

Nuclei in Dense Matter and Equation of State

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Abstract. Correlations are an essential feature of interacting many-body systems such as nuclear and stellar matter. They cause the formation of clusters, i.e. nuclei and scattering resonances, changing the chemical composition and affecting the thermodynamical properties of the system. The equation of state has to be known in a wide range of density, temperature and isospin asymmetry for astrophysical simulations of core-collapse supernovae and compact stars. In a generalized relativistic density functional approach, the properties of dense stellar matter can be described with the correct limits at low densities, the model-independent virial equation of state, and at high densities using a quasiparticle mean-field approach. Effects of correlations and changes of the cluster properties in dilute matter can be studied experimentally in heavy-ion collisions.

1. Introduction

The final stage in the evolution of a massive star ($M_{\text{star}} \gtrsim 8M_{\text{sun}}$) is reached when the growing iron core collapses under its own gravitational attraction since nuclear fusion reactions can no longer provide energy and pressure to stabilize the system. A violent supernovae explosion is launched and the outer shells of the star are ejected leaving a neutron star or a black hole as a remnant [1]. The equation of state (EoS) of dense stellar matter determines the dynamical evolution of such a core-collapse supernova, the properties of the (proto-)neutron star [2] and the thermodynamic conditions and chemical composition of the expanding matter where nucleosynthesis processes are presumed to create heavy elements.

The thermodynamic conditions of the matter are set by three parameters: the mass density ρ (or baryon number density n), the temperature T and the electron fraction Y_e (or neutron-proton asymmetry $\beta = 1 - 2Y_e$). In supernovae simulations these quantities cover large ranges with typical values of $10^{-9} \lesssim \rho/\rho_{\text{sat}} \lesssim 10$ (with the nuclear saturations density $\rho_{\text{sat}} \approx 2.5 \cdot 10^{14} \text{g/cm}^3$), $0.1 \text{ MeV} \lesssim k_B T \lesssim 50 \text{ MeV}$ and $0 \leq Y_e \lesssim 0.6$. At densities below ρ_{sat} , the relevant degrees of freedom in stellar matter are nucleons, nuclei, electrons and thermal photons. An equation of state of dense matter in thermodynamical and chemical equilibrium assuming local charge neutrality can be used in astrophysical simulations because the timescale of the nuclear reactions is much shorter than that of the dynamical evolution of the system. Neutrinos are not included in the EoS because they are not in equilibrium with the matter and therefore treated explicitly in supernovae simulations.

A large variety of EoS can be found in the literature, from simple parametrizations to sophisticated models, often focusing on specific aspects and properties of the matter. For many years, there were only few global EoS available that were used in astrophysical simulations

covering the relevant parameter ranges, e.g. [3, 4, 5], These conventional EoS tables consider only those particles that are also incorporated in astrophysical simulations, i.e. nucleons, electrons, photons, ^4He and a representative heavy nucleus. Recently, several new and extended models were developed [6, 7, 8, 9, 10, 11] including the full distribution of nuclei or more exotic particles such as hyperons or quarks.

At densities around and above nuclear saturation density, nuclear matter is expected to be uniform and mean-field models with nucleons as degrees of freedom are very successful. A quasiparticle approach is used to take the effects of the interaction into account. At lower densities and temperatures below approximately 15 MeV, inhomogeneities will develop with a coexistence of gas and liquid phases on a macroscopic scale. This feature is the origin of the “liquid-gas” first-order phase transition in nuclear matter, a fictitious system where the long-range Coulomb interaction is neglected. However, in realistic stellar matter with Coulomb interaction and electrons under the condition of charge neutrality, low- and high-density regions are separated by various shapes of the interfaces on a microscopic scale that are often described by the so-called “pasta phases”. Solid phases with crystal/lattice structures develop at very low temperatures, e.g. in the outer crust of neutron stars. At very low densities and sufficiently high temperature, light clusters (^2H , ^3H , ^3He , ^4He) have to be considered in the EoS model as explicit two-, three-, . . . , many-body correlations.

The theoretical description of dense matter at nuclear saturation density and (much) below is the focus of this contribution, in particular the formation of clusters due to correlations and their dissolution with increasing density. Different concepts and approaches to describe dense matter with clusters at low densities are discussed in section 2. A specific approach introducing a generalized relativistic density functional (gRDF) is presented in section 3. The model takes the effects of few- and many-body correlations explicitly into account with the correct low- and high-density limits. The change of the properties of light and heavy nuclei due to the medium are discussed in section 4. Conditions from the low-density limit of the model are examined in section 5. In section 6, the abundances of light clusters and the effects of two-nucleon scattering correlations are considered. The behavior of symmetry energy with cluster formation is studied in section 7. Finally, experimental approaches to test the model predictions are mentioned. Details of the material presented in this contribution and further references can be found in references [12, 13, 14]. Natural units are used in order to simplify the equations, i.e. $\hbar = c = k_B = 1$.

2. Theoretical approaches

The chemical composition and thermodynamic properties of dilute matter vary substantially with density, temperature and neutron-proton asymmetry. Theoretical models of the system can be constructed by starting from two different points of view.

A mixture of nucleons and nuclei in chemical equilibrium is considered in the *chemical picture*. The properties of the particles composed of nucleons are assumed to be independent of the medium. Correlation effects can be taken into account by specifying the interaction individually for all pairs of constituents. A major problem is the description of the cluster dissolution at high densities. A simple geometric concept such as the excluded-volume mechanism can be introduced for this purpose.

Only nucleons are considered as fundamental constituents in the *physical picture*. Correlations are consequences of the nucleon-nucleon interaction. It is the only and essential ingredient in this type of approach leading to the formation of bound states and scattering resonances. In order to treat two-, three-, . . . many-body correlations inside the medium, appropriate theoretical methods have to be applied.

In the following, several models are presented that combine both pictures to various degrees considering different effects of the nuclear interaction.

2.1. Low-density models

2.1.1. Nuclear statistical equilibrium model The simplest NSE model assumes an ideal mixture of nucleons (p, n) and nuclei (X) with mass numbers $A = N + Z$ in chemical equilibrium $Zp + Nn \Leftrightarrow \frac{A}{Z}X$ neglecting mutual interactions. Hence the nonrelativistic chemical potentials μ_i of the particles are given by $\mu_X = Z\mu_p + N\mu_n + B_X$ with the binding energies $B_X > 0$ of the nuclei X with mass $m_X = Zm_p + Nm_n - B_X$. For nonrelativistic kinematics and Maxwell-Boltzmann statistics, the grand canonical potential assumes the form

$$\Omega(T, V, \mu_i) = -TV \sum_i \frac{g_i}{\lambda_i^3} \exp\left(\frac{\mu_i}{T}\right) \quad (1)$$

with thermal wavelenghts $\lambda_i = \sqrt{2\pi/(m_i T)}$ and degeneracy factors g_i by summing over protons, neutrons and all nuclei. Excited states of nuclei are usually taken into account by introducing temperature dependent degeneracy factors $g_i(T)$. All thermodynamical properties, e.g. the equations of state, such as $pV = NT$ and $E = 3NT/2$ with $N = V \sum_i n_i$, for a mixture of ideal gases can be derived from Eq. (1). In particular, the individual particle number densities are given by $n_i = -\partial\Omega/\partial\mu_i|_{T, V, \mu_{j \neq i}} = g_i \lambda_i^{-3} \exp(\mu_i/T)$. The densities of the particles species are connected by the law of mass action that is well known in chemistry. The simple NSE model cannot describe the dissolution of nuclei with increasing density. Their abundance can be suppressed with increasing density using an excluded-volume mechanism, see, e.g., reference [6].

2.1.2. Virial equation of state The VEOs can incorporate two-, three-, ..., many-body correlations due to the interaction between the constituents using an expansion of the grand canonical partition function in powers of the particle fugacities $z_i = \exp(\mu_i/T)$. This approximation is only valid for small $z_i \approx n_i \lambda_i^3 \ll 1$, i.e. for low densities and not too low temperatures. The grand canonical potential in the virial expansion is given by

$$\Omega(\mu_i, T, V) = -TV \left(\sum_i b_i \frac{z_i}{\lambda_i^3} + \sum_{ij} b_{ij} \frac{z_i z_j}{\lambda_i^{3/2} \lambda_j^{3/2}} + \dots \right) \quad (2)$$

with the dimensionless cluster (virial) coefficients $b_i = g_i$, b_{ij} , The second, third, ... virial coefficients vanish for independent particles without interaction and the NSE result is recovered.

The second virial coefficient b_{ij} encodes the effect of correlations between particles i and j . In classical mechanics it depends directly on the two-body interaction potential V_{ij} . G.E. Beth and E. Uhlenbeck derived the quantum mechanical generalization of b_{ij} [15, 16] by replacing the integration over phase space by a summation over quantum states. The second virial coefficient

$$b_{ij}(T) = \frac{1 + \delta_{ij}}{2} \left(\frac{m_i + m_j}{\sqrt{m_i m_j}} \right)^{3/2} \int dE D_{ij}(E) \exp\left(-\frac{E}{T}\right) \pm \delta_{ij} g_i 2^{-5/2}. \quad (3)$$

depends on the quantity

$$D_{ij}(E) = \sum_k g_k^{(ij)} \delta(E - E_k^{(ij)}) + \sum_l \frac{g_l^{(ij)}}{\pi} \frac{d\delta_l^{(ij)}}{dE} \quad (4)$$

that can be seen as the difference of the level densities between the correlated and uncorrelated two-body system. There are contributions from bound states at energies $E_k^{(ij)} < 0$ and scattering states with phase shifts $\delta_l^{(ij)}$ in channels k and l , respectively. The last term in equation (3)

is a quantum statistical correction with positive (negative) sign if i and j are identical bosons (fermions).

The low-density behavior of the EoS can be established model-independently [17] since the binding energy of the deuteron and the scattering phase shifts are known experimentally in the two-nucleon system. Thus, the VEOs can serve as a benchmark at low densities but the dissolution of clusters with increasing density cannot be accounted for.

2.1.3. Quantum statistical model In the QS or generalized Beth-Uhlenbeck approach, medium effects on the properties of nuclei in matter can be included. The approach was formulated using thermodynamic Green’s functions [18]. The EoS of an interacting many-body system is derived by expressing the total nucleon number density with the help of the spectral functions $A(j, E)$ for single-particle states $j \equiv (\vec{p}_j, \sigma_j, \tau_j)$. The spectral function depends on the complex valued self-energy that contains the information on the interaction. After expanding $A(j, E)$ for small imaginary parts of the self-energy, the total density splits into two parts $n = n_{\text{free}} + 2n_{\text{corr}}$. The first contribution $n_{\text{free}} = \sum_j f_{FD}[e(j)]$ with the Fermi-Dirac distribution function f_{FD} is the density of free quasiparticles with quasiparticle energies $e(j)$. The correlation density

$$n_{\text{corr}} = \sum_k g_k^{(2)} \sum_{\vec{P}, P > P_{\text{Mott}}} f_{BE}(E_{\text{cont}} - B_k) + \sum_l g_l^{(2)} \sum_{\vec{P}} \int \frac{dE}{\pi} 2 \sin^2 \delta_l \frac{d\delta_l}{dE} f_{BE}(E_{\text{cont}} + E) \quad (5)$$

with the Bose-Einstein distribution function f_{BE} receives contributions from bound states and from scattering states. The form of the correlation density resembles that of the second virial coefficient (3) in the VEOs. However, the properties of the two-body states in the QS model depend on their c.m. momentum \vec{P} with respect to the medium. There are no two-body bound states for P below the Mott momentum P_{Mott} due to the action of the Pauli principle that blocks states by the medium. The energy of the scattering states with zero relative momentum defines the continuum edge E_{cont} . The binding energies B_k and the in-medium scattering phase shifts δ_l depend on the medium properties and \vec{P} . They have to be determined from the in-medium T matrix in general. The additional $2 \sin^2 \delta_l$ factor in the continuum contribution reduces the strength of the explicit scattering correlations since the self-energies contain a part of the correlation effect due to the interaction.

The medium-dependent shift of the nuclear binding energies can be calculated by solving the appropriate in-medium Schrödinger equation for light nuclei with mass number $A \leq 4$ with realistic nucleon-nucleon potentials or with nucleon self-energies taken from phenomenological mean-field models. See subsection 4.1 for results. A different approach is used for heavier nuclei ($A > 4$), see subsection 4.2.

2.2. Intermediate/high-density models

There are two major classes of theoretical approaches that describe nuclear matter successfully near nuclear saturation density. Phenomenological approaches use effective interactions, e.g. nonrelativistic Hartree-Fock models with the Skyrme or Gogny interaction or relativistic mean-field models with meson exchange. Results depend on a small number of parameters that are usually determined by fits to properties of finite nuclei. ‘‘Ab-initio’’ approaches employ realistic nucleon-nucleon interactions that were fitted to two- (and three-) nucleon bound and scattering states, see, e.g., (non-)relativistic Brueckner-Hartree-Fock calculations. The interaction is given from the outset and the quality of the results is determined by the sophistication of the many-body method. Unfortunately, the models have some limitations, e.g., often they can be applied only to uniform nuclear matter.

Phenomenological mean-field models are the main choice in order to obtain quantitatively reasonable results for an EoS for astrophysical applications in a wide range of temperature,

density and neutron-proton asymmetry. Relativistic approaches are the preferred choice, as they avoid, e.g., the problem of a superluminal speed of sound. Despite their virtues, explicit correlations are not considered in mean-field models with only nucleons as basic constituents. The formation and dissolution of nuclei is not incorporated into the description.

3. Generalized relativistic density functional

Nucleons are considered as quasiparticles in conventional relativistic mean-field (RMF) with self-energies that depend on the medium properties and take into account the effects of the interaction. For astrophysical applications when the EoS of charge neutral stellar matter is needed and the Coulomb interaction has to be included, electrons (and myons) can be added. The nuclear/electromagnetic interaction is modeled by an exchange of mesons/photons that couple minimally to the nucleons. The mesons/photon are generally treated as classical fields.

Additional degrees of freedom are introduced in a generalized relativistic density functional (gRDF) approach by including also nuclei as constituents and considering correlations in an effective way. The Lagrangian density of a RMF model was extended in reference [12] by introducing light nuclei (${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$) as spin 1, 1/2 and 0 fields. They interact minimally with the meson and photon fields like the nucleons, however, with rescaled couplings. In addition, it is assumed that the binding energies of the nuclei depend on the medium properties. This dependence enters through functions of the temperature and vector meson fields in the particular approach of reference [12].

In reference [13], nucleon-nucleon scattering correlations, i.e. the nn , np and pp scattering states, were added to the model. The continuum correlation in a particular channel is represented by a single state with an effective ‘‘resonance’’ energy, that depends on the medium properties like the binding energy of nuclei, and a temperature dependent degeneracy factor, similar as excited states of nuclei in NSE models.

Heavy nuclei with mass number $A > 4$ can be considered in a similar way as light nuclei. See subsection 4.2 concerning the medium dependence of their binding energy shifts.

For an quantitative description of nuclear matter and finite nuclei, it is not sufficient to assume minimal nucleon-meson couplings with constant strength Γ_{im} between particles i and mesons m in RMF models. A medium dependence of the effective nuclear interaction has to be considered in the model. The traditional approach uses nonlinear self-interactions of the mesons in the Lagrangian density with a polynomial dependence on various combinations of the meson fields. A more recent and more flexible approach is motivated by results of Dirac-Brueckner calculations of nuclear matter [19]. Here the couplings Γ_{im} depend on the density of the medium with a well controlled asymptotic behavior. The gRDF model assumes such density dependent couplings.

The parameters in phenomenological RMF models are determined in fits to properties of nuclei and nuclear matter. Their number is rather small (≈ 10). When clusters are introduced in the gRDF approach, additional parameters appear that determine the medium dependence of the binding energies. EoS models can be tested by using constraints from nuclear physics and astrophysics, see, e.g., reference [20].

In the gRDF approach a density functional serves as the starting point to derive all equations and thermodynamical quantities. In the particular case of reference [12], the grand canonical thermodynamical potential

$$\Omega = \int d^3r \omega_g(T, \mu_i, \sigma, \delta, \omega_0, \rho_0, \vec{\nabla}\sigma, \vec{\nabla}\delta, \vec{\nabla}\omega_0, \vec{\nabla}\rho_0, \vec{\nabla}A) \quad (6)$$

with the potential density ω_g is chosen. The temperature T , the chemical potentials μ_i of all particles i , the meson and photon fields σ , δ , ω_0 , ρ_0 , A and their derivatives are the independent variables. There are contributions from nucleons, bound states of nuclei and two-nucleon scattering states in the baryonic sector. The particle masses are given by

$m_i = N_i m_n + Z_i m_p - (1 - \delta_{in})(1 - \delta_{ip})B_i^{(\text{vac})}$ with the vacuum binding energy $B_i^{(\text{vac})} \geq 0$ of the composite systems ($B_i^{(\text{vac})} = 0$ for the scattering states). The energy of a baryon or lepton with momentum \vec{k} has the relativistic form $E_i = \sqrt{k^2 + (m_i - S_i)^2} + V_i$ with scalar potentials S_i and vector potentials V_i that depend on the meson fields σ , δ , ω_0 , ρ_0 , the photon field A and the couplings Γ_{im} . The scalar self-energy contains medium dependent binding energy shifts. In the vector self-energy, a rearrangement contribution appears due to the density dependence of the couplings. Neglecting antiparticle and boson condensate contributions, the potential density in equation (6) assumes the form

$$\begin{aligned} \omega_g = & \sum_i \mp g_i T \int \frac{d^3k}{(2\pi)^3} \ln \left[1 \pm \exp \left(-\frac{E_i - \tilde{\mu}_i}{T} \right) \right] - \frac{1}{2} (m_\omega^2 \omega_0^2 + m_\rho^2 \rho_0^2 - m_\sigma^2 \sigma^2 - m_\delta^2 \delta^2) \quad (7) \\ & + \frac{1}{2} \left(\vec{\nabla} \omega_0 \cdot \vec{\nabla} \omega_0 + \vec{\nabla} \rho_0 \cdot \vec{\nabla} \rho_0 - \vec{\nabla} \sigma \cdot \vec{\nabla} \sigma - \vec{\nabla} \delta \cdot \vec{\nabla} \delta + \vec{\nabla} A_0 \cdot \vec{\nabla} A_0 \right) \\ & + (\Gamma'_\omega \omega_0 n_\omega + \Gamma'_\rho \rho_0 n_\rho - \Gamma'_\sigma \sigma n_\sigma - \Gamma'_\delta \delta n_\delta) (n_n + n_p) \end{aligned}$$

where the upper (lower) signs applies to fermions (bosons). The source densities in equation (7) are determined by $n_k = \sum_i g_{ik} n_i$ for $k = \omega, \rho$ and $n_k = \sum_i g_{ik} n_i^s$ for $k = \sigma, \delta$, respectively, with scaling factors g_{ik} , vector densities n_i and scalar densities n_i^s of the individual constituents. The relativistic chemical potentials of the baryonic particles are given by $\tilde{\mu}_i = N_i \tilde{\mu}_n + Z_i \tilde{\mu}_p = \mu_i + m_i$. Hence, there are only two independent chemical potentials due to the condition of chemical equilibrium. The degeneracy factors g_i can depend on the temperature T . Note that n_i and n_i^s have to be considered as functions of the independent variables of Ω . The field equations, derived from ω_g , include rearrangement contributions that ensure the thermodynamic consistency of the model. The set of coupled field equations has to be solved self-consistently for given parameters T , n and Y_e .

4. Medium effects on nuclear binding energies

The medium dependence of the nuclear binding energies is the essential feature of the gRDF approach that causes the reduction of the cluster abundances with increasing density. The shift originates mainly from action of the Pauli principle suppressing the possibility of a cluster formation.

4.1. Light nuclei

For light clusters ($A \leq 4$), results of the QS model are adopted for the medium dependent shifts of the binding energies. In the gRDF approach, they are represented as functions of temperature and an effective density, see reference [12] for details, in particular figure 1. With increasing medium density, the binding energy of a cluster decreases smoothly from its experimental value in the vacuum. Depending on the medium temperature, it finally crosses zero, indicating that the cluster becomes unbound. In reference [13] different extrapolations to high densities were explored. See reference [21] for more recent parametrizations of the energy shifts, including the momentum dependence.

4.2. Heavy nuclei

The binding energy shifts of heavy nuclei are calculated using the relativistic density functional considering nucleons and electrons. For this purpose, the field equations are solved self-consistently in a spherical Wigner-Seitz cell employing an extended Thomas-Fermi approximation with inhomogeneous density distribution of nucleons and electrons under the condition of a vanishing total charge. A heavy cluster is formed in the center of the cell that is surrounded by a low-density gas of nucleons, see figure 11 of reference [14]. Electrons form

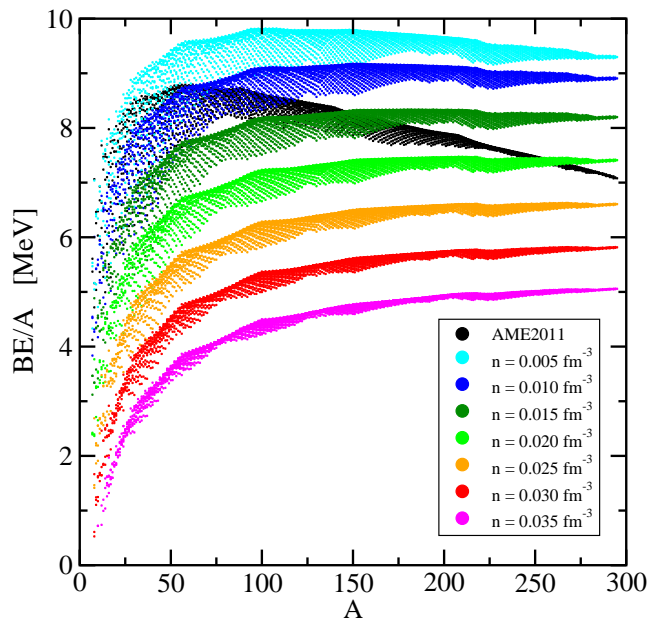


Figure 1. Binding energy per nucleon of nuclei with mass number A in vacuum (AME2011 [22]) and in the medium at temperature $T = 0$ MeV for various densities n . Figure from reference [14].

a highly degenerate Fermi gas resulting in an practically uniform distribution inside the cell. They screen the Coulomb potential of the protons. When light clusters are considered in the calculation in addition to nucleons and electrons, an enhancement of the light cluster abundancies at the surface of the heavy cluster is observed.

When the energies of the Wigner-Seitz cell calculation with uniform and nonuniform density distributions are compared, the binding energy of a nucleus at a certain density of the medium can be determined since it represents the energy gain due to the correlations. The extended Thomas-Fermi approximation does not take into account shell effects and it is more reasonable to extract only relative shifts of the binding energies from the calculations than to extract absolute binding energies in the medium.

In figure 1, experimental binding energies per nucleon [22] as a function of the mass number A are depicted in by black points. The maximum near $A = 60$ and shell effects close to doubly magic nuclei are clearly visible. Taking into account the the binding energy shifts in the medium, the A dependence clearly changes. The screening of the Coulomb field by the electrons causes a stronger binding of the nuclei at low medium densities. At higher medium densities, the binding energies of the nuclei reduce substantially. Nuclei with low mass numbers dissolve at lower medium densities than clusters with larger mass numbers.

In the so-called single-nucleus approximation (SNA), only one representative nucleus is considered in traditional EoS tables for astrophysical applications, see, e.g., Refs. [3, 4]. The full distribution of nuclei is covered only in more recent NSE-type models [6, 7]. The latter approaches usually account for the Coulomb screening effect but not for the reduction of the nuclear binding energies at higher medium densities.

5. Low-density limit

The VEOs represents the correct description of the matter properties at low densities and finite temperatures. The gRDF model in constructed such that the model-independent low-density results of the VEOs are reproduced. Comparing the fugacity expansions of the grand thermodynamical potential Ω in both models, consistency relations can be derived to determine the effective resonance energies and degeneracy factors of the nucleon-nucleon scattering states that both depend on T . Due to relativistic kinematics, there are already corrections to the

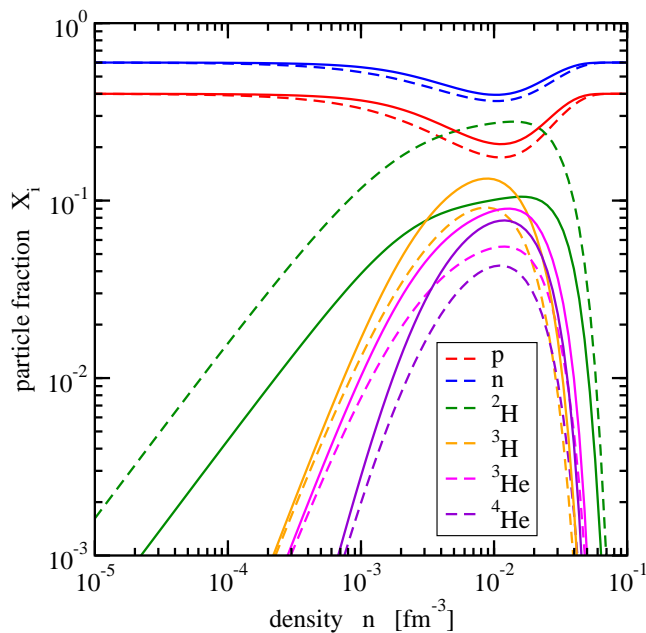


Figure 2. Particle fractions $X_i = n_i/n$ in nuclear matter with temperature $T = 10$ MeV and total proton fraction $Y_p = 0.4$ as a function of the total nucleon density n in the gRDF approach without heavy clusters. Calculation without (dashed lines) and with (full lines) nucleon-nucleon scattering correlations. Heavy clusters with mass number $A > 4$ are not considered in this calculation. Figure adapted from reference [14].

first order coefficient b_i in equation (6) of the VEOs. The effective resonance energies increase smoothly with temperature as depicted in figure 2 of reference [13]. At low temperatures, the contribution of the s-wave channel dominates and analytical results for the effective resonance energies can be obtained using the effective-range expansion of the phase shifts. Consistency relations for the nn , pp and np channels are found from a comparison of the second virial coefficients in the VEOs and gRDF approaches. The occurrence of a term that depends on the meson couplings Γ_k at zero density through the coefficients $C_{\pm} = C_{\omega} - C_{\sigma} \pm C_{\rho} \mp C_{\delta}$ with $C_k = [\Gamma_k(0)]^2/m_k^2$ is the important feature in these relations. The consistency relations serve to define the effective degeneracy factors for the effective two-nucleon scattering states. Again, a smooth dependence on the temperature is found as depicted in figure 3 of reference [13].

In the zero-temperature limit, two conditions are found that connect the differences $C_{\omega} - C_{\sigma}$ and $C_{\rho} - C_{\delta}$ with the scattering lengths of the four relevant nucleon-nucleon s-wave scattering channels, see references [13, 14] for details.

In pure neutron matter, there is no bound state and two-nucleon correlations appear only in the nn scattering channel. In figure 5 of reference [13] the internal energy per neutron E/N is depicted as a function of the total neutron density in different approximations. The gRDF model with the contribution of the effective nn scattering correlation perfectly reproduces the VEOs at low densities.

6. Light clusters and continuum correlations

The composition of matter changes substantially when the parameters T , n and Y_p are varied. In figure 2 the evolution of the particle mass number fractions $X_i = A_i n_i/n$ is depicted as a function of the total nucleon density n for $T = 10$ MeV and $Y_p = 0.4$ for the gRDF model using the parametrization DD2 [12]. At very low densities, nucleons dominate the composition of matter. There is a tiny fraction of deuterons and the abundance of heavier clusters can be neglected. Contributions of clusters to the composition become more important with increasing density and the abundance of free nucleons is reduced. The total cluster fraction reaches a maximum at approximately 1/10 of the nuclear saturation density. A further increase of the medium density causes the reduction of the cluster fraction and finally they disappear. Hence, the gRDF model successfully describes the dissolution of clusters in matter and the Mott effect is observed.

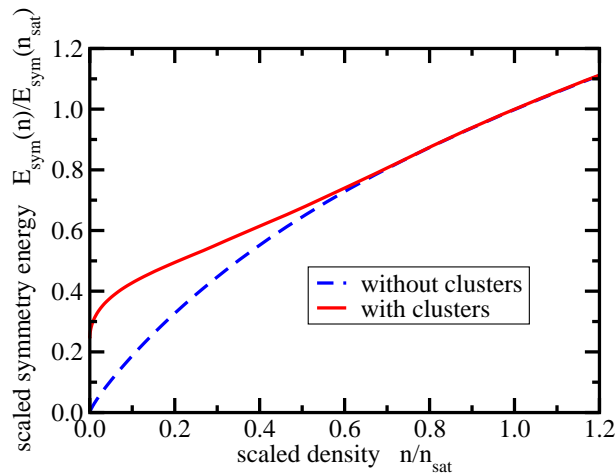


Figure 3. Scaled symmetry energy of nuclear matter at zero temperature as function of the scaled density in the gRDF model with parametrization DD2 without (blue dashed line) and with (red full line) cluster formation taken into account. Figure adapted from reference [14].

Two-nucleon scattering correlations substantially affect the cluster fractions. The number of deuteron-like correlations is reduced and the remaining particle fractions are redistributed. Note that the particle abundancies in the gRDF model represent those of quasiparticles. Some part of the correlation strength is accounted for by the finite self-energies of the quasiparticles and the size of explicit correlations is reduced.

7. Symmetry energy

The symmetry energy $E_{\text{sym}}(n)$ of nuclear matter is currently investigated with much experimental and theoretical effort. See, e.g., references [23, 24] for constraints and the importance in nuclear physics and in astrophysical applications. The appearance of clusters strongly modifies the density dependence of E_{sym} (including Coulomb contributions) at low densities, as depicted in figure 3 for matter at zero temperature. Conventional mean-field models of uniform matter without cluster degrees of freedom predict that $E_{\text{sym}}(n)$ approaches zero in the limit of vanishing density n . With correlations and cluster formation the symmetry energy rises as compared to that in mean-field models without correlations. For vanishing temperature it even approaches a finite value in the limit of zero density. The effects will be less pronounced at higher temperatures, but possibly still detectable in experiments.

The analysis of heavy-ion collisions can help to extract the symmetry energy and to study the properties of fragments in dilute matter by determining the thermodynamical properties of the expanding system. Recent results indicate an increase of the symmetry energy as predicted by models with correlations and cluster formation, see references [25, 26, 27, 28, 29] for details.

8. Conclusions

Correlations strongly affect the thermodynamic properties and composition of an interacting many-body system such as nuclear matter. Clusters can form and dissolve with changing density and temperature of the medium. This aspect has to be considered in theoretical models for the EoS of dense matter with possible consequences in astrophysical applications. The effects can be studied also experimentally in heavy-ion collisions.

In this contribution, a generalized relativistic density functional approach was presented that interpolates between the correct low-density limit, the VEoS, and the high-density quasiparticle description. Nucleons, two-nucleon continuum states and nuclei are considered as degrees of freedom. They are treated as quasiparticles with medium-dependent properties. The description of light clusters was examined in some detail, see references [12, 13]. At high densities, the dissolution of the composite systems is observed. The approach of the gRDF model can be

extended to include heavy clusters in the future. The final aim for the application of the gRDF model is to provide extensive EoS tables of dense matter.

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References

- [1] Janka H-Th, Langanke K, Marek A, Martinez-Pinedo G and Mueller B 2007 *Phys. Rep.* **442** 38
- [2] Glendenning N K 2000 *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (New York: Springer)
- [3] Lattimer J M and Swesty F D 1991 *Nucl. Phys. A* **535** 331
- [4] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 1998 *Nucl. Phys. A* **637** 435
- [5] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 1998 *Prog. Theor. Phys.* **100** 1013
- [6] Hempel M and Schaffner-Bielich J 2010 *Nucl. Phys. A* **837** 210
- [7] Botvina A S and Mishustin I N 2010 *Nucl. Phys. A* **843** 98
- [8] Shen G, Horowitz C J and Teige S 2010 *Phys. Rev. C* **82** 015806
- [9] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 2011 *Ap. J. Suppl. Series* **197**, 20
- [10] Shen G, Horowitz C J and Teige S 2010 *Phys. Rev. C* **83** 035802
- [11] Shen G, Horowitz C J and O’Connor E 2011 *Phys. Rev. C* **83** 065808
- [12] Typel S, Röpke G, Klähn T, Blaschke D, and Wolter H H 2010 *Phys. Rev. C* **81** 015803
- [13] Voskresenskaya M D and Typel S 2012 *Nucl. Phys. A* **887** 42
- [14] Typel S 2012 Clusters in Nuclear Matter and the Equation of State for Astrophysical Applications *Proc. Int. Workshop XII Hadron Physics (April 22 - 27, 2012, Bento Gonçalves, RS, Brazil)* ed M V T Machado (Melville, NY: American Institute of Physics) to be published
- [15] Beth G E and Uhlenbeck E 1936 *Physica* **3** 729
- [16] Beth G E and Uhlenbeck E 1937 *Physica* **4** 915
- [17] Horowitz C J and Schwenk A 2006 *Nucl. Phys. A* **776** 55
- [18] Schmidt M, Röpke G and Schulz H 1990 *Ann. Phys. (N.Y.)* **202** 57
- [19] Typel S and Wolter H H 1999 *Nucl. Phys. A* **656** 331
- [20] Klähn T, *et al.* 2006 *Phys. Rev. C* **74** 035802
- [21] Röpke G 2011 *Nucl. Phys. A* **867** 66
- [22] Audi G and Meng W 2011 private communication
- [23] Tsang M B *et al.* 2012 Constraints on the symmetry energy and neutron skins from experiments and theory *Preprint arXiv:1204.0466*
- [24] Lattimer J M and Lim Y 2012 Constraining the Symmetry Parameters of the Nuclear Interaction *Preprint arXiv:1203.4286*
- [25] Kowalski A *et al.* 2007 *Phys. Rev. C* **75** 014601
- [26] Natowitz J B *et al.* 2010 *Phys. Rev. Lett.* **104** 202501
- [27] Hagel K *et al.* 2012 *Phys. Rev. Lett.* **108** 062702
- [28] Qin L *et al.* 2012 *Phys. Rev. Lett.* **108** 172701
- [29] Wada R *et al.* 2012 *Phys. Rev. C* **85** 064618