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Exploring the Interpretations of Charmonia and $cc\bar{c}\bar{c}$ Tetraquarks in the Relativistic Flux Tube Model

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Abstract: Incited by the scant understanding of unsettled charmonia and newly observed $cc\bar{c}\bar{c}$ tetraquarks, this work aims to explore the canonical interpretations and spectroscopic properties of these fully hidden-charm states. In the framework of a relativistic flux tube model, the centroid masses of the low-lying nL-wave states with $1 \leq n+L \leq 4$ are unraveled. In order to pin down the complete mass spectra, the hyperfine splittings induced by the spin-dependent interactions are incorporated into the final predictions. Accordingly, fourteen charmonia are well identified, including the $\eta_c(1S)$, $J/\psi(1S)$, $\chi_{c0}(1P)$, $h_c(1P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, $\eta_c(2S)$, $\psi(2S)$, $\psi(3770)$, $\psi_2(3823)$, $\psi_3(3842)$, $\chi_{c0}(3915)$, $\chi_{c2}(3930)$, and $\psi(4040)$ states. Additionally, the exotic $T_{\psi\psi}(6400)$, $T_{\psi\psi}(6600)$, $T_{\psi\psi}(6900)$, and $T_{\psi\psi}(7300)$ states are interpreted as the $1S$-wave, $1P/2S$-wave, $1D/2P$-wave, and $2D/3P/4S$-wave $cc\bar{c}\bar{c}$ tetraquarks, respectively. Based on the achieved outcomes, the spin-parity quantum number is imperative to discriminate the nature of the $cc\bar{c}\bar{c}$ structures, pending further experimental measurement in the future.

Keywords: charmonium; flux tube; spectroscopy; tetraquark

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1. Introduction

As a novel type of state beyond the conventional quark model, a number of exotic hadrons with heavy flavors have been discovered by various experiments over the past several decades [1–3]. A remarkable example among them is the $\chi_{c1}(3872)$ state reported in 2003 [1], whose structure is endowed with diverse theoretical interpretations [2–20], including the conventional charmonium $\chi_{c1}(2P)$ state, the hybrid charmonium $c\bar{c}g$ state, the tightly bound hidden-charm tetraquark state, the loosely bound $D\bar{D}^*$ molecular state, the hadrocharmonium state, the mixing state, the threshold cusp, etc. So far, most members of the exotic hadron zoo are the charmonium-like states [1–3], such as the neutral $\chi_{c1}(3872)$, $\chi_{c0}(3915)$, $\chi_{c1}(4140)$, $\chi_{c1}(4274)$, $\chi_{c0}(4500)$, $\chi_{c0}(4700)$, $\psi(4230)$, $\psi(4360)$, and $\psi(4660)$ states and the charged $Z_c(3900)$, $Z_c(4020)$, $Z_c(4050)$, $Z_c(4055)$, $Z_c(4100)$, $Z_c(4200)$, $Z_c(4240)$, $Z_c(4250)$, and $Z_c(4430)$ states. Compared to the charged charmonium-like states, it is challenging to distinguish the neutral ones from the conventional charmonium states. Therefore, investigation of the charmonium spectrum will offer indispensable hints for demystifying the nature of the exotic XYZ states.

The earliest experimentally observed charmonium is the $J/\psi(1S)$ state reported in 1974 [1]. During the following decades, several members of the charmonium family were established by a number of experiments [1], involving the $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $\psi(2S)$, $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, and $h_c(1P)$ states. Analogously, the doubly hidden-flavor states composed of four charm quarks should also exist in the exotic hadron zoo. In 2020, a narrow structure around 6.9 GeV was observed by the LHCb Collaboration in the di-J/ψ invariant mass spectrum [21]. Soon afterwards, it was confirmed by the ATLAS Collaboration [22] and CMS Collaboration [23] individually. Apart from that, two additional structures around 6.6 and 7.3 GeV were detected by both ATLAS Collaboration [22] and CMS Collaboration [23]. Remarkably, a broad peaking structure around 6.4 GeV discovered by the ATLAS Collaboration [22] was not manifestly claimed by the CMS Collaboration [23]. In spite of this, a tiny peak around 6.4 GeV may be found in the CMS data [23]. For the sake of brevity and clarity, these exotic $cc\bar{c}\bar{c}$ states located at 6.4, 6.6, 6.9, and 7.3 GeV are referred to as $T_{\psi\psi}(6400)$, $T_{\psi\psi}(6600)$, $T_{\psi\psi}(6900)$, and $T_{\psi\psi}(7300)$, respectively. The detailed experimental information of all the $T_{\psi\psi}$ states is listed in Table 1.

Table 1. The observed data of the fully charmed tetraquark states (in unit of MeV).

Notation	Mass	Decay Width	Channel	Experiment
$T_{\psi\psi}(6400)_{\mathrm{A}}$	$6410 \pm 80^{+80}_{-30}$	$590 \pm 350^{+120}_{-200}$	J/ψ+J/ψ	ATLAS (Model A) [22]
$T_{\psi\psi}(6600)_{\mathrm{C}}$	$6552 \pm 10 \pm 12$	$124^{+32}_{-26} \pm 33$	J/ψ+J/ψ	CMS (No interference) [23]
$T_{\psi\psi}(6600)_{\mathrm{A}}$	$6630 \pm 50^{+80}_{-10}$	$350 \pm 110^{+110}_{-40}$	J/ψ+J/ψ	ATLAS (Model A) [22]
$T_{\psi\psi}(6600)_{\mathrm{C'}}$	6638^{+43+16}_{-38-31}	$440^{+230+110}_{-200-240}$	J/ψ+J/ψ	CMS (Interference) [23]
$T_{\psi\psi}(6600)_{\mathrm{A'}}$	$6650 \pm 20^{+30}_{-20}$	$440 \pm 50^{+60}_{-50}$	J/ψ+J/ψ	ATLAS (Model B) [22]
$T_{\psi\psi}(6600)_{\mathrm{L'}}$	6741 ± 6	288 ± 16	J/ψ+J/ψ	LHCb (Model II) [21]
$T_{\psi\psi}(6900)_{\mathrm{C'}}$	6847^{+44+48}_{-28-20}	191^{+66+25}_{-49-17}	J/ψ+J/ψ	CMS (Interference) [23]
$T_{\psi\psi}(6900)_{\mathrm{A}}$	$6860 \pm 30^{+10}_{-20}$	$110 \pm 50^{+20}_{-10}$	J/ψ+J/ψ	ATLAS (Model A) [22]
$T_{\psi\psi}(6900)_{\mathrm{L'}}$	$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	J/ψ+J/ψ	LHCb (Model II) [21]
$T_{\psi\psi}(6900)_{\mathrm{L}}$	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	J/ψ+J/ψ	LHCb (Model I) [21]
$T_{\psi\psi}(6900)_{\mathrm{A'}}$	$6910 \pm 10 \pm 10$	$150 \pm 30 \pm 10$	J/ψ+J/ψ	ATLAS (Model B) [22]
$T_{\psi\psi}(6900)_{\mathrm{C}}$	$6927 \pm 9 \pm 4$	$122^{+24}_{-21} \pm 18$	J/ψ+J/ψ	CMS (No interference) [23]
$T_{\psi\psi}(6900)_{\mathcal{A}'}$	$6960 \pm 50 \pm 30$	$510 \pm 170^{+110}_{-100}$	J/ψ+$\psi(2S)$	ATLAS (Model β) [22]
$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7134^{+48+41}_{-25-15}	97^{+40+29}_{-29-26}	J/ψ+J/ψ	CMS (Interference) [23]
$T_{\psi\psi}(7300)_{\mathcal{A}}$	$7220 \pm 30^{+10}_{-40}$	$90 \pm 60^{+60}_{-50}$	J/ψ+$\psi(2S)$	ATLAS (Model α) [22]
$T_{\psi\psi}(7300)_{\mathrm{C}}$	$7287^{+20}_{-18} \pm 5$	$95^{+59}_{-40} \pm 19$	J/ψ+J/ψ	CMS (No interference) [23]

As the landmark of the hadron physics phenomenology, the seminal quark model was proposed by M. Gell-Mann [24] and G. Zweig [25] individually in 1964. On the basis of various sorts of quark potential models, the charmonium spectrum incorporating the spin-dependent hyperfine splitting has been explored by numerous theoretical approaches [26–40], including the nonrelativistic potential model [26–28], the semirelativistic potential model [29,30], the relativized potential model [31–33], the chiral quark model [34,35], the coupled-channel model [36], the screened potential model [37–40], and so forth. According to the heavy antiquark–diquark symmetry (HADS), the heavy antiquark $\bar{Q}$ can be deemed as the doubly heavy diquark QQ with the antitriplet color representation. This means that the HADS achieves the correlation between the charmonium and the $T_{\psi\psi}$ tetraquark by replacing the charm quark pair $(\bar{c})(c)$ with the doubly charmed diquark pair $(cc)(\bar{c}\bar{c})$. Hence, the fully charmed tetraquark spectroscopy can be disentangled based on the existing perception of the charmonium family. At present, the mass spectrum of the exotic $cc\bar{c}\bar{c}$ structure has been surveyed by sundry phenomenological scenarios [41–120]. However, there are still no acknowledged interpretations of these $T_{\psi\psi}$ states [114]. For instance, the mass of the 1S-wave scalar $cc\bar{c}\bar{c}$ state with $J^{PC} = 0^{++}$ predicted by the four-quark potential model [46,63,65,66,71,98] is at least 200 MeV higher than the one in the diquark potential model [49,56,58,59,62,72], chromomagnetic quark model [53,77,89], and QCD sum rules [51,94,97]. Moreover, the pivotal light quark degree of freedom dominating the formation of the hadronic molecular states is absent in the $cc\bar{c}\bar{c}$ structure. Thus, these observed $T_{\psi\psi}$ states are very likely the doubly hidden-charm tetraquarks composed of the diquark cc and antidiquark $\bar{c}\bar{c}$, even though there are several theoretical schemes that propose the likelihood of the molecular configuration [67,79,87].

This manuscript is arranged as follows. First of all, the experimental and theoretical status quo of the charmonium and fully charmed tetraquark is introduced in Section 1. Whereafter, a relativistic flux tube model in the heavy quark limit is elaborated in Section 2. Subsequently, the spectroscopic hyperfine splitting induced by the spin-dependent interaction potential and its application on the charmonium and fully charmed tetraquark are explicated in Section 3. Next, the entire mass spectra of the charmonium and fully charmed tetraquark and a discussion on the discrepancy in the theoretical outcomes between the relativistic flux tube model and other phenomenological approaches are unveiled in Section 4. Finally, a succinct summary of this work is delivered in Section 5.

2. Relativistic Flux Tube Model

As is well known, a variety of excited light hadrons can be well depicted by the famous Chew–Frautschi formula [121], i.e.,

$$L = \alpha' M^2 + \alpha_0, \tag{1}$$

with the orbital angular momentum L, orbitally excited slope α', excited hadron mass M, and orbitally excited intercept α_0. Based on the observed masses [1] and Equation (1), the systematics of hadron states can be embodied as the Regge trajectories in the (L, M^2) plane [121]. In Ref. [122], Y. Nambu proposed a string picture which deemed the $q\bar{q}$ meson state as a dynamical gluon flux tube flanked by the quark q and antiquark $\bar{q}$. If the quark and antiquark are approximately massless, they will be rotating at velocities that can be comparable to the light. Here, the speed of light is set as 1. Thus, the rotating velocity $v(x)$ of the certain position x between the center position O and the endpoint position r_i can be expressed as

$$v(x) = \frac{x}{r_i}, \tag{2}$$

with

$$r_i = \frac{r}{2}. \tag{3}$$

Here, the length of string is set as r. Thereupon, the mass M can be expressed as the energy of a rotating gluon flux tube [123], i.e.,

$$M = \sum_{i=1}^{2} \int_0^{r_i} \tau\gamma(x)dx, \tag{4}$$

with

$$\gamma(x) = \frac{1}{\sqrt{1 - v^2(x)}}. \tag{5}$$

Here, the energy density per unit length is denoted as a constant string tension τ. Moreover, the angular momentum L can be expressed as [123]

$$L = \sum_{i=1}^{2} \int_0^{r_i} \tau x v(x) \gamma(x) dx. \tag{6}$$

By incorporating Equation (4) into Equation (6), the Chew–Frautschi formula can be verified successfully. Accordingly, the slope α' can be expressed as

$$\alpha' = \frac{1}{2\pi\tau}. \tag{7}$$

Hence, the flux tube model effectively corroborated the existence of the Regge trajectories [123]. Currently, the Regge trajectories and the flux tube (QCD string) model have been extensively applied to the spectroscopic inquiries on miscellaneous hadrons [76–78,121–193], involving mesons [121–167], baryons [121,147–164,168–180], tetraquarks [76–78,159–163,165,181–184], pentaquarks [160,184,185], hexaquarks [186,187], hybrids [166,188,189], glueballs [160,161,164,167,189–192], diquarks [162,163], triquarks [193], and so on.

When it comes to the case of massive quarks, a generalized expression of the mass M is proposed by Refs. [123,152], i.e.,

$$M = \sum_{i=1}^{2} \left[m_i \gamma_i + \int_0^{r_i} \tau \gamma(x) dx \right], \tag{8}$$

with

$$\gamma_i = \gamma(r_i). \tag{9}$$

Here, m_i is the mass of (anti)quark i. It is essential to note that the rotating velocity $v(r_i)$ of the endpoint position r_i is unequal to the speed of light due to the massive quarks. Therefore, the rotating velocity $v(x)$ of the certain position x appeared in Equations (2) and (5) is redefined as

$$v(x) = \omega x, \tag{10}$$

where ω is the rotating angular velocity of the whole system. Furthermore, a generalized expression of the angular momentum L is [123,152]

$$L = \sum_{i=1}^{2} \left[m_i r_i v_i \gamma_i + \int_0^{r_i} \tau x v(x) \gamma(x) dx \right], \tag{11}$$

with

$$v_i = v(r_i). \tag{12}$$

In addition, for each side of the flux tube, the string tension τ can be expressed via the relationship with the angular acceleration [152,165], i.e.,

$$\tau = m_i \omega^2 r_i \gamma_i^2. \tag{13}$$

When the masses of quarks are equal ($m_1 = m_2$), by incorporating Equations (11) and (13) into Equation (8), the expression of the mass M can be simplified as

$$M = 2 m_i \gamma_i^{-1} + 2\omega L. \tag{14}$$

In the heavy quark limit, the value of r_i is extremely tiny. Consequently, Equation (14) is expanded up to the second order of r_i, i.e.,

$$\begin{aligned} M &= 2m_i + 2\omega L - m_i\omega^2 r_i^2 + \mathcal{O}\left(r_i^4\right) \\ &\sim 2m_i + 2\omega L - m_i v_i^2. \end{aligned} \tag{15}$$

Analogously, based on the equal quark masses ($m_1 = m_2$) and Equation (13), the expression of the angular momentum L can be simplified as

$$L = \tau\omega^{-2}\left(v_i\gamma_i^{-1} + \arcsin v_i\right). \tag{16}$$

By keeping pace with Equation (15), the aforementioned expression is expanded up to the second order of r_i, i.e.,

$$\begin{aligned} L &= 2\tau\omega^{-1} r_i + \mathcal{O}\left(r_i^3\right) \\ &\sim 2\tau\omega^{-2} v_i. \end{aligned} \tag{17}$$

According to the relationship between the string tension τ and the angular velocity ω in Equation (13), the expression of v_i is recast into

$$v_i = \frac{m_i\omega}{2\tau}\left[-1 + \sqrt{1 + \left(\frac{2\tau}{m_i\omega}\right)^2}\right]. \tag{18}$$

Following Ref. [165], the expression of v_i is expanded up to the second order of $\epsilon = \tau m_i^{-1}\omega^{-1}$, i.e.,

$$\begin{aligned} v_i &= \epsilon + \mathcal{O}\left(\epsilon^3\right) \\ &\sim \tau m_i^{-1}\omega^{-1}. \end{aligned} \tag{19}$$

Next, by interpolating Equation (19) into Equation (17), the angular velocity ω is approximately expressed as the relationship with the string tension τ, the (anti)quark mass m_i, and the angular momentum L, i.e.,

$$\omega \approx \left(\frac{2\tau^2}{m_i L}\right)^{\frac{1}{3}}. \tag{20}$$

Apparently, by associating Equations (19) and (20) with Equation (15), the expression of the mass M can be approximated as the relationship with the angular momentum L [165,182], i.e.,

$$M \approx 2m_i + 3\left(\frac{\tau^2 L^2}{4m_i}\right)^{\frac{1}{3}}. \tag{21}$$

Regge trajectories are not only successful for portraying the hadrons with orbital excitations [121,123], but also for delineating the radially excited mass spectra of multifarious hadrons, such as mesons [131,133,136,138,141–145], baryons [149,158,170,172,175,177–180], multiquarks [76,161–163,181], and glueballs [161,167]. The validity of radial Regge trajectories has been confirmed by the WKB approximation [124,127,149], the Bohr–Sommerfeld quantization [129,137,138], and the AdS/QCD approach [135,139,153]. Stimulated by the linearity and parallelism between the parent and daughter Regge trajectories unveiled by the relativistic flux tube model [125,126], Refs. [142,170] extend the individually (radially or orbitally) excited Regge trajectory into a jointly (radially and orbitally) excited form by dint of superseding L with $\lambda n_r + L$. Here, n_r is defined as

$$n_r = n - 1, \tag{22}$$

where n denotes the principal quantum number. What is more, the jointly excited form of Regge trajectory has been successfully employed into the studies on the mass spectra of light mesons [142], heavy-light mesons [144,146,158], heavy quarkonia [145], singly heavy baryons [158,170,175,177,179,180], and doubly heavy baryons [178]. Therefore, following Refs. [145,170], a generalized form of Equation (21) is expressed as

$$M \approx 2m_i + 3\left(\frac{\tau^2}{4m_i}\right)^{\frac{1}{3}} (\lambda n_r + L)^{\frac{2}{3}}. \tag{23}$$

It is noteworthy that the mass M that appears in this section represents the spin-averaged mass of the certain nL-wave hadron. The spin-dependent interaction terms will be expounded in the next section.

3. Fine and Hyperfine Structure

Needless to say, the spectroscopy unraveled by the relativistic flux tube model can only reflect the properties of spinless hadrons. In order to acquire the complete mass spectrum of the hadron with the definite spin quantum number, it is requisite to bring in the fine and hyperfine splittings induced by the spin-dependent interactions. Hence, the mass M in Equation (23) is modified as

$$M = 2m_i + 3\left(\frac{\tau^2}{4m_i}\right)^{\frac{1}{3}} (\lambda n_r + L)^{\frac{2}{3}} + \Delta M, \tag{24}$$

where ΔM denotes the spin-dependent mass correction term. Concretely, it is composed of three sorts of spin-dependent interactions [145], i.e.,

$$\Delta M = H_{\text{cont}} + H_{\text{ten}} + H_{\text{so}}. \tag{25}$$

Here, H_{cont}, H_{ten}, and H_{so} denote the spin–spin contact hyperfine interaction term, the tensor interaction term, and the spin–orbit interaction term, respectively.

Firstly, the contact term H_{cont} is expressed as [145]

$$H_{\text{cont}} = \frac{32\pi\alpha_s(r)}{9m_i^2}\tilde{f}(r)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2, \tag{26}$$

with

$$\alpha_s(r) = \alpha_c \text{erf}\left(\frac{m_i r}{0.47\pi^2}\right), \tag{27}$$

$$\tilde{f}(r) = \left(\frac{2\kappa}{\pi}\right)^2 \frac{e^{-\sqrt{\kappa r}}}{r}. \tag{28}$$

Here, $\alpha_s(r)$, $\tilde{f}(r)$, and $\text{erf}(x)$ are the running coupling constant, the smearing function, and the error function, respectively. Thereinto, the expression of $\text{erf}(x)$ is

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \tag{29}$$

In terms of the parameter κ, the value is determined as 0.66 GeV by fitting the observed masses of the radially excited S-wave charmonium states. The detailed derivation and physical illustration may be found in Ref. [145]. In order to conciliate with the saturated coupling constant in the bottomonium spectroscopy, the value of α_c is assumed as 0.68

which is adopted by Ref. [145]. However, considering that the 1S-wave $J/\psi - \eta_c$ hyperfine mass splitting [1]

$$M(J/\psi) - M(\eta_c) \quad = \quad 112.8 \text{ MeV},$$

cannot be produced by the coupling constant $\alpha_s(r)$ with the form of $r^{5/2}$ in Ref. [145], this work makes use of the coupling constant $\alpha_s(r)$ with the form of r^1 by mimicking the Godfrey–Isgur model [31]. Remarkably, the 1P-wave $\chi_c - h_c$ hyperfine mass splitting [1]

$$\frac{M(\chi_{c0}) + 3M(\chi_{c1}) + 5M(\chi_{c2})}{9} - M(h_c) \quad = \quad -0.08 \text{ MeV},$$

extremely verges on zero, demonstrating that the spin–spin contact hyperfine interaction contributes very little to the mass spectra of the orbitally excited charmonium states. Consequently, following Refs. [145,165], the contributions made by the contact term H_{cont} to the charmonium and $cc\bar{c}\bar{c}$ tetraquark with orbital excitations are omitted in this work.

Then, there is the tensor interaction term H_{ten}, which possesses the form of [145]

$$H_{\text{ten}} \quad = \quad \frac{4\alpha_s(r)}{m_i^2 r^3}\mathbb{T}, \tag{30}$$

with

$$\mathbb{T} \quad = \quad \frac{(\boldsymbol{S}_1 \cdot \boldsymbol{r})(\boldsymbol{S}_2 \cdot \boldsymbol{r})}{r^2} - \frac{1}{3}\boldsymbol{S}_1 \cdot \boldsymbol{S}_2. \tag{31}$$

Here, $\mathbb{T}$ denotes the operator of the tensor interaction. Lastly, the spin–orbit interaction term H_{so} is expressed as [145]

$$H_{\text{so}} \quad = \quad \frac{1}{m_i^2}\left(\frac{2\alpha_s(r)}{r^3} - \frac{\tau}{2r}\right)\boldsymbol{L} \cdot \boldsymbol{S}, \tag{32}$$

where the string tension τ is employed as the linear confinement potential coefficient. In the case of the S-wave states without orbital excitations, the tensor interaction term H_{ten} and the spin–orbit interaction term H_{so} will vanish due to the tensor and spin–orbit operators ($\mathbb{T}$ and $\boldsymbol{L} \cdot \boldsymbol{S}$) with zero. Hence, following Refs. [145,165], the spin-dependent mass correction term ΔM is simplified as

$$\Delta M \quad = \quad \begin{cases} H_{\text{cont}}, & L = 0, \\ H_{\text{ten}} + H_{\text{so}}, & L > 0. \end{cases} \tag{33}$$

As mentioned by Equations (26), (30) and (32), the flux tube length r plays a crucial role in the fine and hyperfine splittings of hadrons. It is convenient to elicit the expression of r by combining Equations (10) and (18) with Equation (3), i.e.,

$$r \quad = \quad \frac{m_i}{\tau}\left[-1 + \sqrt{1 + \left(\frac{2\tau}{m_i\omega}\right)^2}\right]. \tag{34}$$

It is notable that Equations (3) and (34) only work on the premise of $m_1 = m_2$. For the sake of deducing a relationship between the string length r and the angular momentum L, the expression of r is approximated as [165]

$$r \quad \approx \quad \frac{m_i}{\tau}\left[-1 + \sqrt{1 + \left(\frac{4\tau L}{m_i^2}\right)^{\frac{2}{3}}}\right], \tag{35}$$

by inserting Equation (20) into Equation (34). Subsequently, following the steps in Refs. [145,170], a jointly excited form of r is reaped by replacing L with $\lambda n_r + L$, i.e.,

$$r \approx \frac{m_i}{\tau}\left[-1+\sqrt{1+\left(\frac{4\tau}{m_i^2}\right)^{\frac{2}{3}}(\lambda n_r+L)^{\frac{2}{3}}}\,\right]. \quad (36)$$

With regard to the $1S$-wave ground state hadrons, the flux tube length rendered by Equation (36) is out of order due to the emergence of zero. Manifestly, the value of the ground state string length $r(1S)$ is underestimated by Equation (36). Thus, the $r(1S)$ value utilized in the charmonium calculation is acquired by fitting the $1S$-wave $J/\psi - \eta_c$ hyperfine mass splitting. Apart from that, in the heavy quark limit ($m_i \to \infty$), the expression of r is able to be approximated as [165]

$$r \approx \left(\frac{2}{m_i\tau}\right)^{\frac{1}{3}}(\lambda n_r+L)^{\frac{2}{3}}, \quad (37)$$

by taking into account the leading order expansion of $1/m_i$. Akin to the charmonium, the nonzero $r(1S)$ value of the $cc\bar{c}\bar{c}$ states also cannot be obtained by Equation (36). Nevertheless, the string length $r_{cc\bar{c}\bar{c}}(1S)$ of the $1S$-wave fully charmed tetraquark can be estimated by uniting Equation (37) with the string length $r_{c\bar{c}}(1S)$ of the ground state charmonium, i.e.,

$$\frac{r_{cc\bar{c}\bar{c}}(1S)}{r_{c\bar{c}}(1S)} \approx \left(\frac{m_c}{m_{cc}}\right)^{\frac{1}{3}}. \quad (38)$$

Here, m_c and m_{cc} denote the charm quark mass and the doubly charmed diquark mass, respectively. Concerning the charmonia and $cc\bar{c}\bar{c}$ tetraquarks with radial or orbital excitations, Equation (36) is adopted as the expression of the flux tube length so as to approach the prototype. Additionally, the determination of all parameters including m_c, m_{cc}, τ, and λ will be set forth in the next section.

4. Results and Discussion

The critical step towards spectroscopic results is the determination of parameters. As far as the charmonia and $cc\bar{c}\bar{c}$ tetraquarks are concerned, there are four imperative parameters, i.e., the charm quark mass m_c, the doubly charmed diquark mass m_{cc}, the string tension τ, and the dimensionless coefficient λ. Firstly, according to the experimentally observed data [1], the spin-averaged mass of the $1S$-wave charmonium is

$$\frac{M(\eta_c)+3M(J/\psi)}{4} = 3068.7\ \text{MeV}.$$

Based on Equations (24) and (26), the center of gravity of the $1S$-wave charmonium is expressed as

$$\bar{M}_{c\bar{c}}(1S) = 2m_c. \quad (39)$$

Evidently, the charm quark mass m_c is determined as 1.5344 GeV. Regrettably, there are still no sufficient experimental observations for the fully charmed tetraquark states. Therefore, in order to determine the mass of the doubly charmed diquark, this work takes advantage of the heavy hadron mass relations derived from the heavy quark symmetry (HQS) [194], i.e.,

$$m_{\{cc\}[\bar{u}\bar{d}]} - m_{\{cc\}u} = m_{c[ud]} - m_{c\bar{u}}, \quad (40)$$
$$m_{\{cc\}[\bar{u}\bar{d}]} - m_{c[ud]} = m_{\{cc\}u} - m_{c\bar{u}}. \quad (41)$$

Here, brackets $[qq]$ and braces $\{qq\}$ represent scalar diquarks and axial-vector diquarks, respectively. It is convenient to evaluate the mass gap between the doubly charmed diquark and the charm quark by utilizing the mean value of LHS and RHS of Equation (41) [195], i.e.,

$$m_{cc} - m_c = \frac{1}{2}(m_{\{cc\}[\bar{u}\bar{d}]} - m_{c[ud]} + m_{\{cc\}u} - m_{c\bar{u}}). \quad (42)$$

As a result, the mass of the doubly charmed diquark is determined as 3.1537 GeV by plugging the charm quark mass and corresponding heavy hadron masses into Equation (42). In accordance with the Pauli exclusion principle, the total wave function of a diquark containing the color, flavor, spin, and spatial components ought to be antisymmetric under fermion exchange. Thereupon, the S-wave doubly charmed diquark can exist as the scalar diquark with sextet color representation or the axial-vector diquark with antitriplet color representation. Noteworthily, considering that the positive color factor gives rise to the repulsive quark–quark interaction deterring the formation of the color sextet diquark, only the color antitriplet diquark is employed in the diquark potential model [55–62,72,85,114,195] and the diquark flux tube model [76,78,161–163,181–185]. Furthermore, compared to the states with the sextet–antisextet color configuration, the physical properties of the fully charmed tetraquarks with the triplet–antitriplet color configuration are more close to the charmonia made up of a color triplet quark and a color antitriplet antiquark. Accordingly, this work adopts the color antitriplet/triplet axial-vector doubly charmed diquark/antidiquark as the effective ingredient to decode the $cc\bar{c}\bar{c}$ tetraquark spectroscopy.

Remarkably, there are four sorts of color configurations about the $cc\bar{c}\bar{c}$ structure, including the color triplet–antitriplet tetraquark state, the color sextet–antisextet tetraquark state, the color singlet–singlet molecular state, and the color octet–octet molecule-like state [110,114]. Compared to other possible configurations, the main limitation of the color triplet–antitriplet configuration is that the ground state $cc\bar{c}\bar{c}$ tetraquarks can only be constructed by the axial-vector diquark and exist as the 0^{++}, 1^{+-}, and 2^{++} states [114]. By contrast, the S-wave color sextet–antisextet tetraquark state can only be constructed by the scalar diquark and exists as the 0^{++} state [114]. Concerning the $1S$-wave molecular or molecule-like $cc\bar{c}\bar{c}$ states, there are four types of charmonium combinations, involving the $\eta_c\eta_c$ structure with $J^{PC} = 0^{++}$, the $\eta_c J/\psi$ structure with $J^{PC} = 1^{+-}$, the $J/\psi J/\psi$ structure with $J^{PC} = 0^{++}$, and the $J/\psi J/\psi$ structure with $J^{PC} = 2^{++}$ [110]. Thus, there are possibly two S-wave scalar $cc\bar{c}\bar{c}$ states if the color triplet–antitriplet configuration can coexist with the color sextet–antisextet configuration [114]. However, the existence of the color sextet diquark is still dubious, meaning that it is usually omitted in the diquark models [55–62,72,76,78,85,114,161–163,181–185,195]. Then, the next parameter is the string tension τ. Currently, in terms of the $1P$-wave charmonium states, the spin-averaged value of the observed masses is [1]

$$\frac{M(\chi_{c0}) + 3M(h_c) + 3M(\chi_{c1}) + 5M(\chi_{c2})}{12} = 3525.31 \text{ MeV}.$$

By virtue of Equations (24), (30), and (32), the centroid mass of the $1P$-wave charmonium is expressed as

$$\bar{M}_{c\bar{c}}(1P) = 2m_c + 3\left(\frac{\tau^2}{4m_c}\right)^{\frac{1}{3}}. \quad (43)$$

After interpolating the charm quark mass and the $1P$-wave spin-averaged charmonium mass into Equation (43), the string tension τ is fixed as 0.1471 GeV^2, which is consistent with the value fitted by Ref. [165]. Whereafter, the remnant parameter is the dimensionless coefficient λ, symbolizing the ratio between the radial and orbital excitations. With respect to the first radial excitation of the charmonium, the spin-averaged mass is assessed as

$$\frac{M(\eta_c') + 3M(\psi')}{4} \quad = \quad 3674.0 \text{ MeV}.$$

As mentioned by Equations (24) and (26), the mass center of the 2S-wave charmonium is expressed as

$$\bar{M}_{c\bar{c}}(2S) \quad = \quad 2m_c + 3\left(\frac{\tau^2}{4m_c}\right)^{\frac{1}{3}} \lambda^{\frac{2}{3}}. \tag{44}$$

Thereupon, the dimensionless coefficient λ is determined as 1.5263 by inputting the charm quark mass, the sting tension, and the experimental data of the 2S-wave charmonium. On account of the same color configuration, this work makes use of a set of parameters (τ and λ) to uniformly tackle the mass spectra of the charmonia and color triplet–antitriplet $cc\bar{c}\bar{c}$ states. In the following discussion, the spectroscopic predictions of the charmonium and fully charmed tetraquark and the phenomenological comparison between this work and other theoretical models will be elucidated.

4.1. Charmonium

The entire mass spectra of the low-lying charmonium states predicted by this work, the nonrelativistic potential model [26–28], the semirelativistic potential model [29,30], the relativized potential model [31–33], the chiral quark model [34,35], the coupled-channel model [36], and the screened potential model [37–40] are explicitly enumerated in Table 2. For ease of comparison, the charmonium candidates discovered by various experiments [1] and their corresponding observed masses are also displayed in Table 2. Hitherto, the 1S-, 1P-, and 2S-wave charmonium candidates have been established by the abundant experimental data [1]. This work successfully reproduces the observed masses of the $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $\psi(2S)$, $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, and $h_c(1P)$ states. Our model predicts that the masses of the $\psi(1D)$, $\psi_2(1D)$, $\psi_3(1D)$, and $\eta_{c2}(1D)$ states are 3760, 3791, 3810, and 3794 MeV, respectively, showing consistency with the results offered by the nonrelativistic potential model [26], the semirelativistic potential model [30], the relativized potential model [32], the chiral quark model [34,35], and the screened potential model [37,38]. In spite of this, the predicted results of the $\psi_2(1D)$ and $\psi_3(1D)$ states are about 30 MeV lower than the observed masses of the $\psi_2(3823)$ and $\psi_3(3842)$ states [1]. Analogously, the mass gap of 30 MeV also shows up in the spectroscopic predictions of the 1D-wave charmonium states rendered by the renowned Ebert–Faustov–Galkin model [32]. Hence, the research status quo of the 1D-wave charmonium states is still noncommittal, awaiting the further explorations of experiments and theories. Moreover, this study finds that the masses of the $\chi_{c0}(2P)$, $\chi_{c1}(2P)$, $\chi_{c2}(2P)$, and $h_c(2P)$ states are 3897, 3916, 3919, and 3916 MeV, respectively. Accordingly, the $\chi_{c2}(3930)$ state with the observed mass of 3922.5 MeV can be identified as the charmonium with the $\chi_{c2}(2P)$ assignment. This identification is also endorsed by the coupled-channel model [36] and the screened potential model [37,38]. Generally, there are two main experimental candidates of the $\chi_{c0}(2P)$ charmonium, i.e., the $\chi_{c0}(3860)$ and $\chi_{c0}(3915)$ states. The absolute values of the gaps between their observed masses and our theoretical prediction are approximately 35 and 25 MeV, indicating that the $\chi_{c0}(3915)$ state is more proper to be deemed as the candidate of the $\chi_{c0}(2P)$ charmonium. Further, the $\chi_{c0}(2P)$ assignment of the $\chi_{c0}(3915)$ state is also championed by the nonrelativistic potential model [28], the relativized potential model [31,33], the chiral quark model [35], and the screened potential model [39]. In terms of the second radial excitation of the S-wave charmonium, this work proposes that the masses of the $\eta_c(3S)$ and $\psi(3S)$ states are 4007 and 4037 MeV, respectively. Thereupon, the $\psi(4040)$ state is identified as the $\psi(3S)$ charmonium in light of the relativistic flux tube model, the semirelativistic potential model [30], the relativized potential model [32], the chiral quark model [34], the coupled-channel model [36], and the screened potential model [38–40].

When it comes to the higher radial or orbital excitations, this work systematically investigates the spectroscopy of the $1F$-, $2D$-, $3P$-, and $4S$-wave charmonium states. So far, there are still no definite experimental candidates of the $1F$- and $3P$-wave charmonium states. This work predicts that the masses of the $\chi_{c2}(1F)$, $\chi_{c3}(1F)$, $\chi_{c4}(1F)$, $h_{c3}(1F)$, $\chi_{c0}(3P)$, $\chi_{c1}(3P)$, $\chi_{c2}(3P)$, and $h_c(3P)$ states are 4007, 4020, 4024, 4018, 4226, 4232, 4228, and 4229 MeV, respectively, providing the meaningful clues to the prospective experimental research. As far as the $2D$-wave charmonium is concerned, our study shows that the masses of the $\psi(2D)$, $\psi_2(2D)$, $\psi_3(2D)$, and $\eta_{c2}(2D)$ states are 4122, 4129, 4127, and 4127 MeV, respectively, in agreement with the outcomes derived by the nonrelativistic potential model [26], the semirelativistic potential model [30], the chiral quark model [34], and the screened potential model [37–40]. Nonetheless, as the experimental candidate of the $\psi(2D)$ charmonium, the $\psi(4160)$ state possesses a mass of 4191 MeV, which is about 70 MeV higher than our theoretical prediction. In a like manner, this 70 MeV gap is also present in the predicted mass of the $\psi(2D)$ state acquired by the illustrious Lanzhou group [39]. Thus, it is requisite to censor the nature of the $\psi(4160)$ state via further experimental and theoretical surveys. Concerning the $4S$-wave charmonium, the predicted masses of the $\eta_c(4S)$ and $\psi(4S)$ states are 4312 and 4333 MeV, respectively. With regard to the experimental candidates of the $\psi(4S)$ charmonium, there are two possible options proposed by sundry theories, i.e., the $\psi(4230)$ and $\psi(4415)$ states. In consideration of the conspicuous gaps, 110 and 80 MeV, between their observed masses and our theoretical prediction, both of them are inapposite to be treated as the pure $\psi(4S)$ state. In order to figure out the nature of the $\psi(4415)$ state, it is imperative to explore the spectroscopy of the $3D$-wave charmonium. Our calculation reveals that the masses of the $\psi(3D)$, $\psi_2(3D)$, $\psi_3(3D)$, and $\eta_{c2}(3D)$ states are 4416, 4416, 4410, and 4413 MeV, respectively. Accordingly, the $\psi(4415)$ state can be assigned as the $\psi(3D)$ charmonium. In addition, this assignment is also espoused by the distinguished Salamanca group [35].

Table 2. The charmonium mass spectrum (in unit of MeV).

State		Experiment		Theory															
$n^{2S+1}L_J$	J^{PC}	$c\bar{c}$	PDG [1]	Our	[31]	[34]	[26]	[36]	[29]	[37]	[32]	[27]	[35]	[38]	[28]	[30]	[39]	[40]	[33]
1^1S_0	0^{-+}	$\eta_c(1S)$	2984.1(4)	2984	2975	2990	2982	2979.9	2981.7	2979	2981	2978.4	2990	2984	2989	2995	2981	2986.3	2992.1
1^3S_1	1^{--}	$J/\psi(1S)$	3096.900(6)	3097	3098	3097	3090	3096.9	3096.9	3097	3096	3087.7	3096	3097	3094	3094	3096	3094.1	3104.0
1^3P_0	0^{++}	$\chi_{c0}(1P)$	3414.71(30)	3417	3445	3436	3424	3416.3	3415.2	3433	3413	3366.3	3452	3415	3428	3457	3464	3434.4	3446.4
1^1P_1	1^{+-}	$h_c(1P)$	3525.37(14)	3525	3517	3507	3516	3526.4	3523.7	3519	3525	3526.9	3515	3526	3470	3534	3538	3517.2	3499.3
1^3P_1	1^{++}	$\chi_{c1}(1P)$	3510.67(5)	3506	3510	3494	3505	3511.7	3510.6	3510	3511	3517.7	3504	3521	3468	3523	3530	3514.2	3504.2
1^3P_2	2^{++}	$\chi_{c2}(1P)$	3556.17(7)	3558	3550	3526	3556	3556.9	3556.2	3554	3555	3559.3	3532	3553	3480	3556	3571	3556.2	3532.1
2^1S_0	0^{-+}	$\eta_c(2S)$	3637.7(9)	3638	3623	3627	3630	3639.6	3619.2	3623	3635	3646.9	3643	3637	3602	3606	3642	3633.1	3634.6
2^3S_1	1^{--}	$\psi(2S)$	3686.097(11)	3686	3676	3685	3672	3685.5	3686.1	3673	3685	3684.7	3703	3679	3681	3649	3683	3690.0	3691.3
1^3D_1	1^{--}	$\psi(3770)$	3773.7(7)	3760	3819	3775	3785	3735.1	3789.4	3787	3783	3808.8	3796	3792	3775	3799	3830	3817.1	3751.4
1^1D_2	2^{-+}			3794	3837	3793	3799	3819.2	3822.2	3796	3807	3815.1	3812	3805	3765	3802	3848	3826.3	3809.6
1^3D_2	2^{--}	$\psi_2(3823)$	3823.51(34)	3791	3838	3790	3800	3812.3	3822.1	3798	3795	3820.1	3810	3807	3772	3805	3848	3830.3	3803.4
1^3D_3	3^{--}	$\psi_3(3842)$	3842.71(20)	3810	3849	3802	3806	3854.0	3844.8	3799	3813	3812.6		3808	3755	3801	3859	3829.3	3841.5

Table 2. *Cont.*

State		Experiment		Theory															
$n^{2S+1}L_J$	J^{PC}	$c\bar{c}$	PDG [1]	Our	[31]	[34]	[26]	[36]	[29]	[37]	[32]	[27]	[35]	[38]	[28]	[30]	[39]	[40]	[33]
2^3P_0	0^{++}	$\chi_{c0}(3915)$	3922.1(1.8)	3897	3916		3852	3859.9	3864.3	3842	3870	3842.7	3909	3848	3897	3866	3896	3857.5	3915.1
2^1P_1	1^{+-}			3916	3956	3924	3934	3916.6	3963.2	3908	3926	3941.9	3956	3916	3943	3936	3933	3927.6	3938.1
2^3P_1	1^{++}			3916	3953	3913	3925	3920.7	3950.0	3901	3906	3935.0	3947	3914	3938	3925	3929	3924.7	3951.1
2^3P_2	2^{++}	$\chi_{c2}(3930)$	3922.5(1.0)	3919	3979		3972	3937.4	3992.3	3937	3949	3973.1	3969	3937	3955	3956	3952	3965.2	3965.3
3^1S_0	0^{-+}			4007	4064		4043	3945.9	4052.5	3991	3989	4058.0	4054	4004	4058	4000	4013	4011.9	4061.4
3^3S_1	1^{--}	$\psi(4040)$	4040(4)	4037	4100	4050	4072	4038.0	4102.0	4022	4039	4087.0	4097	4030	4129	4036	4035	4048.2	4099.3
1^3F_2	2^{++}			4007	4092		4029		4049.9		4041	4059.7	4043		3990		4070	4064.8	4000.8
1^1F_3	3^{+-}			4018	4094		4026		4066.9		4071	4040.8			4017		4074	4056.6	4056.1
1^3F_3	3^{++}			4020	4097		4029		4069.0		4068	4047.6			4012		4075	4061.2	4051.6
1^3F_4	4^{++}			4024	4095		4021		4084.3		4093	4024.7			4036		4076	4048.6	4088.8
2^3D_1	1^{--}	$\psi(4160)$	4191(5)	4122	4194	4103	4142		4159.2	4089	4150	4154.4	4153	4095	4188	4145	4125	4123.3	4150.0
2^1D_2	2^{-+}			4127	4208		4158		4196.9	4099	4196	4164.9	4166	4108	4182	4150	4137	4135.3	4177.2
2^3D_2	2^{--}			4129	4208		4158		4195.8	4100	4190	4168.7	4164	4109	4188	4152	4137	4137.5	4176.4
2^3D_3	3^{--}			4127	4217		4167		4218.9	4103	4220	4166.1		4112	4176	4151	4144	4141.8	4197.6
3^3P_0	0^{++}			4226	4292		4202			4131	4301	4207.6	4242	4146	4296	4197	4177	4151.5	4282.7
3^1P_1	1^{+-}			4229	4318		4279			4184	4337	4309.7	4278	4193	4344	4269	4200	4209.9	4291.2
3^3P_1	1^{++}			4232	4317		4271			4178	4319	4298.7	4272	4192	4338	4257	4197	4206.9	4307.7
3^3P_2	2^{++}			4228	4337		4317			4208	4354	4352.4		4211	4358	4290	4213	4242.7	4315.4
4^1S_0	0^{-+}			4312	4425		4384			4250	4401	4391.4		4264	4448	4328	4260	4269.3	4407.3
4^3S_1	1^{--}	$\psi(4415)$	4415(5)	4333	4450	4307	4406		4446.8	4273	4427	4411.4	4389	4281	4514	4362	4274	4294.4	4434.0

4.2. Fully Charmed Tetraquark

On the basis of the color antitriplet–triplet diquark–antidiquark configuration, the complete mass spectra of the low-lying fully charmed tetraquark states predicted by this work and their corresponding experimental candidates are laid out in Table 3. For the ease of the comparison, the predicted outcomes produced by several sorts of diquark–antidiquark scenarios, containing the nonrelativistic potential model [56,57,61,62], the relativized potential model [58–60,62], and the screened potential model [62], are also expressly exhibited in Table 3. Firstly, our model finds that the masses of the 1^1S_0, 1^3S_1, and 1^5S_2 $cc\bar{c}\bar{c}$ states are 6192, 6250, and 6365 MeV, respectively, coherent with the theoretical values 6190, 6271, and 6367 MeV derived by the famed Faustov–Galkin–Savchenko model [59,60]. Further, the predicted mass 6192 MeV of the 1^1S_0 state is also advocated by the chromomagnetic interaction model [53], the nonrelativistic potential model [72], the QCD string model [78], the Bethe–Salpeter equation [90], and the QCD sum rules [94]. Considering the mass interval 6330–6490 MeV of the $T_{\psi\psi}(6400)$ state measured by the ATLAS Collaboration [22], the $T_{\psi\psi}(6400)$ state is prone to be interpreted as the 1^5S_2 $cc\bar{c}\bar{c}$ state with the theoretical mass 6365 MeV. Nonetheless, not all of the diquark potential models champion this assignment. Generally, the spin-averaged mass and contact hyperfine splitting will impact the $1S$-wave predicted outcomes. There is no doubt that the higher diquark mass will cause the larger value of the centroid mass. For instance, the diquark cc mass gap between Refs. [60,61] contributes the difference of almost 200 MeV to the ultimate $cc\bar{c}\bar{c}$ tetraquark mass. In terms of the fully charmed tetraquark with the first orbital excitation, there are seven sorts of

P-wave $cc\bar{c}\bar{c}$ states. This study predicts that the masses of the 1^3P_0, 1^1P_1, 1^3P_1, 1^5P_1, 1^3P_2, 1^5P_2, and 1^5P_3 $cc\bar{c}\bar{c}$ states are 6573, 6666, 6661, 6560, 6688, 6669, and 6710 MeV, respectively. Accordingly, the observed masses offered by the ATLAS and CMS Collaborations are in favor of the 1^3P_1 assignment of the $T_{\psi\psi}(6600)$ state [22,23]. Apart from that, this work suggests that the masses of the 2^1S_0, 2^3S_1, and 2^5S_2 $cc\bar{c}\bar{c}$ states are 6745, 6764, and 6803 MeV, respectively. According to the $T_{\psi\psi}(6600)$ mass scope 6735–6747 MeV detected by the LHCb Collaboration [21], it is possible to deem the $T_{\psi\psi}(6600)$ state as the 2^1S_0 $cc\bar{c}\bar{c}$ tetraquark. Next, there are nine $cc\bar{c}\bar{c}$ states with the second orbital excitation. Our calculation shows that the masses of the 1^5D_0, 1^3D_1, 1^5D_1, 1^1D_2, 1^3D_2, 1^5D_2, 1^3D_3, 1^5D_3, and 1^5D_4 $cc\bar{c}\bar{c}$ states are 6828, 6853, 6840, 6877, 6877, 6862, 6888, 6884, and 6899 MeV, respectively. Taking into account the experimental data rendered by the LHCb, ATLAS, and CMS Collaborations [21–23], the potential candidates of the $T_{\psi\psi}(6900)$ state include the 1^5D_0, 1^5D_1, 1^1D_2, 1^5D_2, 1^5D_3, and 1^5D_4 $cc\bar{c}\bar{c}$ tetraquarks. These candidates stunningly involve five options of quantum numbers, demonstrating that the further experimental measurement for the $T_{\psi\psi}(6900)$ state is indispensable. As for the $2P$-wave $cc\bar{c}\bar{c}$ tetraquarks, the predicted masses of the 2^3P_0, 2^1P_1, 2^3P_1, 2^5P_1, 2^3P_2, 2^5P_2, and 2^5P_3 states are 6959, 6973, 6974, 6958, 6976, 6976, and 6979 MeV, respectively. Based on the $T_{\psi\psi}(6900)$ mass realm 6910–7010 MeV observed by the ATLAS Collaboration in the J/ψ+$\psi(2S)$ channel [22], there is a possibility of interpreting the $T_{\psi\psi}(6900)$ state as the 2^3P_0, 2^3P_1, or 2^3P_2 $cc\bar{c}\bar{c}$ tetraquarks. Then the predicted masses of the 3^1S_0, 3^3S_1, and 3^5S_2 states are 7043, 7053, and 7073 MeV, respectively, well conforming to the calculated outcomes 7031, 7038, and 7054 MeV achieved by the nonrelativistic potential model [62]. By comparing the theoretical prediction with the observed data enumerated in Table 1, there are no proper experimental candidates of the $3S$-wave $cc\bar{c}\bar{c}$ tetraquarks.

In the right part of Table 3, the mass spectra of the $cc\bar{c}\bar{c}$ states with the higher radial or orbital excitations are clearly listed. Initially, our model predicts that the masses of the 1^5F_1, 1^3F_2, 1^5F_2, 1^1F_3, 1^3F_3, 1^5F_3, 1^3F_4, 1^5F_4, and 1^5F_5 $cc\bar{c}\bar{c}$ states are 7034, 7044, 7041, 7054, 7055, 7050, 7059, 7059, and 7064 MeV, respectively, in accordance with the theoretical predictions proposed by the nonrelativistic potential model and the screened potential model [62]. Consequently, the candidates of the $1F$-wave $cc\bar{c}\bar{c}$ tetraquarks have not been discovered by sundry experiments. Subsequently, this work advises that the masses of the 2^5D_0, 2^3D_1, 2^5D_1, 2^1D_2, 2^3D_2, 2^5D_2, 2^3D_3, 2^5D_3, and 2^5D_4 $cc\bar{c}\bar{c}$ states are 7128, 7134, 7132, 7139, 7140, 7137, 7141, 7142, and 7143 MeV, respectively, congruent with the counterparts predicted by the nonrelativistic potential model [57,62] and the relativized potential model [58]. Whereafter, this study finds that the masses of the 3^3P_0, 3^1P_1, 3^3P_1, 3^5P_1, 3^3P_2, 3^5P_2, and 3^5P_3 $cc\bar{c}\bar{c}$ states are 7216, 7220, 7221, 7216, 7220, 7222, and 7221 MeV, respectively, which has a reconciliation with the results attained by the eminent Genoa group [58]. Lastly, the 4^1S_0, 4^3S_1, and 4^5S_2 $cc\bar{c}\bar{c}$ states with the masses of 7285, 7291, and 7304 MeV are predicted by this work. As revealed in Table 1, there are three sets of observed data for the $T_{\psi\psi}(7300)$ state, which possesses a huge mass gap in the vicinity of 150 MeV [22,23]. Noticeably, the inconsistency of the experimental data will bring on the ambiguity of the theoretical judgement. For instance, the $T_{\psi\psi}(7300)$ states with these three experimentally measured mass ranges 7109–7182, 7190–7250, and 7269–7307 MeV are likely to be assigned as the $2D$-, $3P$-, and $4S$-wave $cc\bar{c}\bar{c}$ tetraquarks, respectively. Hence, the precision enhancement of the experimental measurement is vital to decipher the properties of the $T_{\psi\psi}(7300)$ state. Furthermore, the experimental identification of the spin-parity quantum number will also facilitate the interpretation of the $cc\bar{c}\bar{c}$ tetraquarks.

Table 3. The fully charmed tetraquark mass spectrum (in unit of MeV).

State $n^{2S+1}L_J$	J^{PC}	This Work Mass	This Work Candidate	Other Approaches [56]	[58]	[60]	[61]	[62] [1]	[62] [2]
1^1S_0	0^{++}	6192		5969.4	5883	6190	5942	5989	5944
1^3S_1	1^{+-}	6250		6020.9	6120	6271	5989	6115	6001
1^5S_2	2^{++}	6365	$T_{\psi\psi}(6400)_{\mathrm{A}}$	6115.4	6246	6367	6082	6260	6105
1^3P_0	0^{-+}	6573		6480.4	6596	6628	6462	6545	6478
1^1P_1	1^{--}	6666		6577.1	6580	6631	6555	6605	6584
1^3P_1	1^{-+}	6661	$T_{\psi\psi}(6600)_{\mathrm{A;A';C'}}$	6577.4		6634	6556	6604	6584
1^5P_1	1^{--}	6560		6495.4	6584	6635	6461	6544	6495
1^3P_2	2^{-+}	6688		6609.9		6644	6589	6623	6618
1^5P_2	2^{--}	6669		6600.2		6648	6579	6618	6609
1^5P_3	3^{--}	6710		6641.2		6664	6625	6643	6648
2^1S_0	0^{++}	6745	$T_{\psi\psi}(6600)_{\mathrm{L'}}$	6663.3	6573	6782	6644	6644	6667
2^3S_1	1^{+-}	6764		6674.5	6669	6816	6656	6683	6679
2^5S_2	2^{++}	6803		6698.1	6739	6868	6678	6744	6703
1^5D_0	0^{++}	6828	$T_{\psi\psi}(6900)_{\mathrm{C'}}$	6820.4	6827	6899		6831	6826
1^3D_1	1^{+-}	6853		6832.8	6829	6909		6846	6841
1^5D_1	1^{++}	6840	$T_{\psi\psi}(6900)_{\mathrm{A;C'}}$	6828.4	6827	6904		6839	6835
1^1D_2	2^{++}	6877	$T_{\psi\psi}(6900)_{\mathrm{A;C';L'}}$	6847.7	6827	6921		6860	6859
1^3D_2	2^{+-}	6877		6846.4		6920		6860	6860
1^5D_2	2^{++}	6862	$T_{\psi\psi}(6900)_{\mathrm{A;C'}}$	6841.5	6827	6915		6853	6850
1^3D_3	3^{+-}	6888		6855.8		6932		6867	6867
1^5D_3	3^{++}	6884	$T_{\psi\psi}(6900)_{\mathrm{A;C';L'}}$	6855.5		6929		6867	6867
1^5D_4	4^{++}	6899	$T_{\psi\psi}(6900)_{\mathrm{L}}$	6864.0		6945		6875	6876
2^3P_0	0^{-+}	6959	$T_{\psi\psi}(6900)_{A'}$	6866.5	6953	7100	6852	6902	6867
2^1P_1	1^{--}	6973		6944.1	6940	7091	6926	6937	6951
2^3P_1	1^{-+}	6974	$T_{\psi\psi}(6900)_{A'}$	6943.9		7099	6927	6937	6951
2^5P_1	1^{--}	6958		6875.6	6943	7113	6850	6902	6877
2^3P_2	2^{-+}	6976	$T_{\psi\psi}(6900)_{A'}$	6970.4		7098	6952	6949	6977
2^5P_2	2^{--}	6976		6962.1		7113	6945	6946	6970
2^5P_3	3^{--}	6979		6996.7		7112	6983	6963	7002
3^1S_0	0^{++}	7043			6948	7259	7011	6979	7031
3^3S_1	1^{+-}	7053			7016	7287	7018	7001	7038
3^5S_2	2^{++}	7073			7071	7333	7033	7040	7054

State $n^{2S+1}L_J$	J^{PC}	This Work Mass	This Work Candidate	Other Approaches [57]	[58]	[61]	[62] [1]	[62] [2]
1^5F_1	1^{--}	7034					7035	7046
1^3F_2	2^{-+}	7044					7040	7051
1^5F_2	2^{--}	7041					7039	7050
1^1F_3	3^{--}	7054					7046	7056
1^3F_3	3^{-+}	7055					7047	7057
1^5F_3	3^{--}	7050					7045	7055
1^3F_4	4^{-+}	7059					7049	7059
1^5F_4	4^{--}	7059					7050	7060
1^5F_5	5^{--}	7064					7052	7061
2^5D_0	0^{++}	7128	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7120.8	7125		7092	7124
2^3D_1	1^{+-}	7134		7131.8	7128		7103	7137
2^5D_1	1^{++}	7132	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7128.4	7125		7098	7133
2^1D_2	2^{++}	7139	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7146.0	7126		7113	7154
2^3D_2	2^{+-}	7140		7145.0			7113	7155
2^5D_2	2^{++}	7137	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7140.9	7125		7108	7147
2^3D_3	3^{+-}	7141		7154.5			7118	7163
2^5D_3	3^{++}	7142	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7154.5			7118	7163
2^5D_4	4^{++}	7143	$T_{\psi\psi}(7300)_{\mathrm{C'}}$	7163.3			7124	7172
3^3P_0	0^{-+}	7216	$T_{\psi\psi}(7300)_A$		7236	7154	7151	7166
3^1P_1	1^{--}	7220			7226	7222	7175	7240
3^3P_1	1^{-+}	7221	$T_{\psi\psi}(7300)_A$			7221	7175	7239
3^5P_1	1^{--}	7216			7229	7151	7151	7173
3^3P_2	2^{-+}	7220	$T_{\psi\psi}(7300)_A$			7244	7184	7263
3^5P_2	2^{--}	7222				7237	7181	7256
3^5P_3	3^{--}	7221				7272	7194	7283
4^1S_0	0^{++}	7285	$T_{\psi\psi}(7300)_{\mathrm{C}}$		7237		7213	7316
4^3S_1	1^{+-}	7291			7293		7228	7321
4^5S_2	2^{++}	7304	$T_{\psi\psi}(7300)_{\mathrm{C}}$				7257	7333

[1] Results for the modified Godfrey–Isgur model in Ref. [62]. [2] Results for the nonrelativistic potential model in Ref. [62].

5. Summary

Over the past two decades, the burgeoning members of the exotic hadron zoo have enlightened theoretical inquiries on the diverse multiquark configurations. In view of the fact that the light quark degree of freedom is absent in the hadrons composed fully of the heavy (anti)quarks, the fully charmed tetraquarks with the diquark–antidiquark configuration are the most likely construction for the experimentally discovered $cc\bar{c}\bar{c}$ states. On the basis of the heavy antiquark–diquark symmetry (HADS), the spectroscopic investigation of the conventional charmonium will boost the decipherment of the exotic fully charmed tetraquark. Therefore, this work explores the potential assignments of the low-lying charmonia and $cc\bar{c}\bar{c}$ tetraquarks in light of the relativistic flux tube model.

To wrap things up, there are fourteen observed charmonia well identified by this work, involving the $1S$-wave assignments of the $\eta_c(1S)$ and $J/\psi(1S)$ states, the $1P$-wave assignments of the $\chi_{c0}(1P)$, $h_c(1P)$, $\chi_{c1}(1P)$, and $\chi_{c2}(1P)$ states, the $2S$-wave assignments of the $\eta_c(2S)$ and $\psi(2S)$ states, the $1D$-wave assignments of the $\psi(3770)$, $\psi_2(3823)$, and $\psi_3(3842)$

states, the $2P$-wave assignments of the $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$ states, and the $3S$-wave assignment of the $\psi(4040)$ state. Moreover, the status of the $\psi(4160)$ and $\psi(4415)$ states demonstrates that both of them necessitate further experimental and theoretical investigation. As far as the $cc\bar{c}\bar{c}$ tetraquarks are concerned, there are four experimental candidates which have been observed by the LHCb, ATLAS, and CMS Collaborations [21–23], including the $T_{\psi\psi}(6400)$ structure with the $1S$-wave interpretation, the $T_{\psi\psi}(6600)$ structure with the $1P$- and $2S$-wave interpretations, the $T_{\psi\psi}(6900)$ structure with the $1D$- and $2P$-wave interpretations, and the $T_{\psi\psi}(7300)$ structure with the $2D$-, $3P$-, and $4S$-wave interpretations. In addition, the predicted mass spectra for the undetected charmonia and $cc\bar{c}\bar{c}$ tetraquarks will deliver available clues to the projected experimental explorations.

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References

1. Navas, S.; Amsler, C.; Gutsche, T.; Hanhart, C.; Hernández-Rey, J.; Lourenco, C.; Masoni, A.; Mikhasenko, M.; Mitchell, R.; Patrignani, C.; et al. Review of Particle Physics. *Phys. Rev. D* **2024**, *110*, 030001. [CrossRef]
2. Olsen, S.L.; Skwarnicki, T.; Zieminska, D. Nonstandard heavy mesons and baryons: Experimental evidence. *Rev. Mod. Phys.* **2018**, *90*, 015003. [CrossRef]
3. Zhu, F.; Bauer, G.; Yi, K. Experimental Road to a Charming Family of Tetraquarks ... and Beyond. *Chin. Phys. Lett.* **2024**, *41*, 111201. [CrossRef]
4. Hosaka, A.; Iijima, T.; Miyabayashi, K.; Sakai, Y.; Yasui, S. Exotic hadrons with heavy flavors: X, Y, Z, and related states. *PTEP* **2016**, *2016*, 062C01. [CrossRef]
5. Ali, A.; Lange, J.S.; Stone, S. Exotics: Heavy Pentaquarks and Tetraquarks. *Prog. Part. Nucl. Phys.* **2017**, *97*, 123–198. [CrossRef]
6. Lebed, R.F.; Mitchell, R.E.; Swanson, E.S. Heavy-Quark QCD Exotica. *Prog. Part. Nucl. Phys.* **2017**, *93*, 143–194. [CrossRef]
7. Esposito, A.; Pilloni, A.; Polosa, A.D. Multiquark Resonances. *Phys. Rep.* **2017**, *668*, 1–97. [CrossRef]
8. Guo, F.K.; Hanhart, C.; Meißner, U.G.; Wang, Q.; Zhao, Q.; Zou, B.S. Hadronic molecules. *Rev. Mod. Phys.* **2018**, *90*, 015004. Erratum in *Rev. Mod. Phys.* **2022**, *94*, 029901. [CrossRef]
9. Karliner, M.; Rosner, J.L.; Skwarnicki, T. Multiquark States. *Ann. Rev. Nucl. Part. Sci.* **2018**, *68*, 17–44. [CrossRef]
10. Liu, Y.R.; Chen, H.X.; Chen, W.; Liu, X.; Zhu, S.L. Pentaquark and Tetraquark states. *Prog. Part. Nucl. Phys.* **2019**, *107*, 237–320. [CrossRef]
11. Albuquerque, R.M.; Dias, J.M.; Khemchandani, K.P.; Torres, A.M.; Navarra, F.S.; Nielsen, M.; Zanetti, C.M. QCD sum rules approach to the X, Y and Z states. *J. Phys. G* **2019**, *46*, 093002. [CrossRef]
12. Agaev, S.; Azizi, K.; Sundu, H. Four-quark exotic mesons. *Turk. J. Phys.* **2020**, *44*, 95–173. [CrossRef]
13. Yang, G.; Ping, J.; Segovia, J. Tetra- and penta-quark structures in the constituent quark model. *Symmetry* **2020**, *12*, 1869. [CrossRef]
14. Brambilla, N.; Eidelman, S.; Hanhart, C.; Nefediev, A.; Shen, C.P.; Thomas, C.E.; Vairo, A.; Yuan, C.Z. The XYZ states: Experimental and theoretical status and perspectives. *Phys. Rep.* **2020**, *873*, 1–154. [CrossRef]
15. Faustov, R.N.; Galkin, V.O.; Savchenko, E.M. Heavy tetraquarks in the relativistic quark model. *Universe* **2021**, *7*, 94. [CrossRef]
16. Chen, H.X.; Chen, W.; Liu, X.; Liu, Y.R.; Zhu, S.L. An updated review of the new hadron states. *Rept. Prog. Phys.* **2023**, *86*, 026201. [CrossRef]
17. Huang, H.; Deng, C.; Liu, X.; Tan, Y.; Ping, J. Tetraquarks and Pentaquarks from Quark Model Perspective. *Symmetry* **2023**, *15*, 1298. [CrossRef]

18. Bicudo, P. Tetraquarks and pentaquarks in lattice QCD with light and heavy quarks. *Phys. Rep.* **2023**, *1039*, 1–49. [CrossRef]
19. Liu, M.Z.; Pan, Y.W.; Liu, Z.W.; Wu, T.W.; Lu, J.X.; Geng, L.S. Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions. *Phys. Rep.* **2025**, *1108*, 1–108. [CrossRef]
20. Wang, Z.G. Review of the QCD sum rules for exotic states. *arXiv* **2025**, arXiv:2502.11351. Available online: https://arxiv.org/abs/2502.11351 (accessed on 6 June 2025).
21. LHCb Collaboration. Observation of structure in the J/ψ-pair mass spectrum. *Sci. Bull.* **2020**, *65*, 1983–1993. [CrossRef] [PubMed]
22. Aad, G.; Abbott, B.; Abeling, K.; Abicht, N.; Abidi, S.; Aboulhorma, A.; Abramowicz, H.; Abreu, H.; Abulaiti, Y.; ATLAS Collaboration. Observation of an Excess of Dicharmonium Events in the Four-Muon Final State with the ATLAS Detector. *Phys. Rev. Lett.* **2023**, *131*, 151902. [CrossRef] [PubMed]
23. Hayrapetyan, A.; Tumasyan, A.; Adam, W.; Andrejkovic, J.; Bergauer, T.; Chatterjee, S.; Damanakis, K.; Dragicevic, M.; Valle, A.E.D.; CMS Collaboration. New Structures in the $J/\psi J/\psi$ Mass Spectrum in Proton-Proton Collisions at $\sqrt{s} = 13$ TeV. *Phys. Rev. Lett.* **2024**, *132*, 111901. [CrossRef] [PubMed]
24. Gell-Mann, M. A Schematic Model of Baryons and Mesons. *Phys. Lett.* **1964**, *8*, 214–215. [CrossRef]
25. Zweig, G. An SU_3 Model for Strong Interaction Symmetry and Its Breaking; Version 2. CERN-TH-412. Available online: https://cds.cern.ch/record/570209 (accessed on 6 June 2025).
26. Barnes, T.; Godfrey, S.; Swanson, E.S. Higher charmonia. *Phys. Rev. D* **2005**, *72*, 054026. [CrossRef]
27. Cao, L.; Yang, Y.C.; Chen, H. Charmonium states in QCD-inspired quark potential model using Gaussian expansion method. *Few Body Syst.* **2012**, *53*, 327–342. [CrossRef]
28. Soni, N.R.; Joshi, B.R.; Shah, R.P.; Chauhan, H.R.; Pandya, J.N. $Q\bar{Q}$ ($Q \in \{b, c\}$) spectroscopy using the Cornell potential. *Eur. Phys. J. C* **2018**, *78*, 592. [CrossRef]
29. Radford, S.F.; Repko, W.W. Potential model calculations and predictions for heavy quarkonium. *Phys. Rev. D* **2007**, *75*, 074031. [CrossRef]
30. Kher, V.; Rai, A.K. Spectroscopy and decay properties of charmonium. *Chin. Phys. C* **2018**, *42*, 083101. [CrossRef]
31. Godfrey, S.; Isgur, N. Mesons in a Relativized Quark Model with Chromodynamics. *Phys. Rev. D* **1985**, *32*, 189–231. [CrossRef]
32. Ebert, D.; Faustov, R.N.; Galkin, V.O. Spectroscopy and Regge trajectories of heavy quarkonia and B_c mesons. *Eur. Phys. J. C* **2011**, *71*, 1825. [CrossRef]
33. Ahmad, Z.; Asghar, I.; Akram, F.; Masud, B. Strong decays of charmonia. *Phys. Rev. D* **2025**, *111*, 034007. [CrossRef]
34. Vijande, J.; Fernandez, F.; Valcarce, A. Constituent quark model study of the meson spectra. *J. Phys. G* **2005**, *31*, 481. [CrossRef]
35. Segovia, J.; Entem, D.R.; Fernandez, F.; Hernandez, E. Constituent quark model description of charmonium phenomenology. *Int. J. Mod. Phys. E* **2013**, *22*, 1330026. [CrossRef]
36. Eichten, E.J.; Lane, K.; Quigg, C. New states above charm threshold. *Phys. Rev. D* **2006**, *73*, 014014. Erratum in *Phys. Rev. D* **2006**, *73*, 079903. [CrossRef]
37. Li, B.Q.; Chao, K.T. Higher Charmonia and X, Y, Z states with Screened Potential. *Phys. Rev. D* **2009**, *79*, 094004. [CrossRef]
38. Deng, W.J.; Liu, H.; Gui, L.C.; Zhong, X.H. Charmonium spectrum and their electromagnetic transitions with higher multipole contributions. *Phys. Rev. D* **2017**, *95*, 034026. [CrossRef]
39. Wang, J.Z.; Chen, D.Y.; Liu, X.; Matsuki, T. Constructing J/ψ family with updated data of charmoniumlike Y states. *Phys. Rev. D* **2019**, *99*, 114003. [CrossRef]
40. Bokade, C.A.; Bhaghyesh. Conventional and XYZ charmonium states in a relativistic screened potential model. *Phys. Rev. D* **2025**, *111*, 014030. [CrossRef]
41. Iwasaki, Y. A Possible Model for New Resonances-Exotics and Hidden Charm. *Prog. Theor. Phys.* **1975**, *54*, 492. [CrossRef]
42. Chao, K.T. The (cc)-$(\bar{c}\bar{c})$ (Diquark-Antidiquark) States in e^+e^- Annihilation. *Z. Phys. C* **1981**, *7*, 317. [CrossRef]
43. Ader, J.P.; Richard, J.M.; Taxil, P. Do narrow heavy multiquark states exist? *Phys. Rev. D* **1982**, *25*, 2370. [CrossRef]
44. Heller, L.; Tjon, J.A. Bound States of Heavy $Q^2\bar{Q}^2$ Systems. *Phys. Rev. D* **1985**, *32*, 755. [CrossRef]
45. Badalian, A.M.; Ioffe, B.L.; Smilga, A.V. Four-quark states in heavy quark systems. *Nucl. Phys. B* **1987**, *281*, 85. [CrossRef]
46. Lloyd, R.J.; Vary, J.P. All-charm tetraquarks. *Phys. Rev. D* **2004**, *70*, 014009. [CrossRef]
47. Barnea, N.; Vijande, J.; Valcarce, A. Four-quark spectroscopy within the hyperspherical formalism. *Phys. Rev. D* **2006**, *73*, 054004. [CrossRef]
48. Chiu, T.W.; Hsieh, T.-H.; TWQCD Collaboration. $Y(4260)$ on the lattice. *Phys. Rev. D* **2006**, *73*, 094510. [CrossRef]
49. Berezhnoy, A.V.; Luchinsky, A.V.; Novoselov, A.A. Heavy tetraquarks production at the LHC. *Phys. Rev. D* **2012**, *86*, 034004. [CrossRef]
50. Heupel, W.; Eichmann, G.; Fischer, C.S. Tetraquark Bound States in a Bethe-Salpeter Approach. *Phys. Lett. B* **2012**, *718*, 545–549. [CrossRef]
51. Wang, Z.G. Analysis of the $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules. *Eur. Phys. J. C* **2017**, *77*, 432. [CrossRef]
52. Chen, W.; Chen, H.X.; Liu, X.; Steele, T.G.; Zhu, S.L. Hunting for exotic doubly hidden-charm/bottom tetraquark states. *Phys. Lett. B* **2017**, *773*, 247–251. [CrossRef]

53. Karliner, M.; Nussinov, S.; Rosner, J.L. $QQ\bar{Q}\bar{Q}$ states: masses, production, and decays. *Phys. Rev. D* **2017**, *95*, 034011. [CrossRef]
54. Wu, J.; Liu, Y.R.; Chen, K.; Liu, X.; Zhu, S.L. Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration. *Phys. Rev. D* **2018**, *97*, 094015. [CrossRef]
55. Anwar, M.N.; Ferretti, J.; Guo, F.K.; Santopinto, E.; Zou, B.S. Spectroscopy and decays of the fully-heavy tetraquarks. *Eur. Phys. J. C* **2018**, *78*, 647. [CrossRef]
56. Debastiani, V.R.; Navarra, F.S. A non-relativistic model for the $[cc][\bar{c}\bar{c}]$ tetraquark. *Chin. Phys. C* **2019**, *43*, 013105. [CrossRef]
57. Kalamidas, P.; Vanderhaeghen, M. All-charm tetraquark and its contribution to two-photon processes. *Phys. Rev. D* **2025**, *111*, 094033. [CrossRef]
58. Bedolla, M.A.; Ferretti, J.; Roberts, C.D.; Santopinto, E. Spectrum of fully-heavy tetraquarks from a diquark+antidiquark perspective. *Eur. Phys. J. C* **2020**, *80*, 1004. [CrossRef]
59. Faustov, R.N.; Galkin, V.O.; Savchenko, E.M. Masses of the $QQ\bar{Q}\bar{Q}$ tetraquarks in the relativistic diquark-antidiquark picture. *Phys. Rev. D* **2020**, *102*, 114030. [CrossRef]
60. Faustov, R.N.; Galkin, V.O.; Savchenko, E.M. Fully Heavy Tetraquark Spectroscopy in the Relativistic Quark Model. *Symmetry* **2022**, *14*, 2504. [CrossRef]
61. Tiwari, R.; Rathaud, D.P.; Rai, A.K. Spectroscopy of all charm tetraquark states. *Indian J. Phys.* **2023**, *97*, 943–954. [CrossRef]
62. Dong, W.C.; Wang, Z.G. Going in quest of potential tetraquark interpretations for the newly observed $T_{\psi\psi}$ states in light of the diquark-antidiquark scenarios. *Phys. Rev. D* **2023**, *107*, 074010. [CrossRef]
63. Liu, M.S.; Lü, Q.F.; Zhong, X.H.; Zhao, Q. All-heavy tetraquarks. *Phys. Rev. D* **2019**, *100*, 016006. [CrossRef]
64. Wang, Z.G.; Di, Z.Y. Analysis of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with QCD sum rules. *Acta Phys. Polon. B* **2019**, *50*, 1335. [CrossRef]
65. Wang, G.J.; Meng, L.; Zhu, S.L. Spectrum of the fully-heavy tetraquark state $QQ\bar{Q}'\bar{Q}'$. *Phys. Rev. D* **2019**, *100*, 096013. [CrossRef]
66. Gordillo, M.C.; Soto, F.D.; Segovia, J. Diffusion Monte Carlo calculations of fully-heavy multiquark bound states. *Phys. Rev. D* **2020**, *102*, 114007. [CrossRef]
67. Albuquerque, R.M.; Narison, S.; Rabemananjara, A.; Rabetiarivony, D.; Randriamanatrika, G. Doubly-hidden scalar heavy molecules and tetraquarks states from QCD at NLO. *Phys. Rev. D* **2020**, *102*, 094001. [CrossRef]
68. Jin, X.; Xue, Y.; Huang, H.; Ping, J. Full-heavy tetraquarks in constituent quark models. *Eur. Phys. J. C* **2020**, *80*, 1083. [CrossRef]
69. Zhao, J.; Shi, S.; Zhuang, P. Fully-heavy tetraquarks in a strongly interacting medium. *Phys. Rev. D* **2020**, *102*, 114001. [CrossRef]
70. Karliner, M.; Rosner, J.L. Interpretation of structure in the di-J/ψ spectrum. *Phys. Rev. D* **2020**, *102*, 114039. [CrossRef]
71. Lü, Q.F.; Chen, D.Y.; Dong, Y.B. Masses of fully heavy tetraquarks $QQ\bar{Q}\bar{Q}$ in an extended relativized quark model. *Eur. Phys. J. C* **2020**, *80*, 871. [CrossRef]
72. Lundhammar, P.; Ohlsson, T. Nonrelativistic model of tetraquarks and predictions for their masses from fits to charmed and bottom meson data. *Phys. Rev. D* **2020**, *102*, 054018. [CrossRef]
73. Giron, J.F.; Lebed, R.F. Simple spectrum of $c\bar{c}c\bar{c}$ states in the dynamical diquark model. *Phys. Rev. D* **2020**, *102*, 074003. [CrossRef]
74. Wang, Z.G. Tetraquark candidates in LHCb's di-J/ψ mass spectrum. *Chin. Phys. C* **2020**, *44*, 113106. [CrossRef]
75. Zhang, J.R. 0^+ fully-charmed tetraquark states. *Phys. Rev. D* **2021**, *103*, 014018. [CrossRef]
76. Sonnenschein, J.; Weissman, D. Deciphering the recently discovered tetraquark candidates around 6.9 GeV. *Eur. Phys. J. C* **2021**, *81*, 25. [CrossRef]
77. Deng, C.; Chen, H.; Ping, J. Towards the understanding of fully-heavy tetraquark states from various models. *Phys. Rev. D* **2021**, *103*, 014001. [CrossRef]
78. Nefediev, A.V. $X(6200)$ as a compact tetraquark in the QCD string model. *Eur. Phys. J. C* **2021**, *81*, 692. [CrossRef]
79. Wang, Q.N.; Yang, Z.Y.; Chen, W. Exotic fully-heavy $Q\bar{Q}Q\bar{Q}$ tetraquark states in $\mathbf{8}_{[Q\bar{Q}]} \otimes \mathbf{8}_{[Q\bar{Q}]}$ color configuration. *Phys. Rev. D* **2021**, *104*, 114037. [CrossRef]
80. Yang, G.; Ping, J.; Segovia, J. Exotic resonances of fully-heavy tetraquarks in a lattice-QCD insipired quark model. *Phys. Rev. D* **2021**, *104*, 014006. [CrossRef]
81. Zhu, R. Fully-heavy tetraquark spectra and production at hadron colliders. *Nucl. Phys. B* **2021**, *966*, 115393. [CrossRef]
82. Wang, G.J.; Meng, L.; Oka, M.; Zhu, S.L. Higher fully charmed tetraquarks: Radial excitations and P-wave states. *Phys. Rev. D* **2021**, *104*, 036016. [CrossRef]
83. Liu, F.X.; Liu, M.S.; Zhong, X.H.; Zhao, Q. Higher mass spectra of the fully-charmed and fully-bottom tetraquarks. *Phys. Rev. D* **2021**, *104*, 116029. [CrossRef]
84. Li, Q.; Chang, C.H.; Wang, G.L.; Wang, T. Mass spectra and wave functions of $T_{QQ\bar{Q}\bar{Q}}$ tetraquarks. *Phys. Rev. D* **2021**, *104*, 014018. [CrossRef]
85. Mutuk, H. Nonrelativistic treatment of fully-heavy tetraquarks as diquark-antidiquark states. *Eur. Phys. J. C* **2021**, *81*, 367. [CrossRef]
86. Wang, Z.G. Revisit the tetraquark candidates in the $J/\psi J/\psi$ mass spectrum. *Int. J. Mod. Phys. A* **2021**, *36*, 2150014. [CrossRef]

87. Yang, B.C.; Tang, L.; Qiao, C.F. Scalar fully-heavy tetraquark states $QQ'\bar{Q}\bar{Q}'$ in QCD sum rules. *Eur. Phys. J. C* **2021**, *81*, 324. [CrossRef]
88. Zhao, Z.; Xu, K.; Kaewsnod, A.; Liu, X.; Limphirat, A.; Yan, Y. Study of charmoniumlike and fully-charm tetraquark spectroscopy. *Phys. Rev. D* **2021**, *103*, 116027. [CrossRef]
89. Weng, X.Z.; Chen, X.L.; Deng, W.Z.; Zhu, S.L. Systematics of fully heavy tetraquarks. *Phys. Rev. D* **2021**, *103*, 034001. [CrossRef]
90. Ke, H.W.; Han, X.; Liu, X.H.; Shi, Y.L. Tetraquark state $X(6900)$ and the interaction between diquark and antidiquark. *Eur. Phys. J. C* **2021**, *81*, 427. [CrossRef]
91. Chen, X. The Genuine Resonance of Full-Charm Tetraquarks. *Universe* **2021**, *7*, 155. [CrossRef]
92. Dong, X.K.; Baru, V.; Guo, F.K.; Hanhart, C.; Nefediev, A. Coupled-Channel Interpretation of the LHCb Double-J/ψ Spectrum and Hints of a New State Near the $J/\psi J/\psi$ Threshold. *Phys. Rev. Lett.* **2021**, *126*, 132001. Erratum in *Phys. Rev. Lett.* **2021**, *127*, 119901. [CrossRef]
93. Kuang, Z.; Serafin, K.; Zhao, X.; Vary, J.P. All-charm tetraquark in front form dynamics. *Phys. Rev. D* **2022**, *105*, 094028. [CrossRef]
94. Wang, Z.G. Analysis of the X(6600), X(6900), X(7300) and related tetraquark states with the QCD sum rules. *Nucl. Phys. B* **2022**, *985*, 115983. [CrossRef]
95. Majarshin, A.J.; Luo, Y.A.; Pan, F.; Segovia, J. Bosonic algebraic approach applied to the $[QQ][\bar{Q}\bar{Q}]$ tetraquarks. *Phys. Rev. D* **2022**, *105*, 054024. [CrossRef]
96. Zhuang, Z.; Zhang, Y.; Ma, Y.; Wang, Q. Lineshape of the compact fully heavy tetraquark. *Phys. Rev. D* **2022**, *105*, 054026. [CrossRef]
97. Wu, R.H.; Zuo, Y.S.; Wang, C.Y.; Meng, C.; Ma, Y.Q.; Chao, K.T. NLO results with operator mixing for fully heavy tetraquarks in QCD sum rules. *JHEP* **2022**, *11*, 023. [CrossRef]
98. Zhang, J.; Wang, J.B.; Li, G.; An, C.S.; Deng, C.R.; Xie, J.J. Spectrum of the S-wave fully-heavy tetraquark states. *Eur. Phys. J. C* **2022**, *82*, 1126. [CrossRef]
99. Wang, G.J.; Meng, Q.; Oka, M. S-wave fully charmed tetraquark resonant states. *Phys. Rev. D* **2022**, *106*, 096005. [CrossRef]
100. Zhou, Q.; Guo, D.; Kuang, S.Q.; Yang, Q.H.; Dai, L.Y. Nature of the $X(6900)$ in partial wave decomposition of $J/\psi J/\psi$ scattering. *Phys. Rev. D* **2022**, *106*, L111502. [CrossRef]
101. Agaev, S.S.; Azizi, K.; Barsbay, B.; Sundu, H. Exploring fully heavy scalar tetraquarks $QQ\bar{Q}\bar{Q}$. *Phys. Lett. B* **2023**, *844*, 138089. [CrossRef]
102. Ortega, P.G.; Entem, D.R.; Fernández, F. Exploring $T_{\psi\psi}$ tetraquark candidates in a coupled-channels formalism. *Phys. Rev. D* **2023**, *108*, 094023. [CrossRef]
103. Agaev, S.S.; Azizi, K.; Barsbay, B.; Sundu, H. Hadronic molecules $\eta_c\eta_c$ and $\chi_{c0}\chi_{c0}$. *Eur. Phys. J. Plus* **2023**, *138*, 935. [CrossRef]
104. Yan, T.Q.; Zhang, W.X.; Jia, D. Mass spectra of hidden heavy-flavor tetraquarks with two and four heavy quarks. *Eur. Phys. J. C* **2023**, *83*, 810. [CrossRef]
105. Agaev, S.S.; Azizi, K.; Barsbay, B.; Sundu, H. Resonance $X(7300)$: Excited $2S$ tetraquark or hadronic molecule $\chi_{c1}\chi_{c1}$? *Eur. Phys. J. C* **2023**, *83*, 994. [CrossRef]
106. An, H.T.; Luo, S.Q.; Liu, Z.W.; Liu, X. Spectroscopic behavior of fully heavy tetraquarks. *Eur. Phys. J. C* **2023**, *83*, 740. [CrossRef]
107. Kuang, S.Q.; Zhou, Q.; Guo, D.; Yang, Q.H.; Dai, L.Y. Study of $X(6900)$ with unitarized coupled channel scattering amplitudes. *Eur. Phys. J. C* **2023**, *83*, 383. [CrossRef]
108. Yu, G.L.; Li, Z.Y.; Wang, Z.G.; Lu, J.; Yan, M. The S- and P-wave fully charmed tetraquark states and their radial excitations. *Eur. Phys. J. C* **2023**, *83*, 416. [CrossRef]
109. Badalian, A.M. The $X(6550)$, $X(6900)$, $X(7280)$ Resonances as the nS, $cc\bar{c}\bar{c}$ States. *Phys. Atom. Nucl.* **2023**, *86*, 701–708. [CrossRef]
110. Wu, W.L.; Chen, Y.K.; Meng, L.; Zhu, S.L. Benchmark calculations of fully heavy compact and molecular tetraquark states. *Phys. Rev. D* **2024**, *109*, 054034. [CrossRef]
111. Agaev, S.S.; Azizi, K.; Barsbay, B.; Sundu, H. Fully charmed resonance $X(6900)$ and its beauty counterpart. *Nucl. Phys. A* **2024**, *1041*, 122768. [CrossRef]
112. Liu, M.S.; Liu, F.X.; Zhong, X.H.; Zhao, Q. Fully heavy tetraquark states and their evidences in LHC observations. *Phys. Rev. D* **2024**, *109*, 076017. [CrossRef]
113. Chen, Z.Z.; Chen, X.L.; Yang, P.F.; Chen, W. P-wave fully charm and fully bottom tetraquark states. *Phys. Rev. D* **2024**, *109*, 094011. [CrossRef]
114. Anwar, M.N.; Burns, T.J. Structure of $cc\bar{c}\bar{c}$ tetraquarks and interpretation of LHC states. *Phys. Rev. D* **2024**, *110*, 034012. [CrossRef]
115. Wu, W.L.; Zhu, S.L. Fully charmed P-wave tetraquark resonant states in the quark model. *Phys. Rev. D* **2025**, *111*, 034044. [CrossRef]
116. Wu, Y.; Liu, X.; Ping, J.; Huang, H.; Tan, Y. Further study of $c\bar{c}c\bar{c}$ system within a chiral quark model. *Eur. Phys. J. C* **2025**, *85*, 147. [CrossRef]
117. Meng, Y.; Liu, C.; Tuo, X.Y.; Yan, H.; Zhang, Z. Lattice calculation of the $\eta_c\eta_c$ and $J/\psi J/\psi$ s-wave scattering length. *Eur. Phys. J. C* **2025**, *85*, 458. [CrossRef]

118. Eskin, A.V.; Martynenko, A.P.; Martynenko, F.A. Mass spectrum of heavy tetraquarks in variational approach. *arXiv* **2025**, arXiv:2505.05993. Available online: https://arxiv.org/abs/2505.05993 (accessed on 6 June 2025).
119. Li, G.; Shi, C.; Chen, Y.; Sun, W. Tensor Resonance in $J/\psi J/\psi$ Scattering from Lattice QCD. *arXiv* **2025**, arXiv:2505.24213. Available online: https://arxiv.org/abs/2505.24213 (accessed on 6 June 2025).
120. Li, G.; Shi, C.; Chen, Y.; Sun, W. $\eta_c\eta_c$ and $J/\psi J/\psi$ scatterings from lattice QCD. *arXiv* **2025**, arXiv:2505.23220. Available online: https://arxiv.org/abs/2505.23220 (accessed on 6 June 2025).
121. Chew, G.F.; Frautschi, S.C. Regge Trajectories and the Principle of Maximum Strength for Strong Interactions. *Phys. Rev. Lett.* **1962**, *8*, 41–44. [CrossRef]
122. Nambu, Y. Strings, monopoles, and gauge fields. *Phys. Rev. D* **1974**, *10*, 4262. [CrossRef]
123. Johnson, K.; Nohl, C. Simple semiclassical model for the rotational states of mesons containing massive quarks. *Phys. Rev. D* **1979**, *19*, 291. [CrossRef]
124. Cea, P.; Colangelo, P.; Nardulli, G.; Paiano, G.; Preparata, G. WKB approach to the Schrodinger equation with relativistic kinematics. *Phys. Rev. D* **1982**, *26*, 1157. [CrossRef]
125. LaCourse, D.; Olsson, M.G. String potential model: Spinless quarks. *Phys. Rev. D* **1989**, *39*, 2751. [CrossRef]
126. Olson, C.; Olsson, M.G.; Williams, K. QCD, relativistic flux tubes, and potential models. *Phys. Rev. D* **1992**, *45*, 4307–4311. [CrossRef] [PubMed]
127. Sergeenko, M.N. An Interpolating mass formula and Regge trajectories for light and heavy quarkonia. *Z. Phys. C* **1994**, *64*, 315–322. [CrossRef]
128. Dubin, A.Y.; Kaidalov, A.B.; Simonov, Y.A. Dynamical regimes of the QCD string with quarks. *Phys. Lett. B* **1994**, *323*, 41–45. [CrossRef]
129. Brau, F. Bohr-Sommerfeld quantization and meson spectroscopy. *Phys. Rev. D* **2000**, *62*, 014005. [CrossRef]
130. Brisudova, M.M.; Burakovsky, L.; Goldman, J.T. Effective functional form of Regge trajectories. *Phys. Rev. D* **2000**, *61*, 054013. [CrossRef]
131. Anisovich, A.V.; Anisovich, V.V.; Sarantsev, A.V. Systematics of $q\bar{q}$ states in the (n, M^2) and (J, M^2) planes. *Phys. Rev. D* **2000**, *62*, 051502. [CrossRef]
132. Kalashnikova, Y.S.; Nefediev, A.V.; Simonov, Y.A. QCD string in light-light and heavy-light mesons. *Phys. Rev. D* **2001**, *64*, 014037. [CrossRef]
133. Bugg, D.V. Four sorts of meson. *Phys. Rep.* **2004**, *397*, 257–358. [CrossRef]
134. Kruczenski, M.; Zayas, L.A.P.; Sonnenschein, J.; Vaman, D. Regge trajectories for mesons in the holographic dual of large-N_c QCD. *JHEP* **2005**, *6*, 046. [CrossRef]
135. Karch, A.; Katz, E.; Son, D.T.; Stephanov, M.A. Linear confinement and AdS/QCD. *Phys. Rev. D* **2006**, *74*, 015005. [CrossRef]
136. Gershtein, S.S.; Likhoded, A.K.; Luchinsky, A.V. Systematics of heavy quarkonia from Regge trajectories on (n, M^2) and (M^2, J) planes. *Phys. Rev. D* **2006**, *74*, 016002. [CrossRef]
137. Bicudo, P. Large degeneracy of excited hadrons and quark models. *Phys. Rev. D* **2007**, *76*, 094005. [CrossRef]
138. Afonin, S.S. Properties of possible new unflavored mesons below 2.4 GeV. *Phys. Rev. C* **2007**, *76*, 015202. [CrossRef]
139. Gherghetta, T.; Kapusta, J.I.; Kelley, T.M. Chiral symmetry breaking in the soft-wall AdS/QCD model. *Phys. Rev. D* **2009**, *79*, 076003. [CrossRef]
140. Chen, B.; Wang, D.X.; Zhang, A. Interpretation of $D_{sJ}(2632)^+$, $D_{s1}(2700)^{\pm}$, $D^*_{sJ}(2860)^+$, and $D_{sJ}(3040)^+$. *Phys. Rev. D* **2009**, *80*, 071502. [CrossRef]
141. Masjuan, P.; Arriola, E.R.; Broniowski, W. Systematics of radial and angular-momentum Regge trajectories of light nonstrange $q\bar{q}$-states. *Phys. Rev. D* **2012**, *85*, 094006. Reply: *Phys. Rev. D* **2013**, *87*, 118502. [CrossRef]
142. Afonin, S.S.; Pusenkov, I.V. Universal description of radially excited heavy and light vector mesons. *Phys. Rev. D* **2014**, *90*, 094020. [CrossRef]
143. Badalian, A.M.; Bakker, B.L.G. Radial and orbital Regge trajectories in heavy quarkonia. *Phys. Rev. D* **2019**, *100*, 054036. [CrossRef]
144. Jia, D.; Dong, W.C. Regge-like spectra of excited singly heavy mesons. *Eur. Phys. J. Plus* **2019**, *134*, 123. [CrossRef]
145. Chen, B.; Zhang, A.; He, J. Bottomonium spectrum in the relativistic flux tube model. *Phys. Rev. D* **2020**, *101*, 014020. [CrossRef]
146. Pan, J.; Pan, J.H. Remarks on the S-wave masses of singly heavy mesons. *arXiv* **2022**, arXiv:2209.14948. Available online: https://arxiv.org/abs/2209.14948 (accessed on 6 June 2025).
147. Carlson, J.; Kogut, J.B.; Pandharipande, V.R. Hadron spectroscopy in a flux-tube quark model. *Phys. Rev. D* **1983**, *28*, 2807. [CrossRef]
148. Brambilla, N.; Prosperi, G.M.; Vairo, A. Three body relativistic flux tube model from QCD Wilson loop approach. *Phys. Lett. B* **1995**, *362*, 113–122. [CrossRef]
149. Iwasaki, M.; Takagi, F. Mass spectra and decay widths of hadrons in the relativistic string model. *Phys. Rev. D* **1999**, *59*, 094024. [CrossRef]
150. Tang, A.; Norbury, J.W. Properties of Regge trajectories. *Phys. Rev. D* **2000**, *62*, 016006. [CrossRef]

151. Inopin, A.; Sharov, G.S. Hadronic Regge trajectories: Problems and approaches. *Phys. Rev. D* **2001**, *63*, 054023. [CrossRef]
152. Selem, A.; Wilczek, F. Hadron systematics and emergent diquarks. *Proc. Ringberg Workshop New Trends Hera Phys.* ***2006***, 2006, *337–356*. [CrossRef]
153. Forkel, H.; Beyer, M.; Frederico, T. Linear square-mass trajectories of radially and orbitally excited hadrons in holographic QCD. *JHEP* **2007**, *7*, 077. [CrossRef]
154. Glozman, L.Y. Restoration of chiral and $U(1)_A$ symmetries in excited hadrons. *Phys. Rep.* **2007**, *444*, 1–49. [CrossRef]
155. Shifman, M.; Vainshtein, A. Highly excited mesons, linear Regge trajectories, and the pattern of the chiral symmetry realization. *Phys. Rev. D* **2008**, *77*, 034002. [CrossRef]
156. Gutsche, T.; Lyubovitskij, V.E.; Schmidt, I.; Vega, A. Dilaton in a soft-wall holographic approach to mesons and baryons. *Phys. Rev. D* **2012**, *85*, 076003. [CrossRef]
157. Chen, K.; Dong, Y.; Liu, X.; Lü, Q.F.; Matsuki, T. Regge-like relation and a universal description of heavy–light systems. *Eur. Phys. J. C* **2018**, *78*, 20. [CrossRef]
158. Jia, D.; Dong, W.C.; Hosaka, A. Regge-Like Mass Relation of Singly Heavy Hadrons. *JPS Conf. Proc.* **2019**, *26*, 022021. [CrossRef]
159. Nielsen, M.; Brodsky, S.J. Hadronic superpartners from a superconformal and supersymmetric algebra. *Phys. Rev. D* **2018**, *97*, 114001. [CrossRef]
160. Rossi, G.; Veneziano, G. The string-junction picture of multiquark states: An update. *JHEP* **2016**, *06*, 041. [CrossRef]
161. Sonnenschein, J.; Weissman, D. Excited mesons, baryons, glueballs and tetraquarks: Predictions of the Holography Inspired Stringy Hadron model. *Eur. Phys. J. C* **2019**, *79*, 326. [CrossRef]
162. Chen, J.K. Regge trajectory relation for the universal description of the heavy-heavy systems: Diquarks, mesons, baryons and tetraquarks. *Nucl. Phys. A* **2024**, *1050*, 122927. [CrossRef]
163. Chen, J.K. Regge trajectory relations for the universal description of the heavy-light systems: Diquarks, mesons, baryons and tetraquarks. *Eur. Phys. J. C* **2024**, *84*, 356. [CrossRef]
164. Capossoli, E.F.; Contreras, M.A.M.; Li, D.; Vega, A.; Boschi-Filho, H. Hadronic spectra from deformed AdS backgrounds. *Chin. Phys. C* **2020**, *44*, 064104. [CrossRef]
165. Burns, T.J.; Piccinini, F.; Polosa, A.D.; Sabelli, C. The 2^{-+} assignment for the $X(3872)$. *Phys. Rev. D* **2010**, *82*, 074003. [CrossRef]
166. Barnes, T.; Close, F.E.; Swanson, E.S. Hybrid and conventional mesons in the flux tube model: Numerical studies and their phenomenological implications. *Phys. Rev. D* **1995**, *52*, 5242–5256. [CrossRef] [PubMed]
167. Li, D.; Huang, M. Dynamical holographic QCD model for glueball and light meson spectra. *JHEP* **2013**, *11*, 088. [CrossRef]
168. Fiore, R.; Jenkovszky, L.L.; Paccanoni, F.; Prokudin, A. Baryonic Regge trajectories with analyticity constraints. *Phys. Rev. D* **2004**, *70*, 054003. [CrossRef]
169. Klempt, E.; Metsch, B.C. Multiplet classification of light-quark baryons. *Eur. Phys. J. A* **2012**, *48*, 127. [CrossRef]
170. Chen, B.; Wei, K.W.; Zhang, A. Investigation of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture. *Eur. Phys. J. A* **2015**, *51*, 82. [CrossRef]
171. Fernández-Ramírez, C.; Danilkin, I.V.; Mathieu, V.; Szczepaniak, A.P.; JPAC Collaboration. Understanding the Nature of $\Lambda(1405)$ through Regge Physics. *Phys. Rev. D* **2016**, *93*, 074015. [CrossRef]
172. Masjuan, P.; Arriola, E.R. Regge trajectories of excited baryons, quark-diquark models, and quark-hadron duality. *Phys. Rev. D* **2017**, *96*, 054006. [CrossRef]
173. Silva-Castro, J.; Fernández-Ramírez, C.; Albaladejo, M.; Danilkin, I.; Jackura, A.; Mathieu, V.; Nys, J.; Pilloni, A.; Szczepaniak, A.; JPAC Collaboration. Regge phenomenology of the N^* and Δ^* poles. *Phys. Rev. D* **2019**, *99*, 034003. [CrossRef]
174. Jia, D.; Liu, W.N.; Hosaka, A. Regge behaviors in orbitally excited spectroscopy of charmed and bottom baryons. *Phys. Rev. D* **2020**, *101*, 034016. [CrossRef]
175. Jia, D.; Pan, J.H.; Pang, C.Q. A Mixing Coupling Scheme for Spectra of Singly Heavy Baryons with Spin-1 Diquarks in P-waves. *Eur. Phys. J. C* **2021**, *81*, 434. [CrossRef]
176. Cheng, H.Y. Charmed baryon physics circa 2021. *Chin. J. Phys.* **2022**, *78*, 324–362. [CrossRef]
177. Jakhad, P.; Oudichhya, J.; Gandhi, K.; Rai, A.K. Identification of newly observed singly charmed baryons using the relativistic flux tube model. *Phys. Rev. D* **2023**, *108*, 014011. [CrossRef]
178. Song, Y.; Jia, D.; Zhang, W.; Hosaka, A. Low-lying doubly heavy baryons: Regge relation and mass scaling. *Eur. Phys. J. C* **2023**, *83*, 1. [CrossRef]
179. Jakhad, P.; Oudichhya, J.; Rai, A.K. Interpretation of recently discovered single bottom baryons in the relativistic flux tube model. *Phys. Rev. D* **2024**, *110*, 094005. [CrossRef]
180. Pan, J.H.; Pan, J. Investigation of the mass spectra of singly heavy baryons Σ_Q, Ξ'_Q, and $\Omega_Q(Q = c, b)$ in the Regge trajectory model. *Phys. Rev. D* **2024**, *109*, 076010. [CrossRef]
181. Iwasaki, M.; Fukutome, T. Tetraquark particle in the string model. *Phys. Rev. D* **2005**, *72*, 094016. [CrossRef]
182. Cotugno, G.; Faccini, R.; Polosa, A.D.; Sabelli, C. Charmed Baryonium. *Phys. Rev. Lett.* **2010**, *104*, 132005. [CrossRef] [PubMed]

183. Song, Y.; Jia, D. Mass spectra of doubly heavy tetraquarks in diquark−antidiquark picture. *Commun. Theor. Phys.* **2023**, *75*, 055201. [CrossRef]
184. Sonnenschein, J.; Green, M.M. Taming the Zoo of Tetraquarks and Pentaquarks using the HISH Model. *arXiv* **2024**, arXiv:2401.01621. Available online: https://arxiv.org/abs/2401.01621 (accessed on 6 June 2025).
185. Iwasaki, M.; Takagi, F. Pentaquark in the flux tube model. *Phys. Rev. D* **2008**, *77*, 054020. [CrossRef]
186. Deng, C.; Ping, J.; Yang, Y.; Wang, F. Baryonia and near-threshold enhancements. *Phys. Rev. D* **2013**, *88*, 074007. [CrossRef]
187. Andreev, O. Doubly heavy dibaryons as seen by string theory. *Phys. Rev. D* **2024**, *109*, 106001. [CrossRef]
188. Buisseret, F.; Semay, C. Two- and three-body descriptions of hybrid mesons. *Phys. Rev. D* **2006**, *74*, 114018. [CrossRef]
189. Isgur, N.; Paton, J.E. Flux-tube model for hadrons in QCD. *Phys. Rev. D* **1985**, *31*, 2910. [CrossRef]
190. Iwasaki, M.; Nawa, S.I.; Sanada, T.; Takagi, F. Flux tube model for glueballs. *Phys. Rev. D* **2003**, *68*, 074007. [CrossRef]
191. Boschi-Filho, H.; Braga, N.R.F.; Carrion, H.L. Glueball Regge trajectories from gauge-string duality and the pomeron. *Phys. Rev. D* **2006**, *73*, 047901. [CrossRef]
192. Szanyi, I.; Jenkovszky, L.; Schicker, R.; Svintozelskyi, V. Pomeron/glueball and odderon/oddball trajectories. *Nucl. Phys. A* **2020**, *998*, 121728. [CrossRef]
193. Song, H.; Xie, J.Q.; Chen, J.K. Regge trajectories for the triply heavy triquarks. *Eur. Phys. J. C* **2025**, *85*, 482. [CrossRef]
194. Eichten, E.J.; Quigg, C. Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_iQ_j\bar{q}_k\bar{q}_l$. *Phys. Rev. Lett.* **2017**, *119*, 202002. [CrossRef] [PubMed]
195. Dong, W.C.; Wang, Z.G. Hunting for the prospective T_{cc} family based on the diquark-antidiquark configuration. *Nucl. Phys. B* **2025**, *1012*, 116828. [CrossRef]