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Review

CPT Symmetry Searches in the Neutral Meson System

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Abstract: A review of the landscape of CPT symmetry tests is presented, centered around the Standard-Model Extension and focusing on tests in the neutral meson system. A discussion of the relevant theories summarizes original ideas. It is followed by a short transition into phenomenology. A more detailed parameterization is presented. Various experiments are used to deliver an overview of testing CPT from every angle that the theory suggested and that the neutral meson (NM) system could accommodate.

Keywords: particle physics phenomenology; discrete symmetries; CPT violation; neutral mesons

1. Introduction

The Standard Model (SM) of particle physics is invariant under Lorentz transformations and the combined CPT symmetry, where C, P, and T are the discrete symmetries of charge conjugation, parity, and time reversal, respectively. The former corresponds to the proper Lorentz group, while the latter corresponds to the improper Lorentz group. Proper Lorentz transformations are the continuous rotations and boosts, while the CPT discrete symmetries C, P, T cannot be implemented via continuous and infinitesimal transformations or by any combination of the rotations and boosts of the proper Lorentz group. The two symmetries, however, are tightly connected. It is important to understand the representations of the Lorentz group to correctly define the discrete symmetries in quantum field theory [1–4].

To properly define the charge conjugation transformation, a field theory approach is necessary where fields are functions of coordinates. Therefore, in field theory, the Lorentz transformation is a combination of the transformations of the finite dimensional, scalar, vector, and spinor transformations, as well as a coordinate transformation. Since the latter acts on a space of functions, it describes an infinite dimensional representation of the Lorentz algebra [1].

CPT symmetry has sound theoretical proof as the CPT theorem, originally shown by [5–9]. Since then, it was given various other demonstrations and explanations. It guarantees CPT conservation for any local relativistic theory of point-like particles, such as the SM. So far, no experimental signal has ever been observed that would contradict it.

This work is about how the limits of possible CPT violation (CPTV) were tightened in the neutral meson (NM) system. The paper originates in decades of NM system searches and relevant theoretical considerations and has a vast literature. The author sincerely apologizes for not being able to cite all related papers. Citations must be confined to a few which carry highest clarity or most relevance for important points to be made. However, the cited references each contain a focused expanded list in their own citations. The readers can also find an organized Data Table [10] revised yearly with results of searches in all sectors of physics including General Relativity (GR). The Data Table contains detailed references as well.

Since the Sakharov conditions [11] given for the asymmetry of matter and antimatter assert the necessity of at least CP violation (CPV) and the NM systems display such

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violation, interest remained strong to scrutinize it for T and CPTV as well. With CP broken, either T or CPT must also be. CPV is one of the most profound mechanisms in physics; without it, our world as we know it would not "materialize", quite literally. The imbalance of matter and antimatter that makes it possible is not accounted for by SM-based calculations.

CPV was first observed in the neutral kaon system [12], a particle–antiparticle pair that differs only in quark flavor. The neutral kaon contains a quark of one flavor (a down quark) and an antiquark of a different flavor (an anti-strange quark). NM systems are so illuminating because these neutral particle–antiparticle pairs can change into each other via the CP-violating weak force, the force responsible for radioactive decay, but are produced in strong interactions, which is CP-preserving. In a quantum mechanical treatment, the CP-odd weak phase and the CP-even strong phase appear in the interference of decay amplitudes, leading to CPV.

This leads to a remarkable quantum mechanical system of meson–antimeson oscillations. The particle–antiparticle pair combines into two propagating states, mixing and decaying weakly. Production of them from quarkonium states at resonance launches quantum-correlated mesons entangled until one of the two decays. In the case of neutral kaons, features of this otherwise obscure physics can be observed over a macroscopic distance. Decay modes then reveal the discrete symmetries of it. Since this discussion is about CPT symmetry, there is no possibility of covering the complicated but beautiful physics of CPV nor the history of its detection [13]. What is perhaps helpful is to point to the fundamentals that might be a common mechanism between CP and possible CPT symmetry breaking.

T violation was first observed at CPLEAR [14] in the K system and at BaBar [15] in neutral B mesons by the BaBar collaboration [15] allowed by an elegant idea for meson factories [16]. Until then, in decaying neutral mesons, direct time-reversed observation was difficult because the probability of a time-reversed process of unstable particles occurring is practically zero. However, using direct observation and calculations based on the SM allows comparative studies of CP and T symmetries as another way of testing CPTV. Any discrepancy between the T violation predicted by the SM and that observed would suggest the violation of CPT.

This is a novel T and CPT test in transition processes between meson states, using a combination of CP-specific and flavor-specific decays in the identification of T-reversed processes of entangled factory mesons. Direct T tests have been completed in the neutral B and K factories, as well as a CPT test at the K factory [17–19]. It marks another milestone in discrete symmetry investigations.

The *D*∅ Collaboration also conducted such an investigation, comparing different decay rate asymmetries signaling CP and T violation for both the B_d and B_s systems. They evaluated those against SM predictions assuming or relaxing CPT symmetry [20].

General relativity taught us that spacetime is dynamical and could have some nontrivial topological structure. Physicists also found that the naive perturbative vacuum in which all fields have zero expectation values can be unstable. In the "true" vacuum, some fields can acquire nonzero vacuum expectation values such that the ground state has a different symmetry from a Lagrangian describing the propagation and interaction of the particle. The process is named spontaneous symmetry breaking (SSB). These were fundamental new insights.

These new views, the success of the method of thought experiments and mathematical extrapolations, encouraged theoretical explorations, extending the description of observed phenomena. In the rare cases when the Standard Model and General Relativity still fell short of matching observation, physicists armed with abstract mathematics and imagination proceeded to write down extensions beyond the known physics. This was also highly motivated by the effort to unify quantum field theories with the classical field theory of gravity. Beyond Standard Model (BSM) theories also became a necessity for SM parameters not explained from first principles.

One of the strongest deviations relating to the NM system is the discrepancy between the particle–antiparticle asymmetry of the universe and the predicted CPV by the SM [21–23]. One can look for resolution in better understanding and measuring CPV or allowing for CPTV in a BSM context, hence the birth of the phenomenology facilitating CPT tests as the focus of this work.

Due to the connection of CPTV to Lorentz violation (LV), it is logical to look for the source of such violations in spacetime features. This pointed to two strong theoretical approaches: string theory and something in string theory that has an effect on spacetime and its symmetries. As it turns out, the theory of CPV gives two more ingredients for cooking up a CPTV scenario: the importance of a flavor-specific interaction with a nontrivial spacetime background and contributions to the quantum mechanical interference opposite in sign for particle and antiparticle.

In the case of CPV, this has been traced back to a nontrivial scalar spacetime background, three generations of quarks with flavor-specific couplings to it, and a flavor-mixing matrix with complex elements containing a CP-odd phase that cannot be removed. The next section presents the Standard Model Extension (SME) framework, into which one can place the NM system to include it in wide, systematic searches for the validity or breaking of Lorentz symmetry.

In what follows, we assume flat, Minkowski spacetime in low-energy approximations with gravity effects neglected. CPTV could also be a result of a strong gravitational spacetime background. Excitement was high in the 1970s and 1980s when Hawking and others looked at quantum mechanics, particle creation, CPT symmetry, and radiation under the effect of a black hole [24].

This paper is structured as follows. Section 1 gives a discussion of BSM scenarios that could produce CPTV. Section 2 introduces the specific model and the general Lorentz-violating effective quantum field theory framework of the SME. Section 3 bridges over from theory to the specific description of the NM system. Section 4 narrows the phenomenology to parameterizing the CPTV searches. Finally, Section 5 presents relevant experiments.

2. Neutral Meson-Relevant Frameworks of CPTV Theory

To look beyond the CPT theorem, at least one of the assumptions it is based on must be relaxed. This includes various models relaxing the pointlike property of particle, locality, or unitarity of quantum time development. Some models consider C(PT) as ill-defined in a quantum gravity scenario at Planck scale.

In the early 1990s, there were attempts to relate string theory to physical systems and to provide a nonperturbative definition of it. There was also a goal to find a string field theory in which all possible backgrounds could be derived from a single set of degrees of freedom, thereby facilitating vacuum selection for physical systems.

Kostelecky and Samuel investigated the open bosonic perturbative string vacuum and the physical vacuum in string theory. Instabilities of the perturbative vacuum were resolved from nonperturbative effects in a lower-energy vacuum into which the bosonic string condenses. They noticed that the structure of string field theory implied the possibility of spontaneous violation of Lorentz symmetry appearing in the new vacuum [25,26]. In general, SSB occurs when fields acquire a nonzero expectation value in the true vacuum. In a string theory scenario, this can involve tensor fields, resulting in breaking the isotropy of spacetime.

The core of this review is formed by a realistic effective quantum field theory that is based in this spontaneous Lorentz symmetry breaking [27]. This framework preserves the gauge symmetries of the SM and is local and causal, as described in [28]. Similarly to SSB in the context of the Higgs mechanism with a scalar background, it is assumed that in the approximations within which its effective theory is laid out, it does not change the unitarity of quantum processes nor quantum statistics.

It is written down as a combination of Lorentz-violating tensor coefficients contracted with SM fields in such a way that they form a Lorentz scalar of the SME Lagrangian. These will introduce LV coefficients appropriate for vector, spinor, and tensor fields (the finite representations of the Lorentz algebra). The coefficients represent LV couplings to the tensor background. In flat spacetime, the coefficients are constant. It is important to emphasize that, like the Yukawa couplings, they are also flavor-dependent. This foreshadows some crucial insights about CPTV. No matter what origin of CPTV one looks at, there is a mandatory dependence on flavor.

The reader is probably interested in work that further studied the SME tensor-type SSB in flat or curved spacetime. This topic is exciting, and indeed, there are many investigations in that direction. A class of models considering vector-type symmetry breaking is the bumblebee models, but the discussions extends to tensors as well [29–32].

In the SME framework, the theory must remain invariant under coordinate transformations. As indicated above, the Lorentz transformations of fields must transform the scalars, vectors, spinors, or tensors along with the four-vector coordinates, which the fields are a function of. This separates two types of transformations: particle Lorentz transformation and observer transformation. Particle transformation acts on the fields relative to the constant background, which remains unchanged, while observer transformations transform the background as well [27,33]. The Lagrangian terms being Lorentz scalars ensures observer Lorentz symmetry. Some of the SME terms odd under CPT transformations also represent the violation of CPT symmetry. In an effective quantum field theory, CPTV implies LV [34].

The other interesting insight from the SME framework is momentum dependence. If there is a constant tensor background against which the fields transform, then a combination of particle and observer Lorentz transformations is required as one moves from the rest frame, where the CPTV phenomenology is set up, to the observer frame. This reveals a boost and orientation dependence of the CPTV parameter [35,36]. This was tested in all NM experiments. The sidereal variation as the earth rotates also becomes significant when one talks about a nontrivial spacetime vector background.

As a general realistic EFT extension of the SM, the SME must incorporate all effects from specific realistic LV or CPTV models that contain the SM in a limit, such that a nonzero SME coefficient can be assigned in the appropriate sector of the SM. Such identification can be obscured if a model is not set up as a field theory or describes an artifact of a model that is not actually physically observable.

The minimal (mSME) is also power-counting renormalizable [27]. It is extended to higher derivative terms and to GR [37–42]. In the latest research, explicit LV is also considered in the mathematical context of Finsler geometry [43–57]. Due to the generality of the framework, the ensuing publications are many, expanding to all sectors of physics. Tests of CPT and Lorentz violation rooted in the SME sprouted into one of the biggest experimental endeavors in physics [10].

Cosmology, black hole physics, dark energy, and quantum gravity, combined with mathematically consistent ideas in string theory, led to models of a nontrivial behavior of spacetime. String theory presents the necessity of considering non-local interactions. As stated in Ref. [58], non-locality of interaction terms in string theory is not just due to a choice of description but is an inherent characteristic. More generally, non-locality seems to emerge as a general property that all theories were preserving Lorentz invariance as a basic underlying assumption [59,60]. They appear to present a discreteness of Planck-scale spacetime structure. Such BSM models of quantum gravity are, for instance, loop quantum gravity, noncommutative geometries, and spacetime foam theories [33,61–65].

Spacetime foam theories presented a direct relation to the NM system. Spacetime is seen as "fuzzy", deriving from microscopic quantum fluctuations of the metric field. Also, assuming singularities as part of the quantum gravity vacuum, spacetime acquires a topologically non-trivial structure, also christened a "foamy" structure [66]. This opens the usual questions presented by black holes. Black hole studies raised the fundamental question of information loss leading to pure quantum states becoming mixed, and specific quantum numbers becoming ensemble averages.

Specifically in quantum gravity, there is the potential for disruption of communication between Hilbert spaces in and around the microscopic singularities. If quantum-gravity effects mandate a foamy spacetime structure, it becomes difficult to define the coordinate of a point. Most importantly, the unitarity of quantum evolution also came into question. As we will see later, the appropriate quantum physics description of the NM in regard to these models must change compared to the traditional approach.

As a further development of the connection of string theory to physics and understanding of nonperturbative effects, string branes and specifically D-branes have been introduced. Unstable D-branes decay in the tachyon vacuum. This resolves the vacuum instability. Furthermore, the existence of open strings could also be understood as D-brane instability [67]. The foamy, fluctuating spacetime behavior found one explanation in the visualization of D-foam. An illustration of it is given in Figure 2 of [68]. The mechanism behind D-particles affecting spacetime and the metrics are involved, and the details are beyond the scope of this work; see, for example, refs. [67,68]. Of interest here is what phenomenology based on this relates to the NM system. The most important aspect to investigate experimentally is the loss of quantum coherence or the unitarity of quantum evolution.

Here, we mention two approaches that relate closely to NM experiments of CPTV and entanglement. The details are quite involved; the reader is advised to follow up with the references. In the first quantum gravity model, the described interaction of D-particle foam with stringy matter is responsible for a nontrivial spacetime metric. Spacetime fluctuations and the effect of the non-trivial metric are considered to change the quantum behavior of entangled massive particles such as factory neutral mesons [65,68–73]. This mechanism also assumes that there is flavor dependence and the metric changes are expanded on the Pauli matrices for a two-state system to accommodate a flavor structure [68]. The second model studies the loss of coherence within an effective theory where the spacetime-foam background is replaced by a fixed (classical) background, but the interaction of low-energy fields is non-local [64].

In usual textbook treatment and in NM discrete symmetry searches, meson time development is studied in a Weisskopf–Wigner Schroedinger-like approximation. In spacetime foam models, the quantum mechanical system must be considered as open to a quantum gravity environment. This needs to be accounted for in a density matrix description. The details will be discussed in Section 3.

In the density matrix treatment, an additional environmental operator is introduced. This in turn changes the density matrix evolution, the scattering matrix, and unitarity. These changes would provide another mechanism for intrinsic violation of "strong" CPT [68,71].

Unifying gravity and quantum mechanics is still an ongoing point of contemplation for physicists. The SME set out a goal to provide a wide framework into which to fit LV and CPTV searches, compare and tabulate results, and precision-test most sectors of physics.

Since the SME formalism includes all sectors of physics, it is more manageable when applied to a specific area. The area relevant here is the NM system. Building symmetric and asymmetric colliders and highly boosted fixed-target experiments for NM research is one of those play fields where BSM theories can translate back to the physical world from mathematics. The skill and precision of these tests gave the ability to place tight bounds on LV and CPTV.

Both the inherent momentum dependence of Lorentz-violating SME and the disentanglement due to spacetime foam of the correlated meson states have been tested. KLOE shines as the experiment that put impressive bounds on both effects [74–80].

While these models and the relevant SME sectors cover different angles, some common core ideas give some basic questions. How do spacetime features or quantum gravity affect the physics of the NM system? What is the best way to view these effects—as transformation properties of the vacuum, creation and annihilation operators, change in dispersion relation, as shifts in basic properties of particles and antiparticles, or as an influence on the time development of their propagation? The rest of the article is a review of how testable phenomenological parameters were found, measured, and reflected back as bounds of the coefficients of BSM frameworks and models.

In producing observable signals one must be careful. Often in the quantum context, fields and transformations can be shifted by phases which result in physically undetectable quantities. There must be a hierarchy of interactions that take place. All practical theories relating to experimentation are valid at certain energy scales.

With searching in the unknown, it is always wise to check back with known physics. For CPV, one must have a flavor-specific coupling to a scalar background field. There have to be enough different flavors with different couplings. Mixing of flavor states into mass eigenstates must occur such that a complex physical phase is there to change the behavior of particle vs. antiparticle, i.e., a CP-odd phase next to some CP-even one. One must compare this to CPTV behavior.

3. Phenomenology

3.1. SME Phenomenology

It is best to start by looking at the general form of the SME Lagrange density, as presented first in [81]:

$$\mathcal{L}_{SME} \sim \frac{\lambda_i \langle T \rangle}{M_{Pl}^k} \bar{q}_i \Gamma (i\partial)^k q_i, \tag{1}$$

where $\langle T \rangle$ is the tensor expectation value after SSB, λ_i is the coupling to background, Γ represents a gamma structure appropriate to form a Lorentz scalar with SM fields, $\bar{q}_i, q_i$ here denote fermion fields, and $(i\partial)^k$ describes partial derivatives to the kth power. Matching this is a Planck-scale suppression M_{Pl}^k factor raised to the same power. The minimal SME contains only $k = 0, 1$ terms. These are power-counting renormalizable.

$$\mathcal{L}_{SME} = \tfrac{1}{2} i \overline{\psi} \Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \overline{\psi} M \psi. \tag{2}$$

For clarity, the $k = 0$ and $k = 1$ terms are shown separately below in M and Γ^ν, respectively. Note that the coefficients in the two will have different mass dimensions.

$$\begin{aligned} \Gamma^\nu \quad &:= \quad \gamma^\nu + c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu \\ &\qquad + e^\nu + i f^\nu \gamma_5 + \frac{1}{2} g^{\lambda\mu\nu}\sigma_{\lambda\mu}, \end{aligned} \tag{3}$$

and

$$M := m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu + \frac{1}{2} H^{\mu\nu}\sigma_{\mu\nu}. \tag{4}$$

Here, γ_5, γ^μ,$\gamma_5\gamma^\mu$, $\sigma^{\mu\nu}$ are simply conventional gamma matrices. The coefficients a_μ, b_μ, $c_{\mu\nu}$, ..., $H_{\mu\nu}$ combine from the coupling λ_i and the background tensor expectation value $\langle T \rangle$.

The relevant term to NM experiments is the quark term with a_μ coefficients. It can be identified from comparing the Hamiltonian of the NM oscillations to the terms of the SME [82].

$$\mathcal{L}_{\text{SME}} \supset -\bar{q}\, a_q^{(3)\mu} \gamma_\mu q\,, \tag{5}$$

where q is a single quark field and the $(d = 3)$ superscript indicates dimension 3 mSME term. The coefficient a^μ is contracted with the standard Dirac gamma matrix. a^μ is a four-component vector coefficient of mass dimension $(d = 1)$. How the SME coefficients for quarks fit into the NM physics will be presented in Section 4.

For one quark a field, a redefinition could remove any CPTV effect, but a particle containing two quarks of different flavors leads to a rest energy shift difference, which is measurable via its influence on the NM interferometry, as long as the SME couplings are flavor-specific. In terms of the NM relevant coefficient a^μ, this difference appears as

$$\Delta a^{(3)\mu} = a_{q_2}^{(3)\mu} - a_{q_1}^{(3)\mu}, \tag{6}$$

which in turn can be related to the CPTV experimentally observable parameters.

Still, the fact that hadrons are more than just a combination of the two valence quarks is important to address. One must account for gluons and sea quarks, which interact in complex ways in the strong force environment and could also be influenced by the background. Originally, this was dealt with by introducing the r_{q_i} multipliers to correct for those latent processes:

$$\Delta a^{(3)\mu} = r_{q_2} a^{(3)\mu}_{q_2} - r_{q_1} a^{(3)\mu}_{q_1}, \tag{7}$$

where q_1, q_2 signify two quarks of different flavor adjusted by the r constants. Current research expands into the nonminimal SME. It also explores the treatment of the mesons in terms of effective scalar field theory versus the quark-level analysis.

3.2. Mesons at the Hadronic and Quark Level in Nonminimal SME

When addressing the nonminimal SME, it is practical to examine the lowest nonminimal mass dimension. Only odd ones violate CPT, which takes to dimension 5. Conclusions learned can be extrapolated to higher terms [83]. The general fundamental insights remain the same. For a full treatment of nonminimal terms, see [38–41].

At dimension 5, the relevant SME term [41]

$$\mathcal{L}^{(5)fermion}_{\text{SME}} \supset -\tfrac{1}{2} a^{(5)\mu\alpha\beta} \bar{\psi} \gamma_\mu i\partial_\alpha i\partial_\beta \psi + \text{h.c.}, \tag{8}$$

and the corresponding scalar term [83]

$$\mathcal{L}^{(5)scalar}_{\text{SME}} \supset -\tfrac{1}{2} \phi^\dagger (k_a^{(5)})^{\mu\alpha_1\alpha_2} \partial_\mu \partial_{\alpha_1} \partial_{\alpha_2} \phi + \text{h.c.}\,. \tag{9}$$

For simplicity, it is shown for free fields. For interactions, the partial derivative should be replaced by the covariant derivative properly symmetrized. Many aspects of the interactive theory are still open areas of research. Like the a_μ coefficient, the $(k_a^{(5)})^{\mu\alpha_1\alpha_2}$ of the scalar fields is also a real spacetime constant.

The connection between the quark-level and hadron-level approaches has been discussed for dimensions 3 and 4 in a quantum electrodynamics approximation where the light quarks are massless. These are the u and d quarks, with thought given to including the strange quark. The theoretical underpinning is chiral perturbation theory (ChPT) [84,85].

There are other details in such involved models and the best way to deal with modifications is one at a time, eventually making a reasonable argument for which effect outweighs the other. Since modern physics studies span such large energy scales, one must carefully consider the hierarchy of effects and the energy domains in which EFTs apply. This is the core idea of looking at the mesons as elementary Goldstone boson in a strong force (QCD) dynamics or composite particle of two different flavor quarks [84–88].

Notice the expression scalar effective field theory, effective quantum field theory. Theories usually must be restricted to well-thought-out and justified energy domains. Important is that observable calculations directly connect to a possible underlying theory, perhaps having different degrees of freedom.

When building a suitable Lagrangian at the levels of different energy domains, care must be take to be consistent with the symmetries of the underlying theory. This renders the work of connecting hadronic and quark-level approaches in an extension of the SM rather insightful.

In NM physics, it was always important not just to acknowledge that results can differ from a low-energy effective theory and the underlying physics, but to also test them in experiment and put bounds on the results in each domain and on the small differences of those results between the domains via meaningful scale ratios or suppression factors.

3.3. Quantum Gravity-Motivated Phenomenology in CPTV Searches

Finally, a separate phenomenology describes the effective loss of quantum coherence of an entangled meson system. The background interaction in this case is with the quantum-gravitational environment, and the matter quantum system needs to be considered an open system. Such a system does not fit into the usual Schrödinger-type description [70]. The appropriate treatment is the density matrix approach.

$$\partial_t \rho = i[\rho, H] + \delta \not{H} \rho \,, \tag{10}$$

Another omega (ω) parameter is introduced for testing the so called "omega effect". In some quantum gravity scenarios, the proper CPT symmetry becomes ill-defined at a more fundamental level. It is possible that particle and antiparticle keep the same mass, yet there is a departure from the development of standard quantum mechanical states. Particle and antiparticle states are assigned independent subspaces as matter states. One relaxes the notion in NM systems that particle and antiparticle are identical.

Phenomenologically, it is tracked as the perturbation of Bose statistic characteristic for NM states produced in a coherent state. Omega parameterizes this scenario with an extra term added to the entangled form of the meson states [89]. Further details continue in Section 4.3. For further reading on a comprehensive and clear presentation, see [73].

As we will see, Belle searches of entanglement violations introduce an immediate, spontaneous separation of the entangled states. More about this is presented in Section 5.2.

An important takeaway here is that the NM system, which offers such detailed probes of the SME, also proved to be of great use for the quantum gravity foam models with phenomenology developed for all scenarios of disentanglement.

4. The Neutral Meson System

Here, the NM system is described in the Weisskopf–Wigner approximation, as opposed to the density matrix description mentioned above [13,72]. Quantum entanglement studies are conducted with correlated meson at KLOE, BaBar, and Belle. Let us start with

$$i\partial_t \Psi = \Lambda \Psi, \tag{11}$$

where $\Psi(t)$ is the two-component wave function of the flavor-specific, strong eigenstates of meson and antimeson.

Matrix Λ is the effective Hamiltonian. Its mass eigenstates and eigenvalues describe the time evolution of the weakly connected mesons in the meson–antimeson state space.

$$\Psi(t) = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix}. \tag{12}$$

The effective Hamiltonian has several parameterizations. Here, we use the same SME parameterization as in [36,90]. Both papers give a detailed correspondence to other notations. The works in [4,13] are highly recommended to understand the subtleties of phase dependence of the various parameterizations.

$$\Lambda = \frac{1}{2}\Delta\lambda \begin{pmatrix} U + \xi & VW^{-1} \\ VW & U - \xi \end{pmatrix}. \tag{13}$$

Here, U, V, W, and ξ are dimensionless complex coefficients. ξ is flavor diagonal, given by the difference to the two diagonal elements; $\Delta\Lambda = \frac{1}{2}(\Lambda_{11} - \Lambda_{22})$ and describes indirect CPTV (CPTV in mixing), in a phase-independent way [36]. Also important is the eigenvalue difference $\Delta\lambda \equiv \lambda_a - \lambda_b$, which directly determines oscillation and decay characteristics.

Considering a first-order approximation in the coupling constant shown in (1) to the CPTV background [35,36,81], ξ can be expressed as

$$\xi = \frac{\Delta\Lambda}{\Delta\lambda}. \tag{14}$$

The complex parameter $W \equiv w \exp(i\omega)$ introduces CPV into the Hamiltonian. If CP is a good symmetry, its modulus is $w = 1$.

Most of the work using this and equivalent parameterizations of the effective Hamiltonian approximate the linear order in the CPTV parameter in the time development of the mesons. In the parameterization of Equation (13), it is implicitly included. Where justified by important physical consequences, higher-order terms are indicated if necessary. Note, however, that even powers of ξ would be CPT-symmetric, since it comes from a CPT-even (squared) "a" coefficient. However, $\mathcal{O}(\xi^2)$ contributions can have significance for correlated meson phenomenology or for D mesons [90,91].

As mentioned above, the rest energy shift between two quarks of different flavor is what can be tied to the difference of the diagonal elements of the Hamiltonian [27,35,36,82]. Since this is covered extensively in all publications pertaining to the NM system, just enough is included here to show the progression of the description.

$$\Delta\Lambda \approx \Delta a_0 \equiv a_0^{q_1} - a_0^{q_2}. \tag{15}$$

The phenomenological parameter ξ and the formalism that connects it to the meson effective Hamiltonian is set up in the meson rest frame and has the basic expression

$$\xi = \Delta\Lambda/\Delta\lambda = \frac{\beta^\mu \Delta a_\mu}{\Delta\lambda}, \tag{16}$$

where it is shown that there appears to be momentum dependence as one considers different frames. The neutral mesons can be highly boosted and have varied direction of momenta. As described in Section 5, to evaluate the measurement in each respective experimental environment, boost and orientation characteristics to that meson propagation, as well as the influence of sidereal earth rotation, need to be carefully identified [35].

As mentioned above, the eigenstates and eigenvalues of the effective Hamiltonian give the trivial time development for a combination of the strong eigenstate meson and antimeson. In the kaon system, propagating states have been named short- and long-lived due to the significant difference observed in their average decay times.

In the B system, it is denoted L and H, since decay time differences are small. The notation used below is adopted from the B system. The eigenstates $|B_L(t)\rangle, |B_H(t)\rangle$ develop in time as

$$\begin{aligned} |B_L(t)\rangle &= \exp(-i\lambda_L t)|B_L\rangle, \\ |B_H(t)\rangle &= \exp(-i\lambda_H t)|B_H\rangle. \end{aligned} \tag{17}$$

The eigenvalues λ_L, λ_H depend on mass and decay part, corresponding to the Hamiltonian according to

$$\lambda_L \equiv m_L - \tfrac{1}{2}i\gamma_L, \quad \lambda_H \equiv m_H - \tfrac{1}{2}i\gamma_H. \tag{18}$$

To be used in calculations, it is practical to define the sum and difference of the eigenvalues

$$\begin{aligned} \lambda &\equiv \lambda_L + \lambda_H = m - \tfrac{1}{2}i\gamma, \\ \Delta\lambda &\equiv \lambda_L - \lambda_H = -\Delta m - \tfrac{1}{2}i\Delta\gamma, \end{aligned} \tag{19}$$

where

$$\begin{aligned} m = m_L + m_H, &\quad \Delta m = m_H - m_L, \\ \gamma = \gamma_H + \gamma_L, &\quad \Delta\gamma = \gamma_H - \gamma_L. \end{aligned} \tag{20}$$

Expressing the eigenstates as a combination of $B^0, \overline{B}^0$.

$$\begin{aligned} |B_L\rangle &\sim |B^0\rangle + (1-\xi)W/V|\overline{B^0}\rangle, \\ |B_H\rangle &\sim |B^0\rangle - (1+\xi)W/V|\overline{B^0}\rangle. \end{aligned} \quad (21)$$

Properly inverting (21), the time-dependent states $B^0(t)$, $\overline{B^0}(t)$ are shown below as

$$\begin{aligned} \langle B^0(t,\hat{t},\vec{p})| &= (C+S\xi)\langle B^0| + (SVW)\langle\overline{B^0}|, \\ \langle\overline{B^0}(t,\hat{t},\vec{p})| &= (SVW^{-1})\langle B^0| + (C-S\xi)\langle\overline{B^0}|, \end{aligned} \quad (22)$$

where C and S are functions of the eigenvalues and the proper time t of the propagation. They are defined for concise notation as

$$\begin{aligned} C &= \cos(\tfrac{1}{2}\Delta\lambda t)\exp(-\tfrac{1}{2}i\lambda t) \\ &= \tfrac{1}{2}(e^{-i\lambda_L t} + e^{-i\lambda_H t}), \\ S &= -i\sin(\tfrac{1}{2}\Delta\lambda t)\exp(-\tfrac{1}{2}i\lambda t) \\ &= \tfrac{1}{2}(e^{-i\lambda_L t} - e^{-i\lambda_H t}). \end{aligned} \quad (23)$$

Let us also define the following general amplitudes F and $\overline{F}$, decaying into final states f at time t:

$$\langle f_i|T|B^0\rangle = F_i, \qquad \langle f_j|T|\overline{B^0}\rangle = \overline{F}_j. \quad (24)$$

The importance in experimentation of finding the most beneficial decay modes cannot be overstated. As demonstrated in Section 5, every observation pertaining to the neutral mesons requires the production of the mesons and following them from propagation to decay.

Often already at production, examining the side decay products next to the meson allows tagging the flavor of the object of the actual observation. Even if that is not known, it becomes imperative to keep track of the meson decay products themselves.

Two notable decays are differentiated: flavor-specific decays and decays to CP eigenstates. Semileptonic decay, meaning a decay that includes a lepton, is flavor-specific in that the sign of the lepton marks the meson vs. antimeson.

To summarize, so far, the searches within the SME framework mandate the recognition of the momentum spread and boost range of the mesons, as well as the classification of decay products. These can be applied quite differently in mesons produced in an uncorrelated state from those that are generated in a correlated or entangled state.

One must stress again that the topic here is the testing of the combined CPT symmetry, embedded into a Lorentz-violating effective field theory framework. Even if one could let go of the inherent connection between the two, the searches are still about understanding a particularly sensitive, complex quantum system with its interaction with some nontrivial spacetime background.

Changes can affect decay amplitudes directly or propagation and meson oscillation. There is a leaning toward the latter two in SME NM phenomenology. Direct decay amplitude CPTVs are hard to separate from direct CP ones. Both types have observed experimental evidence.

Once the propagation and decay of the mesons are covered, the goal becomes to form appropriate decay rate asymmetries. Appropriate means a comparison of types of particle events that hint at a difference between particle and antiparticle characteristics. Raw asymmetries simply count certain outcomes and subtract the raw number of chosen events.

Of immediate interest was to compare the lifetime and the masses for a particle and its antiparticle; no inequality was ever seen. A more sophisticated step is to form specific normalized ratios of time-dependent decay probabilities. This procedure with its somewhat cumbersome calculations is discussed in many references and should not be repeated here.

After inverting the propagation equations such that they describe the time evolution of the particle and its antiparticle, decay amplitudes can be found for different decay products, followed by determining the corresponding decay rate probabilities. Once that is completed, the asymmetries are simply a normalized subtraction of rates such that parameters of the discrete symmetry violation in question become accessible. Pinpointing the useful decay modes and calculating probability differences is the art of symmetry violation studies.

This must incorporate momentum dependence. Further quantities can be compared by binning in the momentum direction or boost ranges. A similarly advantageous technique is the asymmetries formed based on decay time differences.

4.1. Uncorrelated Mesons

For mesons that are produced without (obvious) correlation with each other, a general expression can be applied with only small adjustments to the respective experiment. From [36]:

$$\mathcal{A}^{uncor}_{CPT}(t,\hat{t},\vec{p}) = \frac{Re\xi \sinh\frac{\Delta\gamma t}{2} + 2Im\xi \sin\Delta m t}{\cosh\frac{\Delta\gamma t}{2} + \cos\Delta m t}. \tag{25}$$

A typical structure can be seen here. The numerator is arrived at by subtraction of decay rate probabilities and hence shows the actual difference occurring and is relevant for obtaining information about the CPTV phenomenological parameter ξ. The denominator, included for normalization, is from a decay rate sum and shows terms that are similar in both decay rate time developments. One can also always identify decay-type terms and oscillation-type terms.

4.2. Correlated Mesons

Let us also define the following general amplitudes F and $\overline{F}$; $a = 1,2$ transform into final states f_1 at time t_1, and into f_2 at time t_2, given as follows:

$$\begin{aligned} A_{f_1f_2} &\equiv A_{f_1f_2}(t_1,t_2,\hat{t},\vec{p}_1,\vec{p}_2) = \langle f_1 f_2|T|i\rangle \\ &= \frac{1}{\sqrt{2}}\big[\langle f_1|T|B^0(t_1,\hat{t},\vec{p}_1)\rangle\langle f_2|T|\overline{B^0}(t_2,\hat{t},\vec{p}_2)\rangle \\ &\quad -\langle f_1|T|\overline{B^0}(t_1,\hat{t},\vec{p}_1)\rangle\langle f_2|T|B^0(t_2,\hat{t},\vec{p}_2)\rangle\big], \end{aligned} \tag{26}$$

with definitions (24), $C_a = C(t_a)$, $S_a = S(t_a)$, $a = 1,2$. The key insight of momentum dependence in a consistent effective field theory extension properly implementing particle and observer transformations is indicated by the the momentum argument in the functions.

The correlation introduces indices 1, 2, corresponding to a system described by two Hamiltonians but entangled in the asymmetric combination of the states in the correlated wave function. Sidereal time dependence is also shown as $\hat{t}$.

The reader might be more familiar with the CPT-preserving amplitude in the BaBar notation widely used in the literature. It is shown here for reminder and clarity. A detailed correspondence of notation and approaches is given in [36,90]

$$\begin{aligned} A_{f_1f_2} &= \frac{1}{\sqrt{2}}\big[(A_1\overline{A}_2)g_+ - (A_2\overline{A}_1)g_+ \\ &\quad + (A_1A_2\frac{p}{q})g_- + (\overline{A}_1\overline{A}_2\frac{q}{p})g_-\big]. \end{aligned} \tag{27}$$

In the SME formalism developed in [35,36,82], this corresponds to

$$\begin{aligned} A_{f_1f_2} &= \frac{1}{\sqrt{2}}\big[(F_1\overline{F}_2)C - (F_2\overline{F}_1)C \\ &\quad + (F_1F_2W^{-1})S + (\overline{F}_1\overline{F}_2W)S\big]. \end{aligned} \tag{28}$$

Only one decay probability function is shown; the rest can be found in [36] to illustrate important points to note for the correlated decay:

$$\begin{aligned} P_{f\bar{f}} &= P_{f\bar{f}}(t, \Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) \\ &= \frac{1}{4}|F\overline{F}|^2\, e^{-\gamma t/2}\big\{\cosh\tfrac{1}{2}\Delta\gamma\Delta t + \cos\Delta m\Delta t \\ &\quad -\mathrm{Re}\,(\xi_1+\xi_2)\sinh\tfrac{1}{2}\Delta\gamma\Delta t \\ &\quad -\mathrm{Im}\,(\xi_1+\xi_2)\sin\Delta m\Delta t \\ &\quad +2\mathrm{Im}\,\big[(\xi_1-\xi_2)\cos(\tfrac{1}{2}\Delta\lambda^*\Delta t)\sin(\tfrac{1}{2}\Delta\lambda t)\big]\big\}. \end{aligned} \tag{29}$$

There is a modification compared to Equation (25). The phenomenological parameter now constitutes the sum and difference of separate parameters for mesons going in separate directions. Due to the momentum dependence, this comes as a natural modification. KLOE is unique in producing a wide range of meson directions and will be presented as the main example for experiments conducted.

Another subtlety is that $\xi_1 + \xi_2$ aligns with the traditional or classical z, ξ, δ parameters for practical purposes and the real new effect on correlated decays comes from the $\Delta\xi$ terms. Intuitively, these carry differences of mesons moving in different directions relative to the constant background: $2\mathrm{Im}\,\big[(\xi_1-\xi_2)\cos(\tfrac{1}{2}\Delta\lambda^*\Delta t)\sin(\tfrac{1}{2}\Delta\lambda t)\big]\big\}$.

In these $\Delta\xi$ terms, one cannot carry out the standard approach of fitting functions and building useful asymmetries based on the decay-time differences. Due to the differential background effect, the time development is best considered from the creation of the particle.

One of the great advantages of the entangled mesons is that, at the time the first decay happens, the Bose statistics guarantees that the opposite-moving meson will also have the opposite flavor. The probability of the two mesons having the same flavor at the same time is excluded by the asymmetrical combination of the states shown in Equation (25).

Let us proceed by looking at the asymmetries. The first one below is the difference of $\Gamma_{f\bar{f}}$ and $\Gamma_{\bar{f}f}$:

$$\mathcal{A}_{\mathrm{CPT},f\bar{f}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) = \frac{\Gamma_{f\bar{f}} - \Gamma_{f\bar{f}}}{\Gamma_{f\bar{f}} + \Gamma_{f\bar{f}}}. \tag{30}$$

Γ signifies time integration over the decay time sum $t_1 + t_2$. Observable quantities are fitted to the decay time differences $t_2 - t_1 = \Delta t$ and are the relevant variable for examining correlated decays. They also imply tagging techniques, which allow the identification of the un-decayed meson at the time of the first decay of the other meson t_1. Between the production and the first decay, one has an observational "blind spot", $t = 0$ to $t = t_1$, so to speak. Expanding Equation (30) in terms of $\xi_1,\ \xi_2,\ \Delta m,\ \Delta\gamma$ and Δt,

$$\mathcal{A}_{\mathrm{CPT},f\bar{f}} = \frac{\mathrm{Re}\,(\xi_1+\xi_2)\sinh\frac{1}{2}\Delta\gamma\Delta t + \mathrm{Im}\,(\xi_1+\xi_2)\sin\Delta m\Delta t}{-(\cosh\frac{1}{2}\Delta\gamma\Delta t + \cos\Delta m\Delta t)}. \tag{31}$$

If that time is not integrated over, the decay rate asymmetries will contain the decay time difference but also t_1. The only terms affected will be the terms with ξ differences. This gives the $\Delta\xi$ term special significance. In ref. [4], this issue has been raised in the context of the quantum gravity environment named the "demise of tagging".

The difference between probabilities $P_{f\bar{f}}$ and $P_{\bar{f}f}$ is shown with a combination of functions $f^R_{f\bar{f}}, f^I_{f\bar{f}}$. These functions contain the usual hyperbolic and regular sine and cosine functions and dependence on characteristic values such as masses and decays of the propagating mesons.

However, these functions now include the time dependence between meson production at $t = 0$ and the time of first decay t_1, as detailed in [90].

$$\mathcal{A}_{CPT,f\bar{f}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) = \frac{P_{f\bar{f}} - P_{\bar{f}f}}{P_{f\bar{f}} + P_{\bar{f}f}}, \tag{32}$$

$$\begin{aligned} \mathcal{A}_{CPT,f\bar{f}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) \quad &= \\ &\times \left[\frac{\mathrm{Im}\,(\xi_1 + \xi_2) \sin \Delta m \Delta t \; + \; \mathrm{Re}\,(\xi_1 + \xi_2) \sinh \frac{\Delta\gamma\Delta t}{2}}{(\cosh \frac{\Delta\gamma\Delta t}{2} + \cos \Delta m \Delta t)} \right] \\ &- \left[\frac{\mathrm{Re}\,\Delta\xi \, f^R_{f\bar{f}} + \mathrm{Im}\,\Delta\xi \, f^I_{f\bar{f}}}{(\cosh \frac{\Delta\gamma\Delta t}{2} + \cos \Delta m \Delta t)} \right]. \end{aligned} \tag{33}$$

Similar function combinations are included for the asymmetries between probabilities of $P_{\bar{f}\bar{f}}$ and P_{ff} of Equation (34) below. Note that traditionally, Equation (34) would give zero result, but not if $\Delta\xi$ is taken into account. This is an SME-specific behavior which separates particle/(field) and observable/(coordinate) transformation in a constant LV background.

$$\begin{aligned} \mathcal{A}_{CPT,ff-\bar{f}\bar{f}}(\Delta t, \hat{t}, \vec{p}_1, \vec{p}_2) \quad &= \\ &\left[\frac{\mathrm{Im}\,\Delta\xi \left(\sin \Delta m \Delta t - f^I_{ff} \right) - \mathrm{Re}\,\Delta\xi \left(\sinh \frac{\Delta\gamma\Delta t}{2} - f^R_{ff} \right)}{(\cosh \frac{\Delta\gamma\Delta t}{2} - \cos \Delta m \Delta t)} \right]. \end{aligned} \tag{34}$$

Some higher-order theoretically significant effects in the CPTV parameter cannot be ignored in this situation. A nonzero probability proportional to $\Delta\xi^2$ of the two mesons decaying to the same flavor mode at the same time is not ruled out. At this point, it is prudent to mention the so-called "ω" effect of the decoherence phenomenology rooted in spacetime fluctuations in a quantum gravity scenario of Section 4.3. As said in the introduction, when substantially different theories converge raising similar questions, there is a good area to be examined further.

One must keep in mind, though, that squaring ξ amounts to squaring Δa, which is CPT-even. It is possible that this is a pattern and such effects cancel in odd cases and are zero by nature for even ones. The SME is an effective field theory confined to an energy domain where it is valid to good approximation. The suppression factors account for differences of scales and may lead to an unobservable, physically insignificant limit where the effective theory needs to merge with a more appropriate approach. This is inherent in combining spacetime physics with quantum physics.

Observations tied to momentum dependence are carried out, however. There is dependence of ξ on the meson's momentum orientation and magnitude. While the $\Lambda - \xi$ formalism is defined in a comoving (rest) frame, the typical mesons of CPT tests are relativistic, boosted, and have various initial momenta. These also change as the lab itself moves, for instance, in a frame centered on the Sun-Centered Celestial Equatorial Frame (SCCEF) [35,36].

The laboratory's position changes as it rotates in the Sun-centered frame. This translates to the scientist in the lab measuring a time dependence on the earth's sidereal frequency $\omega \simeq 2\pi(23\,\mathrm{h}\ 56\,\mathrm{min})^{-1}$ [82]. Such a dependence would certainly turn some heads toward examining the background.

By inspecting the nonminimal SME extension, one finds that higher mass dimensional terms introduce higher harmonics in $\omega\hat{t}$, a higher degree of momentum dependence, but also higher suppression by M_{Pl} [83].

Time-stamped data are recommended for experimental searches. One of the methods popular in directional and time variation is binning. Binning in laboratory regions, momentum magnitude, and sidereal time is one the most applied methods for data handling. Collaborations found optimum bin numbers. These can then be adjusted as needed for higher-dimensional studies.

The production of the mesons and the collision beam direction also carries special meaning in orientation studies. If one allows the Sun-centered frame to be the frame of the constant background, the geographic location of a laboratory, its typical beam line and the momentum spread characteristic of the produced mesons can also be compared to SCCEF.

The mesons' directions can be considered collimated or randomly spread. It is best to review this in terms of the respective set up. Much experimental advantage can also be gained from knowing the relative direction of a pair of mesons produced together. These important data are summarized for each collaboration in Section 5. That section will provide details of the momentum analysis based on the example of the KLOE detector. Once KLOE is well described, it is best to review similar aspects in each respective set up. As can be seen in the Data Tables published [10], the experiments have access to different components of the relevant coefficient.

4.3. Quantum Gravity Decoherence and CPT Violation in Entangled Mesons

As mentioned above, when CPTV scenarios are such that the entangled quantum system is considered open to a quantum gravity fluctuating background where the issue becomes unitarity, i.e., probability conservation from the low-energy observer's view, the Weiskopp–Wigner approximation was found to be less practical than the density matrix description of open quantum systems. Parameterization can be set up in a simple way within the linear Lindbad approach. Since this detour deserves its own set of long articles, the treatment is rather introductory, assuming serious follow-up work on the reader's side.

Energy conservation, as well as a monotonic increase in entropy, is assumed, and the $\Delta Q = \Delta S$ rule applies. Expanding the equation of the density matrix evolution of Equation (10) with Lindbad modification using the mass and decay parameters of neutral kaons:

$$H_{\alpha\beta} = \begin{pmatrix} -\Gamma & -\frac{1}{2} & -Im\Gamma_{12} & -Re\Gamma_{12} \\ -\frac{1}{2}\delta\Gamma & -\Gamma & -2ReM_{12} & -ImM_{12} \\ -Im\Gamma_{12} & 2ReM_{12} & -\Gamma & -\delta M \\ -Re\Gamma_{12} & -2ImM_{12} & \delta M & -\Gamma \end{pmatrix}. \quad (35)$$

This form of parameterization is based on the condition of having a positive definite density matrix for a single kaon as $\alpha, \gamma > 0$ and $\alpha\gamma > \beta^2$ The parameters α, β, γ all violate CPT and CP symmetry.

$$\not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix}. \quad (36)$$

The parameters α, β, γ all violate CPT and CP symmetry. The parameter reduces to γ for the evolution of a correlated kaon pair, where a further condition for the positivity of the density matrix is $\alpha = \gamma$ and $\beta = 0$.

This can only be performed using the same tools, such as decay rate analysis. In density matrix notation, a particle starting at $t = 0$ in a pure K^0 state has a decay rate into final state f: $R(K^0 \to f) = Tr[O_f \rho(t)]$. The tracing comes from the essence of this approach. For the observer at asymptotic infinity, some initial information appears lost in "trapped" degrees of freedom. This the observer traces over. So, the trace is over trapped quantum states outside of observability [73].

The phenomenological parametrization of QG effects can also pertain to the *omega effect* with complex parameter ω; it parameterizes the modification of the Bose statistics, which is a strong quantum mechanical requirement for entangled neutral mesons. It guaranties for the $l = 1$ angular momentum number that a state with two identical bosons is forbidden.

Returning for the phenomenology introduction, we established that if there is an intrinsic CPTV such that the CPT operator is ill-defined, then the antiparticle Hilbert space could have components independent of the corresponding particle Hilbert space [73]. The neutral mesons are practically indistinguishable except for flavor, but that feature could be broken by the ill-defined CPT operator.

The signal of observable consequence would be a small addition of P parity symmetry opposite to the initial state. The parameter ω is introduced as a linear multiplier of the extra term.

$$\begin{aligned} |K\overline{K}, t=0\rangle &= \frac{(|K^0, \vec{p}\rangle|\overline{K}^0, -\vec{p}\rangle - |\overline{K}^0, \vec{p}\rangle|K^0, -\vec{p}\rangle)}{\sqrt{2}} \\ &+ \frac{\omega}{2}\frac{(|K^0, \vec{p}\rangle|\overline{K}^0, -\vec{p}\rangle + |\overline{K}^0, \vec{p}\rangle|K^0, -\vec{p}\rangle)}{\sqrt{2}}. \end{aligned} \tag{37}$$

One of the most useful NM decay modes is 2π decays. An alternative specific way for these types of decays of the "omega effect" is to introduce a phenomenological parameter into the intensity formula of 2π decays. The violation would be indicated in the interference term and the relevant parameter was defined as ζ [92].

$$\begin{aligned} I \quad & (t_1, t_2; \zeta_{SL}) = \frac{1}{2}|\langle\pi^+\pi^-|K_S\rangle|^4|\eta_{+-}|^2 \\ \times \quad & \left(e^{-\Gamma_L t_1 - \Gamma_S t_2} + e^{-\Gamma_S t_1 - \Gamma_L t_2} - 2(1-\zeta_S L)e^{-(\Gamma_S+\Gamma_L)(t_1+t_2)/2}\cos(\Delta(t_1-t_2))\right), \end{aligned} \tag{38}$$

where t_i represents the proper times of the two kaon decays, Γ_S and Γ_L are the decay widths of K_S and K_L, Δm is their mass difference, and

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|K_L\rangle}{\langle\pi^+\pi^-|K_S\rangle} \tag{39}$$

The indices differentiate ζ_{K_S,K_L} in the (K_S, K_L) basis, while $\zeta_{K^0,\overline{K}^0}$ in the $(K^0, \overline{K}^0)$ bases. Experimental results can be found elsewhere, but the best results are also given in Section 5.1 [77,78]. The decoherence parameter ζ can provide a kaon and decay-mode-specific testable indicator of ω [73].

In summary, the relevant CPTV parameters defined by groups outside the SME group and measured or bounded with factory mesons are the γ from the density matrix and ζ and ω from the omega effect. The Belle results are discussed separately as comparisons of functions, not parameters. All searches have the appropriate SME coefficients, specifically the a coefficient for the quark sector and the k coefficients for the scalar sector applied to NM.

The difficult task is now to organize experiments and their results based on the NM phenomenology. The author does not aim for a complete survey. What is more helpful is to learn to navigate the vast information collected over decades.

5. Experiments

The SME's defining goal was to bring BSM theories under scrutiny where they are falsifiable, constrainable, or detectable. In the following, each experiment whose contribution to this review is enlightening is discussed to various levels of depth, piecing together a full picture for the reader. Each collaboration also publishes a detailed description of the experimental construct, method of producing mesons, kinematical analysis, charge identification, detectors used, and statistical analyses.

To have a deeper understanding of tagging and using flavor-specific or CP-specific decays as a means of evaluating discrete symmetries, the reader should consult general source books such as [13] or the source paper of the experimental publication. Because each collaboration aspires to contribute new results or a new angle on earlier ones, a different set of parameters are used for their phenomenology.

It is practical to separate high-boost collimated beam experiments or high-energy colliders with uncorrelated meson production [35,93–96] from the meson factories with low-boost, correlated mesons [76,77,97–100]. The high boost enhances sensitivities for CPTV bounds; however, the low-boost experiments are usually marked by a large number of clean events, low background, excellent particle identification, and kinematical analysis. Of course, this is by no means exclusive. Each collaboration shines with some unique resourceful initiative.

Obviously, what was said in Sections 3.1 and 4 is tightly connected to the experimental details summarized below. The goal is to demonstrate how real physical observation takes shape from all the considerations above. It is instructive in transitioning to specific experimentation to apply the above thought process to a particular detector.

KLOE is an excellent example. They put one of the best bounds on the a_μ coefficient component by component. KLOE presents the most complete geometry for testing momentum-dependent effects beyond the traditional approach. For that, they demonstrated binning techniques in sidereal time and detector regions.

Quantum entanglement and CPTV in the context of quantum mechanics violation was also examined for the quantum gravity phenomenology, with results of specific parameters introduced in Sections 3.1 and 4.

The production of the mesons and the collision beam direction also carries special meaning in orientation studies. The mesons' directions can be considered collimated or randomly spread. It is best to review this at each respective set up.

Much experimental advantage can also be gained from knowing the relative direction of a pair of mesons produced together. These important data are summarized for each collaboration. As can be seen in the Data Tables published [10], the experiments have access to different components or their combination for each coefficient coming from the data gathering and analysis designed.

From the experimental lab's point of view, the direction of the production beam and collimated beam is the main geometrical reference. When those do not coincide with the compass rose directions, like east–west, another constant rotation must be implemented. Once included, these do not play any role in analyzing variations. In general, the colatitude of the experimental facilities and the azimuth or corresponding value of the meson beam is present in the full formalism.

5.1. *Symmetric Collider Meson Factory at KLOE*

The KLOE collaboration will be treated with special attention. KLOE has carried out all searches discussed in this paper, and being a factory, that includes the studies of quantum mechanics [74–80,99].

At KLOE, the neutral mesons are created in a symmetric e^-e^+ collider originally set out to operate at the energy equivalent to the mass of the Φ quarkonium. The decay of the kaons from the Φ is back-to-back created at rest in the center-of-mass frame as pairs of neutral or charged kaons, and they exit back-to-back such that for the two different 3-momenta of Equation (26), one can take $\vec{p}$ and $-\vec{p}$. Their $\gamma = 1.02$, $\gamma\beta = 0.22$ [74,75,77,78]

Important is that DAΦNE produces the kaon and antikaon in a quantum correlated state. While the created pairs can move at full spread, two connected mesons of a particular pair propagate in opposite directions in the same line. A full introduction to the technical details of the lab and detector itself can be found in [77,78].

Here, some features are emphasized to give a picture of what experimenters have at their disposal. The events can be reconstructed using the fact that charged tracks and energy deposited are observed and grouped for the ones happening at the same time [78]. The detector geometry, the trigger conditions, and the apparatus response are all accounted for in a Monte Carlo simulation.

The neutral kaons have a fairly small set of decay modes, and the propagating states have a short- and long-lived version, facilitating separate detection and the observation of flavor oscillation at a macroscopic scale. These propagating states combine flavor eigenstates and decay into CP-specific and flavor-specific modes. The kaon factory at KLOE can produce huge amounts of entangled neutral mesons.

The kaons decay as a correlated pair from $\Phi \to K^0\overline{K}^0$ in a pure $J^{PC} = 1^{--}$ state. Equation (26) shows an amplitude where two kaons decay into mode f_1 at t_1 and the other to f_2 at t_2. The initial state at $t = 0$ looks like

$$|K\overline{K}, t = 0\rangle = \frac{(|K^0, \vec{p}\rangle|\overline{K}^0, -\vec{p}\rangle - |\overline{K}^0, \vec{p}\rangle|K^0, -\vec{p}\rangle)}{\sqrt{2}}. \tag{40}$$

As mentioned before, the laboratories use convenient velocity coordinates relative to the detector, the colliding beam, and the interaction point. At KLOE, these are straightforward spherical coordinates θ, ϕ angles for direction.

For path length, energy momentum data and time of flight are used and the magnitude of the velocity of the kaon is known from the decay kinematics as $\beta = 0.216$ in the ϕ frame and range between $\beta = 0.193$ and 0.239 in the lab. As hinted earlier, to express β and Δa_μ in the same frame, the laboratory coordinates of the velocity are chosen for transfer to the SCCEF. One benefit of this choice is a clear way to see dependence on sidereal time. In all subsections of the experiments, the notation is:

Sidereal time: $\hat{t}$
Sidereal rotation frequency of earth: ω

The SCCEF is denoted by X, Y, Z, using all upper case letters. The Z axis points to the celestial north pole at equinox 2000.0 at declination 90° and the X, Y axes are at declination 0°, with right ascension 0° for X and 90° for Y [101].

To get from the north pole Z axis to the Up direction at the lab's location, one must execute a rotation by the colatitude (90 deg − latitude). At KLOE, the colatitude is $\chi(\hat{Z}, \hat{U}) = 48.2°$.

The expression of the local laboratory Up ($\hat{U}$), and the tangential plane of compass directions, east ($\hat{E}$) and south ($\hat{S}$) in terms of the SCCEF coordinates, is

$$\begin{aligned} \hat{S} &= -\sin\chi \cdot \hat{Z} + \cos\chi \cos\omega\hat{t} \cdot \hat{X} + \cos\chi \sin\omega\hat{t} \cdot \hat{Y}, \\ \hat{E} &= \cos\omega\hat{t} \cdot \hat{Y} - \sin\omega\hat{t} \cdot \hat{X}, \\ \hat{U} &= \cos\chi \cdot \hat{Z} + \sin\chi \cos\omega\hat{t} \cdot \hat{X} + \sin\chi \sin\omega\hat{t} \cdot \hat{Y}. \end{aligned} \tag{41}$$

Beams are colliding in the EWSN plane. The direction of the incoming positron beam axis is named $\hat{Z}'$, about which the lab coordinates are set up, as discussed above. This forms an angle of 50° to the south direction. This is the second rotation by a constant angle, now in the EWNS plane.

The direction perpendicular to the beam directions becomes $\hat{X}'$ and $\hat{Y}'$, which coincides with the Up direction. In South–East–Up coordinates,

$$\begin{aligned} \hat{Z}' &= \sin 40° \cdot \hat{S} - \cos 40° \cdot \hat{E}, \\ \hat{X}' &= \cos 40° \cdot \hat{S} + \sin 40° \cdot \hat{E}, \\ \hat{Y}' &= \hat{U}'. \end{aligned} \tag{42}$$

The velocity expression in the Sun-centered frame is rather visually cumbersome, which can be handled by separating detector-specific geometrical functions and sidereal rotation:

$$\vec{\beta}_{XYZ} = \beta\big[c\hat{Z} + \big(a\cos\omega\hat{t} + b\sin\omega\hat{t}\big)\hat{X} + \big(a\sin\omega\hat{t} - b\cos\omega\hat{t}\big)\hat{Y}\big]. \tag{43}$$

In the detector-specific functions $a(\theta,\phi), b(\theta,\phi), c(\theta,\phi)$, the constant angles are also absorbed:

$$\begin{aligned} a &\equiv \sin\chi\sin\theta + \cos 40^\circ\cos\chi\cos\theta\cos\phi \\ &\quad - \sin 40^\circ\cos\chi\cos\theta\sin\phi, \\ b &\equiv \sin 40^\circ\cos\theta\cos\phi - \cos 40^\circ\cos\theta\sin\phi, \\ c &\equiv \cos\chi\sin\theta - \cos 40^\circ\sin\chi\cos\theta\cos\phi \\ &\quad + \sin 40^\circ\sin\chi\cos\theta\sin\phi. \end{aligned} \tag{44}$$

For the observer,

$$\beta^\mu = \gamma(1, \vec{\beta}_{XYZ}). \tag{44}$$

As we have seen above in the SME framework, correlated kaons form asymmetries which will contain $\xi_1 + \xi_2$ and $\xi_1 - \xi_2$.

For the lab observer,

$$\beta^\mu = \gamma(1, \vec{\beta}_{XYZ}). \tag{45}$$

KLOE measured $\Delta a_{X,Y,Z}$ spatial parameters via detector binning in Θ and sidereal time stamping and binning [74]. The introduction of Ref. [74] is a concise summary including figures that show specifically the sidereal and detector binning. That result is an already admirable 10^{-18} GeV constraint on each component of the "a" coefficient separately.

Important KLOE experimental data is presented below (see also [10]):

Boost: $\gamma = 1.02$; $\beta = 0.193$ to 0.239 in the lab.
Colatitude: 48.2°.
Orientation of direction of the incoming positron beam axis: 50° to South.
Decay used: CP-specific 2π and semileptonic $\pi e \nu$. For an excellent summary, see Section 2.2.2 of ref. [77].

Order-of-magnitude bounds for mSME [75]:

$\Delta a_{T,X,Y,Z} \sim 10^{-18}$ GeV.

The newest results from KLOE are the separate T and CPT measurements based on combining semileptonic and CP-specific decay modes to design an explicit T- and CPT-reserved process for measuring TV and CPTV independently of CPV. This new method, thought to be unrealizable for decaying particles, is a truly elegant addition to SME searches [17–19].

The CPT and quantum mechanics test have parameters γ for the density matrix description. Note that for the entangled kaons, only one parameter remains after imposing conditions of positive definite density matrix [73,77,79].

Order-of-magnitude bounds for decoherence parameters:

From Section 3 of ref. [77]:
$\zeta_{K_S,K_L} \sim 0.003$, $\zeta_{K^0,\overline{K}^0} \sim 10^{-7}$.
From Section 3.4 ref. [77]:
for the density matrix parameter $\gamma \sim 10^{-21}$ and for the ω-effect: $\mathrm{Re}(\omega), \mathrm{Im}(\omega) \sim 10^{-4}$.

5.2. Asymmetric Collider Meson Factories, BaBar, and Belle

There is a notable difference between symmetric colliders such as KLOE and asymmetric ones such as BaBar or Belle. In a symmetric collider, such as the kaon factory, the sum of velocities of the off-coming mesons is zero and signals are related to the velocity difference. In the asymmetric colliders such as the B factories, the off-coming meson velocities are in a narrow cone. The vector sum of their velocities is close in magnitude to the scalar sum of the absolute magnitudes of each velocity vector. The velocity vector differences are small and neglected in B factory experiments.

The dependence on the sum of the velocities aligns more with the earlier searches and relates to the respective phenomenological parameters introduced by each collaboration, which neglect any momentum dependence. The dependence on the difference of the velocities is an entirely SME-specific realization of effects that point to important insights about possible more subtle but also more fundamental insights about quantum processes placed into a nontrivial spacetime [90].

BaBar

BaBar went to all four stages in the ever-developing NM searches. First, BaBar set the formalism for traditional correlated searches. Second, they measured ξ. The third step offered bounds in real and imaginary combination as well as sidereal analysis consistent with SME. BaBar also carried out the transition studies and established experimentally separate T violation as well as testing for separate CPTV.

BaBar and Belle are both asymmetric electron proton colliders produced from the Υ quarkonium in correlated form as $\Upsilon(4S) \to B\overline{B}$ decays. Typical decay mode: inclusive dilepton events. Being B particles, the propagating states are denoted Light and Heavy [98]. We have shown the $z \equiv \xi$ equivalence and the basic notation match above. The leading-order SME CPTV contribution in this notation is given as

$$z \approx \frac{\beta^{\mu}\Delta a_{\mu}}{\Delta m - i\Delta\Gamma/2}. \tag{46}$$

Counting semileptonic events for the decay rate asymmetries, one obtains 4 percent of all inclusive dilepton events of the Υ decays. This a large sample in a factory to study CPTV in mixing. The semileptonic decays can be $b \to Xl\nu$. "l" denotes the lepton, which could be an electron or muon or their antiparticles. In direct semileptonic decays, the leptons ensure flavor tagging.

The correlated state is a P-wave. BaBar uses the well-practiced tagging method to use the first decay as an indicator of the second B's flavor at that time. For a pair of B's, three lepton pair outcomes can happen: l^+l^-, l^-l^-, or l^+l^+. A B^0 gives an l^+, and the time at which it happens is t^+. For the antimeson, the signs are negative. The difference of decay times is $\Delta t = t^+ - t^-$ as a choice of definition. This value can be positive or negative.

The number of decays with one positive and one negative lepton is proportional to

$$\begin{aligned} N^{+-} \quad &\propto \quad e^{-\Gamma|\Delta t|}\Big\{\cosh(\Delta\Gamma\Delta t/2) + \cos(\Delta m\Delta t) \\ &+ \quad 2Imz\sin(\Delta m\Delta t) - 2Rez\sinh(\Delta\Gamma\Delta t/2)\Big\}, \end{aligned} \tag{47}$$

leading to the familiar expression

$$\begin{aligned} A_{CPT}(|t|) = \frac{P(B^0 \to B^0) - P(\overline{B}^0 \to \overline{B}^0)}{P(B^0 \to B^0) + P(\overline{B}^0 \to \overline{B}^0)} \quad &= \\ \frac{N^{+-}(\Delta t > 0) - N^{+-}(\Delta t < 0)}{N^{+-}(\Delta t > 0) + N^{+-}(\Delta t < 0)} \quad &\simeq \\ 2\frac{Imz\sin(\Delta m\Delta t) - Rez\sinh(\Delta\Gamma\Delta t/2)}{\cosh(\Delta\Gamma\Delta t/2) + \cos(\Delta m\Delta t)}. \end{aligned} \tag{48}$$

BaBar was time-stamped in sidereal time, and a linear function fit for sidereal variation was carried out:

$$z = z_0 + z_1 cos(\Omega\hat{t} + \Phi). \tag{49}$$

It should be noted that, in the Babar and Belle paper, the asymmetric collider launches the two B particles into a narrow cone. Usually, this justifies using only $\xi = \xi_1 + \xi_2$ as a beam and the sum of the velocities as the beam's direction. While the B factories cannot produce the wide spatial spread the K factory does, since options are limited, one should analyze for $\Delta\xi$ as well. Details of that are contemplated in [90].

The direction of this narrow cone at BaBar is 37.8° East of South, giving $\cos\chi = \hat{Z} \times \hat{z} = 0.628$.

Even though it would raise concerns about tagging and both the omega effect of quantum gravity background and the momentum difference of the SME phenomenology-predicted signal as at most proportional to the square of the respective parameters, looking for decay modes that violate the Bose symmetry and produce the same leptonic flavor tag at the same time is worthwhile. Its detection would open a profound way to think further about the issue this paper is all about.

An interesting late addition at BaBar was an improved bound on CPTV based on $c\bar{c}K^0$ decays, which are CP eigenstates, not semileptonic flavor-specific states. This gives a means to give a bound on Re(z). Due to the smallness of $\Delta\Gamma$ in the B system, a bound on Re(z) was not easy to establish from the semileptonic asymmetries [97]

$$\beta^\mu \Delta a_\mu = \gamma \Delta a_0 - \gamma \beta_Z \Delta a_Z = Re(z) \times \Delta m. \tag{50}$$

BaBar important experimental data (see also [10]).

Boost: $\langle \beta\gamma \rangle \simeq 0.55$, $\gamma \simeq 1.14$.
Colatitude: 52.6°.
Orientation of beam direction: 37.8° East of South.
Decay used: semileptonic decays can be $b \to Xl\nu$,
and $c\bar{c}K^0$ decays; CP eigenstates.

Order-of magnitude bounds for mSME:
from ref. [98]:

$$\begin{aligned} \Delta a_0 - 0.30 \Delta a_Z &\sim 10^{-15} GeV, \\ \sqrt{(\Delta a_X)^2 + (\Delta a_Y)^2} &\sim 10^{-15} GeV, \end{aligned} \tag{51}$$

from ref. [97]:

$$\Delta a_0 - 0.30 \Delta a_Z \simeq 10^{-14}. \tag{52}$$

Belle

Belle is also an asymmetric energy electron–positron collider still very active and updating to produce increasingly large amounts of $B\,\overline{B}$, excellent particle identification, and kinematics analysis, even for neutral particles. Currently, it is the leading B factory.

Belle important experimental data [100]:

Boost: $\langle \beta\gamma \rangle \simeq 0.28 - 0.425$.
Colatitude: 54°.
Decay used:$J/\psi K^0$.

Belle physics and detector details can be found in [102]. Results for the imaginary and real part of the phenomenological parameter $z \equiv \xi$ are published without sidereal analysis in [100]. The geometrical details in the lab and in the SCCEF are given in [90].

Since the mesons are correlated in these experiments as well, every CPTV framework discussed above could be tested. A separate work was indeed devoted to searches with quantum correlation [103].

Notable in this study is the recognition of the importance of a time development of the entangled mesons as opposed to time development if they disentangle. The work separates spontaneous disentanglement (SD) right after decay, from the correlated quantum mechanical propagation, where the flavor identified upon decay of the first meson marks the flavor of the undecayed meson as opposite.

It is interesting to point out that in the SME, where the mesons interact in a momentum-dependent manner with the CPTV background, the situation is between these two scenarios.

However, the way this is discussed in ref. [103] is highly illuminating. In the spontaneous disentanglement, right at production, the $\overline{B}^0$ and B^0 assume independent time development tracked by two time variables t_1 and t_2. The asymmetry is a function of $t = t_1 + t_2$ as well as of $\Delta t = |t_1 - t_2|$, giving

$$A_{SD} = \cos(\Delta m t_1)\cos(\Delta m t_2) = \frac{1}{2}[\cos(\Delta m(t_1 + t_2)) + \cos(\Delta m \Delta t)], \tag{53}$$

where Δm is the mass difference between the $\overline{B}^0$ and B^0 mass eigenstates and $\Delta t = |t_1 - t_2|$ is the proper time difference of the decays. For the B system, it is a good assumption to neglect the lifetime differences and CPV in mixing. The entangled quantum mechanical asymmetry is formed by the same flavor and opposite flavor normalized rate differences with flavor-specific decays observed, leading to an expression:

$$A_{QM} = \cos(\Delta m \delta t). \tag{54}$$

Please see ref. [103] for an analysis based on Δt-dependent functions comparing two SD models and the quantum mechanical time development.

It must be pointed out that neither corresponds to the SME scenario, which is a more involved analysis of specific time dependence without assumption of disentanglement at production. One thing is in clear correspondence, and that is the dependence on proper time sum as well as the proper time difference. This translates into the challenge of observing meson behavior in that time "blind spot" discussed in Section 4.2. For details, see [90]. The Belle investigation returned results consistent with the quantum mechanical correlated description.

It should be added that Belle also led by measuring the neutral B particle antiparticle mass difference [104]

$$\frac{|m_{B^0} - m_{\overline{B^0}}|}{m_{B^0}} \lesssim 10^{-14}. \tag{55}$$

5.3. Discussion of Quantum Entanglement and Spacetime Coupling in Factory Experiments

There is a notable difference between symmetric colliders such as KLOE and asymmetric ones such as Belle.

In a symmetric collider, such as the kaon factory, the vector sum of 3-velocities of the off-coming mesons is zero and spatial bounds for $\Delta a_X, a_Y, a_Z$ signals are related to the velocity difference, which we take in first-order approximation in CPTV to align with the 3-momenta.

This gives the KLOE results a new interpretation. Bounds on the spatial components relate to the momentum difference of the kaons produced together and that is a unique feature of the momentum-dependent behavior derived from the SME. Constraints on Δa_T, however, are related to the sum of the momenta, hence to $\xi_1 + \xi_2$, which aligns with the traditional parameterization.

In the asymmetric colliders, such as the B factories, the off-coming meson velocities are in a narrow cone. The vector sum of their velocities is close in magnitude to the sum of the magnitudes of the velocity vectors. The velocity vector differences are small, so much so that it is considered one beam direction even after the B-s are produced. However, if it is taken into account, all components of Δa remain in $\Delta\xi$ and $\xi = \xi_1 + \xi_2$ and the full factory analysis can be performed [90].

The dependence on the sum of the velocities aligns more with the earlier searches and relates to the respective phenomenological parameters introduced by each collaboration, which neglect any momentum dependence. The dependence on the difference of the velocities is an entirely SME-specific realization of effects that point to important insights.

There is an effect on the tagging by $\Delta\xi^2$, but its probability of producing a false tag is small enough to be neglected. Interestingly, both the ω effect and the coupling to the LV background with a momentum difference found second-order possible effects in the respective parameter [4,90]. These are likely to be unobservable and/or do not fit into the domain assumed for the effective field theory in question.

However, both factories, B or K, could carry out a search for the forbidden scenario by Bose statistics of the same flavor decay at the same time since such a signal would have profound meaning. Unfortunately, the small likelihood in this case also seems prohibitive.

The subtlety of spacetime background influences at the Planck scale or of a constant very weak background at large scales remains an interesting open question. Using the effective field theory approach, extra care must be taken to examine whether such effects fit into the domain assumed for the effective field theory of the SME.

5.4. *D∅*

D∅ is a proton–antiproton collider of center-of-mass energy 1.8 TeV located at the Fermilab Tevatron collider. It is notable for finding the top quark, studies of W mass, gauge–boson couplings, jet production, leptoquarks, and supersymmetry studies, as well as CPTV in both B_d^0 and B_s^0 mesons. The Tevatron produces b quarks primarily via $b\overline{b}$ pairs. The mesons are not correlated. D∅ conducted CPTV symmetry analysis for B_s^0 particles [20,93,105]. These oscillate into antiparticles many times before decaying, as opposed to the B_d^0.

The collaboration measured the charge asymmetry of like-sign dimuon events due to semileptonic b-hadron decays. Semileptonic decays are popular in discrete symmetry research, since they mark the flavor of the decaying particle. The dimuon charge asymmetry is

$$A_b^{ssdimuon} = \frac{(N_b^{\mu^+\mu^+} - N_b^{\mu^-\mu^-})}{(N_b^{\mu^+\mu^+} + N_b^{\mu^-\mu^-})}, \tag{56}$$

where "ssdimuon" denotes same-sign dimuon asymmetry.

The muon is signaling a semileptonic decay and the muon charge specifies whether the decay came from B or $\overline{B}$. For a pair produced, two muons are observed. If they are the same sign, then one of the b or $\overline{b}$ must have oscillated into the $\overline{b}$ or b, respectively. It is assumed that after correcting for background, the asymmetry is due to neutral B-meson mixing.

There is another class of decays, inclusive wrong-sign single-muon (marked as "smuon") decays, which signal B meson oscillation through the sign of the muon.

$$A_b^{smuon} = \frac{(\overline{B} \to \mu^+ X - B \to \mu^- X)}{(\overline{B} \to \mu^+ X + B \to \mu^- X)}. \tag{57}$$

The second asymmetry and background analysis leads to minimizing the $A_b^{ssdimuon}$ dimuon asymmetry uncertainty. This corrected measurement of the asymmetry can be compared to the SM prediction. D∅ found a 3.2 standard deviation from SM prediction, showing anomalous CPV pointing beyond the SM.

The safer assumption is taken to be a CPT-invariant scenario where T and CPV need to find a BSM source. However, the SME provides just the framework to use this as sensitivity for CPT breaking. For the B_s^0, it represented the first.

While the details are for the reader to follow up on, attention can be drawn to the discussion in the D∅ publication of the connection of $(\Delta a^K)_\mu, (\Delta a^{B_d})_\mu, (\Delta a^{B_s})\mu$. Since only differences of quark coefficients are measurable in a meson system, and each meson contains a combination of the same quarks, a certain constraint makes sense:

$$(\Delta a^K)_\mu + (\Delta a^{B_d})_\mu + (\Delta a^{B_s})\mu \cong 0. \tag{58}$$

The boost and orientation dependence for D∅ is based on uncorrelated meson pairs.

$$\mathcal{A}_{CPT} = -\frac{\Delta\Gamma\gamma^{D\varnothing}}{\Gamma_s \Delta m_s}[\Delta a_T - (\cos\alpha\sin\chi)\beta_z\Delta a_z + \sqrt{(\cos^2\alpha\cos^2\chi + \sin^2\alpha)}\sin(\omega\hat{t} + \phi)\beta_z\Delta a_\perp]. \quad (59)$$

When the mesons are substantially boosted along the beam, their a^μ components are constrained in the perpendicular sidereal time-dependent part $\Delta a_\perp$ and the T and Z combination that does not depend on sidereal time $\Delta a_T - (\cos\alpha\sin\chi)\beta_z\Delta a_z$, including a constant geometrical factor from the latitude of the experiment and the orientation of the beam relative to the compass rose of the tangential plane at that latitude; for $D\varnothing$, it is 0.396.

$D\varnothing$ important experimental data (see also [10]):
Boost: $(\beta\gamma)^{D\varnothing} \cong 4.1$;
Colatitude: $\chi = 48.2°$;
Orientation of $p\overline{p}$ beam: $\alpha = 219.5°$;
Decay used: $B_s^0 \to \mu^+ D_s^- X$.

Appropriately, the bound returned from the $D\varnothing$ collaboration is

$$\Delta a_\perp < 1.2 \times 10^{-12} GeV, \quad -0.8 < \Delta a_T - 0.396\beta_z\Delta a_z < 3.9 \times 10^{-13}\text{GeV}. \quad (60)$$

5.5. KTeV, E773

The best and earliest bounds for neutral mesons came from a realization that highly collimated beams with strong boost can deliver better limits. One of these was KTeV at Fermi National Laboratory. KTeV also gave the leading bound on the most direct figure of merit, the particle–antiparticle mass difference:

$$\frac{|m_{K^0} - m_{\overline{K^0}}|}{m_{K^0}} \lesssim 10^{-19}. \quad (61)$$

Kaons were produced by a proton beam incident on a fixed beryllium oxide target; hence, it is referred to as a fixed-target experiment. The non-kaon components are reduced and the remainder is collimated into two beams. This is important for momentum dependence, since one works with known kaon beam direction in the lab frame in the NSWE plane at the latitude of Fermilab. KTeV produces two K_L beams, one of which passes through a regenerator, i.e., through nuclear matter. Since K^0 and $\overline{K}^0$ interact differently with the nucleus, the short-lived K_S regenerates through rescattering with nuclear matter [13].

The KTeV collaboration measured the direct CPV parameter and the lifetime and decay constant of the K_S. In the CPV domain, it measured the difference between the CPV parameter phase and the superweak phases, as well as relative phases between the CPV and CPI decay amplitudes for ϕ^{+-} and ϕ^{00}. These phase differences allow tight constraints on the $K^0\,\overline{K}^0$ mass difference of the order of $10^{-19}\frac{GeV}{c^2}$. For the CPTV parameter, it also reached $10-22$ sensitivities. E773 was an order of magnitude lower [35,94,95].

KTeV and E773 important experimental data (see also [10]):
Boost: $\beta\gamma \approx 100$.
From latitude and beam direction: $\cos\alpha\sin\chi \cong 0.6$.
Decay used: $K_S^0 \to \pi\pi$.

Order-of-magnitude bounds returned from the $E773$ collaboration:

$$|\Delta a_0 - 0.6\Delta a_Z| \lesssim 5 \times 10^{-21}\text{GeV}, \quad (62)$$

and bound for KeTV

$$\Delta a_T - 0.6\beta_z \Delta a_z \lesssim 10^{-20} m_K. \quad (63)$$

5.6. CPLEAR

One of the earliest results for the complex CPTV parameter δ was given by the CPLEAR [106] collaboration measuring the real part of it to the accuracy of 10^{-4}. Note that the relationship between ξ and δ is $2\delta = \xi$ [36].

The experiment was carried out for K mesons produced in proton–antiproton annihilation. The strangeness (the flavor) of the produced kaon is known at production by the sign of the charged kaon produced along with it. The strangeness at decay can be tagged by the sign of the decaying lepton.

In this work, we focus on CERN's LHCb NM searches, which aimed to deliver improved bounds for neutral D and B mesons and carried out a full SME analysis.

5.7. LHCb

LHC is a circular accelerator colliding protons at 14 TeV center-of-mass energy. It also studies $b\bar{b}$ pairs. In the strong interactions, b quarks are always produced in pairs. The LHCb operates at lower energy, about 7–8 TeV center-of-mass energy, than, for instance, ATLAS.

Experiments at the LHCb thrive for a complete analysis of any testable NM phenomenon. It is a large collaboration, correspondingly ambitious. It performs separate study of direct CP and CPT asymmetries as opposed to effects in mixing ($\mathcal{A}^{dir}, \mathcal{A}^{mix}$). Decays to CP eigenstates and to flavor-specific modes are both considered. The analysis also takes into account the respective characteristics of the meson system, orientation and boost dependence, and features stemming from an internally consistent phenomenological set up. CPT searches pursue the traditional approach, where the CPTV parameter is taken as constant, and also perform their evaluation in the SME framework.

The difference between direct CPV and CPTV and violation in mixing should be clarified. Direct violation means that there is an instantaneous decay rate difference for particle and antiparticle as $A_f \neq \overline{A}_{\overline{f}}$. Direct CPVs and CPTVs are hard to distinguish. Violation in mixing occurs as the particle–antiparticle oscillate into each other during propagation. One of the strengths of LHCb's work is that discrete symmetry violations are considered in all possible manifestations.

Let us define decay rate asymmetries for CPT and CP in a general form:

$$\mathcal{A}_{CPT}(t) = \frac{\overline{P}_{\overline{f}}(t) - P_f(t)}{\overline{P}_{\overline{f}}(t) + P_f(t)},$$
$$\mathcal{A}_{CP}(t) = \frac{\overline{P}_f(t) - P_{\overline{f}}(t)}{\overline{P}_f(t) + P_{\overline{f}}(t)}, \quad (64)$$

where $f, \overline{f}$ indicate decay modes and their conjugates, and $P, \overline{P}$ indicate a neutral meson and its antiparticle.

These refer to tagged asymmetries. In the LHCb treatment, the production of the meson–antimeson pairs is taken as uncorrelated. Since tagging in a hadron collider is relatively poor, it is necessary to also specify an untagged asymmetry for flavor-specific decays.

$$\mathcal{A}_{untagged}(t) = \frac{[P_f(t) + P_{\overline{f}}(t)] - [P_{\overline{f}}(t) + \overline{P}_{\overline{f}}(t)]}{[P_f(t) + P_{\overline{f}}(t)] + [P_{\overline{f}}(t) + \overline{P}_{\overline{f}}(t)]}. \quad (65)$$

LHCb can be taken as the example of uncorrelated decay rate asymmetries with

$$\mathcal{A}^{uncor}_{CPT}(t, \hat{t}, \vec{p}) = \frac{Re(z)\sinh\frac{\Delta\gamma t}{2} + 2Im(z)\sin\Delta mt}{(1 + |z^2|)\cosh\frac{\Delta\gamma t}{2} + (1 - |z^2|)\cos\Delta mt}, \quad (66)$$

where "z" used at LHCb is equal to "ξ" used in the SME formalism. Note also that, at this stage, the formalism can still be taken as traditional with a constant $\xi \equiv z$ value.

Since, in the SME phenomenology, flavor-specific decays are favored, in this article, this remains the case as well. The focus is on CPTV in mixing. Note that, for decay modes to CP eigenstates where $f = \overline{f}$, a combined CPV and CPTV expression can be given [91,96].

For the analysis of the orientation and boost dependence:

For mesons moving, the real part of the CPTV parameter $z \equiv \xi$ along the beam axis,

$$\begin{aligned} Re(z) &\approx \frac{\gamma}{\Delta m}[(\Delta a_0 + \cos(\chi)\Delta a_Z \\ &+ \sin(\chi)[\Delta a_Y \sin(\omega\hat{t}) + \Delta a_X \cos(\omega\hat{t}))]. \end{aligned} \tag{67}$$

LHCb important experimental data (see also [10]):

Boost $\langle\beta\gamma\rangle \cong 15 - 20$;
Beam East of North 236.3°;
Colatitude $\chi = 112.4°$;
Decay used $B_d^0 \to J/\Psi K_S^0$;
Decay used $B_s^0 \to J/\Psi K^+ K^-$.

We define

$$\begin{aligned} \Delta a_{\parallel} &= \Delta a_0 + \cos(\chi)\Delta a_Z = \Delta a_0 - 0.38\Delta a_Z, \\ \Delta a_{\perp} &\equiv 0.38\Delta a_0 + \Delta a_Z. \end{aligned} \tag{68}$$

The LHCb component-by-component bounds for the "a" coffiecient for B_d^0 are as follows:

$$\begin{aligned} \Delta a_{\parallel}^{B_d^0} &= (-0.10 \pm 0.82(stat) \pm 0.54(syst)) \times 10^{-15}\text{GeV}, \\ \Delta a_{\perp}^{B_d^0} &= (-0.20 \pm 0.22(stat) \pm 0.04(syst)) \times 10^{-13}\text{GeV}, \\ \Delta a_X^{B_d^0} &= (+1.97 \pm 1.30(stat) \pm 0.29(syst)) \times 10^{-15}\text{GeV}, \\ \Delta a_Y^{B_d^0} &= (= 0.44 \pm 1.26(stat) \pm 0.29(syst)) \times 10^{-15}\text{GeV}. \end{aligned} \tag{69}$$

The D∅ limits can be improved by the LHCB using the CP eigenmode $B_s^0 \to J/\psi\phi$ decay and the average boost of $\langle\beta\gamma = 15\rangle$ to give B_s^0 bound from LHCb:

$$\Delta a_0 - 0.38\Delta a_Z \approx 10^{-14}. \tag{70}$$

LHCb also targeted improved bounds and advanced analysis of D^0 particles.

5.8. FOCUS

FOCUS at Fermilab tests neutral D mesons. The charm particles are produced by interacting high-energy photons with a BeO target [104]. The key asymmetry is the same as for uncorrelated meson systems shown in Equation (66). The D^0 contains up-type quarks. The suppression in the SM of neutral D mixing gave a slow start to these experiments.

One can use the time dependence of D^0 decays for CPT studies. The useful decay modes are $D^0 \to K^-\pi^+$ and $\overline{D}^0 \to K^+\pi^-$. The flavor at production is marked by the soft pion charge in the $D^{*+} \to D^0\pi^+$ decay. The kaon charge lends itself for the flavor tagging at decay.

Between production and decay, the mixing can be followed by analyzing right-sign decays $D^0 \to K^-\pi^+$ vs. wrong sign decays when mixing occurs between D^0 and $\overline{D}^0$; $D^0 \to \overline{D}^0 \to K^+\pi^-$. Direct CPV/CPTV is neglected, which is supported by experimental evidence. The phenomenological expression of the asymmetry implements mixing parameters $y = \Delta\gamma/\gamma$, $x = 2\Delta m/\gamma$ such that Equation (66) takes the form

$$\mathcal{A}^{uncor}_{CPT} = (Re\xi y - Im\xi x)\Gamma t, \tag{71}$$

According to a condition set on the real and imaginary parts of the complex ξ parameter based on the fact that the SME coefficients relating to it are real, the above expression is zero unless the asymmetry is expanded to

$$\begin{aligned} \mathcal{A}_{CPT} &= \frac{Re(x^2+y^2)(t/\tau)^2}{2x}\Big[\frac{xy}{3}(t/\tau) \\ &+ \sqrt{R_{DCS}}(x\cos\delta + y\sin\delta)\Big]. \end{aligned} \tag{72}$$

In general, one must also introduce a third parameter, a doubly Cabibbo-suppressed (DCS) to Cabibbo-favored δ relative phase. This phase is included in expression (72), appropriate for the D^0 system.

In the SME, ξ depends on the lab momentum, orientation, and sidereal modulation of the CPTV signal. At FOCUS, the D meson beam is highly collimated, and the mesons and antimesons are uncorrelated. Binning is performed in sidereal time in four-hour bins.

At LHCb, more $D^0 \to K^-\pi^+$ to work with. LHCb also took into account the constraint of the real and imaginary values of ξ stemming from the characteristics that the LV coefficients are real. Regarding that, further expansion is required for a proper SME-based evaluation. This delivers an improvement in neutral D research [91].

FOCUS important experimental data [104] (see also [10]):

Boost $\gamma\beta \cong 39$;
Beam East of North 236.3°;
Colatitude $\chi = 53°$;
Decay used $D^0 \to K^-\pi^+$.

We define $N(x,y,\delta) = \Big[\frac{xy}{3}(t/\tau) + 0.06(x\cos\delta + y\sin\delta)\Big]$. The general results have the form

$$\begin{aligned} N(x,y,\delta)(\Delta a_0 + 0.6\Delta a_Z) &\sim 10^{-16}\text{GeV}, \\ N(x,y,\delta)\Delta a_X &\sim 10^{-16}\text{GeV}, \\ N(x,y,\delta)\Delta a_Y &\sim 10^{-16}\text{GeV}. \end{aligned} \tag{73}$$

6. Conclusions

CPT and Lorentz symmetries are specifically spacetime symmetries. The NM system is one of the most involved realizations of quantum mechanics. While the mesons also contain all type of forces and particles, they are acting in the context of spacetime symmetries.

In this work, the possibility of minute CPTV was taken under scrutiny through its influence on the quantum mechanical propagation, mixing, and quantum numbers of the NM system. It is demonstrated that no signal beyond Standard Model predictions was seen.

The significant experimental endeavors are summarized with necessary characteristics of the experimental environment and the ensuing constraints placed on CPT violation and on disentanglement of correlated meson wave functions.

A deep theoretical undertaking lies at the foundation of these searches, both in quantum mechanics and in spacetime symmetry research. Similarly, the connection between discrete CPT symmetries and Lorentz symmetry produces puzzles that in the case of neutral mesons were projected into experimentally feasible physical predictions.

These reach all the way to Planck-scale theories, which are often mathematical models that struggle to manifest in a unified physical landscape. Testing such physics was thought to be impossible, but the above listed searches come close to reaching that level. It provides means to verify or falsify models.

The reader can follow some of these specific scenarios in a theoretical undertaking of Planck-scale spacetime behavior and its connection to quantum physics. Hopefully, this review serves as an encouraging and thought-provoking foundation for solving still-demanding problems.

On the phenomenological side, there could be directions not explored, even with the seemingly complete effort to cover all visible angles on it. For instance, in the case of disentanglement, decay products considered forbidden would provide signals of fundamental importance.

In the SME, the search on the meson topic expands to the nonminimal SME of higher derivative terms. An example is the theory of noncommutative spaces, which do not produce mSME effects but could influence higher-order terms.

While the SME started with the idea of spontaneous symmetry breaking, some feature of explicit symmetry-breaking scenarios fell in parallel with the mathematical field of Finsler geometries and already produced notable additions to LV research.

Another new endeavor entails learning about the LV framework by studying the mesons at the quark level and as scalar particles. Development is ongoing on relating SME coefficients of the two levels. Aspiring to achieve that consistently is already producing insights and connections between various other significant symmetries and their violation with the LV theories. Any such connection carries the promise of simplification and unification residing in the heart of the art of physics.

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