: the cc would be positive in the broken phase if it is zero in the symmetric phase. Hence, while this by itself does not solve the cc problem, this can be a source of dark energy.

Dark matter: If we consider a small, positive deviation of P' from zero then the homogeneous part of the energy density is proportional to a^{-3} and behaves like cold dark matter (CDM). Inhomogeneous linear perturbations around the homogeneous deviation also behaves like CDM. However, at this moment it is not clear whether we can replace the CDM with ghost condensate. We need to see if it clumps properly. Ref. [2] can be thought to be a step towards this direction.

Cosmological perturbation: By using the formalism of the cosmological perturbations developed in [4], the theory of ghost condensation can be tested by dynamical information of large scale structure in the universe such as cosmic microwave background anisotropy, weak gravitational lensing and galaxy clustering.

4 UV completion via gauged ghost condensation

To realize the ghost condensation without fine-tuning, we need to spontaneously break the 4-dimensional diffeomorphism invariance times a global shift symmetry down to the 3-dimensional spatial diffeomorphism invariance times an unbroken global shift symmetry. The latter global shift is a combination of the former global shift and the time shift. However, it is generally believed that all symmetries in string theory are gauged. Therefore, it seems more plausible to obtain the ghost condensation as the neutral limit of the gauged version of the ghost condensation, i.e. the gauged ghost condensation [5]. To obtain the gauged ghost condensation from the ghost condensation we replace the global shift symmetry with a minimal gauge symmetry, i.e. $U(1)$ gauge symmetry, so that no global symmetry is needed. The ghost condensation can be obtained from the gauged ghost condensation if we can fine-tune the gauge coupling to a sufficiently small value. Thus, it is likely that, without fine-tuning, the gauged ghost condensation is the simplest Higgs phase of gravity in string theory. An attempt to realize the gauged ghost condensation in string theory has been made in ref. [6].

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