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A brief tour of gauge CPT

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Abstract. We outline the gauge theory of CPT transformations. Gauge CPT provides an alternative to the dark matter hypothesis by unveiling a natural extension of general relativity. Additionally, the new dynamical degrees of freedom revealed by gauge CPT should be essential for any approach to quantum gravity. Even though it may seem absurd to entertain the notion of gauging a discrete symmetry because there are no continuous parameters, we show otherwise via a "back door".

1. Introduction

The outstanding success of the gauge paradigm provides hope that it can again be used to solve some of the current issues in physics such as an alternative to dark matter and supplying missing ingredients required for quantum gravity. Actually, we do not have much of a choice other than the CPT symmetry if we are to stick with experimentally verified symmetries. The use of vierbein and the arbitrary choices of *where* (on the continuous spacetime manifold) one wants to apply the local CPT symmetry transformations allow for the gauging procedure.

The actual physics consists of four parts: motivation, fundamental local CPT transformations, choice of Lagrangian, and experimental/observational explanations and predictions. A good introduction to gauge CPT can be found in "The Baryonic Tully-Fisher Law and the Gauge Theory of CPT Transformations" [1]. A simplified mathematical approach is in "A free-field Lagrangian for a gauge theory of the CPT symmetry" [2].

2. Motivation

There are two components of the motivation to gauge CPT - quantum gravity and astrophysical issues. Briefly, we argue that gauging the CPT symmetry is necessary for any approach to quantum gravity for two reasons: First, PT (together) is a proper Lorentz transformation just like the continuous Lorentz rotations (boosts and spatial rotations), λ, used to obtain the metric spin connection formulation of general relativity (GR). Therefore, it would seem logical to include local PT transformations with local Lorentz rotations in order to complete the gauging of the entire group of proper Lorentz transformations. Inclusion of C is required to preserve the universal symmetry status introduced by the Lorentz rotations. Second, the CPT symmetry is born from the eminently successful *union* of special relativity with quantum theory - a "bridge" between the two theories. Because GR can be obtained by gauging global Lorentz rotations, should not CPT be brought along for the ride, too? Perhaps the new physical dynamical degrees

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of freedom unveiled by gauge CPT can be used to resolve at least some of the renormalization problems encountered when trying to make GR a quantum field theory? This last point is analogous to expanding the early nonrenormalizable weak interaction theories by the renormalizable $SU(2) \otimes U(1)$ electroweak theory.

We now turn to the astrophysical motivation for gauge CPT. The starting point is the flat rotation curves of spiral galaxies and the excessive gravitational lensing produced by galaxies and galactic groups. That particles (matter and photons) are not moving the way they should be *are the observational facts.* Dark matter and modified gravity theories *are not.* Even though dark matter sounds too much like the 19th century aether, it is certainly worth examining. However, the absence of its detection and the well-known theoretical issues such as the core-cusp problem should add to the motivation to examine the logical alternative of modified or extended gravity theories.

Because accelerations are caused by forces, and all known forces can be obtained by gauging select global symmetries, we search for a candidate symmetry to be gauged. That CPT is an experimentally verified symmetry is sufficient reason to attempt gauging it and examine the consequences. Also, because CPT is a universal symmetry containing Lorentz transformations, there is the intuition of obtaining the required mass independent accelerations of galactic rotation curves.

3. Fundamental local CPT transformations

The two fundamental local CPT transformations of spacetime (vierbein) and Dirac wavefunction are perhaps the most difficult part of gauge CPT; different descriptions of the derivation are found in [1], [2], [3], [4]. In the usual gauging procedure we first introduce "arbitrariness," then introduce locality in a closely intertwined procedure. For example, in the $U(1)$ gauging process, we first pick an arbitrary phase ϕ from the continuum of choices in $0 \leq \phi < 2\pi$ and apply it to a wavefunction $\psi \rightarrow e^{i\phi}\psi$. Then, we make it local by replacing ϕ with $\phi(x)$: $\psi \rightarrow e^{i\phi(x)}\psi$. In gauging CPT, the two steps are more distinct and reversed.

We make note that in Minkowski spacetime, the coordinate axes are flipped, and the origin is mapped to itself under the global PT transformation - these two properties are of crucial importance in defining local CPT transformations in curved spacetime! We assume that there is no nontrivial C operation acting on spacetime. In other words, we assume that there is no such thing as antispacetime. A discussion of why this is an important concept to consider and an attempt to find a nontrivial C spacetime operation can be found in [3]. We also note that the Dirac spinor wavefunction, ψ, transforms under Minkowski global CPT as $\psi(x_\mu) \rightarrow i\gamma^5\psi(x_\mu)$.[1] These two points are the essence of defining local CPT transformations.

The first crucially important step in the CPT gauging procedure is to make the CPT transformation local. We start by introducing a vierbein field, ${e_a}^\mu(x)$, where μ are the manifold coordinates and a are local inertial coordinates at the point x. As described in [1], we view ${e_a}^\mu(x)$ as tiny Minkowski coordinate axes centered at x_μ. When the free falling observer at x_μ applies a CPT transformation, she sees that her origin stays put ($x_\mu \rightarrow x_\mu$), her coordinate

[1] We use Bjorken-Drell conventions except for σ^{ab}. In this paper $\sigma^{ab} = \frac{1}{4}\left[\gamma^a, \gamma^b\right]$.

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axes flip ($e_a^{\ \mu}(x) \rightarrow -e_a^{\ \mu}(x)$), and a Dirac wavefunction at her origin picks up a factor of $i\gamma^5$ ($\psi(x_\mu) \rightarrow i\gamma^5\psi(x_\mu)$). This is the definition of local CPT transformations applied at the point x_μ in curved spacetime.

To complete the gauging procedure, we have to introduce arbitrariness. This may seem difficult because there are only two choices at a given point: do nothing, or do the above local CPT transformations. To complete gauging our discrete transformations, we introduce a real, differentiable scalar function $f(x)$ defined everywhere on our manifold. The *choice* of $f(x)$ is arbitrary. Wherever $f(x) \leq 0$, we do nothing to the vierbein and Dirac fields. Wherever $f(x) > 0$, we apply the local CPT transformation to the vierbein and Dirac fields. *We emphasize that $f(x)$ is NOT a physical field!* The use of $f(x)$ is the "back door" to introduce continuous parameters - the manifold coordinates - into the gauging procedure for the discrete CPT symmetry. Just as $\phi(x)$ must disappear in the final field equations of the $U(1)$ gauge theory, $f(x)$ must also disappear from the field equations of gauge CPT. Finally, we also introduce local Lorentz rotations[2] $\lambda(x)$, with the local CPT transformation for two reasons. First, as stated before, we are gauging the entire proper Lorentz group of spacetime transformations (induced by gauging $CPT\lambda$ on the Dirac field). Second, this forces us to introduce the metric spin connection, $\omega_{\mu ab}$. If the transformation of $\omega_{\mu ab}$ can compensate for the variations induced by local $CPT\lambda$, then nothing new is happening, i.e., there is no need to introduce another gauge field.

Putting everything together, we get the two fundamental local $CPT\lambda$ transformations acting at a point x_μ [4]:

$$e_a^{\ \mu} \rightarrow \Theta[-f]\, e_a^{\ \mu} + \Theta[f]\left(-e_b^{\ \mu}\right)\Lambda_a^{\ b}, \tag{1}$$

$$\psi \rightarrow \Theta[-f]\,\psi + \Theta[f]\, i\gamma^5\Lambda_\psi\psi, \tag{2}$$

where $\Lambda_a^{\ b}$ and Λ_ψ are the spacetime vector and Dirac spinor representations of the proper Lorentz rotations, $\lambda(x)$, respectively. We define the Heaviside step function, $\Theta[y]$, as $\Theta[y] = 0$, if $y \leq 0$; and $\Theta[y] = 1$, if $y > 0$. From these two transformations, one easily obtains the transformations of $\overline{\psi}$ and $\omega_{\mu ab}$:

$$\begin{aligned} \overline{\psi} &\rightarrow \Theta[-f]\,\overline{\psi} + \Theta[f]\, i\overline{\psi}\gamma^5\Lambda_{\overline{\psi}}, \\ \omega_{\mu ab} &\rightarrow \Theta[-f]\,\omega_{\mu ab} + \Theta[f]\,\widetilde{\omega}_{\mu ab} + \delta[f]\,\Theta[-f]\,\varsigma_{\mu ab} + \delta[f]\,\Theta[f]\,\widetilde{\varsigma}_{\mu ab}, \end{aligned}$$

where $\delta[\cdots]$ is the Dirac delta functional. Explicit expressions for $\widetilde{\omega}_{\mu ab}$, $\varsigma_{\mu ab}$, and $\widetilde{\varsigma}_{\mu ab}$ are found in the appendix from [4]. We are using the metric spin connection:

$$\omega_{\mu ab} = \frac{1}{2}\left[e_a^{\ \nu}\left(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}\right) - e_b^{\ \nu}\left(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}\right) - e_a^{\ \rho} e_b^{\ \sigma}\left(\partial_\rho e_{r\sigma} - \partial_\sigma e_{r\rho}\right) e^r_{\ \mu}\right].$$

Also, the volume element transforms as $ed^4x \rightarrow (\Theta[f] + \Theta[-f])\, ed^4x$, where $e = \det\left(e^a_{\ \mu}\right)$. Clearly, these transformations are well defined in curved spacetime.

The appearance of the delta functionals, arising from the differentiation of the transformed vierbein and spinors, need to be in a definite integral in order to be defined. The obvious choice is the action integral because of its fundamental role in physics. *Therefore, we use the action integral as the fundamental arena for deriving the necessary mathematics used to gauge the CPT symmetry and introduce the new gauge field.* The derivation of the mathematics used to handle the various ensuing products of the step functions and delta functionals and their differentiation is found in the appendix from [4].

[2] In most of the author's previous work, Λ, denotes Lorentz rotations. To avoid confusion with the hypothetical cosmological constant, we no longer use Λ.

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4. Consequences of the transformations

Do these transformations make sense? In other words, can we recover the global CPT transformations in Minkowski spacetime from these local transformations? Yes, our starting point is the Hermitian Dirac action in Minkowski spacetime, S_D, and the extension of the Dirac action to curved spacetime by use of vierbein and the spin connection ($\partial_\mu\psi \rightarrow \partial_\mu\psi + \frac{1}{2}\omega_{\mu ab}\sigma^{ab}\psi$), S_ω [4]:

$$S_D = \int \left\{ \frac{i}{2} \left[\overline{\psi}\gamma^a e_a{}^\mu \partial_\mu \psi - e_a{}^\mu \partial_\mu \overline{\psi}\gamma^a \psi \right] - m\overline{\psi}\psi \right\} ed^4x, \quad (3)$$

$$S_\omega = \frac{i}{4}\int e_a{}^\mu \omega_{\mu bc} \overline{\psi} \left\{ \gamma^a, \sigma^{bc} \right\} \psi ed^4x. \quad (4)$$

We now apply the transformations to S_D noting that the transformation of the volume element, ed^4x, only introduces removable singularities where $f = 0$ which we can ignore in a definite integral:

$$\begin{aligned} S_D \rightarrow S'_D = & \int \Theta\left[-f\right] \left\{ \frac{i}{2} \left[\overline{\psi}\gamma^a e_a^\mu \partial_\mu \psi - e_a^\mu \partial_\mu \overline{\psi}\gamma^a \psi \right] - m\overline{\psi}\psi \right\} ed^4x \quad (5) \\ & + \int \Theta\left[f\right] \left\{ \frac{i}{2} \left(\partial_\mu \overline{\psi}\Lambda_{\overline{\psi}} + \overline{\psi}\partial_\mu \Lambda_{\overline{\psi}} \right) e_b^\mu \Lambda_a^b \gamma^a \Lambda_\psi \psi \right\} ed^4x \\ & - \int \Theta\left[f\right] \left\{ \frac{i}{2} \overline{\psi}\Lambda_{\overline{\psi}} \gamma^a e_b^\mu \Lambda_a^b \left(\partial_\mu \Lambda_\psi \psi + \Lambda_\psi \partial_\mu \psi \right) \right\} ed^4x \\ & + \int \Theta\left[f\right] m\overline{\psi}\psi ed^4x \\ & + \frac{i}{2}\int \delta\left[f\right] \partial_\mu f \left\{ \Theta\left[-f\right] i e_a^\mu \overline{\psi} \left(\gamma^a \Lambda_\psi + \Lambda_{\overline{\psi}} \gamma^a \right) \gamma^5 \psi \right\} ed^4x \\ & - \frac{i}{2}\int \delta\left[f\right] \partial_\mu f \left\{ \Theta\left[f\right] i e_b^\mu \Lambda_a^b \overline{\psi} \left(\Lambda_{\overline{\psi}} \gamma^a + \gamma^a \Lambda_\psi \right) \gamma^5 \psi \right\} ed^4x. \end{aligned}$$

By setting $f < 0$ everywhere, we recover our original S_D. Setting $f > 0$ and λ constant everywhere gives us the Minkowski Dirac action transformed by global $CPT\lambda$. So, we assert that the local $CPT\lambda$ transformations make sense. We note that S_D is not invariant under global CPT transformations - it changes sign. This has no effect on the Dirac equation but raises some subtle points when using variational calculus with gauge CPT [4].

To complete the introduction of the local $CPT\lambda$ transformations, we need δS_D and δS_ω. This is almost the same as $\delta S_D = S'_D - S_D$, $\delta S_\omega = S'_\omega - S_\omega$ but for the subtle issues regarding the sign change. However, the use of the simplified mathematical approach in [2] avoids the issues and leads to the same δS_D as in [4]. Upon substitution of the local $CPT\lambda$ transformations into S_D and S_ω, along with various manipulations, one obtains the variation of the Dirac action in curved spacetime, $\delta S = \delta S_D + \delta S_\omega$ [4]:

$$\delta S_D = -\frac{1}{4}\int \delta\left[f\right] \partial_\mu f \left\{ e_a^\mu \overline{\psi}\gamma^5 \left(\left[\gamma^a, \Lambda_{\overline{\psi}} \right] + \left[\Lambda_\psi, \gamma^a \right] \right) \psi \right\} ed^4x, \quad (6)$$

and

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$$\begin{aligned}\delta S_\omega \;=\;& \frac{i}{16}\int \delta\left[f\right]\partial_\mu f\overline{\psi}\left\{\gamma^a,\sigma^{bc}\right\}\left[\eta_{ad}e_b^{\ \mu}\left(\Lambda_c^{\ d}+\Lambda^d_{\ c}\right)-\eta_{bd}e_c^{\ \mu}\left(\Lambda^d_{\ a}+\Lambda_a^{\ d}\right)\right.\\ &\left.+\eta_{bd}e_a^{\ \mu}\left(\Lambda^d_{\ c}-\Lambda_c^{\ d}\right)\right]\psi e d^4x\\ &+\frac{i}{16}\int \delta\left[f\right]\partial_\mu f\overline{\psi}\Lambda_{\overline{\psi}}\left\{\gamma^a,\sigma^{bc}\right\}\Lambda_\psi\eta_{bd}\left[e_i^{\ \mu}\Lambda_a^{\ i}\left(\Lambda_c^{\ d}-\Lambda^d_{\ c}\right)\right.\\ &\left.-e_i^{\ \mu}\Lambda_c^{\ i}\left(\delta^d_a+\Lambda_a^{\ d}+\Lambda^d_{\ a}\right)+\Lambda^d_{\ i}e^{i\mu}\left(\eta_{cj}\Lambda_a^{\ j}+\eta_{aj}\Lambda_c^{\ j}\right)\right]\psi e d^4x.\end{aligned} \tag{7}$$

We now show that the introduction of the local CPT transformations unveils new physical phenomena distinct, yet coupled, to general relativity. This discussion is slightly modified and abridged from [4]. If $\delta S_D = 0$ identically, then nothing new is going on other than defining the CPT symmetry locally on a curved manifold. If $\delta S_D \neq 0$ but $\delta S_D + \delta S_\omega = 0$, then the local CPT transformations are just a part of general relativity without any new physics. If $\delta S_D \neq 0$ and $\delta S_D + \delta S_\omega \neq 0$, then general relativity cannot accommodate the local CPT symmetry. We are then *forced* to introduce a new gauge field X_μ in order to arrive at an expanded action invariant under local $CPT\lambda$ transformations. The proof that there exists at least one transformation such that $\delta S_D \neq 0$ and $\delta S_D + \delta S_\omega \neq 0$ is done by explicit construction using a simple choice for local Lorentz rotations corresponding to a velocity boost along the x-axis of Minkowski spacetime [4]. It is straightforward to show that such a λ applied in regions where $f > 0$ satisfies the above criteria that new physical phenomena is unveiled by gauge $CPT\lambda$. Under this transformation one obtains:

$$\delta S_D = -\sinh\frac{\omega}{2}\int \delta\left[f\right]\partial_\mu f\overline{\psi}\left(e_0^{\ \mu}\gamma^5\gamma^1+e_1^{\ \mu}\gamma^5\gamma^0\right)\psi e d^4x, \tag{8}$$

and

$$\delta S_\omega = \frac{3}{64}\sinh\omega\left(\cosh\omega-1\right)\int \delta\left[f\right]\partial_\mu f\overline{\psi}\left(e_2^{\ \mu}\gamma^5\gamma^3-e_3^{\ \mu}\gamma^5\gamma^2\right)\psi e d^4x, \tag{9}$$

where $\omega = \tanh^{-1}\left(\frac{v}{c}\right)$, v and c being the velocities of the boost and light respectively. Because the $\gamma^5\gamma^a$ are linearly independent and ψ is arbitrary, we see that $\delta S_D \neq 0$, $\delta S_\omega \neq 0$, and $\delta S_D + \delta S_\omega \neq 0$ for this choice of transformation. Also see [2] for a simplified proof. The fact that $\delta S_\omega \neq 0$ means that general relativity is not invariant under local $CPT\lambda$ transformations; however, the Einstein-Hilbert action remains invariant but allows for an extension as shown in [2]. Therefore, the new gauge field X_μ must also compensate for the inhomogeneous (i.e. $\delta\left[f\right]$) terms arising from the transformation of $\omega_{\mu ab}$ under local $CPT\lambda$ transformations. Thus, we see from variational arguments that a new gauge field coupled to GR is required in order to accommodate local $CPT\lambda$ transformations.

5. Introduction of the new gauge field

We now introduce the new gauge field, X_μ, subject to only one assumption - minimal coupling. There are three reasons for minimal coupling: simplicity, the author's preference of viewing gauge fields as "compensating" fields, and a quick way to get rid of the $\Theta\left[\pm f\right]\delta\left[f\right]$ terms in the transformation of $\omega_{\mu ab}$.

This last reason motivates the fundamental necessary requirement for any Lagrangian allowed under gauge $CPT\lambda$ transformations: absence of $\delta\left[f\right]$ in the transformed Lagrangian. Because CPT is a discrete symmetry, singularities must occur when CPT is made local. The arena of the action integral determines which singularities are allowed and which are fatal. For

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example, terms with the coefficient $(\Theta[-f] + \Theta[f])$ have removable singularities at $f = 0$, and the coefficient can be simply replaced by 1. Terms containing $\delta[f]$ produce surface integrals at $f = 0$. These are finite but arbitrary because the choice of f is arbitrary; therefore, one cannot uniquely extremize the action in order to obtain field equations. So, any potential Lagrangian densities cannot contain $\delta[f]$ after local $CPT\lambda$ transformations are applied. The following derivation of the structure and transformation of X_μ is taken directly from [4] abridged with slight changes.

The first step in determining the transformation of X_μ is to notice that both δS_D and δS_ω are boundary integrals, i.e. they contain the terms $\delta[f]\,\partial_\mu f$. Also, the transformation of $\omega_{\mu ab}$ contains the terms $\Theta[-f]\,\delta[f]\,\varsigma_{\mu ab}$ and $\Theta[f]\,\delta[f]\,\widetilde{\varsigma}_{\mu ab}$ which need to be cancelled out by the transformation of X_μ under local $CPT\lambda$ transformations. Hence, the transformation of X_μ under local $CPT\lambda$ transformations will be of the form:

$$X_\mu \to \Theta[-f]\,X_\mu + \Theta[f]\,\widetilde{X}_\mu + \Theta[-f]\,\delta[f]\,Y_\mu + \Theta[f]\,\delta[f]\,\widetilde{Y}_\mu. \tag{10}$$

We introduce the covariant derivatives, $D_\mu\psi$, $D_\mu\overline{\psi}$:

$$D_\mu\psi = \partial_\mu\psi + \frac{1}{2}\omega_{\mu ab}\sigma^{ab}\psi + \beta X_\mu\psi, \tag{11}$$

and

$$D_\mu\overline{\psi} = \partial_\mu\overline{\psi} - \frac{1}{2}\omega_{\mu ab}\overline{\psi}\sigma^{ab} + \beta^*\overline{\psi}\gamma^0 X_\mu^\dagger\gamma^0, \tag{12}$$

where β is the coupling constant. The first order theory [3], [5] unveiled only the γ^5 components of X_μ. However, from δS_D and δS_ω, we see that other components are also needed. So, we treat X_μ as a matrix: $X_\mu = x_{\mu n}\Gamma^n$, where the $x_{\mu n}$ are the dynamical components of X_μ; and the Γ^n are the 16 linearly independent matrices I, γ^5, γ^a, $\gamma^5\gamma^a$, and σ^{ab}.

The replacement of $\partial_\mu\psi$ and $\partial_\mu\overline{\psi}$ in S_D by $D_\mu\psi$, $D_\mu\overline{\psi}$ results in an expanded action, $S_{D\omega X}$, which will determine Y_μ and $\widetilde{Y}_\mu$ upon requiring $\delta S_{D\omega X} = 0$. The form of $\widetilde{X}_\mu$, the transformation of X_μ under global $CPT\lambda$ transformations, is determined by requiring the $\Theta[f]$ term of the transformed $S_{D\omega X}$ to change sign, just as in S'_D.

We determine Y_μ and $\widetilde{Y}_\mu$ by requiring the transformation of the expanded action to have no terms containing $\delta[f]$. By simply substituting the transformation equations into $S_{D\omega X}$ and setting the sums of all terms containing $\Theta[-f]\,\delta[f]$ and $\Theta[f]\,\delta[f]$ separately to zero, we can straightforwardly solve for Y_μ and $\widetilde{Y}_\mu$. We obtain:

$$Y_\mu = \beta^{-1}\left[\partial_\mu f\left(I - i\gamma^5\Lambda_\psi\right) - \frac{1}{2}\varsigma_{\mu ab}\sigma^{ab}\right], \tag{13}$$

and

$$\widetilde{Y}_\mu = \beta^{-1}\left[\partial_\mu f\left(-I - i\gamma^5\Lambda_{\overline{\psi}}\right) - \frac{1}{2}\widetilde{\varsigma}_{\mu ab}\sigma^{ab}\right]. \tag{14}$$

We note that $\gamma^5\Lambda_\psi$ and $\gamma^5\Lambda_{\overline{\psi}}$ are linear combinations of I, γ^5, and σ^{ab}; so we see that only eight of the possible 16 Γ^n are needed. We assume that X_μ has only these eight $x_{\mu n}$ dynamical components: $X_\mu = x_{\mu I}I + x_{\mu 5}\gamma^5 + x_{\mu ab}\sigma^{ab}$.

We now turn our attention to finding $\widetilde{X}_\mu$ by again substituting the transformation equations into the expanded action. As mentioned above, we require that the $\Theta[f]$ terms in the expanded action change sign and cancel out the corresponding unvaried terms of the expanded action in the regions where $f > 0$. We obtain from the $\Theta[f]$ terms:

$$\begin{aligned} & e_b^{\ \mu}\Lambda_a^{\ b}\overline{\psi}\Lambda_{\overline{\psi}}\gamma^a\left(\left(\partial_\mu\Lambda_\psi\right)\psi+\Lambda_\psi\partial_\mu\psi+\frac{1}{2}\widetilde{\omega}_{\mu cd}\sigma^{cd}\Lambda_\psi\psi+\beta\gamma^5\widetilde{X}_\mu\gamma^5\Lambda_\psi\psi\right) \\ & -e_b^{\ \mu}\Lambda_a^{\ b}\left(\partial_\mu\overline{\psi}\right)\Lambda_{\overline{\psi}}\gamma^a\Lambda_\psi\psi-e_b^{\ \mu}\Lambda_a^{\ b}\left(\overline{\psi}\partial_\mu\Lambda_{\overline{\psi}}-\frac{1}{2}\widetilde{\omega}_{\mu cd}\overline{\psi}\Lambda_{\overline{\psi}}\sigma^{cd}\right)\gamma^a\Lambda_\psi\psi \\ & -e_b^{\ \mu}\Lambda_a^{\ b}\beta^*\overline{\psi}\gamma^5\Lambda_{\overline{\psi}}\gamma^0\widetilde{X}_\mu^\dagger\gamma^0\gamma^5\gamma^a\Lambda_\psi\psi \\ = \ & e_a^{\ \mu}\overline{\psi}\gamma^a\left(\partial_\mu\psi+\frac{1}{2}\omega_{\mu cd}\sigma^{cd}\psi+\beta X_\mu\psi\right) \\ & -e_a^{\ \mu}\left(\partial_\mu\overline{\psi}-\frac{1}{2}\omega_{\mu cd}\overline{\psi}\sigma^{cd}+\beta^*\overline{\psi}\gamma^0X_\mu^\dagger\gamma^0\right)\gamma^a\psi. \end{aligned} \tag{15}$$

The discussion on use of this equation to find $\widetilde{X}_\mu$ is found in [4]; the result is:

$$\widetilde{X}_\mu = \gamma^5\Lambda_\psi X_\mu\Lambda_{\overline{\psi}}\gamma^5 = \Lambda_\psi X_\mu\Lambda_{\overline{\psi}}. \tag{16}$$

We temporarily retain the γ^5 to emphasize that the transformations are $CPT\lambda$ and not merely λ.

6. The free-field Lagrangian

Now that the transformation of X_μ has been determined, we can construct a Lagrangian density comprised of the expanded Dirac Lagrangian, free-field Lagrangian for X_μ, and a density containing the Einstein-Hilbert term of GR, κR, where $\kappa = (-16\pi G_N)^{-1}$ (this discussion is slightly modified from [1]). The total Lagrangian must satisfy the following requirements [2], [3], [4], [5]:

1) gauge covariance under local CPTλ transformations,

2) absence of any Dirac delta functionals, $\delta[...]$, in the Lagrangian under local CPTλ transformations,

3) terms containing $x_{\mu ab}$ which are not solely constrained to appear within the combination $\left(\frac{1}{2}\omega_{\mu ab}+\beta x_{\mu ab}\right)\sigma^{ab}$, and

4) some components of X_μ appearing in both types of free-field Lagrangians used in GR and the standard model (SM).

The first requirement is obvious. The second prevents pathological variations of the expanded action under local CPTλ transformations as mentioned earlier. The third ensures that $x_{\mu ab}$ has physical significance and is not just a fancy way to ignore the delta functionals appearing in the transformation of $\omega_{\mu ab}$. The fourth reflects that CPT arises from both GR and quantum theory (SM). To this end, we note that the minimal coupling term acting on matter, $\left(\frac{1}{2}\omega_{\mu ab}+\beta x_{\mu ab}\right)\sigma^{ab}\psi$, implies replacing κR by $\kappa R_{X\omega}$, where $\kappa R_{X\omega}$ is the modified Einstein-Hilbert curvature term formed by replacing the metric spin connection $\omega_{\mu ab}$ in R by $\omega_{\mu ab}+2\beta x_{\mu ab}$. We obtain [2] for the Hermitian action, S:

$$\begin{aligned} S \;=\; & \int \left\{ \frac{\mathrm{i}}{2} e_a{}^{\mu} \overline{\psi} \gamma^a \left(\partial_\mu \psi + \frac{1}{2} \omega_{\mu bc} \sigma^{bc} \psi + \beta X_\mu \psi \right) - m \overline{\psi} \psi \right\} |e| \, d^4 x \\ & - \int \left\{ \frac{\mathrm{i}}{2} e_a{}^{\mu} \left(\partial_\mu \overline{\psi} - \frac{1}{2} \omega_{\mu bc} \overline{\psi} \sigma^{bc} + \beta \overline{\psi} \gamma^0 X_\mu^\dagger \gamma^0 \right) \gamma^a \psi \right\} |e| \, d^4 x \\ & + \int \left\{ \frac{1}{4} Tr \left(H_{\mu\nu} H^{\mu\nu\dagger} \right) + \frac{\kappa}{2} \left(R_{X\omega} + R_{X\omega}^\dagger \right) \right\} |e| \, d^4 x, \end{aligned} \tag{17}$$

where

$$H_{\mu\nu} = \frac{\beta}{2} \left(\omega_{\mu ab} \left[\sigma^{ab}, X_\nu \right] - \omega_{\nu ab} \left[\sigma^{ab}, X_\mu \right] \right) + \beta^2 \left[X_\mu, X_\nu \right] + \beta \left(\partial_\mu X_\nu - \partial_\nu X_\mu \right). \tag{18}$$

The term $\frac{1}{4} Tr \left(H_{\mu\nu} H^{\mu\nu\dagger} \right)$ contains both a Yang-Mills term for X_μ - reflecting the quantum (SM) contribution to the origin of the CPT symmetry - and part of the coupling of GR with gauge CPT. We note that a mass term for X_μ, $mTr \left(X_\mu X^{\mu\dagger} \right)$, is not gauge covariant and not allowed [2], [3], [4], [5]. The proof that the Lagrangian density is gauge covariant is found in [2].

The Euler-Lagrange variation of S with respect to $x_{\mu ab}$, $e_a{}^{\mu}$, $\omega_{\mu ab}$, ψ, $x_{\mu I}$, and $x_{\mu 5}$ respectively gives the following field equations [1], [4], [6]:

$$\begin{aligned} & 4\beta D_\nu \left(\partial^\nu x^\mu_{cd} - \partial^\mu x^\nu_{cd} \right) Tr \left[\left(\sigma^{ab} \right)^\dagger \sigma^{cd} \right] + 2\beta \left\{ \left(\omega_{\nu rs} + 2\beta x^*_{\nu rs} \right) \right. \\ & \left. \times \left(\partial^\nu x^\mu_{cd} - \partial^\mu x^\nu_{cd} \right) Tr \left[\left[\sigma^{ab}, \sigma^{rs} \right]^\dagger \sigma^{cd} \right] \right\} \\ & + 2\beta D_\nu \left(2\beta x^\nu_{cd} x^\mu_{rs} + \omega^\nu_{cd} x^\mu_{rs} + \omega^\mu_{rs} x^\nu_{cd} \right) Tr \left[\left(\sigma^{ab} \right)^\dagger \left[\sigma^{cd}, \sigma^{rs} \right] \right] \\ & - \left\{ \beta \left(\omega_{\nu cd} + 2\beta x^*_{\nu cd} \right) \left(2\beta x^\mu_{mk} x^\nu_{rs} + \omega^\mu_{mk} x^\nu_{rs} + \omega^\nu_{rs} x^\mu_{mk} \right) \right. \\ & \left. \times Tr \left[\left[\sigma^{ab}, \sigma^{cd} \right]^\dagger \left[\sigma^{mk}, \sigma^{rs} \right] \right] \right\} \\ & + 8\kappa \eta^{bc} \left(e^{a\mu} e^{n\rho} - e^{n\mu} e^{a\rho} \right) \left(\omega_{\rho cn} + 2\beta x^*_{\rho cn} \right) \\ & + 8\kappa \eta^{ac} \left(e^{b\mu} e^{n\rho} - e^{n\mu} e^{b\rho} \right) \left(\omega_{\rho nc} + 2\beta x^*_{\rho nc} \right) \\ = \; & 4\mathrm{i} \overline{\psi} \sigma^{ab} \gamma^\mu \psi + 8\kappa D_\nu \left(e^{a\nu} e^{b\mu} - e^{a\mu} e^{b\nu} \right). \end{aligned} \tag{19}$$

The modified GR equation,

$$\frac{1}{2} \left\{ \left(R^{\mu\nu}_{X\omega} + R^{\mu\nu\dagger}_{X\omega} \right) - \frac{1}{2} g^{\mu\nu} \left(R_{X\omega} + R^\dagger_{X\omega} \right) \right\} = \frac{1}{2\kappa} T^{\mu\nu}, \tag{20}$$

where

$$\begin{aligned} T^{\mu\nu} \;=\; & \frac{g^{\mu\nu}}{4} Tr \left(H_{\rho\sigma} H^{\rho\sigma\dagger} \right) + \frac{\mathrm{i}}{2} \left[\left(D^\mu \overline{\psi} \right) \gamma^\nu \psi - \overline{\psi} \gamma^\nu D^\mu \psi \right] \\ & + g^{\mu\nu} \left[\frac{\mathrm{i}}{2} \left(\overline{\psi} \gamma^\rho D_\rho \psi - \left(D_\rho \overline{\psi} \right) \gamma^\rho \psi \right) - m \overline{\psi} \psi \right]. \end{aligned} \tag{21}$$

The Palatini variation with respect to $\omega_{\mu ab}$,

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$$
\begin{aligned}
& \kappa\eta^{bc}\left(e^{n\mu}e^{a\rho}-e^{a\mu}e^{n\rho}\right)\left(\omega_{\rho cn}+\beta x^{*}_{\rho cn}+\beta x_{\rho cn}\right) \\
& +\kappa\eta^{ac}\left(e^{n\mu}e^{b\rho}-e^{b\mu}e^{n\rho}\right)\left(\omega_{\rho nc}+\beta x^{*}_{\rho nc}+\beta x_{\rho nc}\right) \\
& +\frac{i}{4}\overline{\psi}\left\{\sigma^{ab},\gamma^{\mu}\right\}\psi+\frac{\beta^{2}}{4}\left\{x^{*}_{\nu rs}\left(-\partial^{\nu}x^{\mu}_{cd}+\partial^{\mu}x^{\nu}_{cd}\right)\right. \\
& \times Tr\left[\left[\sigma^{ab},\sigma^{rs}\right]^{\dagger}\sigma^{cd}\right]+x_{\nu rs}\left(-\partial^{\nu}x^{*\mu}_{cd}+\partial^{\mu}x^{*\nu}_{cd}\right) \\
& \times Tr\left[\left[\sigma^{ab},\sigma^{rs}\right]\left(\sigma^{cd}\right)^{\dagger}\right]+\frac{1}{2}x_{\nu rs}\left(2\beta x^{*\mu}_{mk}x^{*\nu}_{cd}\right. \\
& \left.+\omega^{\mu}_{mk}x^{*\nu}_{cd}+x^{*\mu}_{mk}\omega^{\nu}_{cd}\right)Tr\left[\left[\sigma^{ab},\sigma^{rs}\right]\cdot\left[\sigma^{mk},\sigma^{cd}\right]^{\dagger}\right] \\
& +\frac{1}{2}x^{*}_{\nu rs}\left(2\beta x^{\mu}_{mk}x^{\nu}_{cd}+\omega^{\mu}_{mk}x^{\nu}_{cd}+x^{\mu}_{mk}\omega^{\nu}_{cd}\right) \\
& \left.\times Tr\left[\left[\sigma^{ab},\sigma^{rs}\right]^{\dagger}\cdot\left[\sigma^{mk},\sigma^{cd}\right]\right]\right\} \\
= \; & \kappa D_{\nu}\left(e^{a\mu}e^{b\nu}-e^{a\nu}e^{b\mu}\right),
\end{aligned}
\tag{22}
$$

the Dirac equation,

$$
ie_{a}^{\ \mu}\gamma^{a}\left(\partial_{\mu}\psi+\frac{1}{2}\omega_{\mu bc}\sigma^{bc}\psi+\beta X_{\mu}\psi\right)-m\psi=0,
\tag{23}
$$

and finally

$$
i\overline{\psi}e_{a}^{\ \mu}\gamma^{a}\psi \;=\; 4\beta\left(\partial^{\mu}x^{\nu}_{I}-\partial^{\nu}x^{\mu}_{I}\right)_{;\nu} \text{ and} \tag{24}
$$

$$
i\overline{\psi}e_{a}^{\ \mu}\gamma^{a}\gamma^{5}\psi \;=\; 4\beta\left(\partial^{\mu}x^{\nu}_{5}-\partial^{\nu}x^{\mu}_{5}\right)_{;\nu} \text{ (the chiral terms of } X_{\mu}\text{).} \tag{25}
$$

We note that chiral and gravitational anomalies have not been taken into account.

7. Weak field gauge CPT extension of gravity

The ultimate judge to see if gauge CPT has anything to do with reality is experimental and astrophysical predictions. The arguments leading to the point neutrino emitting source, weak field gauge CPT force law are found in [1], [6]. However, early discussions of experimental predictions appear in [3], [4]. The weak field law is used to explain the Tully-Fisher law as well as four experimental predictions [1], [6]. Due to space limitations, we simply state the law:

$$
a_{X}=k\frac{(I_{\nu})^{\frac{1}{2}}}{r},
\tag{26}
$$

where a_X is the acceleration a point mass feels due to the (anti)neutrino source, I_ν is the total (anti)neutrino luminosity (power) of the point source, r is the distance of the point mass from the (anti)neutrino source, and k is the physical constant ($k = 4.25 \times 10^{-8}\frac{m}{(kg.s)^{\frac{1}{2}}}$) associated with gauge CPT analogous to Newton's gravitational constant, G_N.

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8. Conclusion

We have gauged the CPT symmetry and used it to derive a new gauge field with some components being a natural extension of GR. Further work remains, for example, the physical meaning of the $x_{\mu I} \pm x_{\mu 5}$ terms and the relationship to anomalies. Rigorous point and plane wave neutrino source solutions to the field equations are required to determine the attractive or repulsive nature of the forces as well as strictly proving the inverse square behavior. Of course, another step is to incorporate Gauge CPT with GR to see if a renormalizable quantum field theory can be obtained.

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