LIFETIME AND BEAM SIZE IN ELECTRON STORAGE RINGS

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Experiments have been going on in the past months with the small storage ring AdA [1]; after preliminary tests in Frascati [2]; operation at higher intensity became possible at Orsay where an electron linac provides better injection performances. We got a certain amount of information on stored electron beams during this time and we think that a number of conclusions relevant to the problem of building such a kind of machine can now be drawn. So, we want to extrapolate from AdA all those considerations we believe are of general interest in this field. Since the injection method and the design of the machine we used are quite peculiar to AdA [1], we will not recall them.

We want to summarize the effects we are aware of, both experimentally and theoretically, concerning the properties of the beams in on e^-e^- or e^+e^- storage ring. Since reactions between colliding beams are our main task we will discuss only those properties affecting the reaction rates; that is «target» properties. The connection between detection rates and cross sections is given by a factor that we write in the form

$$\dot{n} = \frac{f}{k} L\sigma\eta. \qquad (1)$$

Here $\dot{n}$ is the detected number of events per unit time, per interaction region; kf is the frequency of the RF cavity, k is the harmonic number; σ is the total cross section (where it exists) and η is a geometrical efficiency factor of the apparatus detecting the reaction products (so, for instance, for a two body reaction, $\sigma\eta$ is the differential cross section integrated over the solid angle defined by the detector of the final particles). The main quantity we are concerned with is the luminosity L, an inverse area, defined as

$$L = \int n_1(x, z)\, n_2(x, z)\, dx\, dz, \qquad (2)$$

where $n_{1,2}$ is the two dimensional transverse density of beam 1 (2) averaged over its longitudinal extent. Often, a quantity S is introduced having the significance of common transverse area of two colliding beams $\frac{1}{S} = \frac{L}{N_1 N_2}$, where $N_{1,2} = \int n_{1,2}(x, z)\, dx\, dz$ is the total number of particles in beam 1 (2).

The computation of S could be complicated by the circumstance that the beams are artificially required to cross at an angle (by means of electric fields); we will refer in the following to the simple case of head-on collisions in order to avoid tedious geometrical considerations while emphasizing the more physical aspects of the problem.

As far as we know, the luminosity L is determined mainly by the following effects:

α — interactions with the residual gas in the vacuum chamber,

β — interactions with the radiation field (synchrotron radiation and its fluctuations),

γ — interactions of one beam with the other,

δ — interactions between particles in the same beam (more properly, in the same bunch),

ε — interactions with steady or RF transverse electric fields.

All of these topics can be easily treated or found in the literature; we just summarize them here adding a few comments.

α — The residual gas produces mainly lifetime effects — the e-atom scattering could also produce betatron amplitude spreading but in every practical case this process occurs in the single-scattering regime [3] and does not appreciably affect the luminosity.

Lifetime effects are due to: bremsstrahlung, single scattering, inelastic collisions. For each one we introduce a cross section for destructive processes σ defined as the cross section integrated over all processes of that kind removing a particle from the beam; then the lifetime τ will be given by

$$\frac{1}{\tau} = c\Sigma N_i \sigma_i, \tag{3}$$

where N_i stays for the number of atoms of species i per unit volume in the residual gas. Often air is assumed as the residual gas in the doughnut but one has to be careful on this point.

The σ's are the following:

Bremsstrahlung

$$\sigma_i(B) = \Phi_i\left(\ln\frac{E}{\varepsilon} - \frac{3}{4}\right)$$

Scattering

$$\sigma_i(S) = \frac{4r_e^2 Z_i^2}{\gamma^2 \theta_0^2}$$

Inelastic collisions

$$\sigma_i(I) = 2\pi r_e^2 Z_i \frac{mc^2}{\varepsilon}$$

Here

$$\Phi_i = 8\ln\left(\frac{183}{Z_i^{1/3}}\right)\frac{Z_i^2 r_e^2}{137}$$

E — energy of the particle $= \gamma \cdot mc^2$,

$\pm\,\varepsilon$ — RF energy acceptance,

Z_i — atomic number for species i,

r_e — Lorentz radius of the electron.

$1/\theta_0^2$ is the integral of $1/\theta^4$, θ being the scattering angle, over all those angles associated with betatron amplitudes larger than the size of the vacuum chamber — when the doughnut is elliptical and the machine a weak focusing one

$$\frac{1}{\theta_0^2} = \frac{\pi}{2}\left(\frac{\lambda_h^2}{a^2} + \frac{\lambda_v^2}{b^2}\right).$$

The $2\pi\lambda$'s are batatron wavelenghts and a, b are semiaxes of the doughnut. However, the injection system sometimes complicates the definition of the shape of the actual obstruction leading to scattering losses; this is not very important since the scattering lifetime is usually long as compared with the bremsstrahlung one. Furthermore, radiative corrections make $\sigma(S)$ even smaller since in $\sigma(B)$ we already accounted for some of the scattering processes. A very useful numerical formula occurring in all of these pressure-dependent effects is the one relating N_i to the partial pressure p_i at 300° K: $N_i = 3.55\times 10^{16} p_i$ cm^{-3}, p_i being measured in mm Hg.

Measurements of lifetime in AdA at low stored intensity (where the gas bremsstrahlung should dominate) have given a value of 50 h at a pressure 3.5×10^{-10} mm Hg (on the Alpert gauge, placed near the pump); the lifetime seems to be quite insensitive to the energy as expected but the agreement with the assumption that the residual gas is mainly air is poor (the lifetime is a factor of 2 shorter).

β-Interactions with the radiation field influence both the lifetime and the size of the transverse areas of the beams. Multiple effects dominate here and the calculations are not so easy to perform as in the previous cases. A formula for the lifetime is well known and has been reasonably confirmed by measurements [4] (a trouble is connected with the practical difficulty of measuring accurately RF peak voltages to which the lifetime is very sensitive). The lifetime is given by

$$\frac{1}{\tau_{\rm RF}} = \varrho\,\frac{\varepsilon}{U_{\rm ex}}\,e^{-\frac{\varepsilon}{U_{\rm ex}}},$$

where ε is the energy acceptance as in (α), ϱ is the damping constant of the phase oscillations and $U_{\rm ex}$ is an excitation energy associated with radiation fluctuations. The ratio

$$\frac{\varepsilon}{U_{\rm ex}} = \frac{(4-\alpha)\Lambda}{k\alpha}\,\frac{E}{E_1}\,H(\Phi_s),$$

where α is the momentum compaction, Λ the circumference factor,

$$H(\Phi_s) = 2\,{\rm cotg}\,\Phi_s - \pi + 2\Phi_s$$

$$\sin\Phi_s = \frac{eV}{E}$$

V is the peak RF voltage; and $E_1 = \frac{55\sqrt{3}}{64}\times\frac{hc}{r_e} = 104$ MeV.

This lifetime effect is a minor trouble provided the RF power is big enough for the maximum energy of operation of the ring. Also, the beam size is influenced by radiation fluctuations: we believed till recently that this was the only cause for the beams to spread around the synchronous particle (at least when space charge is unrelevant) but there is experimental evidence confirming only the prevision on radial dimensions; whereas the vertical dimension seems to be larger than expected. We will come back later on to this point and give now the rms linear sizes as derived from radiation fluctuations alone [5]:

$$d_v = \left\{\frac{55\sqrt{3}}{24}\,\beta_{v\max}\,\overline{\Phi}^2\,\Lambda\,\frac{\langle\beta_v K^3\rangle}{\langle K^2\rangle}\right\}^{\frac{1}{2}}\gamma, \qquad (4)$$

$$d_r = \left\{\frac{55\sqrt{3}}{24}\,\beta_{r\max}\frac{\Lambda F_1}{\langle K^2(1+2nK\psi-K\psi)\rangle}\right\}^{\frac{1}{2}}\gamma,$$

$$d_s = \psi_{\max}\left\{\frac{55\sqrt{3}}{24}\,\frac{\Lambda\,\langle K^3\rangle}{\langle K^2(2-2nK\psi+K\psi)\rangle}\right\}^{\frac{1}{2}}\gamma,$$

$d_{v,r,s}$ are the rms vertical, radial betatron, radial synchrotron dimension; β's and ψ are conventional betatron amplitude functions and closed orbit functions as given by Courant and Snyder [6]; K is the local curvature of bending magnets, n the field index: F_1 is defined by

$$F_1 = \left\langle\left[\psi^2 + \left(\frac{1}{2}\beta_r'\psi - \beta_r\psi'\right)^2\right]\frac{K^3}{\beta_r}\right\rangle$$

(β' means derivative with respect to the arc length). Eventually $<\varphi^2>^{1/2}$ is the rms angle of emission of a photon with respect to the parent electron direction, projected on the vertical plane; $<\varphi^2>^{1/2}$ is proportional to $1/\gamma$ so that d_v is energy independent.

One must remember that the distributions of the invariants of the oscillations are gaussian whereas the distributions of displacements are not gaussian; this has some relevance in determining the target factor through formula (2). As a rule, the vertical rms dimensions as derived from (4) are in the micron range, whereas the radial rms dimensions (obtained by folding betatron and synchrotron contributions) are in the millimiter range.

γ — Interactions of one beam with the other can give also lifetime and size effects. Lifetimes are now dependent on stored particle number, so it is better to use the loss rate for each beam as a significant quantity. Although these effects are small, they could be important since the loss rates are just proportional to the same target factor occurring in formula (1). The two relevant effects are single scattering and bremsstrahlung; the rates can be given as in formula (1)

$$R_{1,2} = \left(\frac{dN_{1,2}}{dt}\right)_\sigma = -2fL\sigma,$$

where now a factor 2 appears in place of $\frac{1}{R}$ since every interaction region along the ring contributes. σ is the cross section for the destructive process contributing to $\frac{dN_{1,2}}{dt}$: scattering

$$\sigma_S = 4\,\frac{r_e^2}{\gamma^2\,\theta_0^2}$$

bremsstrahlung

$$\sigma_B = \frac{16}{3}\frac{r_e^2}{137}\left\{\left(\ln 4\gamma^2 - \frac{1}{2}\right)\left(\ln\frac{E}{\varepsilon} - 1\right) + \frac{1}{2}\left(\ln\frac{E}{\varepsilon}\right)^2 + \frac{\pi^2}{6}\right\}.$$

Symbols are the same as in previous formulae. When the pressure is low enough to make the scattering from residual gas negligible, this beam-beam scattering process could be used to determine the luminosity via lifetime measurements as a function of internal diaphragm aperture. We have not yet tried this procedure requiring long and accurate lifetime measurements.

The influence of beam-beam interactions on the beam size is very complicated to describe: on one hand there are coherent effects of the space-charge type and these can be described by self consistent methods connecting the electromagnetic fields to the current-charge distributions producing them and on which they react. On the other hand there are incoherent effects of the multiple scattering type.

Actually the problem is the same however it is viewed from two extreme approximations. The space charge has been studied by numerical methods using a computer [7]. Limitations appear on the luminosity mainly because of beam-beam separation. A simple and reasonably good approximation has been given by Amman and Ritson [8] showing the main features of the effect: when the transverse surface density in a beam is bigger than a certain limit the luminosity deviates from proportionality to N_1N_2. The limit depends on complicated machine parameters and we will not give here the results. The intensity at which AdA usually works is far below this space charge limit. The other extreme, the multiple scattering case (completely incoherent) should also give rise to size effects more important than the radiation fluctuations. The result of the calculations on it can be expressed as

$$d_{v\,1,2} = a \frac{N_{2,1}^{2/3}}{N_{1,2}^{1/3}}, \qquad (5)$$

where $d_{v\,1,2}$ is the rms height of beam 1 (2) and

$$a^3 = \frac{(137)^2}{4} \frac{r_e^2 \lambda_v^2}{d_h \gamma^2} \frac{f}{\varrho_v} \log \frac{\theta_0}{\theta_{min}},$$

d_h is the rms radial width of the beams (unaffected by this process), ρ_v the vertical damping constant; other symbols are as before. θ_{min} is a screening angle we estimated to be of the order of 137 r_e/d, where d is the interparticle average distance in a bunch (as a matter of fact, d is of the same order of the vertical rms size!). The reason why this effect can be competitive with radiation fluctuations is that it occurs (compare the gas scattering case) in the multiple scattering regime due to the smallness of the screening angle determining the total cross section: this is also a possible weakness of the result we quoted (form. 5) since the multiple scattering condition

$$\frac{(137)^2}{4\gamma^2} \frac{r_e^2}{\theta_{min}^2} \frac{N_{1,2}}{\omega d_{v\,1,2}} f \gg \varrho_v$$

is very sensitive to the badly known θ_{min}. Once again, we had no experimental evidence on this effect; we will come back later on this point.

δ — Interactions of particles in the same beam seem to give the main lifetime effect at high intensities. The process we refer to is the Coulomb scattering of two electrons or positrons in a bunch in which momentum is transferred from the radial to the longitudinal mode [9]. As a result, a pair of particles redistribute their own energies and they can happen to exceed the RF acceptance and go lost. Calculations would be very simple to do were it not for some averages of powers of inverse relative momenta of two particles requiring the use of a computer. A simple order of magnitude estimate neglecting relativistic effects and using approximate (gaussian) momentum distributions gives for the rate of particle loss [9]:

$$\frac{dN_{1,2}}{dt} = -\frac{\sqrt{\pi} r_e^2 c}{R\Omega} \left(\frac{mc^2}{\varepsilon}\right)^2 \frac{mc}{\delta q} l\left(\frac{\varepsilon}{\gamma \delta q}\right). \qquad (6)$$

Here Ω is the volume of a bunch as defined from the inverse of the average particle density; $1/\delta q$ is the average value of the inverse relative momentum and $l(x)$ is defined by

$$l(x) = \log \frac{1}{x} - \frac{c}{2} - \frac{3}{4},$$

where $C = 0.577...$ is the Euler constant.

We have now more refined calculations [10] showing the substantial correctness of this formula within some ten percent over a wide energy range around GeV energies. Multiple effects have not yet been carefully evaluated but they seem not to contribute appreciably in this range. From the experimental point of view, the situation has still some uncertainties we ascribe essentially to the poor knowledge of the vertical size of a single beam: now it is perhaps the right point to discuss this matter. As it is well known, the natural collimation of synchrotron radiation sets a limit on the source size we can measure by optical means; this limit seems to be generally far bigger than the vertical height we can anticipate on the basis of radiation fluctuations and beam-beam multiple scattering (form. 4.5). On the other hand, we have some manageable effects depending on the same parameters which determine the luminosity and thus the vertical size: beam-beam reactions at low momentum transfer (such as 2γ annihilations, e^+e^- scattering) and the lifetime effect due to Coulomb interactions in a bunch that we just mentioned in this paragraph (form. 6). The latter is presently the best known one from the experimental point of view, and the conclusions we can draw from those measurements are as follows: the lifetime is longer than expected both on the basis of radiation fluctuations and the crude formula (5) for multiple scattering effects. Moreover, the energy dependence of this effect disagrees with what we could foresee, because the measured values behave as if the vertical size depended linearly on the energy; finally the results seem to be independent on whether the lifetime is measured with a single stored beam or two beams (remember formula 5 gives sizes depending on $N_{1,2}$).

Our present interpretation is that there is, in AdA, an effective radial-vertical coupling producing the observed behaviour. But this now is a conclusion involving machine parameters quite difficult to account for and we cannot generalize to the behaviour of other storage rings. As a matter of fact, the luminosity in AdA seems to be dominated by this coupling effect; it reduces the possibility of observing beam-beam reactions of the clean type like 2γ annihilations at forward angles, since the rate is considerably lower (by the way, we recognized that 2γ bremsstrahlung, that is $e^+ + e^- \rightarrow e^+ + e^- + 2\gamma$, has comparable rates with 2γ annihilations and both processes contribute to 2γ events due to limited resolving power of γ detectors). To get an independent and statistically significant measurement of beam height we must perform observations on some reaction having a faster rate than we needed before the discovery of the main lifetime effect: as we anticipated, e^+e^- scattering seems the most promising one and we are oriented on this field with AdA. We are in fact trying to detect scattered particles first and later on we would like to measure lifetime effects of the beam-beam single scattering type if possible.

ε — The presence of electric fields could influence the luminosity since they can separate the paths of the two beams (either for e^+ and e^- in the same ring of for e^-e^- in two tangent rings). In fact this is an useful effect allowing for beam crossing at an angle when needed. Sources of these fields can be provided by introducing suitable plates in the vacuum chamber but we refer here to unwanted accidental electric asimmetries influencing the luminosity. Vertical fields are more likely to be dangerous by displacing the median plane by an amount comparable to d_v, the beam size. These fields could be due to various sources:

1. DC fields occurring because of poor shielding. We are almost sure that no such field can exist in the vacuum chamber of AdA since there are no insulated components in it. However, one must remember that even a field of the order of 10 V/cm can give displacements of the order of ten microns (in AdA at 200 MeV).

2. RF transverse fields due to cavity misalignments. This too we believe is not a very serious problem (in AdA, the synchronous phase is fairly small and this greatly helps to reduce the effect).

3. Fields due to ions in the vicinity of the beam path. If ions were present due to inelastic collisions of the beams with the residual gas, they could make appreciable effects only if trapped by the beams themselves. This would produce an increase of the local pressure and as a consequence the lifetime due to the residual gas should be both pressure and intensity dependent. We did not observe any such effect in AdA.

4. Image effects on the metal environment of the beams. These are not specifically electric effects since image currents can contribute a magnetic part. A crude calculation easily shows that these effects are at least an order of magnitude smaller than observable. There is also a somewhat sophisticated effect due to retarded images [11], providing a mechanism for the blow up of a single beam via the compensation of the betatron damping; but we did not see any effect of this kind with AdA, probably because of the inherent non-linearity of the magnetic field.

As a conclusion, we want to say that the list of effects we have given in the preceeding paragraph could be incomplete: it reproduces our present knowledge of beam properties as can be extrapolated from experiments with AdA. The main restriction of experimental information is the limited energy range (from 50 to 200 MeV); other limitations are the maximum intensity attained (4×10^7 particles in a beam) and the weak focusing field. We did not report detailed numerical results (that can be found in the quoted literature) because these are more relevant to the small ring AdA than to high energy machines. Eventually, we believe that the consistency of the data we dispose of at present is fairly good and encourages in proceeding further with storage ring machines since these data do not show any failure «in principle» of the main idea.

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DISCUSSION

A. N. Lebedev

I would like to make a remark about the report of Dr. Bernardini concerning the so-called natural beam height subjected to the action of quantum fluctuations. All calculations leading to quantities of the order of several microns are based on the assumption of the ideality of the accelerator field; whereas, the vertical beam dimensions turn out to be very sensitive to distortions of the median surface if they depend upon the radius. This takes place not so much as a result of the connection between the oscillations which may be controlled by shifting the operating point, but also of the appearance of the excitation mechanism which is the same as for radial oscillation. Rough estimates given for the FIAN synchrotron lead to values for the height of the order of tenths of a millimeter which is in qualitative agreement with the experiment.

C. Bernardini

I agree with you, but the discrepancy one has to explain is rather big; the vertical size seems to be even two orders of magnitude larger than expected. Possibly the effects you refer to easy to calculate in the AdA case since the weak focusing allows for the use of simple perturbative methods.

S. A. Kheĭfets

I think that there is still one possibility such that the vertical beam dimensions become sufficiently large. Namely, if the frequency of the betatron oscillations turn out to be close to some resonance, then, as a consequence of the dependence of the frequency on energy, the different particles turn out to be at different distances from the resonance. In this case, the effective beam dimension may also turn out to be not very small.

C. Bernardini

I do not think a resonance can play any role since we had a good agreement with the lifetime we could predict (in the coupling hypothesis) at every energy and we know that the field index changes very much with the field.

A. A. Sokolov

Were the polarization properties of synchrotron radiation and the effect of spin and AdA storage investigated?

C. Bernandini

I believe spin effects on the synchrotron radiation are too small to be detected with AdA.

A. N. Didenko

I would like to make a remark in connection with the report of Dr. Bernardini. Evidently, the transverse components of the RF field are not able to increase the beam dimensions since neither the external RF field nor the natural field of the beam leads to damping or oscillation of the betatron or phase oscillations, which means its transverse dimensions also do not change.

C. Bernardini

RF transverse fields do not increase the vertical size; I said that they just separate the trajectories of e^+ and e^-.