Evolution of Nuclear Structure along the chain of Zr-isotopes: Shell model Approach

S. Biswas[1*] and A. Chakraborty[2†]
[1]*Department of Physics, Murshidabad College of Engineering and Technology, Banjetia, Berhampore 742 102, West Bengal and*
[2]*Department of Physics, Siksha Bhavana, Visva-Bharati, Santiniketan 731 235, West Bengal*

Introduction

Experimental results suggest that the chain of Zr-isotopes (with A $\sim$ 80 - 108) show a variety of nuclear structure as the number of neutrons vary from a mid-open-shell deformed region ($^{80}Zr_{40}$), through a closed neutron shell ($^{90}Zr_{50}$), to a closed neutron subshell ($^{96}Zr_{56}$), and then to a sudden reappearance of deformation ($^{100}Zr_{60}$), which persists to a mid-open-shell region ($^{108}Zr_{68}$). This behavior is unprecedented anywhere on the nuclear mass surface. From the theoretical point of view, it would be interesting to see whether the trend of this variation of nuclear structure as a function of neutron number can be reproduced within the limit of common theoretical platform: *that is to say* that when the calculations are to be carried out through shell model approach, importance to be given to the use of the same model space and interaction. However, because of the computational complexity, quite often different types of truncation schemes are adopted. Here, we are bringing out the results of our shell model calculations using a modest model space. The calculations are mainly motivated by the availability of new and revised experimental lifetime data for the semi-magic ^{94}Zr nucleus at low-spin, low-excitation regime [1].

Shell model calculations and results

Shell model calculations were carried out using the NuShellX code [2]. $^{88}_{38}Sr_{50}$ was used as the inert core with the protons filling the $(2p_{1/2}, 1g_{9/2})$ orbitals and neutrons the $(3s_{1/2}, 2d_{5/2})$ orbitals. Two body interaction matrix file *gl* was used. With this model space, reasonable agreement between the experimental and theoretical level energies up to 4_2^+ states were obtained for the even-even $^{90-96}$Zr isotopes (see Fig. 1). Calculations could not be extended for the isotopes with A $>$ 96 because of the restricted model space.

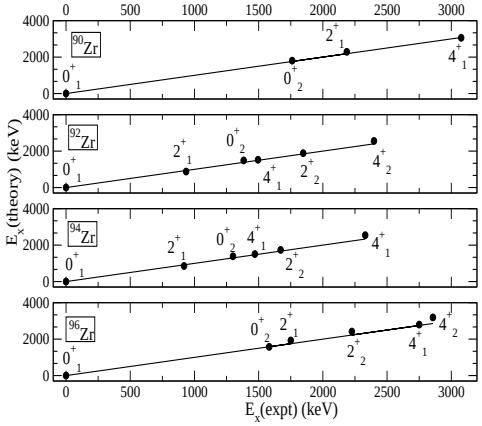

FIG. 1: Comparison between the experimental and theoretical positive-parity low-lying level energies for even-even Zr-isotopes. The theoretical level energies have been plotted against the experimental level energies. The solid line with the slope of unity have been drawn to test the goodness of agreement between the experimental and theoretical results.

The structure of 0_1^+ and 0_2^+ states are found to be quite different in ^{94}Zr. It is found that the main configurations in the wavefunctions of 0_1^+ state are consisting of pair of neutron excitation from $3s_{1/2}$ to $2d_{5/2}$ orbital (about

*Electronic address: saumyajitbiswas@gmail.com
†Electronic address: anagha.chakraborty@gmail.com

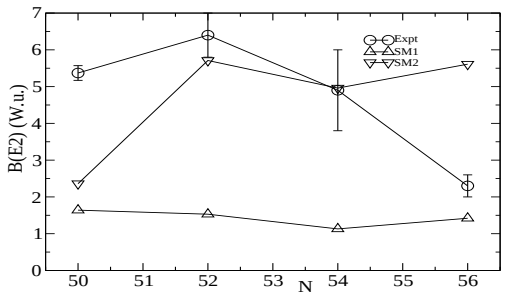

FIG. 2: Comparison of $B(E2)$ values for the even-even Zr-isotopes corresponding to the transition decaying from 2_1^+ to 0_1^+ state. Experimental values have been compared with theoretical predictions (SM1 and SM2).

TABLE I: Comparison of $B(E2)$ strength for the transitions decaying to the first and second 0^+ states in ^{94}Zr. The quoted B(E2) values are in W.u. . Numbers in parentheses indicate uncertainties in the last digit of the quoted experimental values.

$J_i^\pi \rightarrow J_f^\pi$	$B(E2)_{exp}$	$B(E2)_{SM1}$	$B(E2)_{SM2}$
$2_1^+ \rightarrow 0_1^+$	4.9(11)	1.1	5.0
$2_2^+ \rightarrow 0_1^+$	3.9(3)	1.5	3.2
$2_2^+ \rightarrow 0_2^+$	19(2)	4.0	9.6

58%) with no proton excitation from $2p_{1/2}$ to $1g_{9/2}$ orbital. On the otherhand, pair of proton excitation from $2p_{1/2}$ to $1g_{9/2}$ orbital, with no neutron excitation, play the dominant role (about 48%) in generating the 0_2^+ state. To further test the quality of the wavefunctions, experimental $B(E2)$ values for a few transitions are compared with the theoretical results (see Table I). Fig. 2 illustrates the variation of $E2$-strength of even-even Zr-isotopes with A = 90 - 96 corresponding to $2_1^+ \rightarrow 0_1^+$ transition. SM1 calculations were carried out using the standard set of effective charge of e_p = 1.5 and e_n = 0.5 (in the unit of electronic charge); whereas the calculations with e_p = 1.8 and e_n = 1.2 are called SM2. The effective charge used in SM2 have been obtained by adjusting the theoretical $B(E2)$ with that of the experimental value for the $2_1^+ \rightarrow 0_1^+$ transition in ^{94}Zr. It is obvious from Fig. 2 that SM2 gives reasonable agreement with experimental results in most of the cases; although it appears to be the absence of essential configurations in the adopted model space which give rise to the need for the large value of effective charges. On the otherhand, the calculations using SM1 underpredict the experimental values in all the cases. Although the $B(E2)$ strength for the transitions involving yrast states are reproduced within the reasonable agreement (with SM2), the $E2$ strength for the transitions decaying from 0_2^+ states could not be reproduced in the present shell model calculations. In fact, the experimental $B(E2)$ values for the $0_2^+ \rightarrow 2_1^+$ transitions in ^{92}Zr and ^{94}Zr are found to be an order of magnitude larger than those predicted by the present calculations. This indicates that the calculations with the restricted model space is insufficient to predict the observed collective strength of the excited 0^+ states, suggesting the absence of large number of mixing configurations. Many of these discrepancies could not be sorted out even with the extended shell model calculations using ^{78}Ni core [3]. Thus, for better representation, it is quite likely to perform the calculations using ^{56}Ni inert core with the possibility of incorporating the proton excitations from the fp shells to the orbits above $1g_{9/2}$, along with the neutron excitations from the $1g_{9/2}$ orbital to higher shells beyond $3s_{1/2}$ and $2d_{5/2}$. In this regard, rigorous calculations are planned using the upcoming high efficiency computational facilities.

Acknowledgments

This work was carried out as a part of the Collaborative Research Scheme of UGC-DAE CSR, Kolkata (No. UGC-DAE-CSR-KC/CRS/13/NP04/ & UGC-DAE-CSR-KC/CRS/13/NP04/02). Help from Dr S.S. Ghugre at the initial phase of installation of the code is gratefully acknowledged.

References

[1] E.E. Peters *et al.*, Phys. Rev. C **88**, 024317 (2013); A. Chakraborty *et al.*, Phys. Rev. Lett. **86**, 064314 (2012).
[2] www.nscl.msu.edu/ brown/nushellx.zip
[3] K. Sieja *et al.*, Phys. Rev. C **79**, 064310 (2009).