10th Int. Conf. on Topics in Astroparticle and Underground Physics (TAUP2007) IOP Publishing
Journal of Physics: Conference Series **120** (2008) 022002 doi:10.1088/1742-6596/120/2/022002

The impact of non-linear evolution of cosmological matter power spectrum on the measurement of neutrino masses

Shun Saito[1], **Masahiro Takada**[2] **and Atsushi Taruya**[3]

[1] Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
[2] Astronomical Institute, Tohoku University, Sendai 980-8578, Japan
[3] Research Center for the Early Universe, School of Science, The University of Tokyo, Tokyo 113-0033, Japan

E-mail: `ssaito@utap.phys.s.u-tokyo.ac.jp`

Abstract. Next-generation galaxy redshift surveys will open up an exciting opportunity for precision determinations of neutrino masses. Here, we discuss the prospects for measuring the neutrino masses, including properly the non-linear gravitational evolution of matter power spectrum for a mixed dark matter model (neutrinos plus cold dark matter). Based on the perturbation theory, the effect of non-linearity is quantified. Moreover, using Fisher matrix analysis, we present how the neutrino masses will be determined for future galaxy redshift survey.

1. Introduction

One of the most important recent discoveries in physics is that neutrinos have non-zero masses, since neutrinos are massless in the Standard Model. Massive neutrinos necessarily involve new physics, and we need to have improved knowledge of neutrino masses. Intriguingly, the total mass of neutrinos can be determined by the cosmological data through two effects. One is the effect on background expansion of the universe, and another is suppression of matter perturbation. The second effect comes from the fact that the neutrino perturbations affect the structure formation by suppressing the growth of matter density fluctuations at smaller scales below the free-streaming scale owing to their large velocity dispersion. In fact, the most stringent upper bound on the total neutrino mass is obtained from cosmology as $\sum m_\nu \lesssim 0.6\mathrm{eV}(95\%\mathrm{CL})$ [1], compared to the results of the terrestrial experiment limit $\sum m_\nu \lesssim 2\mathrm{eV}$. It is interesting to note that the neutrino's free-streaming scale is comparable to the scale of baryon acoustic oscillation (BAO), $\approx 100\mathrm{Mpc}$, which is aimed by the future wide-field galaxy redshift surveys such as Wide-Field Fiber-fed Multi-Object Spectrograph (WFMOS) in order to reveal the nature of dark energy [2]. The observation around BAOs or neutrino's free-steaming scale requires accurate theoretical template of the matter power spectrum including the non-linear gravitational evolution.

Here, we investigate the non-linear evolution of matter power spectrum including the effect of massive neutrinos for the first time [3]. Based on the fluid approximation of mixed dark matter model (baryon, cold dark matter, and neutrinos) and the perturbation theory, we derive the next-to-leading order correction of gravitational non-linear evolution. Moreover, we demonstrate how

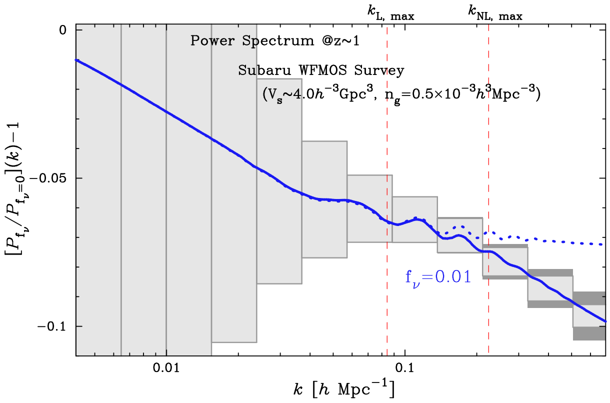

Figure 1. The ratio of matter power spectrum to one without massive neutrinos for the fiducial model. The cases for linear theory, $P^{\mathrm{L}}_{f_\nu\neq0}/P^{\mathrm{L}}_{f_\nu=0}$, (*dashed*) and for the non-linear theory, $P^{\mathrm{NL}}_{f_\nu\neq0}/P^{\mathrm{NL}}_{f_\nu=0}$, (*solid*) are plotted. The *vertical* lines represent the validity of linear theory, $k_{\mathrm{Lmax}} = 0.084h\mathrm{Mpc}^{-1}$ and our non-linear theory, $k_{\mathrm{NLmax}} = 0.213h\mathrm{Mpc}^{-1}$, respectively.

neutrino masses are constrained by future galaxy redshift survey based on Fisher information matrix formalism.

2. Perturbative Approach

Here we treat the evolution of mass distribution for a mixed dark matter model which consists of the cold dark matter (CDM), the baryon, and the neutrinos. One may write the fluctuation of mass density as $\delta_{\mathrm{m}} \equiv (\delta\rho_{\mathrm{cdm}}+\delta\rho_{\mathrm{b}}+\delta\rho_\nu)/(\rho_{\mathrm{m}}) = f_{\mathrm{cb}}\delta_{\mathrm{cb}}+f_\nu\delta_\nu$, where the coefficients, f_{cb} and f_ν, are the fraction of each matter component: $f_{\mathrm{cb}} = (\Omega_{\mathrm{cdm}}+\Omega_{\mathrm{b}})/\Omega_{\mathrm{m}}$ and $f_\nu = \Omega_\nu/\Omega_{\mathrm{m}}$. The matter power spectrum is evaluated as $P_{\mathrm{m}}(k) = f^2_{\mathrm{cb}}P_{\mathrm{cb}}(k) + 2f_{\mathrm{cb}}f_\nu P_{\mathrm{cb},\nu}(k) + f^2_\nu P_\nu(k),$. Since the tiny fraction of massive neutrinos gives a negligible contribution to the non-linear growth and the non-linear gravitational potential contributed from CDM plus baryon does not affect the neutrino's density contrast, it would be sufficient to include the non-linear correction to the term of CDM plus baryon, $P_{\mathrm{cb}}(k)$, only.

For our interest on large-scales, the CDM plus baryon component can be regarded as a pressure-less fluid. From standard perturbation theory (PT), next-to-leading order corrections for the solutions of fluid equations called one-loop corrections can be calculated in a similar way to the cases without massive neutrinos [4]. Note that the effect of neutrino's free-streaming is encoded entirely in $P^L_{\mathrm{cb}}(k;z) = D_{\mathrm{cb}}(k;z)P^L_{\mathrm{cb}}(k;z_{\mathrm{d}})$ through the scale-dependent linear growth factor, $D_{\mathrm{cb}}(k;z)$.

In Fig.1, our PT results for matter power spectra for $f_\nu = 0.01$ with fiducial parameters. We plot the ratio of power spectrum with neutrinos. $f_\nu = 0.01$, to one without neutrinos, $f_\nu = 0$, for linear (*dashed*) and PT (*solid*) at $z = 1$. Here, we adopt the following cosmological parameters as fiducial model parameters: $\Omega_{\mathrm{b}}h^2 = 0.0223$, $\Omega_m h^2 = 0.1277$, $h = 0.73$, $w = -1$, $\Delta^2_{\mathcal{R}} = 2.35\times10^{-9}$, $n_s = 1$, no running spectral index and $N_\nu = 3$. As one can see from Fig.1, the suppression of non-linear power spectra becomes significant due to the non-linear gravitational evolution. Notice that PT may invalid at some large-k. We define the upper limit of validity of our prediction based on the N-body results without neutrinos [5]; $\Delta^2(k_{\mathrm{NLmax}};z) \equiv k^3P^{f_\nu=0}_{\mathrm{m}}(k_{\mathrm{NLmax}};z)/2\pi^2 = 0.4$. Further, applicability of linear theory is restricted to the range within the 1% relative difference between linear and PT results. The dashed vertical lines in Fig.1 indicate these limits, k_{Lmax} and k_{NLmax}. The suppression of non-linear matter power spectrum is noticeable compared to the linear theory even at $k \lesssim k_{\mathrm{NLmax}}$.

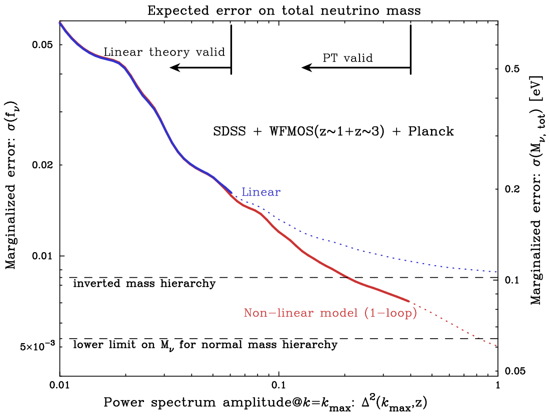

Figure 2. Expected 1σ error on total neutrino mass obtained from linear and non-linear theory for future galaxy redshift survey.

In Fig.1 we also plot the statistical error, $\Delta P(k)/P(k)$, expected from WFMOS survey, for linear (*dark* box) and PT results (*light* box). For PT prediction, the statistical error becomes relatively smaller than linear theory due to the reduced shot-noise term. The relative enhancement of the suppression and reduction of error shown in Fig.1 lead to the improvement of the neutrino mass constraint as discussed below.

3. Parameter forecast with Fisher matrix analysis

Finally, we investigate how the difference of suppression between linear and non-linear theory affect the determination of neutrino masses for future galaxy redshift survey with Fisher matrix analysis developed in [6]. Here we assume the linear biasing and linear redshift distortion, but include non-linearity of gravitational clustering empirically by setting the parameter β as free parameter. The survey parameters can be found in [6]. Then, we consider a set of parameters, $p_\alpha = \{\Omega_{\rm m}, \Omega_{\rm m}h^2, \Omega_{\rm b}h^2, \Delta^2_{\mathcal{R}}, n_s, \alpha, w_0, f_\nu, N_\nu, b_1(z_i), \beta\}$. Here, we adopt the fiducial model discussed above with $f_\nu = 0.01$ and fix the parameters, $N_\nu = 3$ and $b_1(z_i)$ for each redshift slice. As for the maximum wavenumber, we adopt the value of $k_{\rm max}$ so that $\Delta^2(k_{\rm max}, z_i) = x$, and obtain the result against the value of x. Moreover, we assume Planck prior for CMB data.

In Fig.2, we show the 1σ error on total neutrino mass, f_ν, or equivalently $\sum m_\nu$ for reference. The *red* and *blue* lines show the expected 1σ errors of neutrino masses. We adopt the validity range for both theories conservatively; for non-linear theory, $\Delta^2(k_{\rm NLmax}, z_i) \leq 0.4$, and for linear theory, 3% accuracy between linear and non-linear theory. The specific values on constraint on neutrino masses are 0.195eV for linear theory and 0.085eV for non-linear theory, which is about factor of two improvement. This result is remarkable, since it suggest that improvement from non-linear theory may lead to potential to distinguish the difference of mass hierarchy. Note that our result may be optimistic because we ignore several non-linear effects. Nevertheless, we stress that, since the effect of neutrino on matter power spectrum gives characteristic suppression, constraint derived here would be achievable.

References

[1] D. N. Spergel *et al.*, Astrophys. J. Suppl. **170**, 377 (2007),
[2] J. A. Peacock, *et al.*, arXiv:astro-ph/0610906. A. Albrecht *et al.*, arXiv:astro-ph/0609591.
[3] S. Saito, M. Takada and A. Taruya, in prep (2007).
[4] N. Makino, M. Sasaki, and Y. Suto, Phys. Rev. D **46**, 585 (1992).
[5] D. Jeong and E. Komatsu, Astrophys. J. **651**, 619 (2006).
[6] M. Takada, E. Komatsu and T. Futamase, Phys. Rev. D **73**, 083520 (2006).