

# Lateral Distribution Function Estimation of Electrons and Muons using Nishimura-Kamata-Greisen Function

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**Abstract.** The density of charged particles in extensive air showers reaching the surface of the Earth was calculated by estimating the lateral distribution function (LDF) of different primary particles at high energies. LDF simulation was performed using the AIRES system (version 19.04.10), a set of programs and subroutines designed to simulate ultra-high-energy air showers resulting from the interaction of cosmic rays with the Earth's atmosphere. This system includes algorithms for fast simulation and output data management, and can simulate particle showers realistically and manage the related data efficiently. Its aim is to contribute to research on high-energy cosmic ray interactions for two charged particles, like muons and electrons, at very high energies ( $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV) and taking into account the effect of particles, such as protons, helium nuclei, and iron nuclei, and primary energies and zenith angles ( $0^\circ$ ,  $20^\circ$ , and  $40^\circ$ ). The LDF was also calculated using the Nishimura-Kamata-Greisen function, and good agreement was found with the results produced by the AIRES system for high-energy muons and electrons created by primary particles.

## 1. Introduction

An extensive air shower (EAS) is a burst of ionized particles and electromagnetic radiation that occurs in the atmosphere as a result of primary cosmic rays (CRs) interacting with an atom's nucleus there. This process produces a lot of secondary particles, such as electrons, neutrons, muons, X-rays, and alpha particles [1]. It has been noted that high-energy CRs can be identified by examining the sequence of events within an EAS. Protons and electrons are examples of high-energy charged particles and nuclei that are classified as CRs. Victor Hess made the discovery of them in 1912, and they come from the sun and several other astronomical sources. generated around the circle of the atmosphere due to the fact that certain elementary particles must be investigated indirectly based on the shower and various measurement techniques because they cannot be directly detected [2]. CRs strike the atmosphere at  $10^3$  particles per square meter per second and are characterized by their high energy. Most CRs are relativistic and have energy similar to or somewhat greater than their mass [3,4].

The study of CRs is one of the most important topics in the field of astrophysics [5,6]. Before high-energy CRs hit an atom in the Earth's atmosphere, a series of secondary particles are produced. These particles then interact further to produce even more secondary particles before reaching the Earth's surface [7]. This fact was discovered by the French physicist Pierre Victor Auger [8]. The primary particles' kinetic energy is transformed into mass-energy to a lesser extent. The leftover kinetic energy is then dispersed via the shower, and this process of particle production continues until the energy of the EAS particles is no longer sufficient to generate new particles in successive collisions. We refer to this phase of the shower's development as the maximum shower. A small fraction of the kinetic energy of the primary particles is converted into mass-energy, after which the remaining kinetic energy can be distributed through the shower, and the particle-production process continues until the energy of the



EAS particles is insufficient to produce more particles in sequential collisions. This stage of the development of the shower is called the maximum shower [8, 9]. Beyond the maximum, the shower particles are gradually absorbed with an attenuation length of 200 g/cm, so that immediate detection of CRs is possible and the resulting air showers evolve enough to become observable from the ground. The numbers of particles in the cascade develop sufficiently that the (LDF) for the atmospheric depth, defined at the maximum, forms a smooth curve [10,11].

This reaction process is complex [12]. During the shower interaction, which is often called a cascade particles lose energy when their energy falls below their critical energy, and repeatedly colliding particles instead of other radiation processes frequently waste this standard. [13]. The LDF in the EAS for the charged particle is the quantity used to detect and observe Earth's CRs, usually obtained from EAS observations [5]. The coefficients for the Nishimura–Kamata–Greisen (NKG) function serve as the parameters which dictate the shape of the lateral density [14, 15]. In order to identify the features of the CRs that generated an EAS, numerical simulations of the system are required. These actions require a sophisticated computational framework to understand and simulate since the quantity of high-energy charged particles of CRs can be quite huge, sometimes surpassing  $10^{10}$  particles. Hence, because the formation of the shower is a complicated random process, the LDF depends on numerous independent factors [16, 17]. Thus, every shower particle's transit and interaction process in an EAS is simulated using Monte Carlo computer simulation [5].

In this paper, the LDF for charged secondary particles that reach the Earth's surface, like electrons and muons, were estimated by performing simulations using the Monte Carlo Air shower Extended Simulations (AIRES) system within the limits of high energies ( $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV). LDFs for different formulas of the NKG function were also calculated and the results were compared with those obtained from the AIRES simulation; good agreement was observed.

## 2. Lateral distribution and the NKG function

One of the key EAS principles that can be precisely determined with ground shower arrays is the density of charged particles at various distances from the center. Ever since EASs were discovered, experimental and theoretical study has relied heavily on the lateral distribution function  $\rho(r)$  for various particle kinds produced in an EAS [18, 19]. In order to examine empirical data on air showers and develop new techniques for obtaining dependable findings at such distances, it became important to tackle the problem of quick and adequate LDF calculations for experimental data at vast distances from the shower hub ( $r > 1$  km) [20].

The most common method, using high-energy EAS simulation, grants an experimental preview of an electromagnetic shower depending on different modifications. Thus, knowing the LDF and rebuilding the shower core are essential, as well as the shower path. Additionally, this can be related to the elementary mass details model calculations, yielding insightful data. The energy, mass, and distribution of primary particles can all be calculated using the LDF of an EAS, which is fundamentally significant for understanding the phenomenon of air shower. This allows for the correlation of the particle's primary energy with its mass through the use of more accuracy algorithms, which require a thorough simulation of the air shower [21, 22]. The LDF characterizes the shower at various altitudes in the terrestrial atmosphere [23,24].

Through a relationship between the Nishimura–Kamata function and the theory of B cascades, Nishimura and Kamata produced analytical equations for the angular and lateral dispersion of air shower particles in 1950. A popular approximation of the Greisen-proposed computation result is the NKG function, sometimes known as the NKG-function. This is a summary of research on the average characteristics of multidimensional cascades in very high-energy CRs, primarily photon and electron ones [25].

In 1958, Kamata and Nishimura found a solution to Landau's equations for the LDF for electrons and photons in the theory of B cascades. The solution, assessed in units of the Molière radius, is a function of shower age and characterizes the cascade evolution [14,15]:

$$F(R/R_M) \propto \left(\frac{R}{R_M}\right)^{s-2} \left(1 + \frac{R}{R_M}\right)^{s-4.5} \quad (1)$$

where  $R$  represents the distance from the core of the shower,  $s$  is the age of the shower, and  $R_M$  is the Molière radius. There are electrons, hadrons, photons, and muons in an EAS, so the NKG function (lateral distribution form) cannot be applied directly, is the energy function and zenith angle. Therefore, it can become the distance that represents the basic shower period for the specific basic energy. The participation of hadrons and muons in the measured density of charged particles is small and can be ignored, and the LDF is almost just a function of the age of the shower. The LDF coefficients of the electromagnetic cascade can be determined using the NKG function [14,15]:

$$\rho_e(R) = \frac{N_e}{2 \times 3.14 R_M^2} C(s) \left(\frac{R}{R_M}\right)^{(s-2)} \left(\frac{R}{R_M} + 1\right)^{(s-4.5)} \quad (2)$$

Here,  $C(s)$  represents the normalization factor given by [26]

$$C(s) = 0.366 s^2 (2.07 - s)^{1.25} \quad (3)$$

In addition,  $\rho_e(R)$  is the density of electrons as a function of the distance from the core of the shower,  $R_M$  is the molier radius, 78 m at sea level,  $s$  is the shower age the NKG function is useful for  $0.8 < s < 1.6$ , and  $N_e$  represents the total number of electrons in the EAS, which be expressed as follows [16]:

$$N_e = 10^6 \left[ \frac{E_0}{10^{15} \text{ eV}} \right]^{1.03} \quad (4)$$

Several charged particles are produced by the electromagnetic cascade. Thus, in an EAS, shower energy is dissipated through electron ionization [27]. The outcome of the analytical coefficients from the Monte Carlo calculations and the solution of the LDF cascade equations for electrons in the air shower depend on the NKG function [28].

The NKG function has been used with coefficients developed over several years to describe LDFs for various contents of EASs. The LDF for electrons is described by the following equation [16]:

$$\rho_e(R) = N_e C(s) \left(\frac{r}{r_M}\right)^{s-2} \left(1 + \frac{r}{r_M}\right)^{s-4.5} \quad (5)$$

where [16]

$$C(s) = \frac{\Gamma(4.5 - s)}{2 \pi r_M^2 \Gamma(s) \Gamma(4.5 - 2s)}$$

is the constant of normalization,  $r_M$  is the Moliere radius (80 min for electrons in the atmosphere), and  $s$  is the age parameter [16]. The LDF for muons is as follows:

$$\rho_\mu(R) = N_e C(s) \left(\frac{r}{r_M}\right)^{s-2} \left(1 + \frac{r}{r_M}\right)^{s-4} \quad (6)$$

where [16]

$$C(s) = \frac{\Gamma(4 - s)}{2 \pi r_M^2 \Gamma(s) \Gamma(4 - 2s)}$$

$C(s)$  is the constant of normalization,  $r_M$  is the Moliere radius (420 min for muons in the atmosphere), and  $s$  at the age parameter.

### 3. The LDF Simulation

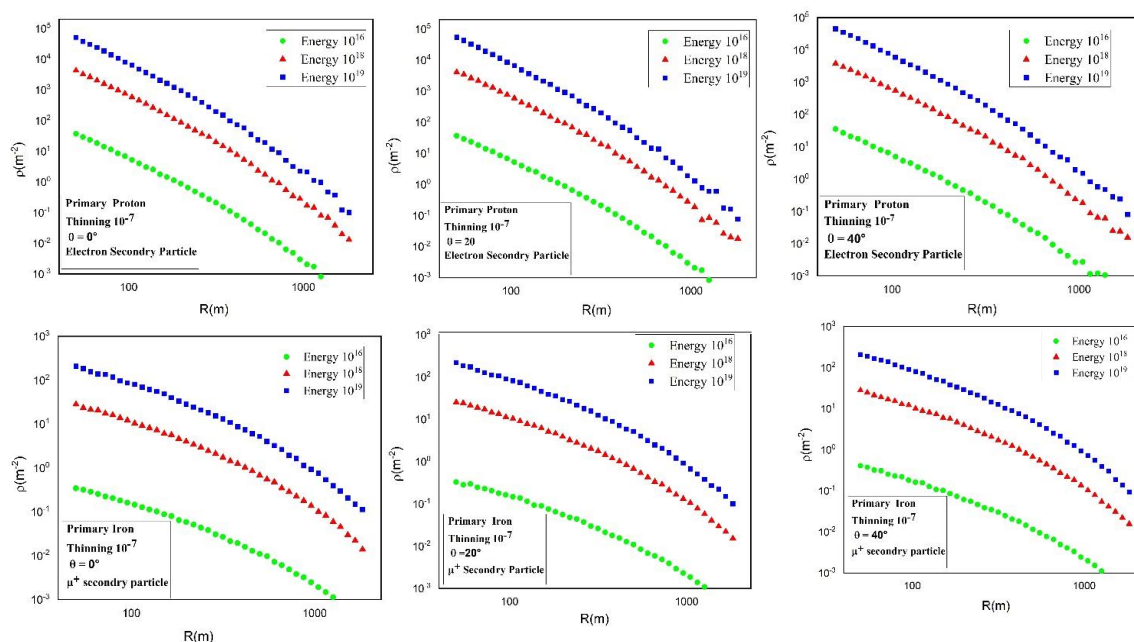
Simple analytical modeling is unable to adequately determine the nuances of the shower's evolution due to their complexity. Rather, to record the contact and transport of every particle, Monte Carlo simulation is required. Lately, the AIRES system has been utilized to simulate EASs using Monte Carlo programs. Air shower simulation programs consist of a variety of connected actions that work on several data and an unlimited number of records and reduce or increase the data size according to predefined rules. The AIRES simulation engine's internal control processes look at particles that land or travel over designated observation surfaces on the ground. From there, the number of showers is

calculated. Next comes the main particle that interacts with atmosphere's atoms and its energy. Next, the electron/muon's kinetic energy and the name of the chosen option are defined. After determining thinning energy and, the zenith angle we select the noticing levels for the group that will be employed [29, 30].

The AIRES system considers electron, muon, gamma-ray, and positron calculations, among other secondary particle simulations. For energies that can surpass  $10^{21}$  eV, protons, helium nuclei, iron nuclei, and other elements fixed in the AIRES index could be among the elementary particles dropping in the EAS [31]. This fact was confirmed based on a plot of the air shower's density against the distance from the core, for showers in the atmosphere with certain energy values ( $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV). The results were determined based on the data received from the simulations carried out through the AIRES system that was used to study the production of primary particles produced by the air shower, and the conclusions of the LDF generated by CRs for the high energies that interacted with the atmosphere, using a thinning of  $10^{-7}$  [32].

#### 4. Results and Discussion

Figure 1 displays the outcomes of LDF simulations for numerous secondary particles based on the distance from the earth's surface to the shower core. Primary particles in the AIRES system, which consists of protons, helium, and iron nuclei, is used, and a thinning energy of  $10^{-7}$  is applied when there is an unusually high particle count at the showers. The impact of primary energy fluctuation and zenith angle on the density of charged secondary particles leading to EASs was considered. It is evident from the figure that as one gets farther away from the shower axis  $R$ , the LDF of various charged particles rapidly drops. This is due to the fact that the particles lose energy and change course as a result of their ongoing interactions with the atmosphere's molecules. It was found that the LDF of the electron is greater than that of the muon in all the processes at different energies since there is less dispersion of muons compared to electrons. As a result, they have a higher probability of reaching the surface of the earth. before electrons because they are heavier and more impacted by the magnetic field, which causes them to losing less energy to atmospheric particles. The image also illustrates how the energy of the main particle affects the LDF, which rises as the primary particle's energy does because higher-energy particles interact with the atmosphere to produce more secondary particles [31].



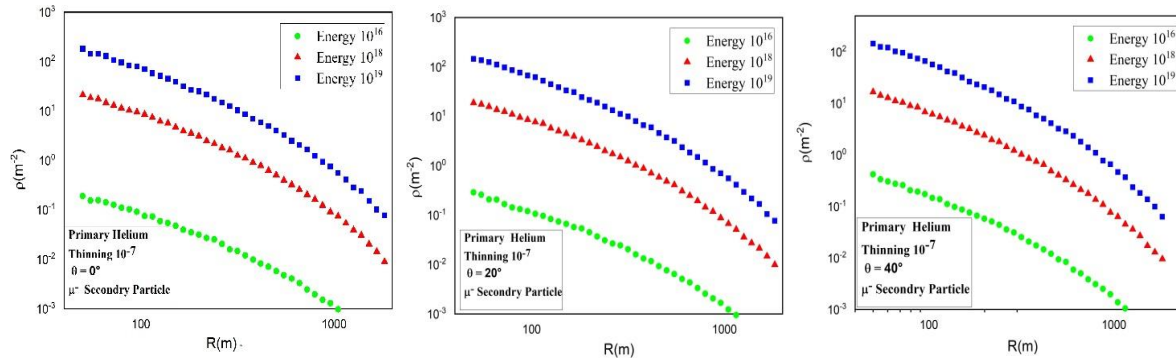
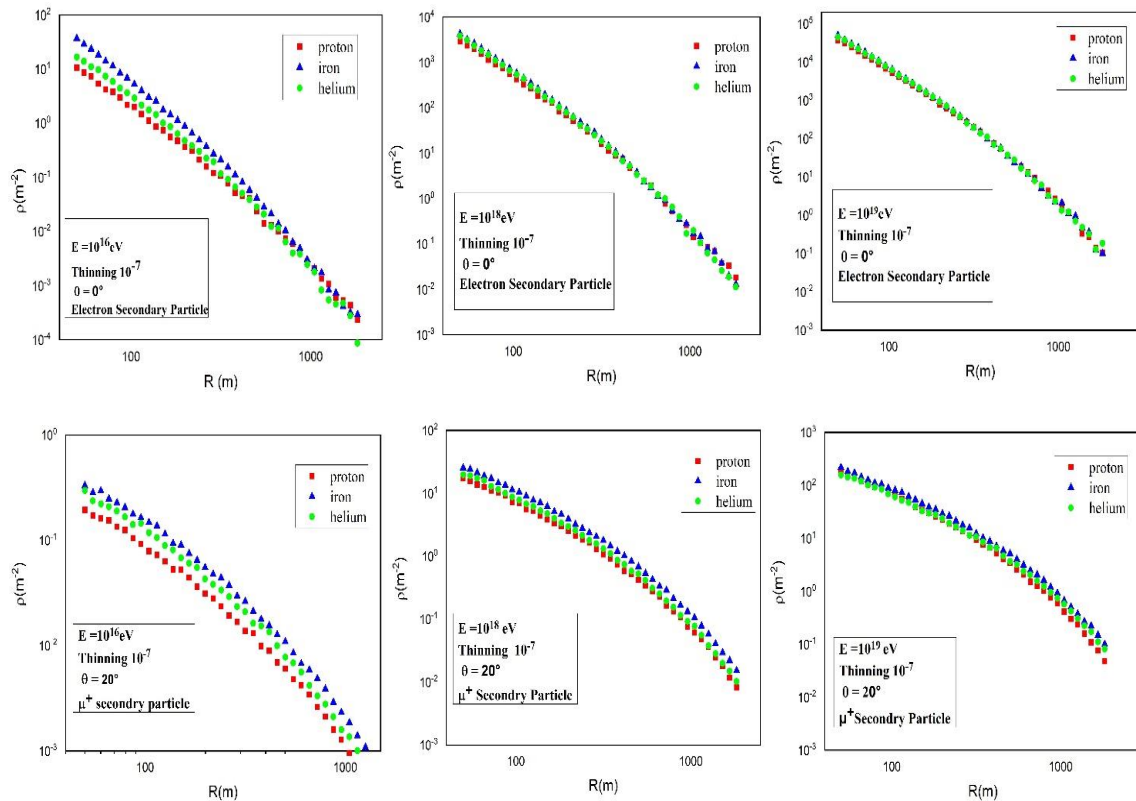


Figure 1 Primary energy effects of He, Fe, and p on density of secondary particles ( $e$ ,  $\mu^+$ , and  $\mu^-$ ) at various zenith angles ( $0^\circ$ ,  $20^\circ$ , and  $40^\circ$ ).

### 5. Comparison of LDF simulations of protons, iron nuclei, and helium nuclei

Figure 2 shows a comparison of the LDF for protons, iron nuclei, and helium nuclei, with thinning energy  $10^{-7}$ , obtained using the AIRES system. We can conclude from the figure that all charged secondary particles created by the primary particles (helium, iron, or proton) at energies  $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV and zenith angles  $0^\circ$ ,  $20^\circ$ , and  $40^\circ$  are very close to each other. We observed that when one gets farther away from the shower's axis, the LDF of the particles produced by the reaction in the atmosphere drops, and that it is proportional to the primary energy. In addition,  $R$  increases with energy and is smaller for heavy nuclei, for which showers develop faster than those produced by protons or helium nuclei. This is due to the larger cross-section of the iron nucleus, and the main reason is that Coulomb scattering, which scatters particles away from the shower axis, causes the heavy nucleus to behave like the overlay of its constituent nucleons and energy ( $E/A$ ) [33,34].



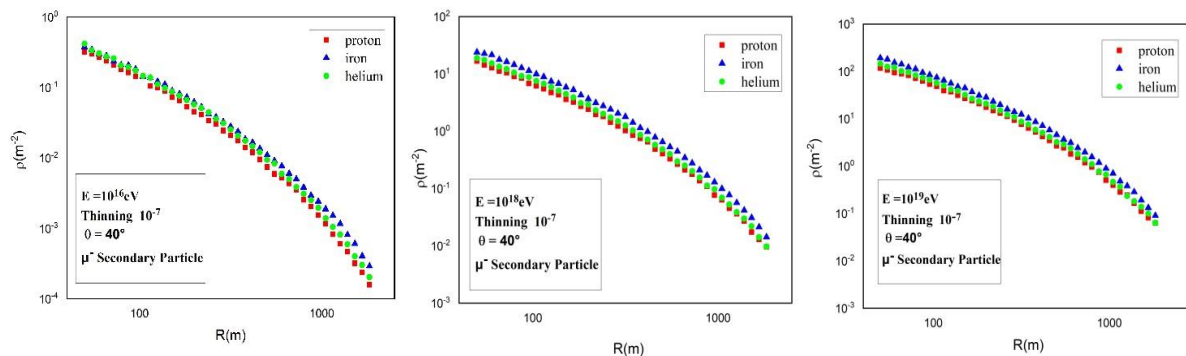
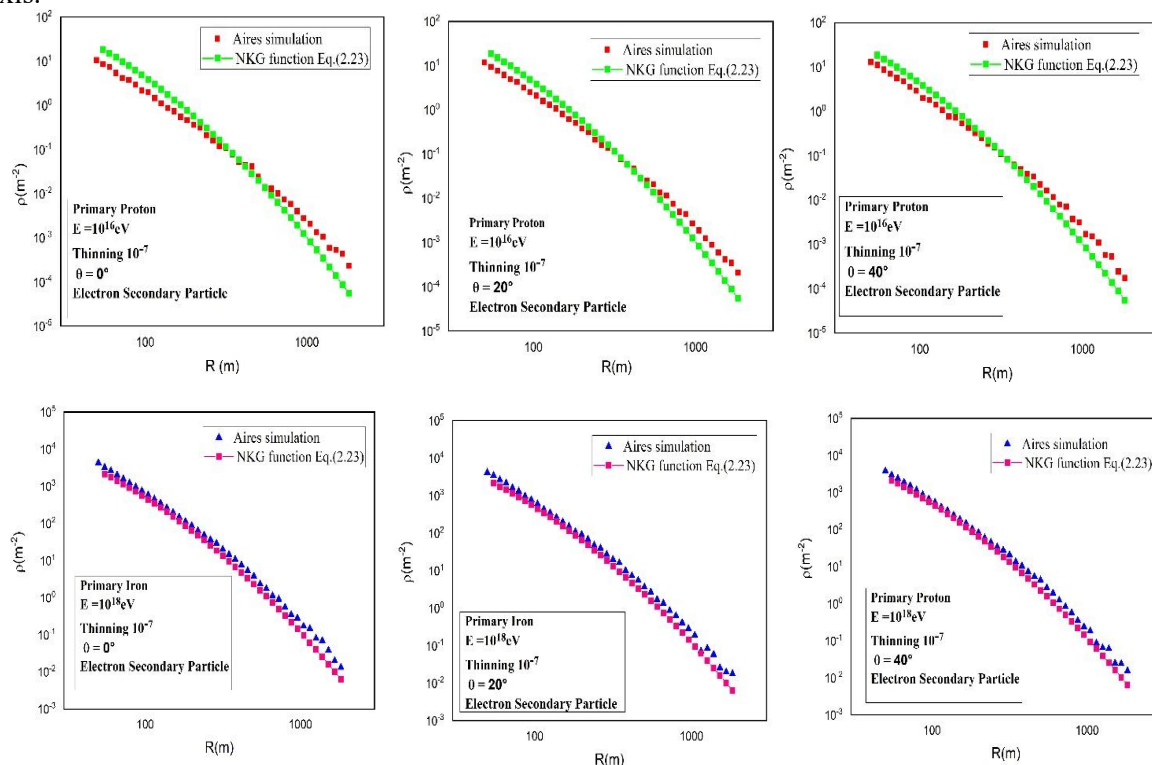


Figure 2 A comparison of the LDF of secondary particles at energies of  $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV at zenith angles of  $0^\circ$ ,  $20^\circ$ , and  $40^\circ$ , initiated by primary particles p, Fe, and He.

### 6. Comparing LDF simulations with the NKG function

Figures 3 and 4 show a comparison between the LDF functions found by using the AIRES system and those obtained using the NKG equation (Eq. 5 and Eq. 6) for electrons and muons. The NKG function's results and the data that the AIRES system simulated may be observed to be in good agreement with one another., for secondary particles produced by primary particles Fe, p, and He at elevated energies ( $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV) and for thinning energy  $10^{-7}$ . One can see from the figures that both the AIRES and NKG curves reflect a function of the shower's distance from its center of the shower. that gradually decreases with increasing distance, and that there is a slight deviation between these curves of electrons and muons, due to the different types of primary particles, energies, and zenith angles that were used in the simulation or inaccuracies in the AIRES simulation at short distances. The LDF for electrons is steeper because electrons are lighter and interact more with the atmosphere than muons, and this leads to greater energy loss and dispersion away from the shower axis.



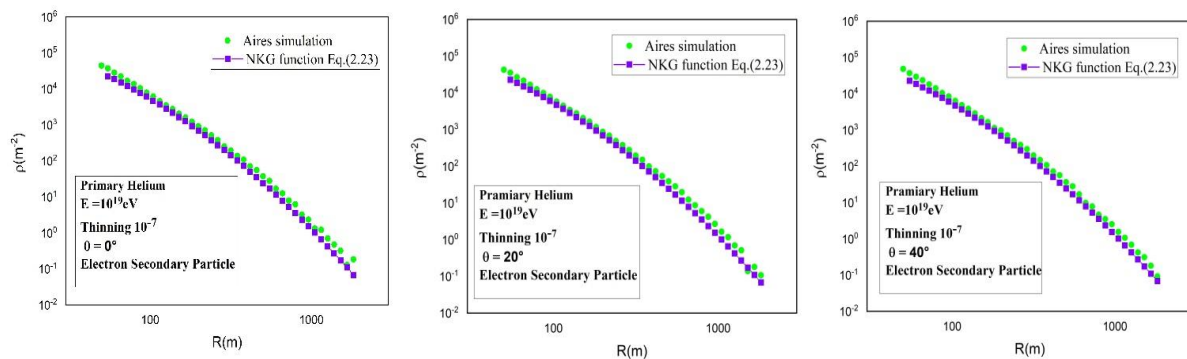


Figure 3 Comparison between the results gotten using the NKG function and the simulated LDF for charged particles of electrons at energies  $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV and zenith angles  $0^\circ$ ,  $20^\circ$ , and  $40^\circ$ .

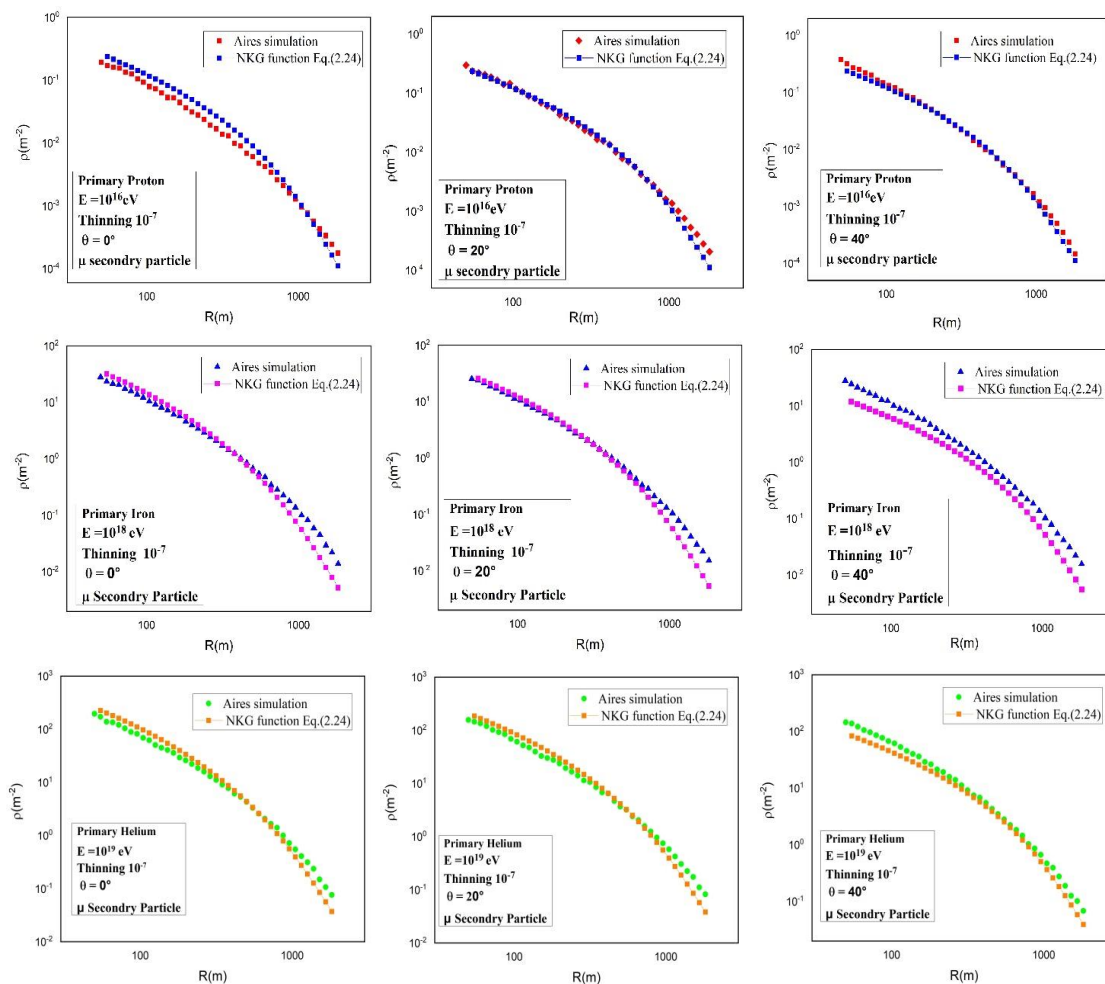


Figure 4 Comparison between the results gotten using the NKG function and the simulated LDF for charged particles of muons at energies  $10^{16}$ ,  $10^{18}$ , and  $10^{19}$  eV and zenith angles  $0^\circ$ ,  $20^\circ$ , and  $40^\circ$ .

## 7. Conclusions

The AIRES system was used to model the LDF of charged particles (electrons and muons) reaching the Earth's surface for high-energy primary particles p, Fe, and He. It was considered how zenith angles, energy, and primary particles affected lateral density. The LDF function was found to progressively decrease with increasing distance from the shower core and to increase with primary

particle energy because high-energy particles interact with the atmosphere to produce more secondary particles. Since muons are below impacted by the magnetic field than electrons and lose less energy to particles in the atmosphere, the lateral distribution function for muons is bigger than that of electrons at different energies. The LDF was also calculated using the NKG function and the results were compared with those obtained using AIRES simulation, showing good agreement. This indicates that both methods effectively explain the particle distribution in an EAS and that both AIRES and NKG are able to modulate the behaviour of different types of particles within the shower and use them to study EAS development.

## References

- [1] A.N. Cillis, S.J. Sciutto, Phys. Rev. D 64, 013010 (2001).
- [2] E.M. Holt, J. Phys. Conf. Ser. 718, 052019 (2016).
- [3] Gaisser T.K., Engel R., Resconi E. Cosmic rays and particle physics, Cambridge University Press (2016).
- [4] Simpson J. Annual Review of Nuclear and Particle Science, 33, 323 (1983)
- [5] M. Roth for the Auger Collaboration, in: Proc. 28th ICRC, Tsukuba (Japan), arXiv:astro-ph/0308392, (2003).
- [6] M. Risse, Acta physica Polonica, vol. 35 (2004)
- [7] A. Aab, P. Abreu, M. Aglietta et al., J. Cosmol. Astropart. Phys. 2019, 022 (2019).
- [8] S. Bhatnagar, Extensive Air Shower High Energy Cosmic Rays (II), Physics Education, 249. (2009).
- [9] M.S. Longair, High energy astrophysics, cambridge university Press, (2011).
- [10] S.P. Swordy and D.B. Kieda, "Elemental Composition of Cosmic Rays near the Knee by Multiparameter Measurement of Air Showers" Astropart. Phys. 13, pp.137. (2000).
- [11] J.W. Fowler, L.F. Fortson, C.C.H. Jui, et al." A Measurement of the Cosmic Ray spectrum and Composition at the Knee" Astroparticle Physics, Vol. 15, No. 1, pp. 49-64. (2000).
- [12] K.F. Fadhel, A.A. Al-Rubaiee, H.A. Jassim, I.T. Al-Alawy, J. Phys. Conf. Ser. 1879, 032089 (2021).
- [13] S.P. Knurenko, A.A. Ivanov, M.P. Pravdin, A.V. Sabourov, I.Y. Sleptsov, Nucl. Phys. B Proc. Suppl. 175-176, 201 (2008).
- [14] K. Kamata, J. Nishimura, Progress of Theoretical Physics Supplement, 6, 93 (1958).
- [15] Greisen K. Cosmic ray showers, Annual Review of Nuclear Science, 10, 63 (2005).
- [16] Matthews J. Astroparticle Physics, 22, 387 (2005).
- [17] Gorbunov D., Rubtsov G. & Troitsky S.V. Physical Review D, 76, 043004 (2007).
- [18] A.A. Lagutin, R.I. Raikin, N. Inoue, A. Misaki, J. Phys. G Nucl. Part. Phys. 28, 1259 (2002).
- [19] A.A. Ivanov, Proc. Sci. 301, 550 (2017).
- [20] H.A. Jassim, A.A. Al-Rubaiee, I.T. Al-Alawy, Indian J. Public Heal. Res. Dev. 9, 1307 arXiv:2007.16004v1. (2018).
- [21] D. Barnhill, P. Bauleo, M.T. Dova et al., for the Pierre Auger Collaboration, Proceedings of 29th ICRC, Pune (India), arXiv:astro-ph/0507590, p. 101. (2005).
- [22] KASCADE Collaboration T. Antoni et al., Astropart. Phys. 14, 245 (2001).
- [23] Hassanen Abdulhussain Jassim, Al-Rubaiee A. A. & Iman Tarik Al-Alawy Experimental and Theoretical Nanotechnology Journal, 4 (3), 257 (2020).
- [24] Al-Rubaiee A., Al-Douri Y., Ibraheem A., Hashim U. & Lazem T. 2014, 2nd International Conference on Electronic Design (ICED), pp. 465-467: IEEE (2014).
- [25] Filonenko A. Technical Physics, 45, 1362 (2000).
- [26] Hayakawa S. Cosmic ray physics. Nuclear and astrophysical aspects, Interscience Monographs and Texts in Physics and Astronomy, New York: Wiley- Interscience (1969).
- [27] Hovsepyan, G 29th International Cosmic Ray Conference, Pune, India, pp.97 (v.6). Aug (2005).
- [28] A A Lagutin, R I Raikin & N Inoue. Journal of Physics G: Nuclear and Particle Physics, 28, 1259 (2002).

- [29] S. Ostapchenko, Phys. Rev. D 83, 014018 (2011).
- [30] T. Pierog, I. Karpenko, J.M. Katzy, E. Yatsenko, K. Werner, Phys. Rev. C 92, 034906 (2015).
- [31] S. Sciutto, ARES User's Manual and Reference Guide, version 2.6, in, (2002).
- [32] K.F. Fadhel, A.A. Al-Rubaiee, H.A. Jassim, I.T. Al-Alawy, J. Phys. Conf. Ser. 1879, 032089 (2021).
- [33] A. A. Al-Rubaiee, Y. Al-Douri, A. S Ibraheam, A Study of Extensive Air Shower Characteristics by Estimating Depth of Shower Maximum Using Heitler Toy Model, 2nd International Conference on Electronic Design (ICED14). August (2014).
- [34] Gaisser T. K. and A. M. Hillas, prroc. 15th Int. Cosmic Ray Conf. (Plovdiv) Vol 8, pp. 353. (1977).