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COMMENTS ON THE HIGH-FREQUENCY BEHAVIOR OF THE COUPLING OF AN ACCELERATOR BEAM TO ITS ENVIRONMENT

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INTRODUCTION

The short bunch lengths and the associated high frequencies found in the latest designs of linear colliders, superconducting linacs, FEL drivers, damping rings, and synchrotron light sources have heightened the importance of understanding the high-frequency behavior of the interaction of an accelerator beam with its environment. This parametric domain is at the limits of both the numerical and analytical tools which have been developed to date, and consequently there exists some question as to the correct asymptotic frequency dependence. The resulting uncertainty in the coupling of a particle beam to vacuum chamber discontinuities has hindered evaluation of bunch lengthening in storage rings and transverse beam blowup in linacs, and limits confidence in assessments of beam quality in proposed designs. Another high-frequency phenomenon, which is of particular concern in damping and storage rings, is the synchotron radiation process in the presence of conductive boundaries. Estimates[1] have indicated that this effect can provide the dominant limit on peak beam current in small, smooth-walled machines, but this earlier work does not take into account fully the complex, finite-Q resonance structure which is present. The charge to the August 1987 Impedance Beyond Cutoff Workshop at Lawrence Berkeley Laboratory was to investigate these issues in some depth and to provide clarification of the main features of the beam coupling impedance at frequencies well above the lowest propagation frequency of the beam pipe. Subsequent papers in this volume present the detailed results of the workshop participants. In this note the motivation for this effort, an overview of the progress made, and a few remarks on remaining questions are offered.

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1. BASIC NOTIONS OF IMPEDANCE AND WAKE POTENTIALS

A charged particle beam passing a discontinuity in its vacuum chamber can deposit electromagnetic energy. Alternatively, a charged particle beam passing through a bending magnet can synchrotron-radiate, again depositing energy. The source term in either case can be the macroscopic charge distribution of a bunched beam or the microscopic random currents, at essentially arbitrarily high frequency (Schottky noise), of incipient beam instabilities. These beam-induced electromagnetic fields act on the beam and create a potentially unstable feedback loop which may limit beam current through instability and phase space dilution. The notions of wake potential and coupling impedance provide a major tool in the analysis of these processes. Consider a charged particle beam passing down the center of a cylindrical beam pipe which has an isolated cavity-like structure. The longitudinal current $I(z, t)$ will generate a longitudinal electric field $E_z(z, t)$ which for a localized, time-independent structure will be of the form

$$E_z(z, t) = \int dz'\, dt' \int dk\, dk'\, d\omega G(k, k', \omega) e^{ikz - ik'z' - i\omega(t-t')} I(z', t') \tag{1}$$

where $G(k, k', \omega)$ is the Fourier-transformed Green function which must satisfy causality and relativistic locality. (In general, there is an additional term describing the contribution of the charge density which will not be discussed here.) Although it is this Green function $G(k, k', \omega)$ which enters into a complete beam stability calculation, if the motion of the particles is well approximated by constant velocity trajectories during transit through the localized structure, the simpler notions of impedance and wakefield provide sufficient information for a sound analysis. Consider a test charge moving at a constant velocity v along a trajectory $r = 0$, $z = -s + vt$ of the cylindrical beam pipe. The integrated longitudinal field $\hat{W}(s)$ seen by the test charge is

$$\hat{W}(s) = \int dz\, dt\, \delta\left(\frac{z + s}{v} - t\right) E_z(z, t) \tag{2}$$

On inserting (1) into (2) and integrating, we have

$$\hat{W}(s) = (2\pi)^3 \int dk\, dk' G(k, k', kv) \tilde{I}(k', kv) e^{-iks} \tag{3}$$

where $\tilde{I}(k, \omega)$ is the Fourier transform of the longitudinal current I. The time dependence of the beam current is generated both through the gross motion of the nonuniform spatial distribution of charge in the beam and through changes in that distribution. For a quasi-stationary distribution of charge moving at a velocity v—that is, when the transit time is short compared to the characteristic time for changes in the distribution—the primary time dependence will be given by $I(z, t) = I_0(z - vt)$. With this approximation

$$I(k, \omega) = \tilde{I}_0(k)\, \delta(\omega - kv) \tag{4}$$

Inserting Eq. (4) into Eq. (3) yields

$$\hat{W}(s) = \frac{(2\pi)^3}{|v|} \int dk e^{-iks} G(k, k, kv) \tilde{I}_0(k) \tag{5}$$

The wake potential $W(s)$ is defined by (5) for a delta-function exciting current; that is,

$$W(s) = \frac{(2\pi)^2}{|v|} \int dk e^{-iks} G(k, k, kv) \tag{6}$$

The wake potential is the effective Green function for interaction with a vacuum chamber component in the quasi-static limit. The Fourier conjugate of the wake potential is the coupling impedance, which is given by the relation

$$Z(\omega) = \frac{(2\pi)^3}{|v|} G(k, k, kv)\bigg|_{kv=\omega} \tag{7}$$

A current $I(\omega)$ yields a voltage

$$V(\omega) = I(\omega) Z(\omega) \tag{8}$$

when averaged over the structure in the quasistatic limit. Similar considerations are applicable for transverse coupling, where a transverse effective Green function (the transverse wake potential) and conjugate impedance can be defined.

If the motion of both the source current and test particle is not simply linear (for example, with synchrotron oscillations or betatron oscillation) the relation $kv = \omega$ between wave number and frequency breaks down. For example, with synchrotron oscillations at ω_s the kernel $G(k, k', kv + \mu\omega_s)$ for integer μ is needed. If for the strongest sidebands $k + \mu\omega_s = k' + \nu\omega_s$ implies that $k = k'$, $\mu = \nu$, then only $G(k, k, kv + \mu\omega_s)$ are necessary for analysis. In the limit of an infinitesimally short structure at position x, the (k, k') dependence is simply $e^{((k-k')x)}$ for finite (k, k'), and $G(k, k, kv + \mu\omega_s)$ follows immediately from the knowledge of $G(k, k, kv)$ for all k.

The full knowledge of $G(k, k', \omega)$ is required in situations in which the perturbation of trajectories by induced fields during transit across the impedance-generating structure is essential to the correct physical picture. An example of where the simplest notions of impedance and wake potential are inadequate is found in the phenomenon of regenerative beam breakup[2] induced by a single transverse mode in an extended structure. Consider a bunch passing through the center of a cavity which has a low-level excitation of a transverse deflecting mode. The associated longitudinal electric field rises linearly from zero on axis. The transverse deflection produces a finite, albeit small, deflection into a region of longitudinal electric field which can couple energy out of the bunch longitudinal motion and into the mode field energy. The time dependence of the trajectory perturbation smears the relation between ω and k, and in turn allows coupling of $k \neq k'$. The feedback loop formed can go unstable at sufficiently high current, when the excitation from the orbit perturbation exceeds inherent mode damping. Although order-of-magnitude estimates of threshold current can be obtained from knowledge of the transverse impedance, a correct treatment requires detailed information on the field pattern and phases of the mode. Since the trajectory varies from linear in transit, the usual straight trajectory wakefields do not provide sufficient information. In fact, the functional form appropriate inside the extended structure is not the usual wakefield expansion.[3] Similar considerations may apply to instability driven by the synchrotron radiation impedance, discussed in a subsequent section, where the fields and beam interact throughout the vacuum chamber. Furthermore, transverse variation of the coupling across the beam may be of significance.

2. PHENOMENA DRIVEN BY HIGH-FREQUENCY IMPEDANCES

The impedance of a variety of particle accelerators has been found in practice to begin to roll off at frequencies of the order of the lowest waveguide cutoff, typically a few gigahertz. Because of this, the dominant current limits for an unbunched, continuous beam, which can be excited in a very narrow frequency band, are dominated by antidamping modes of relatively low frequency content. The very short bunched beams found in a number of current accelerator designs, however, present a quite different picture. Consider the excitation of a localized structure by coherent internal oscillations of a bunch of rms length τ. Because of the finite length, the frequency spectrum offered by an arbitrary perturbation of the bunch has width of $1/2\pi\tau$ and is centered about the typical frequency of the perturbation. For example, a 1 mm bunch generates a corresponding frequency bandwidth of about 50 GHz. Therefore, any successful model of internal bunch stability for these short-bunch designs will include significant frequency smearing over a range where there is considerable variation in the coupling impedance and over frequencies well above typical cutoff frequencies of a beam pipe.

Internal bunch instabilities, both transverse and longitudinal, have provided a fundamental limitation in the design of short-pulse-length synchrotron light sources, high-phase-space-density damping rings, and single-pass FEL drivers. Although several formalisms have been developed to describe this class of beam instability, they share in a common structure.[4] A set of basis states (possibly degenerate) is chosen which describes perturbations of the bunch phase space and current, with the higher states corresponding roughly to shorter-wavelength internal ripples. For each mode there is an associated eigenfrequency. The impedance generates an additional interaction between the states, and the determination of stability reduces to an infinite-dimensional eigenvalue problem. The fundamental matrix is formed from the unperturbed eigenfrequency spectrum and expectation values of the product of the impedance and beam current with the basis set. Since the basis set represents modes on a bunch of finite length τ, the expectation values effectively average the impedance over a frequency range $1/\tau$. In general, reactive impedance can couple a basis mode to itself, yielding a frequency shift. On the other hand, resistive impedance provides the primary coupling between neighboring states and acts to induce instability.

Determination of the threshold current for longitudinal and transverse instability requires solution of an infinite-dimensional-matrix eigenvalue problem. In practice, the matrix is truncated, and certain general features which determine instability onset are observed. Heuristically, the off-diagonal matrix elements (through the resistive component) provide a potential growth rate; the reactive component yields frequency shifts which can either increase or decrease the eigenfrequency spacing for basis states which are of the correct class to couple. Instability (antidamping eigenfrequencies) is observed when the potential growth rate exceeds the mode spacing. A reactive impedance that is large when averaged over the mode spectrum can reduce mode spacing and allow a relatively small resistive coupling to induce instability. As the current is increased the modes can cross and stability can be restored, yielding a stopband structure in current. Therefore, the threshold for this instability becomes a sensitive function of the averaged reactive impedance. For short bunches this average is carried from the low-frequency inductive impedance through to the high-frequency capacitive impedance of the tail, and estimates of stability can become extremely sensitive to both the assumed value of the transition (i.e., cut-off) frequency between inductive and capacitive behavior and the functional form in frequency of the high-frequency rolloff. Longitudinal impedance models invoking so-called "Spear scaling" (with an implicit $\omega^{-0.7}$ dependence) and a "$Q=1$ resonator" (with an implicit ω^{-1} dependence) have been widely used. As will be described later in more detail, the primary discussion of the Impedance Beyond Cutoff Workshop centered about whether the high frequency rolloff of the longitudinal coupling impedance is dominantly $\omega^{-1/2}$ or $\omega^{-3/2}$. For short bunches the choice of model can significantly affect stability estimates. Similarly, assumptions with regard to the 'cutoff' angular frequency where rolloff begins—for example, at c/a or $2.4\,c/a$ (the TM cutoff in a circular pipe of radius a)—can yield either bunch lengthening or shortening in some parameter regimes.

The maintenance of beam quality for the short, highly charged bunches found in proposed linear colliders,[5] multipass superconducting beauty factories, and FEL drivers is a second issue which is intimately tied to the high frequency behavior of the transverse and longitudinal coupling impedances. Since the longitudinal wake potential is related to the coupling impedance by a Fourier transform, an $\omega^{-1/2}$ asymptotic form implies that the δ-function wake $W(s)$ diverges at $s=0$ as $1/\sqrt{s}$ whereas an $\omega^{-3/2}$ dependence yields a finite limit. The functional dependence of the transverse wake varies typically as the integral of the longitudinal wake, which implies $s^{1/2}$ or s behavior, respectively, in the neighborhood of $s=0$.

The longitudinal loss factor $k_\ell(\sigma)$ is defined by the relation

$$Q^2k_\ell = \int_{-\infty}^{+\infty} d\tau I(\tau) \int_{-\infty}^{\tau} dt I(t) W(\tau - t), \tag{9}$$

and Q^2k_ℓ gives the total energy loss of a bunch of charge Q for a current distribution I. For reasonable charge distributions, $2Qk_\ell$ gives the approximate head-to-tail energy variation induced by the longitudinal wake. The transverse loss factor k_t is defined by

$$Q^2k_t = \int_{-\infty}^{+\infty} dt I(\tau) \int_{-\infty}^{\tau} dt I(t) W_t(\tau - t) \tag{10}$$

where $W_t(\tau)$ is the transverse wake potential and Qk_t gives the averaged induced transverse kick. If $\omega^{-1/2}$ asymptotic behavior as discussed above is assumed, then for a Gaussian bunch of sufficiently small rms length σ,

$$k_\ell \propto \sigma^{-1/2} \tag{11}$$

and

$$k_t \propto \sigma^{1/2}. \tag{12}$$

If, on the other hand, $\omega^{-3/2}$ behavior is assumed, then

$$k_\ell \propto \text{constant} \tag{13}$$

and

$$k_t \propto \sigma. \tag{14}$$

As is clear from Eqs. (11–14), in extrapolating either measurements performed with relatively long bunches or numerical estimates at the limits of computer capacity with shorter bunches, one again is faced with substantial differences which depend on the asymptotic form of the high-frequency coupling impedance and can strongly affect the evaluation of performance.

3. EARLIER RESULTS FOR HIGH-FREQUENCY ROLLOFF

The behavior of the longitudinal impedance at very high frequencies has been investigated by several authors. Two models which have been used extensively are the diffraction model of Lawson[6] and the optical resonator model. In the diffraction model, the power lost by a charge traveling along a beam pipe which opens to form a resonator is estimated. For a relativistic particle, the field looks very much like a plane wave, and the approximation is made that diffraction of this wave occurs at the pipe edge. The energy that is diffracted outside the beam pipe radius is reflected at the far side of the resonator and is lost. The primary result, from the point of view of this workshop, is that the energy loss of a point particle increases as $\gamma^{1/2}$. The relativistic distortion of the electric field to an opening angle $1/\gamma c$ provides a high frequency cutoff of order $c\gamma/a$ of the field spectrum of a point charge at the pipe radius a. Thus, the $\gamma^{1/2}$ dependence of the loss factor in the diffraction model translates into an $\omega^{-1/2}$ asymptotic behavior in frequency.

The optical resonator model provides an alternative description of energy loss based on the work of Vainshtein.[7,8] The analogy is drawn between a set of infinite plates with circular holes and the pair of circular mirrors with infinite reflections of the optical resonator. In this model, the energy loss for large γ is found to be independent of γ, and indicates that the asymptotic form of the impedance at high frequencies must be fast enough to yield convergent integrals. Detailed analysis of this model yields an asymptotic dependence of $\omega^{-3/2}$.

Both models describe the energy loss mechanism in terms of diffraction; the fundamental distinction is that the Lawson diffraction model treats a single, isolated cavity, whereas the optical resonator model more immediately addresses a periodic array. Keil's[9] work which numerically evaluates the losses in an infinitely long sequence of accelerating cavities suggests that the distinction drawn between single, isolated structures versus periodic structures is of particular significance. The work finds that the energy loss is strongly γ-dependent at low energies, but γ-independent at high energies. Since, as the energy is increased, higher frequencies are generated, this result would indicate the validity of the optical resonator model for truly periodic structures. At lower energies, the frequency spectrum has not entered the asymptotic regime. The work of Hazeltine, Rosenbluth, and Sessler[9] for the energy loss of a charged rod which moves at a constant speed past an infinite set of parallel semi-infinite conducting plates shows an even more benign behavior for a periodic structure, with the energy loss ultimately falling with increasing γ. However, the semi-infinite geometry itself reduces the dimensionality of the problem and may provide additional regularization of the beam-structure coupling.

4. WORKSHOP CONJECTURES

The efforts of the theory group of the Impedance Beyond Cutoff Workshop entered on two primary issues. First, whether $\omega^{-1/2}$ was indeed the correct asymptotic behavior for an isolated cavity, and secondly, for how long (if ever) must a structure repeat before the $\omega^{-3/2}$ behavior characteristic of the optical resonator model sets in. Results for an isolated pillbox cavity are presented in papers of the workshop proceedings by Dôme, Heifets and Kheifets, Bane and Sands, and Henke. In addition, Palmer presents a diffractive model in the spirit of Lawson's work which indicates the possible nature of the transition between the single cavity and periodic limits.

Dôme's model is based on the assumption that, for a pillbox cavity with beam pipe of radius a, the field pattern within the cavity at radii greater than a are undistorted from the closed cavity solutions. With this approximation and summing over modes with appropriate time delays, he obtains an $\omega^{-1/2}$ behavior. The work of Heifets and Kheifets provides an iterative solution of Maxwell's equations for a pillbox with beam pipe. The leading term agrees with the result of Dôme. In addition, it is shown that the next term in the expansion is "small" with respect to the leading term. Thus, although convergence is not assured, there is evidence that the iteration is well behaved. Bane and Sands have investigated the high-frequency behavior using Weiland's TBCI and have compared these results with their version of the Lawson diffraction model. For short bunches the TBCI computations are found to approach the predictions of this model, and are therefore consistent with $\omega^{-1/2}$ rolloff. Finally, in the work of Henke the field problem is solved with a mode matching technique. It is found from numerical solution that the longitudinal impedance for a radial line behaves as $\omega^{-1/2}$. In summary, a variety of independent techniques, including analytic iterative methods, time-domain and frequency-domain numerical solution to Maxwell's equations, and diffractive approximations agree on the asymptotic form of the longitudinal impedance for an isolated cavity excited by an infinite-energy beam. Of course, iterative methods may not converge, and truncation of matrices and finite mesh size may introduce spurious behavior, but the preponderance of evidence from this workshop points to an asymptotic rolloff of $\omega^{-1/2}$. Unfortunately, a rigorous result, without approximation, for some closed geometry with beam pipe has yet to be achieved. Palumbo, however, does give an analytic solution for a single step which shows a rolloff that is even slower than $\omega^{-1/2}$.

The discussion of the second issue—the nature of transition between single-cell and periodic behavior—yielded a scale-length conjecture which followed from a rather simple interference picture. Consider a length of structure L formed by a series of many cavities with beam pipe of radius a. When viewed from the axis, the path length difference between the first and the last cavity is given by

$$\delta = L - L\cos\theta \approx L\frac{\theta^2}{2}, \tag{15}$$

with θ approximately equal to a/L. The condition for maximal coherence of the diffracted radiation on axis is when δ is of the order of the wavelength or

$$L \approx ka^2, \tag{16}$$

where $k = \omega/c$. Palmer's paper derives an equivalent scale factor from his diffractive model. Thus, for a given k, there is a minimal length of structure required before the structure looks infinitely periodic. There appears to be both a high and low frequency bound to this periodic behavior. For small enough k, L may be less than a single cavity length, and the system is not well modeled by multiple interference. For high enough frequency, on the other hand, the structure is not long enough for full coherence to be established. Thus from this argument it would appear that for any finite length structure, at sufficiently high frequency, $\omega^{-1/2}$ behavior can be expected, but that typically there would be a middle region which would mimic $\omega^{-3/2}$ behavior. In the limit of an infinitely long structure, this middle region would extend out to infinite frequency. It is important to recognize that this picture represents a conjecture of the workshop made with the hope of stimulating a deeper investigation of the nature of the transition, if it indeed does occur, between single-cavity and periodic-structure rolloff. A multiple of length scales appear in the problem—k^{-1}, total length, aperture radius, cavity radius, cell length, and cell separation—all of which may be involved in the determination of asymptotic behavior.

5. SYNCHROTRON RADIATION IMPEDANCE

For small storage rings there appears to be another important source of interaction of the beam with its environment—the synchrotron radiation process. The effect of synchrotron radiation in a bend of radius ρ, and angle θ may be expressed in terms of a machine impedance of magnitude[11]

$$|Z(n)| = 354\left(\frac{n\rho}{R}\right)^{1/3}\left(\frac{\theta}{2\pi}\right)\ \text{ohms}. \tag{17}$$

at harmonic n relative to the machine circumference $2\pi R$. However, the synchotron radiation in the bend magnets is suppressed at frequencies below a cutoff value many times the TM-mode cutoff. For a "vacuum" chamber consisting of two, infinite parallel plates separated by $2h$, the synchrotron radiation will be fully unshielded only for harmonics n satisfying

$$n > \frac{R}{\rho}\left(\frac{\pi\rho}{2h}\right)^{3/2}. \tag{18}$$

The peak value of the coupling resistive component of the coupling impedance is found to be well-approximated by[12]

$$Re(Z(n)) \simeq 300\,\frac{h}{R}. \tag{19}$$

For small machines (radius less than 100 m) this effect can apparently provide the dominant source of high-frequency impedance. Random currents (Schottky noise) which exist at arbitrarily high frequencies on a bunched beam can in principle self-couple through this mechanism and generate internal bunch instabilities. However, the parallel plate geometry for which Eqs. (17) and (18) apply is open and does not exhibit the full resonant structure that would be found in a closed, toroidal vacuum chamber. Thus, although it can be expected that Eq. (19) holds in some averaged sense, there is the need to clarify the resonance structure, including widths. This analysis is the topic of the contribution of Warnock and Morton. Lee addresses the transverse counterpart and shows the importance of chromatic effects. It should be noted that application of the longitudinal impedance should not be naively applied to standard bunch-lengthening formulas because the frequency, phase, and spatial character of the synchrotron-radiation impedance are quite different from those which have generated instabilities in existing rings—rings which are less smooth than those that are now being proposed. In particular, since the structure is not short compared to the wavelengths of interest, stability analysis is entirely in a transient regime.

6. FUTURE DIRECTIONS

Although the results of this workshop indicate strongly that the longitudinal impedance of an isolated cavity-like structure has an $\omega^{-1/2}$ rolloff, a rigorous proof has yet to be achieved. Finite-length systems as presented by this isolated cavity problem or by bunched-beam stability analysis have proven intractable in the exact sense, with most work relying on truncation of an essentially infinite-dimensional problem. Any progress in this area would not only yield possible confirmation of the various approximate results of this workshop proceedings, but also would offer a powerful tool with which to address a variety of accelerator beam-dynamics questions. An issue which was only addressed in passing during the workshop is the effect of tapering on reducing the coupling of the beam to vacuum-chamber discontinuities. To date there have been no clear analytic results on the scaling, with bunch length and taper angle, of the impedance reduction offered by tapering.

The results of Warnock and Morton and also of Ng (in this Proceedings) clearly indicate that, in a closed geometry, there is a self-interaction of the beam through the synchrotron radiation process which is not of negligible strength. In fact, the estimated impedance values estimated demand further study to ensure that the phase-space densities desired in damping rings for linear colliders and high-brightness synchrotron light sources are obtained. This work should include both theoretical beam-dynamics calculations and experiments on small electron storage rings. Unfortunately, the combination of discontinuity cleanliness and small radius required to observe synchrotron-radiation-induced instability may be hard to find in the older generation of machines, and an small experimental machine dedicated to this study may be needed. Such a device would also be of use in evaluating component impedances (cavities, bellows, steps, slotted vacuum chambers) at frequencies too high for confident wire or bead pull measurements.

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