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Permanent Magnet Edge-Field Quadrupoles As Compact Focussing Elements For Single-Pass Particle Accelerators *

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Abstract

A previously proposed orthogonally asymmetric arrangement of permanent magnet (PM) material can be made to generate a highly approximate quadrupole field distribution in the vicinity of its symmetry axis. If a small ($\approx$ 1 cm) device gap is permitted, a relatively small amount of commerically available PM material can generate focussing gradients in excess of 100 T/m. In this article some of the general characteristics of this configuration are examined and applications to the design and construction of ultra-long undulators on single-pass machines are considered.

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1. Introduction

The properties of axisymmetric quadrupole field distributions as focussing elements in linear or recirculating machines are well known [1,2]. The current-driven magnetic quadrupole configured on a steel yoke has proven to be particularly important, and is perhaps the preponderant optical element employed in present-day accelerators and circular machines. Nevertheless, there are areas of field design in which disadvantages or design complications can arise from utilizing permeable material (henceforth referred to as iron), and alternative methods based on either pure currents or pure permanent magnets would generally be considered to be preferable, provided comparably useful flux densities could be attained. One area in which such considerations are currently relevant is that of insertion devices, particularly undulators. A broad base of experience exists in the design and construction of devices both with and without iron, and a large amount of study has been done on defining the circumstances under which alternative technological approaches to undulator construction are suitable [3,4,5].

In recent work, a 60 m long pure-PM undulator with a superimposed iron-based quadrupole focussing/defocussing (FODO) lattice has been designed for Free-Electron Laser (FEL) applications at SLAC [6] (see Figure 1). Analytical studies have shown [7] that the quadrupole yoke facets need to be kept significantly far away from the PM lattice surrounding the undulator axis in order to prevent undesirable modifications of the on-axis field amplitude. At the minimum full aperture (12 cm), a significantly large power consumption by the quadrupoles is necessary in order to attain the required focussing gradients (~15 T/m). In view of this, attention has been given to the possible use of PM structures out of which an alternative FODO lattice could be configured.

In this paper the use of a previously-disclosed PM structure [8] as a possible candidate for this purpose is examined. As indicated in Figure 2, it consists of four identical pieces which, if distributed evenly in the azimuthal direction, would generate a quadrupole field essentially equivalent to that generated along the bore of an ordinary iron-based structure. The basic difference lies in the contiguity of the pieces (and their edge-fields) in the upper and lower PM-pairs, which intuitively can be seen to underlie the maximization of the B_y field gradient in the x-direction. It should be evident that attempting a similar rearrangement of the pole facets in an iron-based quadrupole would be ineffectual; it would, in fact, decrease the on-axis field gradients by helping to channel the flux lines directly from one of the contiguous poles into the other. In the sequel, the proposed PM edge-field configuration will be shown to exhibit a number of properties and potential advantages that could in principle make it useful for replacing iron-based quadrupoles for certain applications. In particular, examples will be given of: 1) the possible use of the proposed element for configuring compact focussing lattices along the axes of hybrid (iron/PM) undulators, and 2) some possible ways of improving the economy and performance of the pure-PM undulator structure shown in Fig. 1.

2. Field distributions in the PM edge-field quadrupole

Expressions for the $B_x(0,y,0)$ and $B_y(x,0,0)$ components of the structure shown in Fig. 2 (for $L \to \infty$) are easily derived:

$$B_y(x) = \frac{2B_r}{\pi}\left[2\tan^{-1}\left(\frac{x}{g/2}\right) - 2\tan^{-1}\left(\frac{x}{h + g/2}\right) - \tan^{-1}\left(\frac{x+w}{g/2}\right) + \tan^{-1}\left(\frac{x+w}{h+g/2}\right) - \tan^{-1}\left(\frac{x-w}{g/2}\right) + \tan^{-1}\left(\frac{x-w}{h+g/2}\right)\right]; \quad (1)$$

$$B_x(y) = \frac{B_r}{\pi}\left[\ln\left(\frac{w^2 + (y-g/2)^2}{(y-g/2)^2} \times \frac{(y+g/2)^2}{w^2 + (y+g/2)^2}\right) - \ln\left(\frac{w^2 + (y-(h+g/2))^2}{(y-(h+g/2))^2} \times \frac{(y+(h+g/2)^2}{w^2 + (y+(h+g/2))^2}\right)\right]. \quad (2)$$

In the vicinity of the axis, $(|x|, |y| \ll g/2)$, and for w sufficiently large, the approximate gradients of B_x and B_y can be derived from the above formulas by

$$\frac{\partial B_x(0,y,0)}{\partial y} \approx -\frac{16B_r}{\pi g}\left(\frac{\left(\frac{h}{g}\right)}{1+\left(\frac{2h}{g}\right)}\right)\left\{1 + 12\left(\frac{y}{g}\right)^2\left(\frac{1 + 2\left(\frac{h}{g}\right) + \frac{4}{3}\left(\frac{h}{g}\right)^2}{\left(1 + \left(\frac{2h}{g}\right)\right)^2}\right) + \ldots\right\} \quad (3)$$

and

$$\frac{\partial B_y(x,0,0)}{\partial x} \approx \frac{16B_r}{\pi g}\left(\frac{\left(\frac{h}{g}\right)}{1+\left(\frac{2h}{g}\right)}\right)\left\{1 - \left(\frac{12\left(\frac{x}{g}\right)^2}{1+\left(\frac{2h}{g}\right)}\right)\left[1 + \left(\frac{4\left(\frac{h}{g}\right)^2}{3\left(1+\left(\frac{2h}{g}\right)\right)}\right)\right] + ..\right\} . (4)$$

It is evident that to 0th order the gradients in the above approximation are both constant and identical, as holds for an ordinary quadrupole. From the asymmetry of the PM structure, however, it is clear that this condition cannot persist too far away from the axis. In order to indicate the extent to which the vertical and horizontal gradients remain sufficiently similar away from the axis for quadrupole dimensions of practical interest, the higher-order terms in equ's. (3-4) must be examined for practical values of w. For $w \geq g/2$, numerical studies indicate that the vertical vs. horizontal gradients remain equal to within ≤4% across an axis-centered diameter of ≤ 0.1g. If this region of "gradient homogeneity" is associated with typical sizes or occupation diameters of particle beams in linear or circular machines, it becomes evident from equ's. (3) and (4) that extremely compact and effective focussing elements could be prepared from commercially available PM materials. To indicate the remarkable effectiveness of the edge-field quadrupole, if we choose the parameter set (g=1cm, h=3mm, w=8cm, L=40cm, B_r=1T), we find that focussing gradients in excess of 100 T/m can be generated in the center of the device.

A natural comparison to make at this point is with an orthogonally symmetrized arrangement of the pieces depicted in Fig. 2, i.e., with each rectangular permanent magnet placed at

90° azimuthal increments about the z axis, as the poles of an ordinary quadrupole. For the magnets placed as close as possible to each other to maximize the field gradients, viz., with $g=w$, and assuming $h \ll g$, it is straightforwardly shown that the symmetrized PM quadrupole gradients are approximately given by $8B_r h/\pi g^2$, i.e., about half that of the edge-field quad (cf. eq. (4)).

3. PM edge-field quadrupole focussing lattices

It is of interest to examine the focussing performance of the edge-field quadrupole (quad) vis-a-vis a conventional structure. To do this both individually, and in a FODO lattice arrangement (see Fig. 3), it will be convenient to refer to the thin-lens approximation to the quadrupole and the associated phase advance per unit cell of the betatron oscillation induced by the depicted lattice [2].

Designating the absolute value of either of the two gradients in equ's. (3) and (4) by G[T/m], the focal length f of the edge-field quad in the thin-lens (and near-axis) approximation is given simply by

$$f[m] \approx \frac{3.35E[GeV]}{L[m]G[T/m]} \approx \frac{2E(g^2 + 2gh)}{3B_r Lh} \qquad (5)$$

The phase advance per unit cell is given approximately by

ϕ[rad]≈D/f, with the total phase advance for the entire betatron wavelength defined to be 2π rad.

If we now note that the range of values of the term G in the denominator of in eq. (5) can by simple parameter selection be made 1-2 orders of magnitude greater for the PM edge-field quad than for iron-based quads with restricted apertures, we immediately recognize the corresponding increase in design flexibility and economy implied by the use of the PM option. For example, for the referenced FEL design, we can estimate that the 40 cm long 15 T/m quads in Fig. 1 could in principle be replaced with 6 cm long 100 T/m edge-field quads utilizing a total volume of magnetic material (per quad) of less than 10 cm^3. In designs requiring shorter betatron wavelengths (as would be typical, for example, for shorter-period FELs operated at lower energies [9]), the limitation on the phase advance per unit cell that would result from the restricted focal parameters of an iron quad could also be extended by corresponding factors with the edge-field structure. More generally, for any given iron lattice, it is apparent that an alternative PM structure could easily be configured with either fewer or significantly more unit cells per betatron wavelength, the latter option being especially attractive when the variation of the β parameter along the insertion device length needs to be minimized. For reference, a tabulation of attainable field gradients and focal lengths for a broad range of quad dimensions is given in Table 1.

Referring again to Fig. 3, we can also note a significant mechanical design advantage permitted by the symmetric field distribution about the axis of the edge-field PM quad, namely, the ability to arrange all the lattice sections with all the magnet-pairs in two parallel planes, providing maximum lateral access to the insertion device gap.

4. Selected applications

In this section we examine some alternative focussing lattice configurations for both hybrid and pure-PM undulator structures and discuss some of their implications.

We first consider the notion of imbedding the lattice of Fig. 3 into the gap of a hybrid (i.e., iron/PM) undulator. It is intuitively evident that for a given D the gap fields of the individual quads would be aperiodically modulated by the proximity of the iron pole faces of an undulator with a non-matching period. In order to enhance the regularity of the imbedded lattice fields as much as possible, it thus appears evident that an optimal arrangement is one in which the FODO period 2D is made identical to some integral multiple of the undulator period λ_u (see Fig. 4) and the PM quads are centered over the pole faces. Based on the observations of the prior section, it is evident that the PM quad structure provides us with the freedom of matching D to a wide and continuous range of undulator periods, implying that virtually any two selected structures could be optimized relatively independently to complement one another.

The remaining issue concerns the actual effect of the pole faces on the quad fields. For simplicity, we will assume that the pole faces are flat, horizontal, and sufficiently far from saturation. Taking $W \gg 2w$, $T \gg L$, and $w \gg g_u$, an approximate expression for the modification of the free-space gradients G of the edge-field quad by the first set of images is easily derived, viz.,

$$G \approx \frac{16|B_r|h}{\pi}\left(\frac{1}{(g^2 + 2gh)} + \frac{g_u^2/2 + g^2/8 + hg/4}{(g_u^2 - (g/2)^2)(g_u^2 - (h + (g/2))^2)} + \ldots\right). \qquad (6)$$

Obviously, for undulator structures with wedged or curved pole faces (e.g., Paladin [10]), the derived expressions will in general be different, but if the appropriate symmetry exists in the iron geometry, the loading of the quad fields should perturb their free-space distributions also symmetrically, helping to maintain their usefulness, to first order, as focussing elements. To examine the magnitude of a typical image-loading effect under the above approximations, if we choose (g_u=5cm, g=2cm, h=5mm), eq. (6) predicts a 15% change in the PM quad's free-space field. For a concavely curved pole face, such as on Paladin, it should be possible to make the magnitude of the loading significantly smaller. We note, in passing, the evident importance of being able to attain small dimensions with the edge-field quad in reducing the analytical and mechanical complexity of matching it to a given hybrid device.

We now consider some of the implications of replacing the iron quad lattice in Fig. 1 with PM edge-field optics. In the top two structures of Fig. 5 we show two alternative placements of the quad lattice, one inside, and the other outside the undulator magnet array gap. The latter option, while requiring a substantially larger quantity of material, has the advantage of generating a proportionately larger "homogeneous" quadrupole field, as may be required for machines or transport systems with large dynamic apertures. Based on the above general formulas, we can readily verify that both the attainable gradients and materials volume even for the out-of-gap quad are still extremely favorable for the referenced FEL structure of Fig. 1 ($g_u \approx 1.5$cm; $H \approx 1$cm).

A potentially interesting possibility that could accrue from using either of the two structures in the top of Fig. 5 is the tuning of the undulator on-axis field amplitude B_0 (and hence the K-parameter) by induced image fields. Two possible configurations for doing this are schematized on the bottom of Fig. 5. Again choosing the left one for simplicity and assuming sufficiently wide magnets, we readily derive an approximation to the modified on-axis field amplitude B_0' as a function of the undulator gap g_u, the permanent magnet block height H, and the vertical gap s between the iron plates:

$$B_0' \approx B_0 \left\{1 + e^{-2\pi(s - (g_u + H))/\lambda_u} + \ldots\ldots \right\} . \qquad (7)$$

Assuming s is closed down to its minimum size, g+2H, we see, for example, that using undulator PM blocks with H=1cm and λ_u=8cm could enable the modulation of B_0 by up to approximately 40%. In either tuning configuration, it is evident that the image tuning method will also simultaneously modify the quadrupole lattice fields according to eq. (6). For a particular application, one should of course attempt to optimize the combined tuning of both lattices to facilitate the required undulator performance. Should this strategy prove unrealizable, additional (electrical or mechanical) degrees of freedom will need to be introduced for independently adjusting either the quad or undulator lattice fields. Apart from these observations, it should be apparent that the tuning methods of Fig. 5 could, under suitable conditions, be implemented significantly less expensively than alternative methods involving movement of the undulator jaws. This might, for example, prove to be particularly relevant in the case of ultra-long undulators requiring tuning ranges of an octave or less.

5. Discussion

A number of general issues related to the introduction of the edge-field quad merit attention. In this section, we focus briefly on field quality, radiation damage, field tuning, some additional possible applications, and selected recommendations for further study.

It has generally been accepted that recent technology is able to attain somewhat higher field quality with iron-based structures than with pure permanent magnets. The quality of commercially available PM material, however, is still in a stage of ongoing improvement [11], and it can be argued that with special, still-evolving fabrication techniques based on ultraprecise measurements and sorting, PMs could be made to deliver field quality at least comparable to iron-based designs. Thus, for example, the four monolithic magnets depicted in Fig. 2 could in principle be assembled from sets of much smaller, sorted rectangular pieces. Here it also seems likely that the extremely simple planar structure of the edge-field quad could prove to be more conducive to enhancing the implementation and control of such techniques than the more complicated orthogonally symmetric geometry. Furthermore, it is to be noted that the effectiveness of the PM quad, even with rather low values of B_r, makes possible the use of a wide range of PM materials, further enhancing the likelihood that construction techniques and components for engineering sufficiently high-quality fields can be found.

A well-known limitation of PMs in accelerator applications is the sensitivity of their field quality to radiation damage. Since linear machines, especially at high energies, can be intense generators of noise radiation, it is evident that in many cases the product of the minimum distance of approach of a PM quad to the beam axis times its maximum operating lifetime will be limited. Here we note that the small heights of the quad poles, in particular in our referenced FEL structure, imply exposures not significantly worse that those experienced by the structure's primary (dipole) magnets. In alternative configurations (e.g., see Fig. 5), we have also seen that the edge-field quad poles could be moved to rather large distances from the axis without seriously compromising economy or performance. It is also reasonable to expect that increasingly effective engineering techniques will continue to be developed to help minimize the radiation damage or its effects in a given configuration.

Perhaps one of the more obvious questions with regard to the edge-field structure relates to its potential for tunability. Equ's. (3-4) show that of the three primary parameters determining G, only the gap is easily varied. For completeness, we may also note that an additional tuning parameter (both for varying G and the size of the region of gradient homogeneity) could be introduced by pulling the contiguous magnet poles evenly apart in the horizontal direction. Both this and g are obviously mechanical tuning motions, which could be expected to be adequate in terms of precision and response time for a large number of applications. An alternative method of mechanical tuning, discussed above, is the image-field approach, which could possibly attain faster response times due to the potentially smaller mass of its tuning element (see Fig. 5). If the PM quad fields need only to be varied by small amounts about a constant value, then an arrangement using electrically controlled "cos2θ" currents would, in principle, be able to attain even faster response times than an ordinary (current-controlled) iron quad.

In considering further applications, perhaps the most natural question is whether the proposed PM quad would be usable on recirculating machines (e.g., storage rings). In this regard, it is clear that, apart from the issue of tuning, consideration must be given to the size of the region of gradient homogeneity. Regarding mandatory tuning (as on machines that need to change their lattice parameters during injection or ramping), the possibility of separating the quad lattice into tunable-iron, pure-PM, and tunable-PM components could possibly lead to more economical designs. Ostensibly, the most dramatic economization would be attainable on fixed-energy machines that inject at full energy. Regarding the region of field gradient homogeneity, the dynamic aperture of recirculating machines could set a strong upper limit to the smallest permissible gap size of an edge-field quad. This may be limited even further by the higher-order asymmetries in the horizontal vs. vertical field distributions. We might, however, note that even at substantially large regions of gradient homogeneity and correspondingly larger gaps (see Table 1), the edge-field quad can still be designed to deliver impressive and economical focussing performance. An additional possibility, warranting further study and analysis, for mitigating the effects of field asymmetries could be to employ the alternative lattice configuration using alternately-rotated quads as described in the caption to Fig. 3. Assuming satisfactory resolutions of the above issues, it is evident that an intriguing future direction in the evolution of circular machines, namely the development of ultra-compact, or "mini" rings based on non-superconducting technology could be facilitated.

Some final possible applications of the PM edge-field quad that deserve mention are: 1) the potentially easy insertion of specially-configured sections into storage rings to modulate the lattice functions for special applications ,e.g., to change the momentum compaction factor [12], or to modulate the β function amplitude at desired locations around the ring [13]; and 2) the use of the quad's intense focussing gradients to construct mini-β sections, either on linear or recirculating machines, for accomodating small-gap devices, such as, e.g., micropole undulators [14].

With regard to subsequent research, a more detailed analytical and numerical study of the edge-field quad as an optical element would seem to be warranted. We recall that even for quads with symmetric field distributions, a focussing/defocussing doublet can exhibit a high degree of astigmatism [2]. In this regard, the asymmetric field distributions in the edge-field quad could perhaps be utilized to advantage for these and certain other multi-element configurations, particularly if the "alternately rotated" configuration could be employed. The same applies to extended lattices in machines featuring natural asymmetries between the vertical vs. horizontal emittance functions. In these and perhaps other cases, the asymmetries in the higher-order field components of the edge-field quad could prove to be of practical interest.

6. Acknowledgements

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Table 1

Selected focussing gradients G and focal lengths f attainable in edge-field quadrupoles composed of PM material with B_r=1T.E=5GeV

g[cm]	0.2g*	h[cm]	2w[cm]	L[cm]	G[T/m]	f[m]
0.2	0.04	0.04	0.4	2	364	2.3
0.2	0.04	0.08	0.4	2	566	1.5
0.5	0.10	0.10	1.0	5	146	2.3
0.5	0.10	0.20	1.0	5	226	1.5
1.0	0.20	0.20	2.0	10	73	2.3
1.0	0.20	0.40	2.0	10	113	1.5
2.0	0.40	0.40	4.0	20	36	2.3
2.0	0.40	0.80	4.0	20	56	1.5
4.0	0.80	0.80	8.0	40	18	2.3
4.0	0.80	1.60	8.0	40	28	1.5
8.0	1.60	1.60	16.0	80	9	2.3
8.0	1.60	3.20	16.0	80	14	1.5

* diameter over which vertical vs. horizontal gradients remain approximately equal.

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Figure Captions

Figure 1. Pure PM undulator design with a superimposed quadrupole lattice field generated by iron-based quadrupoles.

Figure 2. Schematic of the permanent magnet edge-field quadrupole. All four pieces have their easy axes parallel to the y axis and are identically magnetized, with the field vectors directed as shown.

Figure 3. Parameters of a focussing/defocussing (FODO) lattice composed of PM edge-field quadrupoles. An alternative arrangement would be to first set up a similar lattice in which the segments in each section were identically oriented, and then rotate every second quad by 90° about the z-axis.

Figure 4. Arrangement of an edge-field quadrupole FODO lattice in the gap of an insertion device with permeable poles. By centering the quads over the iron pole faces and keeping the same periodicity as the undulator structure (or some multiple of it), virtually identical modulation of each quad's field by the induced image fields is ensured.

Figure 5. Front views of pure-PM undulator structures. The large rectangles represent the undulators' primary dipole

arrays. In the upper two figures, alternative placements of an edge-field quadrupole's pole pieces in the vicinity of the primary dipole arrays is shown. In the bottom left figure, a method for tuning the on-axis field of a pure-PM undulator with a superimposed pure-PM quadrupole lattice is shown. By changing the gap height s between the iron plates, the on-axis field is modified by the equivalent image charges induced in the plates. An alternative realization of this method is schematized in the bottom right figure.

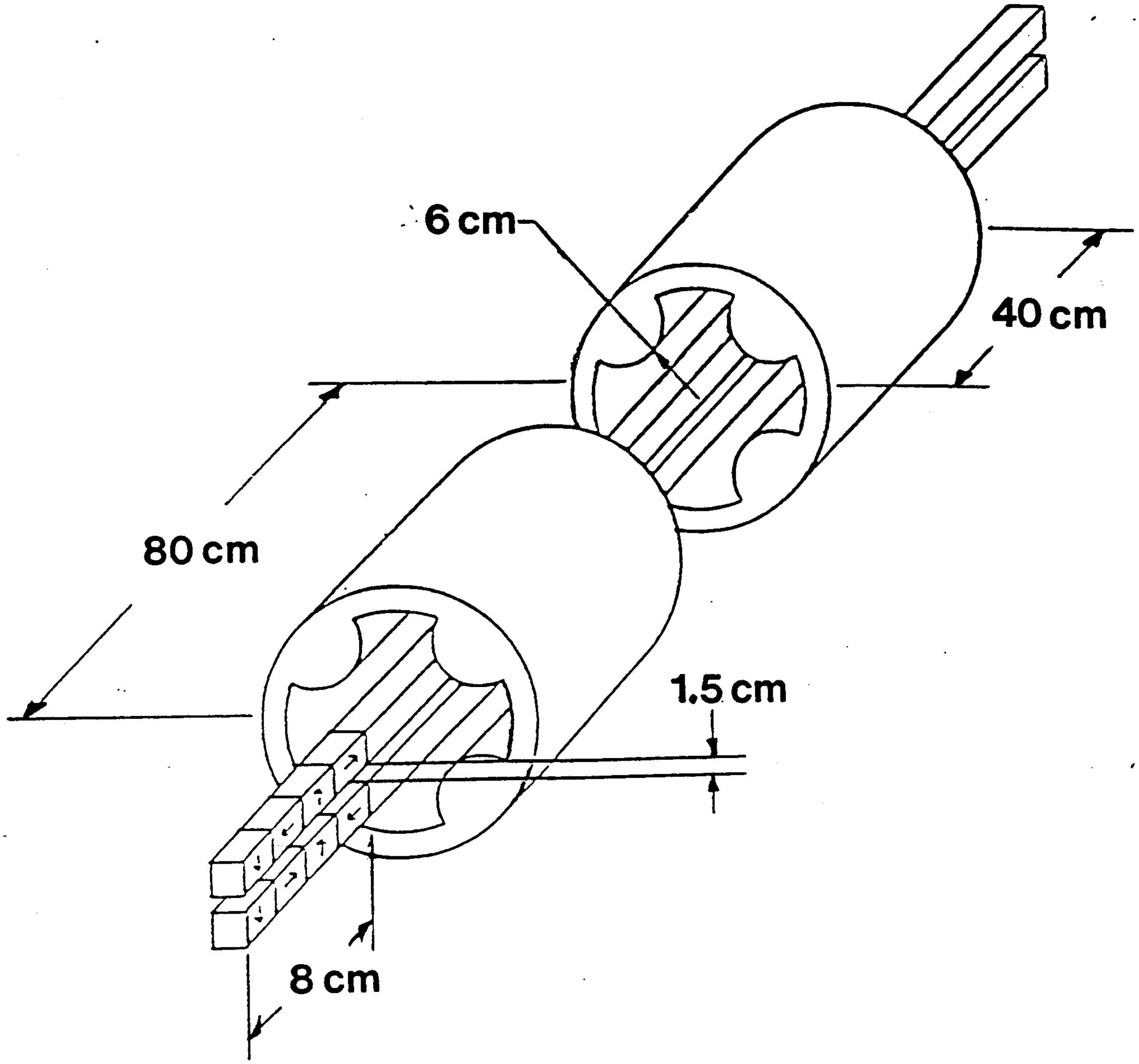

Fig. 1

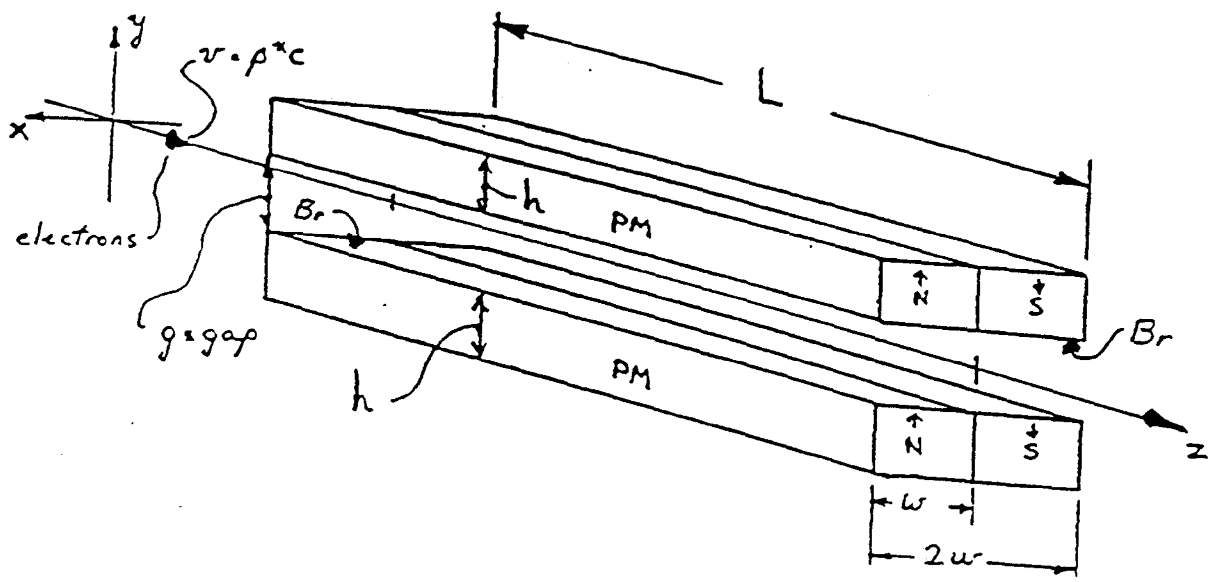

Fig. 2

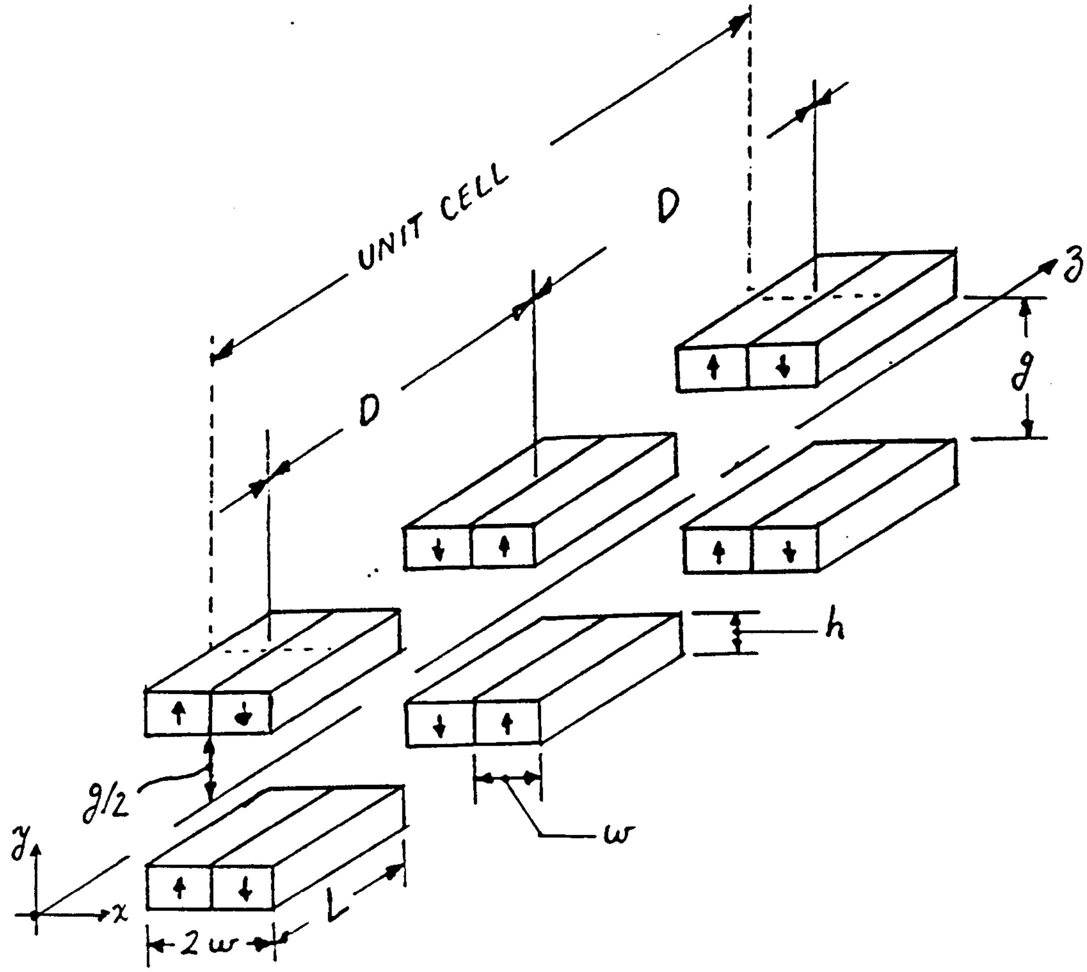

Fig. 3

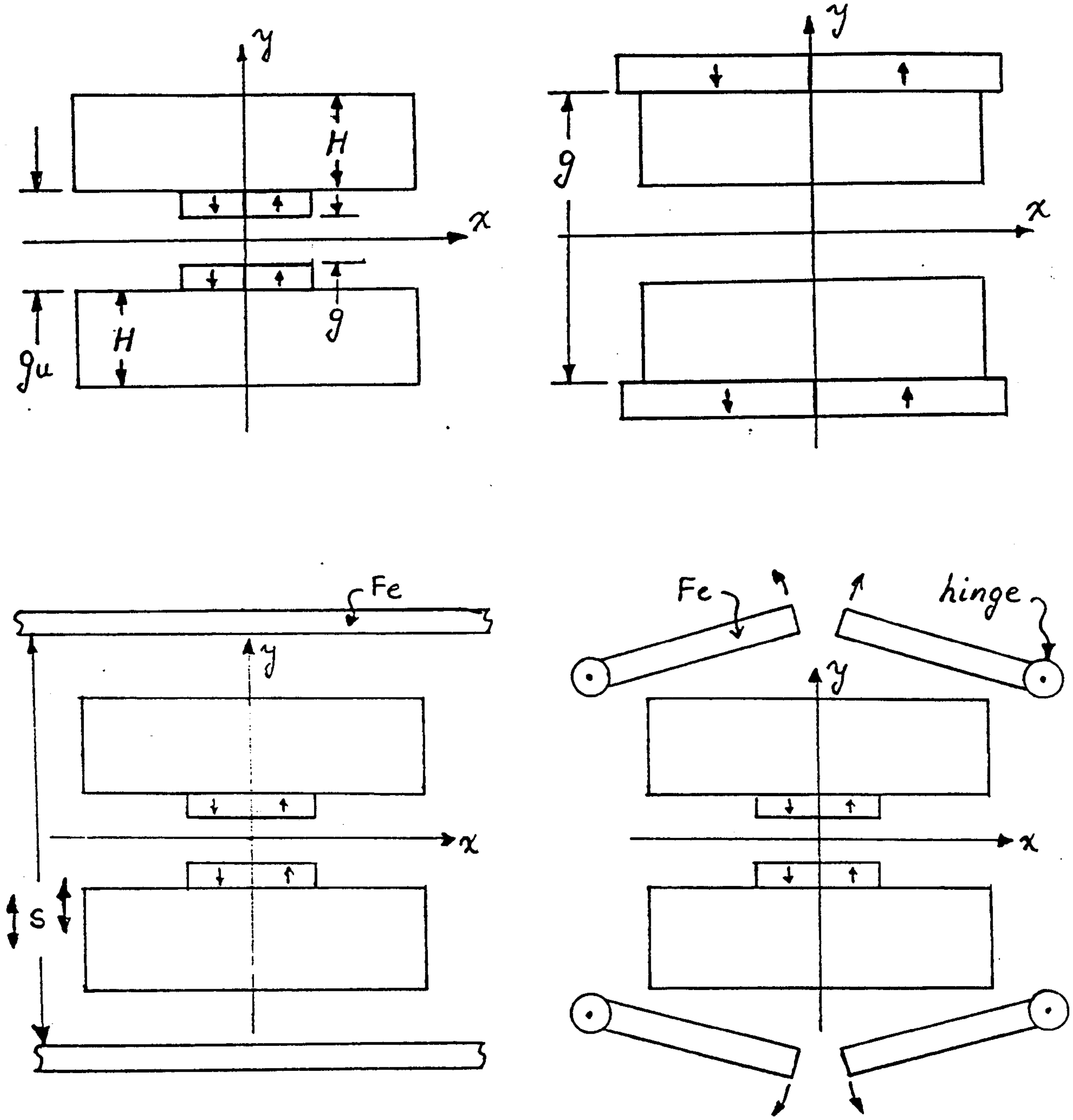

Fig. 5