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Possible role of Berry phase in inflationary cosmological perturbations

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Abstract. Here we have derived a cosmological analogue of Berry phase by obtaining the corresponding wavefunction for the system of inflationary cosmological perturbations solving the Schrodinger equation. We have further shown that cosmological Berry phase can be related inflationary observable parameters. As a result one can, atleast in principle, establish a supplementary probe of inflationary cosmology through the measurement of the associated Berry phase. But we do not make any strong comment on this.

1. Introduction

The original motivation behind inflation [1] was to resolve the puzzles associated with *Standard Big Bang* scenario but the most exciting aspect of this scenario comes from the fact that they provide natural explanation for the quantum origin of classical cosmological fluctuations [2] observed in the large scale structure of the matter and in the cosmic microwave background. The accelerated expansion converts the initial vacuum quantum fluctuations into macroscopic cosmological perturbations. So, measurement of any quantum property which reflects on classical observables serve as a supplementary probe of inflationary cosmology, complementing the well-known CMB polarization measurements [3]. This has led us to investigate for the potentiality of Berry phase [4] in providing a measurable quantum property which is inherent in the macroscopic character of classical cosmological perturbations, thereby serving as a supplementary probe to CMB in inflationary cosmology.

Here our intention is to demonstrate the effect of the curved spacetime background in the dynamical evolution of the quantum fluctuations during inflation through the derivation of the associated Berry phase and search for the possible consequences via observable parameters. To this end, we first find an exact wavefunction for the system of inflationary cosmological perturbation by solving the associated Schrodinger equation. The relation between the dynamical invariant [5, 6, 7] and the geometric phase has then been utilized to derive the corresponding Berry phase [8, 9]. For slow roll inflation the total accumulated phase gained by each of the modes during sub-Hubble oscillations (adiabatic limit) is found to be a new parameter made of corresponding (scalar and tensor) spectral indices. Since tensor spectral index is related to the tensor to scalar amplitude ratio through the consistency relation, the Berry phase can indeed be utilized to act as a supplementary probe of inflationary cosmology.

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2. Inflationary Cosmological perturbations as parametric oscillators

The total action for the system of single scalar field driven inflation is given by

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left[M_P^2 R + g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)\right] \tag{1}$$

where R is the Ricci scalar, ϕ the inflaton and $V(\phi)$ being corresponding potential. The explicit expression for the perturbations quantities can be calculated by varying the above action. The single scalar field driven inflation produces both the *scalar* and *tensor* perturbations, but in the linear theory they evolve independently as a result we can study their evolution separately. It can be shown that for the scalar perturbation everything can be reduced to the study of a single variable v known as Mukhanov-Sasaki variable defined by, $\underline{v \equiv a\delta\phi_{GI} + z\Phi_B}$ with $z = a\frac{\phi'}{\mathcal{H}}$, $\delta\phi_{GI}$ gauge invariant inflaton fluctuation and Φ_B Bardeen's Potential [10]. For the tensor perturbation it is the variable u defined by, $\underline{h_{ij} = \frac{u}{a}Q_{ij}}$ where Q_{ij} Eigentensors of the Laplacian operator.

The actions for the scalar and tensor perturbations can be cast into the following form [11]

$$\begin{aligned}
\mathbf{S}^S &= \frac{1}{2}\int d\eta d\mathbf{x}\left[\Pi^2 + \frac{z'}{z}\left(\Pi v - v\Pi\right) - \delta^{ij}\partial_i v\partial_j v\right], \text{ where } \Pi \equiv \frac{\partial \mathbf{L}}{\partial v'} = v' - \frac{z'}{z}v \\
\mathbf{S}^T &= \frac{1}{2}\int d\eta d\mathbf{x}\left[\pi^2 + \frac{a'}{a}\left(\pi u - u\pi\right) - \delta^{ij}\partial_i u\partial_j u\right], \text{ where } \pi \equiv \frac{\partial \mathbf{L}}{\partial u'} = u' - \frac{a'}{a}u
\end{aligned} \tag{2}$$

The variable v is related to the most fundamental perturbation quantity, comoving curvature perturbation $\mathcal{R}$, via the relation $v = -z\mathcal{R}$. Promoting the fields to operators and taking the Fourier decomposition, the Hamiltonian operators corresponding to the above actions (2) is found to be

$$\hat{\mathbf{H}}_{\mathbf{k}} \equiv \sum_{j=1}^{2}\hat{\mathbf{H}}_{j\mathbf{k}} = \sum_{j=1}^{2}\frac{1}{2}\left[k^2\hat{q}_{j\mathbf{k}}^2 + Y(\eta)\left(\hat{p}_{j\mathbf{k}}\hat{q}_{j\mathbf{k}} + \hat{q}_{j\mathbf{k}}\hat{p}_{j\mathbf{k}}\right) + \hat{p}_{j\mathbf{k}}^2\right] \tag{3}$$

here we have decomposed $v_{\mathbf{k}}$, $u_{\mathbf{k}}$, $\Pi_{\mathbf{k}}$, $\pi_{\mathbf{k}}$ into their real and imaginary parts. Also $\hat{q}_{j\mathbf{k}} = \hat{v}_{j\mathbf{k}}$, $\hat{u}_{j\mathbf{k}}$; $\hat{p}_{j\mathbf{k}} = \hat{\Pi}_{j\mathbf{k}}$, $\hat{\pi}_{j\mathbf{k}}$ and $Y = \frac{z'}{z}$, $\frac{a'}{a}$ for the scalar and tensor modes respectively and $j = 1, 2$ with the frequency given by $\omega = \sqrt{k^2 - Y^2}$. So for each of the scalar and tensor modes, the Hamiltonian is a sum of two parametric oscillators, each of them having the form (3).

3. The Berry phase in inflationary cosmological perturbations

To calculate the Berry phase we require the wavefunction of the system. To this end we first solve the associated Schrdinger equation using the Dynamical Invariant Operator Method (DIOM) [7] for the system of inflationary cosmological perturbations. Following the usual technique [5, 6], after some straightforward but tedious algebra we found

$$\Psi_{n_1,n_2} = \frac{e^{i\alpha_{n_1,n_2}(\eta)}\bar{H}_{n_1}\left[\frac{q_{1\mathbf{k}}}{\rho_k}\right]\bar{H}_{n_2}\left[\frac{q_{2\mathbf{k}}}{\rho_k}\right]}{\sqrt[4]{\pi^2 2^{2(n_1+n_2)}(n_1!n_2!)^2\rho_k^4}}\exp\left[\frac{i}{2}\left(\frac{\rho_k'}{\rho_k} - Y(\eta) + \frac{i}{\rho_k^2}\right)\left(q_{1\mathbf{k}}^2 + q_{2\mathbf{k}}^2\right)\right] \tag{4}$$

where ρ_k is a time dependent real function satisfying the Milne-Pinney equation

$$\rho_k'' + \Omega^2(\eta, k)\rho_k = \frac{1}{\rho_k^3(\eta)}, \text{ where } \Omega^2 = \omega^2 - \frac{dY}{d\eta}. \tag{5}$$

Once the wavefunction of the system is known it can then be used to derive the associated Berry phase and in the present context we have found

$$\gamma_{n_1,n_2,k} = -\frac{1}{2}(n_1+n_2+1)\int_0^{\Gamma}\left(\frac{1}{\rho_k^2}-\rho_k^2\omega^2-(\rho_k')^2\right)d\eta \tag{6}$$

where it has been assumed that the invariant is Γ periodic and its eigenvalues are non-degenerate. Now in the adiabatic limit Eqn.(5) can be solved [6] by a series of powers in adiabatic parameter, so at the ground state of the system the Berry phase for a particular perturbation mode can be evaluated upto the first order in adiabatic parameter and is given by

$$\gamma_k^{(S,\ T)} = \frac{1}{2}\int_0^{\Gamma}\frac{Y'}{\sqrt{k^2-Y^2}}d\eta \tag{7}$$

where the superscripts S and T stand for scalar and tensor modes respectively.

4. Berry Phase interms of cosmological observables

Now to link this Berry phase with the cosmological observables we need to fix the value of the parameter Γ. To this end we shall calculate the total Berry phase accumulated by each mode during inflationary sub-Hubble oscillations which is

$$\gamma_{k\ sub}^{S,T} = -\frac{1}{2}\lim_{\eta'\to-\infty}\int_{\eta'}^{\eta_0^{S,T}}\frac{Y'}{\sqrt{k^2-Y^2}}d\eta \tag{8}$$

where $\eta_0^{S,T}$ is the conformal time which satisfies the relation $k^2 = \left[Y(\eta_0^{S,T})\right]^2$ and guarantees that the modes are within the horizon and oscillating with real frequency.

The *accumulated Berry phase* during sub-Hubble evolution of the scalar modes, in terms of the slow-roll parameters, turns out to be

$$\gamma_k^S = \frac{1}{2}\lim_{\eta'\to-\infty}\int_{\eta'}^{-\frac{\sqrt{1+6\epsilon_1-2\epsilon_2}}{k}}\frac{\frac{z''}{z}-\left(\frac{z'}{z}\right)^2}{\sqrt{k^2-\left(\frac{z'}{z}\right)^2}}d\eta \approx -\frac{\pi}{4}\frac{1+3\epsilon_1-\epsilon_2}{\sqrt{1+6\epsilon_1-2\epsilon_2}} + O(\epsilon_1^2,\epsilon_2^2,\epsilon_1\epsilon_1) \tag{9}$$

Similarly for the tensor modes we have

$$\gamma_k^T = \frac{1}{2}\lim_{\eta'\to-\infty}\int_{\eta'}^{-\frac{\sqrt{1+2\epsilon_1}}{k}}\frac{\frac{a''}{a}-\left(\frac{a'}{a}\right)^2}{\sqrt{k^2-\left(\frac{a'}{a}\right)^2}}d\eta \approx -\frac{\pi}{4}\frac{1+\epsilon_1}{\sqrt{1+2\epsilon_1}} + O(\epsilon_1^2,\epsilon_2^2,\epsilon_1\epsilon_1) \tag{10}$$

where ϵ_1, ϵ_2 are the usual potential slow-roll parameters. As, during inflation the slow-roll parameters do not evolve significantly from their initial values so in the above estimates for $\gamma_k^{S,T}$ we can consider ϵ_1 and ϵ_2 as their values at horizon crossing without committing any substantial error. Also at the horizon exit the fundamental observable parameters can be expressed in terms of slow-roll parameters, so we found the *accumulated Berry phase* during inflationary *sub-Hubble* oscillations of the modes to be given by

$$\gamma_k^S \approx -\frac{\pi}{8}\frac{3-n_S(k)}{\sqrt{2-n_S(k)}},\quad n_S(k)\approx 3-\frac{8\gamma_k^S}{\pi}\left(\frac{4\gamma_k^S}{\pi}-\sqrt{\frac{16[\gamma_k^S]^2}{\pi^2}-1}\right) \tag{11}$$

$$\gamma_k^T \approx -\frac{\pi}{8}\frac{2-n_T(k)}{\sqrt{1-n_T(k)}},\quad n_T(k)\approx 2-\frac{8\gamma_k^T}{\pi}\left(\frac{4\gamma_k^T}{\pi}-\sqrt{\frac{16[\gamma_k^T]^2}{\pi^2}-1}\right) \tag{12}$$

where n_S and n_T are the scalar and tensor spectral indices respectively. So the Berry phase for sub-Hubble oscillations of the perturbation modes during inflation can be completely envisioned through the observable parameters.

5. Discussion and Summary

In this article we have demonstrated how the exact expression for the wave function of the quantum cosmological perturbations can be analytically obtained by solving the associated Schrodinger equation following the dynamical invariant technique. This helps us to derive an expression for cosmological analogue of Berry phase. The classical cosmological perturbation modes (both scalar and tensor) having quantum origin picks up a phase during their advancement through the curved space-time background that depends entirely on the background geometry and can be estimated quantitatively by measuring the corresponding spectral indices. So the Berry phase for the quantum counterpart of the classical cosmological perturbations endow us with the measure of spectral index.

Any attempt towards the measurement of cosmological Berry phase may reflect observational credentials of this parameter in inflationary cosmology. So, cosmological Berry phase may have the potentiality to play some important role in inflationary cosmology. However, we are yet to figure out this quantitative feature in a more concrete language. So, we do not make any strong comment on this. At this point, all we would like to point out is that an analogue of Berry phase appears in inflationary cosmology, which is a measurable observable of the quantum property of the cosmological perturbations and which may presumably give some insight on observable parameters as well.

So far as the detection of cosmological Berry phase is concerned, we are far away from quantitative measurements. A possible theoretical aspect of detection [12] of the analogue of cosmological Berry phase may be developed in squeezed state formalism [11]. For a quantum harmonic oscillator, when a squeezing Hamiltonian is switched on, and the squeezing parameter is varied, we can find a detectable Berry phase. As the inflationary perturbations can be studied in the squeezed state formalism, we hope to put forward our analysis on the detection of the geometric phase in near future.

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