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# Model Building on Asymmetric $Z_3$ Orbifolds: Non-supersymmetric Models

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**Abstract**

Four-dimensional string models arising in the asymmetric $Z_3$ orbifold compactifications of the heterotic string are studied. I present a mechanism for supersymmetry breaking that gives rise to chiral models in four dimensions, and discuss some typical models. A formalism for calculating one-loop partition functions in $Z_3$ models is developed. One partition function may correspond to a non-supersymmetric, tachyon-free theory, with a vanishing cosmological constant as a consequence of Atkin-Lehner symmetry.

## 1. Introduction

Recently there has been much interest in constructing four-dimensional heterotic superstring [1] theories. The formalism of four-dimensional strings [2,3,4] is well understood, although none of these theories has yet emerged as a leading candidate for the theory unifying gravity with strong and electro-weak interactions.

The absence of low-energy supersymmetry is one important experimental fact that must be incorporated in constructing four-dimensional models. When supersymmetry is broken at scales not too far beyond the present experimental bounds, what enters is the zero-slope limit of superstring theories, so field-theoretical mechanisms for supersymmetry breaking are appropriate. If supersymmetry is broken at scales comparable to the Planck scale, the problem of supersymmetry breaking must be considered within the string framework, by the construction of non-supersymmetric string theories. In this paper the second possibility is considered. Previous works [5,6,7,8] have also considered supersymmetry breaking in compactified superstring theories.

I work in the asymmetric orbifold formulation [4] of four-dimensional string theories, since this formulation is particularly intuitive and physically transparent. In this formalism, modular invariance is ensured by simple level-matching conditions, gauge groups are easily identified, the number of massless fermions readily determined, and so on. I choose $Z_3$ orbifolds [9], as they give rise to physically interesting gauge groups, like $SU_3$ and $E_6$, although my results can be generalised readily to the case of arbitrary $Z_n$.

The paper is organised as follows. In section 2, I set up the formalism. A simple way of constructing $Z_3$ symmetric lattices is presented. In section 3, a new mechanism for supersymmetry breaking is presented. This mechanism, which is based on the use of two $Z_3$ symmmetries, produces chiral models. In section 4, a formalism for calculating the one-loop partition functions is developed, and the Atkin-Lehner symmetry [10], which may be responsible for the vanishing of the cosmological constant in the absence of supersymmetry, is discussed. I present one partition function that corresponds to a tachyon-free model with the vanishing vacuum energy. The results are summarized in section 5. In the Appendix, the properties of theta functions with third-integer characteristics are discussed.

## 2. Formalism

A four-dimensional asymmetric orbifold model is fully defined by specifying:

- an even self-dual (16+6,6)-dimensional lorentzian lattice [11],
- the action of a discrete lattice automorphism group on left-moving and right-moving string degrees of freedom,
- discrete torsion [12].

There is a particularly simple way of constructing [1] even, self-dual lattices. Begin with a lower dimensional lattice, symmetric under the desired discrete group, and build up the full lattice by taking the direct sum of sublattices in a way that ensures self-duality and evenness. With $Z_3$ symmetry, natural building blocks are provided by the two-dimensional $SU_3$ root lattice $R$, and its weight lattices $W$ and $W^*$, for the fundamental and conjugate fundamental representations, respectively. By the

[1] I am grateful to K.S. Narain for bringing this construction to my attention.

weight lattice, I mean the root lattice shifted by the weight vector. I adopt the normalisation corresponding to root vectors of $(length)^2 = 2$ and weight vectors of $(length)^2 = \frac{2}{3}$. These lattices are symmetric under $\frac{2\pi}{3}$ rotations. The procedure described here allows lattice theta functions to be written in terms of the $SU_3$ lattice partition functions given in the Appendix.

To illustrate this procedure, I construct two well known lattices. The (8,0)-dimensional $E_8$ root lattice [1] can be constructed by using the $(SU_3)^4$ decomposition of the adjoint representation of $E_8$:

$$\begin{aligned}\Gamma_8 = RRRR \;&+\; RWWW^* + WRW^*W^* + WWRW + WW^*WR \\ &+\; RW^*W^*W + W^*RWW + W^*W^*RW^* + W^*WW^*R\,. \end{aligned} \tag{1}$$

Using the transformation properties of $SU_3$ lattice theta functions under modular transformations, given in the Appendix, one can check that the Eisenstein series corresponding to $\Gamma_8$ is modular invariant. Another useful example is the (2,2)-dimensional $SU_3$ root-weight lattice [4]:

$$\Gamma_{2,2} = R\overline{R} + W\overline{W} + W^*\overline{W}^*\,. \tag{2}$$

It is easy to verify modular invariance in this case. Many other examples, such as the $E_6$ root-weight lattice, etc., can also be discussed.

In the asymmetric orbifold formulation of four-dimensional string theories, the actions of a discrete symmetry group on left-moving and right-moving string excitations are considered separately. I assume that the discrete groups contain a number of $Z_3$ factors, consisting of rotations and translations on the (16+6,6)-dimensional momentum lattice. I focus my attention on the asymmetric orbifolds obtained from the lattice $(\Gamma_8)^2(\Gamma_{2,2})^3$. The momentum vector is represented by:

$$(P_1; P_2; p_1, \tilde{p}_1; p_2, \tilde{p}_2; p_3, \tilde{p}_3),$$

where $P_1$ denotes the component in the first $\Gamma_8$, $P_2$ the component in the second $\Gamma_8$, and $(p_i, \tilde{p}_i)$, $i = 1, 2, 3$, the components in three $\Gamma_{2,2}$ lattices. I use the Green-Schwarz [13] formulation for right-moving fermions, which is convenient for studying the problem of supersymmetry breaking.

I note the physical interpretation of discrete torsion. Let $\alpha$ and $\beta$ be nontrivial elements of two different $Z_3$ groups. The discrete torsion $\epsilon(\alpha, \beta) = 1$, $e^{\frac{2\pi}{3}i}$, or $e^{-\frac{2\pi}{3}i}$, defines the $\alpha$ charge of the vacuum in the sector twisted by $\beta$. In order for this choice to be consistent with modular invariance, it must satisfy the conditions derived in ref.[12].

## 3. Non-supersymmetric models

The mechanism often used for supersymmetry breaking in string theories employs $2\pi$ rotational symmetry of Minkowski space-time, or equivalently, the fermion number $F$ conservation. A twist with the $Z_2$ symmetry operator $(-1)^F$ breaks supersymmetry by eliminating the zero modes of Green-Schwarz fermions [5]. For $Z_3$ symmetry, the analogous operator $\beta$ leaves no fermionic ground state invariant under its action. The action of $\beta$ on Green-Schwarz fermions is given by:

$$\tilde{\beta}_{GS} = e^{2\pi i \frac{2}{3} J_1} = \mathrm{diag}\left(\tfrac{2}{3},\ \tfrac{2}{3},\ \tfrac{2}{3},\ \tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}, -\tfrac{2}{3}\right), \tag{3}$$

where, to be specific, I choose the angular momentum operator $J_1$ to generate rotations in the first $\Gamma_{2,2}$ lattice. The same rotations must be performed on the right-moving bosonic oscilators and the right-moving momenta:

$$\tilde{\beta} : (P_1; P_2; p_1, \tilde{p}_1; p_2, \tilde{p}_2; p_3, \tilde{p}_3) \rightarrow (P_1; P_2; p_1, e^{2\pi i \frac{2}{3} J_1} \tilde{p}_1; p_2, \tilde{p}_2; p_3, \tilde{p}_3), \qquad (4)$$

as required by world-sheet supersymmetry. The operator $\beta$ can be supplemented with some extra $\frac{2\pi}{3}$ rotations on the right-movers and shifts on the momentum lattice (a shift must correspond to one-third of a lattice vector, as dictated by the $Z_3$ symmetry). By checking all possibilities, it is possible to show that up to such generalisations, there is no other supersymmetry breaking $Z_3$ operator satisfying the level matching conditions in the twisted sectors.

The operator $\beta$ is similar to $(-1)^F$ in another respect. A twist with $(-1)^F$ gives back the heterotic superstring in ten dimensions. Similarly, a twist with $\beta$ alone gives a four-dimensional $N = 4$ supersymmetric model, with the gravitinos reemerging from the twisted sectors. This is the reason why, in order to break supersymmetry, the action of $\beta$ must be accompanied by some shifts on the momentum lattice, and/or by rotations on the right-movers. In this process, particular care must be taken to avoid tachyons, but the analogy ends here. While it is possible to generate chiral theories in ten dimensions [14], by using $(-1)^F$ combined with some shifts, a similar procedure applied to $\beta$ gives left-right symmetric models in four dimensions. The reason is that the eigenvalue $\frac{2}{3}$ (or $-\frac{2}{3}$) Green-Schwarz fermions correspond to two helicity plus and two helicity minus states in four dimensions. The way to avoid this problem is to introduce an extra $Z_3$ symmetry, which leaves invariant either one eigenvalue $\frac{2}{3}$ eigenstate of the operator $\beta$, or else two eigenstates of the same helicity. The action of the operator $\alpha_1$, which leaves invariant only one eigenstate, is given by:

$$\tilde{\alpha}_{1GS} = e^{2\pi i(\frac{2}{3}J_1+\frac{2}{3}J_2+\frac{2}{3}J_3)} = \mathrm{diag}\left(0,\ \tfrac{2}{3},\ \tfrac{2}{3},-\tfrac{2}{3},0,-\tfrac{2}{3},-\tfrac{2}{3},\ \tfrac{2}{3}\right), \tag{5}$$

$$\tilde{\alpha}_1 : (P_1;P_2;p_1,\tilde{p}_1;p_2,\tilde{p}_2;p_3,\tilde{p}_3) \rightarrow (P_1;P_2;p_1,e^{2\pi i\frac{2}{3}J_1}\tilde{p}_1;p_2,e^{2\pi i\frac{2}{3}J_2}\tilde{p}_2;p_3,e^{2\pi i\frac{2}{3}J_3}\tilde{p}_3), \tag{6}$$

The action of the operator $\alpha_2$, which leaves invariant two eigenstates, is given by:

$$\tilde{\alpha}_{2GS} = e^{2\pi i(\frac{2}{3}J_2+\frac{2}{3}J_3)} = \mathrm{diag}\left(-\tfrac{2}{3},0,0,\ \tfrac{2}{3},\ \tfrac{2}{3},0,0,-\tfrac{2}{3}\right), \tag{7}$$

$$\tilde{\alpha}_2 : (P_1;P_2;p_1,\tilde{p}_1;p_2,\tilde{p}_2;p_3,\tilde{p}_3) \rightarrow (P_1;P_2;p_1,\tilde{p}_1;p_2,e^{2\pi i\frac{2}{3}J_2}\tilde{p}_2;p_3,e^{2\pi i\frac{2}{3}J_3}\tilde{p}_3), \tag{8}$$

The $Z_3$ symmetry generated by either one of these two operators, imposed in addition to the $Z_3$ symmetry generated by the operator $\beta$, results in a chiral spectrum of massless fermions. This is provided that its action, eqs.(5-8), is accompanied by shifts on the momentum lattice (and/or by rotations on the right-movers), which is necessary in order to prevent the mirror fermions from reemerging in the twisted sectors.

In the rest of this section, I consider $(Z_3)^2$ groups, consisting of nine elements of the form $\alpha^i\beta^j,\ i,j = 0,1,2$. These models contain the untwisted sector and eight twisted sectors. As explained before, $\alpha$ and $\beta$ are supplemented with some shifts and rotations. Additional rotations increase the number of fixed points, and produce models with too many massless generations, so I restrict myself to pure shifts. In this case, tachyons are absent provided that the shift vector, supplementing the action of $\beta$, has $(length)^2 \geq 2$.

For the purpose of illustration, I construct a simple $(Z_3)^2$ model, with the operators $\alpha = \alpha_2$, eqs.(7,8), and $\beta$, eqs.(3,4), modified to $\alpha'$ and $\beta'$. These operators act the same on Green-Schwarz fermions, whereas:

$$\tilde{\alpha}' : (P_1; P_2; p_1, \tilde{p}_1; p_2, \tilde{p}_2; p_3, \tilde{p}_3) \to (P_1; P_2; p_1 + w_1, \tilde{p}_1; p_2, e^{2\pi i \frac{2}{3} J_2} \tilde{p}_2; p_3, e^{2\pi i \frac{2}{3} J_3} \tilde{p}_3) , \tag{9}$$

$$\tilde{\beta}' : (P_1; P_2; p_1, \tilde{p}_1; p_2, \tilde{p}_2; p_3, \tilde{p}_3) \to (P_1 + W_1; P_2 + W_2; p_1, e^{2\pi i \frac{2}{3} J_1} \tilde{p}_1; p_2, \tilde{p}_2; p_3 + w_3, \tilde{p}_3) . \tag{10}$$

$W_1$, $W_2$ and $w_1$ denote the weight vectors; $W_1$ ($W_2$) in one of the $SU_3$ subplanes of the first (second) $\Gamma_8$ lattice, and $w_1$ ($w_3$) in the first (third) $\Gamma_{2,2}$ lattice.

The spectrum of the untwisted sector does not depend on the choice of discrete torsion, so I discuss it first. The massless bosons are: graviton, dilaton, $(E_6)^2(SU_3)^5$ gauge bosons, and scalars in the representations $\mathbf{27}_1 W_1$ and $\mathbf{27}_2 W_2$, where I adopt a self-explanatory notation for the $E_6$ and $SU_3$ representations. There are two generations of chiral fermions: $2 \cdot \mathbf{27}_1 W_1$ and $2 \cdot \mathbf{27}_2 W_2$.

The massless particle content of eight twisted sectors depends on the choice of discrete torsion, or in other words, on the vacuum charge $\epsilon(\alpha, \beta) \equiv \epsilon$. Below, I list massless fermions ($F$) and massless scalars ($S$) from the twisted sectors.
From the sectors twisted by $\alpha$ and $\alpha^2$:

| | | |
|---|---|---|
| $\epsilon = 1$ | $F$: $(w_1 + w_1^*)(w_2 + w_2^*)w_3^*$ | $S$: nothing |
| $\epsilon = e^{\frac{2\pi}{3} i}$ | $F$: $w_1(w_2 + w_2^*)w_3$ | $S$: $2 \cdot w_1(w_2 + w_2^*)w_3^*$ |
| $\epsilon = e^{-\frac{2\pi}{3} i}$ | $F$: $w_1^*(w_2 + w_2^*)w_3$ | $S$: $2 \cdot w_1(w_2 + w_2^*)w_3$ |

From the sectors twisted by $\beta$ and $\beta^2$:

$$\epsilon = 1 \qquad F\colon\ 2\cdot W_1W_2w_3 \qquad S\colon\ W_1W_2(w_1+w_1^*+w_3+w_3^*),\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*$$

$$\epsilon = e^{\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*w_3^* \qquad S\colon\ W_1W_2(w_1+w_1^*+w_3^*),\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*$$

$$\epsilon = e^{-\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*w_3^* \qquad S\colon\ W_1W_2(w_1+w_1^*+w_3^*),\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*$$

From the sectors twisted by $\alpha^2\beta$ and $\alpha\beta^2$:

$$\epsilon = 1 \qquad F\colon\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*,\ W_1^*W_2^*w_3 \qquad S\colon\ W_1^*W_2^*(w_1^*+w_2+w_2^*)$$

$$\epsilon = e^{\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*(w_1^*+w_2+w_2^*) \qquad S\colon\ W_1^*W_2^*(w_1+w_3^*)$$

$$\epsilon = e^{-\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*(w_1+w_3^*) \qquad S\colon\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*,\ W_1^*W_2^*w_3$$

From the sectors twisted by $\alpha\beta$ and $\alpha^2\beta^2$:

$$\epsilon = 1 \qquad F\colon\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*,\ W_1^*W_2^*w_3 \qquad S\colon\ W_1^*W_2^*(w_1+w_2+w_2^*)$$

$$\epsilon = e^{\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*(w_1^*+w_3^*) \qquad S\colon\ \mathbf{27}_1W_2^*,\ \mathbf{27}_2W_1^*,\ W_1^*W_2^*w_3$$

$$\epsilon = e^{-\frac{2\pi}{3}i} \qquad F\colon\ W_1^*W_2^*(w_1+w_2+w_2^*) \qquad S\colon\ W_1^*W_2^*(w_1^*+w_3^*)$$

This list illustrates the rich massless particle spectrum of the models under consideration. It is easy to check that three models, corresponding to different choices of the discrete torsion, are free of anomalies, as expected from the modular invariance [15]. The Higgs sectors contain many scalars, whose vacuum expectation values may reduce the rank of the gauge group.

The $(Z_3)^2$ models discussed above can be generalised by considering further $Z_3$ symmetries. In future work [16], I intend to study, in greater detail, the phenomenological aspects of $Z_3$ asymmetric orbifold models.

## 4. Partition functions and Atkin-Lehner symmetry

In non-supersymmetric string theories one expects that string loop corrections give rise to a vacuum energy of order $(\hbar G)^{-2}$, which is approximately hundred twenty orders of magnitute larger than the experimental bounds on the cosmological constant.

At least at one string loop level, the cosmological constant problem can be avoided by introducing a supersymmetry breaking scale $M_S$, which is less than the Planck scale $M_P$, and ensuring that the numbers of massless fermionic and bosonic string excitations are equal [8]. In such a situation, the cosmological constant is suppressed by the factor $exp(-\frac{M_P}{M_S})$. The supersymmetry breaking scale is related here to the radii of compact dimensions. Such a mechanism is not suitable for asymmetric orbifolds, whose radii are fixed by the requirement of modular invariance.

Recently, Moore [10] has proposed a new way of ensuring vanishing vacuum energy in non-supersymmetric string theories. His mechanism employs so-called Atkin-Lehner symmetry. I explain how this works in the $Z_3$ models under consideration.

At one loop, the four-dimensional string vacuum energy is given by:

$$E_{vac} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^3} P(q,\bar{q}) \tag{11}$$

where $P(q,\bar{q})$ is the one loop partition function, depending on the modular parameter $q = e^{2\pi i\tau}$ for the torus, and $\mathcal{F}$ is the fundamental domain for the modular group. The modular group is generated by the transformations $S : \tau \rightarrow -1/\tau$ and $T : \tau \rightarrow \tau + 1$. It can be decomposed as $\Gamma_0(3) \cup \Gamma_0(3)S \cup \Gamma_0(3)ST \cup \Gamma_0(3)ST^{-1}$, where the subgroup $\Gamma_0(3)$ is generated by the transformations $T$ and $ST^3S$. Correspondingly, the partition function of a $Z_3$ model can be decomposed as the sum of a $\Gamma_0(3)$ form $Z(\tau)$ and its transforms with the elements $S$, $ST$ and $ST^{-1}$ of the right transversal. We will call $Z(\tau)$ the generating function for the partition function. By using the coset decomposition, one can rewrite the vacuum energy, eq.(11), as the integral of $(Im\tau)^{-3}Z(\tau)$ over the fundamental domain of the subgroup $\Gamma_0(3)$. It is not difficult to prove that this integral vanishes, provided that $Z(\tau)$ is eigenvalue $-1$ eigenfunction of the Atkin-Lehner operator:

$$AL: \quad Z(\tau) \to Z'(\tau) = 3\,|\tau|^2\, Z(-1/3\tau) \tag{12}$$

The explanation given above is rather technical and refers to the properties of the partition function; a deeper physical understanding of the Atkin-Lehner symmetry is still missing. The only way known so far to determine whether an orbifold model has a vanishing vacuum energy is to calculate its partition function, and then to check how it transforms under Atkin-Lehner symmetry. Conversely, it is possible to construct an Atkin-Lehner symmetric partition function, and then identify the models which it corresponds to.

Partition functions for $Z_3$ models can be expressed in terms of four theta functions with third-integer characteristics;[2] their definitions and properties are listed in the Appendix. In order to examine transformation properties of a generating function under Atkin-Lehner symmetry, it helps to express theta functions in terms of the functions $\delta$, $\rho$ and $\eta$, which are defined in the Appendix.

As an example I calculate the generating function for the models considered in the previous section. The generating function can be written as the sum of two

[2] See, for example, ref.[17].

$\Gamma_0(3)$ modular forms, $Z_1$ and $Z_2$:

$$Z_1 = \tfrac{1}{9}\{(\beta,1) + (\beta^2,1)\}, \tag{13}$$

$$\begin{aligned} Z_2 &= \tfrac{1}{9}\epsilon\{(\alpha,\beta)+(\alpha\beta,\beta)+(\alpha\beta^2,\beta)+(\alpha^2,\beta^2)+(\alpha^2\beta^2,\beta^2)+(\alpha^2\beta,\beta^2)\} \\ &+ \tfrac{1}{9}\epsilon^2\{(\alpha,\beta^2)+(\alpha\beta^2,\beta^2)+(\alpha\beta,\beta^2)+(\alpha^2,\beta)+(\alpha^2\beta,\beta)+(\alpha^2\beta^2,\beta)\}, \end{aligned} \tag{14}$$

where $(\alpha,\beta)$ denotes contribution from the sector twisted by $\alpha$ in the $t$ direction and $\beta$ in the $\sigma$ direction. The factor $\frac{1}{9}$ takes into account normalization of the projection operators. I obtain:

$$\begin{aligned} Z_1 &= \tfrac{2}{9}\,3^{1/2}\eta^{-24}R^3(R^3-W^3)^2(R\bar{R}+2W\overline{W})(R\bar{R}-W\overline{W})\,\overline{\delta^3\eta^{-9}} \\ &= \tfrac{2}{9}\,\rho^3\eta^{-9}\delta^{-9}\left(|\rho\eta^{-1}\delta^{-1}|^2+2\,|\delta^3\eta^{-1}|^2\right)\left(|\rho\eta^{-1}\delta^{-1}|^2-|\delta^3\eta^{-1}|^2\right)\overline{\delta^3\eta^{-9}}, \end{aligned} \tag{15}$$

$$\begin{aligned} Z_2 &= r\,\tfrac{2}{9}\,3^{9/2}\eta^{-24}R^3W^5(R^3-W^3)\,\overline{\eta\delta^{-3}} \\ &= r\,2\,\delta^9\rho^3\eta^{-23}\,\overline{\eta\delta^{-3}}, \end{aligned} \tag{16}$$

where the factor $r = 2$ or $-1$, depending on the choice of the discrete torsion $\epsilon = 1$ or $e^{\pm\frac{2\pi}{3}i}$, respectively. The formulae listed in the Appendix were used to derive these results. It can be shown that the generating functions $Z = Z_1 + Z_2$ of these models are not Atkin-Lehner eigenfunctions.

In general, the generating function $Z(\tau)$ has the form:

$$Z(\tau) = \sum_{\{\mu,\nu\}} \mu(q)\bar{\nu}(\bar{q}). \tag{17}$$

The individual contributions, $\mu$ from the left movers and $\nu$ from the right movers, are $\Gamma_0(3)$ modular forms of weights $wt(\mu) = wt(\nu) = -1$, possibly with multiplier systems, such that the phases cancel between left and right movers. In eq.(17), the sum includes contributions from a number of twisted sectors; these are chosen so that $Z(\tau)$, plus its coset transforms, fill out all nonvanishing contributions to the partition function. In the orbifold models, with the twist operators restricted to $Z_3$ shifts and rotations, the forms $\mu$ and $\nu$ can be written as linear combinations of the terms proportional to $\rho^x\delta^y\eta^z$; the integer powers $x$, $y$ and $z$ satisfy $4x+y+z=-2$, as dictated by $wt(\mu)=wt(\nu)=-1$, and $x \geq 0$. Restriction to shifts and rotations imposes additional constraints on the powers $x$, $y$ and $z$, since left and right moving mass levels are bounded from below in all twisted sectors:

$$\mu: \ \frac{x_\mu}{6}+\frac{y_\mu}{8}+\frac{z_\mu}{24} \geq -1\,, \qquad \nu: \ \frac{x_\nu}{6}+\frac{y_\nu}{8}+\frac{z_\nu}{24} \geq -\frac{1}{3}. \tag{18}$$

The eigenvalue $-1$ orbifold-like eigenfunctions of the Atkin-Lehner operator are found as follows. If an eigenfunction contains a term proportional to $\rho^x\delta^y\eta^z$, it must also contain its Atkin-Lehner transform, proportional to $\rho^x\delta^z\eta^y$. This condition leads to two more inequalities, obtained from ineqs.(18) by interchanging $y$ with $z$. After taking all of these constraints into account, what remains are the contributions of the right movers $\nu$, restricted to $(y_\nu, z_\nu) = (1,-3)\,,(0,-2)\,,(-1,-1)\,,(-2,0)\,,$ $(-3,1)$ or $(-3,-3)$. The requirement of $\Gamma_0(3)$ modular invariance yields the following conditions on the respective left-movers contributions $\mu$ (see eq.(17) and Appendix):

$$y_\mu = y_\nu \ mod\ 12\,, \qquad z_\mu = z_\nu \ mod\ 12\,. \tag{19}$$

Eq.(19), together with the previously mentioned constraints, allows construction of all orbifold-like, eigenvalue $-1$, Atkin-Lehner eigenfunctions. They are:

$$a \ = \ \delta^9\eta^{-11}\,\overline{\eta\delta^{-3}} \ - \ 81\ \eta^9\delta^{-11}\,\overline{\delta\eta^{-3}} \tag{20}$$

$$b \ = \ \rho^3\delta^{-3}\eta^{-11}\,\overline{\eta\delta^{-3}} \ - \ 3\ \rho^3\eta^{-3}\delta^{-11}\,\overline{\delta\eta^{-3}} \tag{21}$$

$$c = \delta\eta^{-3}\,\overline{\delta\eta^{-3}} - 9\,\eta\delta^{-3}\,\overline{\eta\delta^{-3}} \tag{22}$$

$$d = \eta^{-2}\,\overline{\eta^{-2}} - 3\,\delta^{-2}\,\overline{\delta^{-2}}. \tag{23}$$

The functions $b$, $c$ and $d$, eqs.(21,22,23), correspond to tachyonic models. This leaves us with only one generating function, $Z(\tau)$ proportional to $a(\tau)$ of eq.(20), which may correspond to a tachyon-free non-supersymmetric model with a vanishing one-loop vacuum energy.

I performed an extensive search for a $(Z_3)^n$ orbifold model, with the partition function generated by the function $a$ of eq.(20). The result is rather disappointing. Although I am not able to prove it rigorously, experimentation with models suggests that there is no model with such a partition function. It would be very important to understand if there is some basic obstruction preventing the function $a$ from generating a partition function of a four-dimensional string theory.

## 5. Conclusions

In this paper I studied four-dimensional heterotic superstring theories in the asymmetric orbifold formulation. A mechanism for supersymmetry breaking, that gives rise to chiral spectrum of massless fermionic string excitations, was presented. Several $Z_3$ models were considered. Typically, these models have very rich massless spectra, and phenomenologically interesting Higgs structures. In the future work [16] I intend to study some phenomenological aspects of the asymmetric orbifold models.

A formalism for calculating one-loop partition functions in $Z_3$ models was developed. This formalism can be applied in calculating one-loop string amplitudes. I constructed one partition function, that may correspond to a non-supersymmetric, tachyon-free orbifold model, with the one-loop vacuum energy vanishing due to Atkin-Lehner symmetry. I am not able, though, to construct the corresponding model. This means that so far, there are no known examples of four-dimensional Atkin-Lehner models in existence.[3]

Even if there is some basic reason why Atkin-Lehner symmetry does not work in four dimensions, the idea – that some string symmetries may be responsible for vanishing of the cosmological constant in the present day, non-supersymmetric Universe – is certainly worth pursuing. Such a symmetry would have to manifest itself in the effective (zero string slope) field-theoretical Lagrangean in the form of some non-renormalizable terms, whose vacuum expectation values cancel the vacuum energy. From this point of view, cancellation of the cosmological constant would be quite similar to the cancellation of axial anomalies by Wess-Zumino terms in situations [19] when some of the fermions involved in the anomaly cancellation are rendered infinitely heavy.[4] Perhaps quadratically divergent corrections to Higgs masses could also be cancelled by string symmetries. It may be that non-supersymmetric string theories provide a completely new solution to the naturalness problem.

[3] G. Moore [18] has agreed that the four-dimensional $(Z_2)^4$ model of ref.[10] is not modular invariant.

[4] I am grateful to M. Mangano for a discussion on this point.

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## Appendix. Theta functions

Theta functions with third-integer characteristics are defined as:

$$\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix} = \sum_{n=-\infty}^{\infty} e^{i\pi\tau(n+\frac{1}{6})^2+\frac{1}{3}i\pi(n+\frac{1}{6})}$$

$$= e^{\frac{\pi}{18}i}\, q^{\frac{1}{72}} \prod_{n=1}^{\infty}(1-q^n)(1-q^{n-\frac{1}{3}}e^{-\frac{2\pi}{3}i})(1-q^{n-\frac{2}{3}}e^{\frac{2\pi}{3}i})\,, \qquad (A1)$$

$$\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix} = \sum_{n=-\infty}^{\infty} e^{i\pi\tau(n+\frac{1}{6})^2-\frac{1}{3}i\pi(n+\frac{1}{6})}$$

$$= e^{-\frac{\pi}{18}i}\, q^{\frac{1}{72}} \prod_{n=1}^{\infty}(1-q^n)(1-q^{n-\frac{1}{3}}e^{\frac{2\pi}{3}i})(1-q^{n-\frac{2}{3}}e^{-\frac{2\pi}{3}i})\,, \qquad (A2)$$

$$\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{1}{2}\end{bmatrix} = \sum_{n=-\infty}^{\infty} e^{i\pi\tau(n+\frac{1}{6})^2+i\pi(n+\frac{1}{6})}$$

$$= e^{\frac{\pi}{6}i}\, q^{\frac{1}{72}} \prod_{n=1}^{\infty}(1-q^n)(1-q^{n-\frac{1}{3}})(1-q^{n-\frac{2}{3}})\,, \qquad (A3)$$

$$\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{6}\end{bmatrix} = \sum_{n=-\infty}^{\infty} e^{i\pi\tau(n+\frac{1}{2})^2+\frac{1}{3}i\pi(n+\frac{1}{2})}$$

$$= e^{\frac{\pi}{6}i}\, q^{\frac{1}{8}} \prod_{n=1}^{\infty}(1-q^n)(1-q^{n}e^{-\frac{2\pi}{3}i})(1-q^{n-1}e^{\frac{2\pi}{3}i})\,, \qquad (A4)$$

where $q = e^{2\pi i\tau}$. They satisfy the following cubic identities:

$$\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{2}\end{bmatrix} = \vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix} - \vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix}\,, \qquad (A5)$$

$$\vartheta^3\begin{bmatrix}\frac{1}{2}\\ \frac{1}{6}\end{bmatrix} = e^{\frac{\pi}{3}i}\,\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix} + e^{-\frac{\pi}{3}i}\,\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix}\,. \qquad (A6)$$

These identities can be derived by using standard techniques [20] of the theory of modular forms.

I define the functions:

$$\eta = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1-q^n)\,, \qquad (A7)$$

$$\delta \;=\; \vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{6}\end{bmatrix} = \sqrt{3}\,\eta(3\tau)\,, \tag{A8}$$

$$\rho \;=\; \vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{6}\end{bmatrix}\{e^{-\frac{\pi}{3}i}\,\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix} \;+\; e^{\frac{\pi}{3}i}\,\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix}\}\,. \tag{A9}$$

Under the $\Gamma_0(3)$ modular transformations, these functions transform as modular forms of weights $wt(\eta) = wt(\delta) = \frac{1}{2}$, $wt(\rho) = 2$, with the following multiplier systems [20]:

$$v(T) \qquad \eta : \tfrac{1}{24} \qquad \delta : \tfrac{1}{8} \qquad \rho : \tfrac{1}{6}\,, \tag{A10}$$

$$v(ST^3S) \qquad \eta : \tfrac{1}{8} \qquad \delta : \tfrac{1}{24} \qquad \rho : \tfrac{1}{6}\,. \tag{A11}$$

From eqs.($A5$,$A6$) it follows that the functions $\eta$, $\delta$ and $\rho$ satisfy:

$$\rho^3 \;=\; \delta^{12} \;+\; 27\,\eta^{12} \tag{A12}$$

Atkin-Lehner transformation $\tau \to -1/3\tau$ acts in the following way on a modular form $f$ of weight $wt(f) = k$:

$$AL: \quad f(\tau) \to f'(\tau) = (\sqrt{3}\,\tau)^{-k} f(-1/3\tau) \tag{A13}$$

It is not difficult to show that:

$$\eta^4 \;\to\; -\tfrac{1}{3}\,\delta^4\,, \tag{A14}$$

$$\delta^4 \;\to\; -3\,\eta^4\,, \tag{A15}$$

$$\rho \;\to\; -\rho\,, \tag{A16}$$

$$\eta\delta \;\to\; \eta\delta\,. \tag{A17}$$

For completeness, I express the theta functions of eqs.($A1$-$A4$) in terms of the functions $\eta$, $\delta$ and $\rho$:

$$\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix} = \frac{i}{\sqrt{3}}\left(e^{-\frac{\pi}{3}i}\rho\delta^{-1} - e^{\frac{\pi}{3}i}\delta^3\right), \tag{A18}$$

$$\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix} = -\frac{i}{\sqrt{3}}\left(e^{\frac{\pi}{3}i}\rho\delta^{-1} - e^{-\frac{\pi}{3}i}\delta^3\right), \tag{A19}$$

$$\vartheta^3\begin{bmatrix}\frac{1}{6}\\ \frac{1}{2}\end{bmatrix} = \frac{i}{\sqrt{3}}\left(\rho\delta^{-1} - \delta^3\right), \tag{A20}$$

$$\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{1}{6}\end{bmatrix}\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{5}{6}\end{bmatrix}\vartheta\begin{bmatrix}\frac{1}{6}\\ \frac{1}{2}\end{bmatrix}\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{6}\end{bmatrix} = \sqrt{3}\,e^{\frac{\pi}{6}i}\eta^4. \tag{A21}$$

In addition to the oscillators contributions, one-loop partition functions contain lattice theta functions. A theorem by Schoeneberg [21] allows the lattice theta functions to be computed simply. $SU_3$ root and weight lattices correspond to the partition functions:

$$R = \frac{\rho}{\sqrt{3}\,\eta\delta}, \tag{A22}$$

$$W = W^* = \frac{\delta^3}{\sqrt{3}\,\eta}. \tag{A23}$$

$SU_3$ lattice theta functions transform in the following way under the modular transformations:

$$T: \quad R \to R; \qquad W \to e^{\frac{2\pi}{3}i}W, \tag{A24}$$

$$S: \quad R \to -i\tau\frac{1}{\sqrt{3}}(R + 2W); \qquad W \to -i\tau\frac{1}{\sqrt{3}}(R - W). \tag{A25}$$

## References

[1] D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1985) 502; Nucl. Phys. B256 (1985) 253, B267 (1986) 75

[2] H. Kawai, D.C. Lewellen and S.-H.H. Tye, Phys. Rev. Lett. 57 (1986) 1832; Nucl. Phys. B288 (1987) 1; Cornell preprint CLNS 87/69 (1987)
I. Antoniadis, C. Bachas and C. Kounnas, Nucl. Phys. B289 (1987) 87;
I. Antoniadis and C. Bachas, CERN preprint TH.4767/87 (1987)

[3] W. Lerche, D. Lüst and A.N. Schellekens, Nucl. Phys. B287 (1987) 477

[4] K.S. Narain, M.H. Sarmadi and C. Vafa, Nucl. Phys. B288 (1987) 551

[5] R. Rohm, Nucl. Phys. B237 (1984) 553

[6] S. Ferrara, C. Kounnas and M. Porrati, U.C. Berkeley preprint UCB/PTH/87/25 (1987);
N.C. Tsamis, CERN preprint TH.4777/87 (1987)

[7] V.P. Nair, A. Shapere, A. Strominger and F. Wilczek, Nucl. Phys. B287 (1987) 402;
P. Ginsparg and C. Vafa, Nucl. Phys. B289 (1987) 414

[8] H. Itoyama and T.R. Taylor, Phys. Lett. 186B (1987) 129

[9] L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678, B274 (1986) 285;
L.E. Ibañez, H.P. Nilles and F. Quevedo, Phys. Lett. 187B (1987) 25

[10] G. Moore, Nucl. Phys. B293 (1987) 139

[11] K.S. Narain, Phys. Lett. 169B (1986) 41;
K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. B279 (1987) 369

[12] C. Vafa, Nucl. Phys. B273 (1986) 592

[13] M.B. Green and J.H. Schwarz, Phys. Lett. 109B (1982) 444

[14] L. Dixon and J. Harvey, Nucl. Phys. B274 (1986) 93;
L. Alvarez-Gaumé, P. Ginsparg, G. Moore and C. Vafa, Phys. Lett 171B (1986) 155

[15] A.N. Schellekens and N. Warner, Nucl. Phys. B287 (1987) 317

[16] T.R. Taylor, in preparation.

[17] K. Ito, Phys. Lett. 187B (1987) 33

[18] G. Moore, private communication (1987)

[19] E. D'Hoker and E. Farhi, Nucl. Phys. B248 (1984) 59, 77

[20] R. Rankin, *Modular Forms and Functions*, Cambridge University Press, 1977

[21] B. Schoeneberg, *Elliptic Modular Functions*, Springer, 1974