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Isospin amplitudes of $B \to K\pi\pi$ decays

G. Calderón[a], Cristian H. García-Duque[b]

[a]Facultad de Ingeniería Mecánica y Eléctrica, Universidad Autónoma de Coahuila, C.P. 27276, Torreón, Coahuila, México
[b]Departamento de Física, Universidad del Tolima A. A. 546, Ibagué, Colombia

Abstract

We obtain two isospin amplitude decompositions of the $B \to K\pi\pi$ decays. We use the method of addition of three isospin vectors, and prove the equivalency of the two isospin amplitude decompositions obtained. We only have considered contributions of the Hamiltonian to the transitions with change of isospin $\Delta I = 0$ and $\Delta I = 1$. The symmetry of isospin allows to relate the charged $B^+ \to K^+\pi^+\pi^-$, $B^+ \to K^+\pi^0\pi^0$, and $B^+ \to K^0\pi^0\pi^+$ channels with the neutral $B^0 \to K^0\pi^+\pi^-$, $B^0 \to K^0\pi^0\pi^0$, and $B^0 \to K^+\pi^0\pi^-$ channels. Additionally, we obtain equivalent triangular relations in the charged and neutral channels.

Keywords: Isospin symmetry, $B \to K\pi\pi$ decays

1. Introduction

The quark mixing and CP violation are parametrized in the Standard Model by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Many B decays are an excellent place to measure phases of the CKM matrix.

Most of the research have done on two-body B decays, but the three-body B decays are an alternative. They have the advantage that the final state are stable particles and do not introduce the problems associated with the unstable particles in the final state.

Recently the LHCb Collaboration [1] realized measurements of the CP violation in the phase space of the $B^\pm \to K^\pm\pi^+\pi^-$ and $B^\pm \to K^\pm K^+K^-$ decays.

The isospin amplitudes decomposition have been used in $B \to \bar{D}^{(*)}D^{(*)}K$ decays to obtain the strong phases with experimental data available, see Ref. [2, 3].

In this work we propose to obtain the isospin amplitude decompositions and to realize a general isospin analysis of the $B \to K\pi\pi$ decays.

Email address: german.calderon@uadec.edu.mx (G. Calderón)

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2. Hamiltonian

The $B \to K\pi\pi$ decays considered here, where B is either a B^0 or B^+, K is either a K^0 or K^+ and π is π^+, π^0, or π^-, at the quark level proceed through the $b \to u\bar{u}s$ transition.

The low energy $\Delta S = 1$ non-leptonic weak Hamiltonian is given by

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{ub}[\bar{s}\gamma^\mu(1-\gamma_5)u][\bar{u}\gamma_\mu(1-\gamma_5)b] + h.c. \quad (1)$$

The first term $[\bar{s}u]$ is the isospin state $|1/2, 1/2\rangle$ and the second term $[\bar{u}b]$ is the isospin state $|1/2, -1/2\rangle$. From isospin addition we have

$$|1/2, 1/2\rangle|1/2, -1/2\rangle = \frac{1}{\sqrt{3}}|1, 0\rangle + \frac{1}{\sqrt{2}}|0, 0\rangle. \quad (2)$$

The $\mathcal{H}_W$ weak Hamiltonian can be decomposed in two parts with defined isospin contributions $\mathcal{H}_W = \mathcal{H}_1 + \mathcal{H}_0$, i.e., an isotriplet $\mathcal{H}_1$ and an isosinglet $\mathcal{H}_0$ parts.

The amplitude of probability for the $B \to K\pi\pi$ decay considered is

$$\mathcal{A}(B \to K\pi\pi) \quad = \quad \langle K\pi\pi|\mathcal{H}_1|B\rangle + \langle K\pi\pi|\mathcal{H}_0|B\rangle.$$

The probability amplitude expressed in terms of the isospin amplitudes is

$$\mathcal{A}(B \to K\pi\pi) = \sum A_I, \tag{3}$$

where the isospin amplitudes are $A_I = \langle I_K + I_\pi + I_\pi | I_B + I_H \rangle$, with isospin quantum number I.

The state $|I_B + I_H\rangle$ is build by the addition of two isospins, where I_B is the isospin of the B meson and I_H plays the role of the isospin required by the Hamiltonian to make the transition, i.e.. $\mathcal{H}_1$ and $\mathcal{H}_0$ are $|1,0\rangle$ and $|0,0\rangle$, respectively.

The state $|I_K + I_\pi + I_\pi\rangle$ is the result of the addition of three isospin vectors, corresponding to the isospin of the three final mesons in the $B \to K\pi\pi$ decay.

3. Addition of isospins

In order to add two isospin vectors, the isospin of the initial particle and the isospin assigned to the Hamiltonian, and to sum three isospin vectors, corresponding to the three final particles, we use the formalism from quantum mechanical addition of angular momentum applied to isospin, see [4].

In the case of addition of two isospins $I = I_1 + I_2$, we have two sets of mutually commuting operators. The set $(I_1^2, I_2^2, I_{1z}, I_{2z})$, which gives origin to the base denoted by their quantum numbers $|j_1 j_2 m_1 m_2\rangle = |j_1 m_1\rangle |j_2 m_2\rangle$, it is called the uncoupled representation. And the other set of commuting operators (I_1^2, I_2^2, I^2, I_z) with the base $|j_1 j_2 jm\rangle$, which is called the coupled representation. These representations are connected by a unitary transformation

$$|j_1 j_2 jm\rangle = \sum_{m_1 m_2} \begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix} |j_1 j_2 m_1 m_2\rangle, \tag{4}$$

where the quantity on square brackets is known as the Clebsch-Gordon coefficient.

For the addition of three isospins $I = I_1 + I_2 + I_3$, we have the uncoupled representation $|j_1 j_2 j_3 m_1 m_2 m_3\rangle = |j_1 m_1\rangle |j_2 m_2\rangle |j_3 m_3\rangle$, which is generated by the set $(I_1^2, I_2^2, I_3^2, I_{1z}, I_{2z}, I_{3z})$ of commuting operators.

To build the coupled representation is necessary first sum two isospins and then sum the third isospin, i.e., $I_{12} = I_1 + I_2$ and $I = I_{12} + I_3$ and the other case, $I_{23} = I_2 + I_3$ and $I = I_1 + I_{23}$. For the two different ways to sum three isospins we have two set of commuting operators $(I_1^2, I_2^2, I_3^2, I_{12}^2, I^2, I_z)$ and $(I_1^2, I_2^2, I_3^2, I_{23}^2, I^2, I_z)$. These sets of operators give origin to two coupled representations, denoted by their quantum numbers $|j_{12} j_3; jm\rangle$ and $|j_1 j_{23}; jm\rangle$. These coupled representations are connected to the uncoupled representation by the unitary transformations

$$\begin{aligned} |j_{12} j_3; jm\rangle &= \sum_{m_1 m_2} \begin{bmatrix} j_1 & j_2 & j_{12} \\ m_1 & m_2 & m_{12} \end{bmatrix} \\ &\quad \begin{bmatrix} j_{12} & j_3 & j \\ m_{12} & m_3 & m \end{bmatrix} |j_1 m_1\rangle |j_2 m_2\rangle |j_3 m_3\rangle, \end{aligned} \tag{5}$$

and

$$\begin{aligned} |j_1 j_{23}; jm\rangle &= \sum_{m_1' m_2'} \begin{bmatrix} j_2 & j_3 & j_{23} \\ m_2' & m_3' & m_{23}' \end{bmatrix} \\ &\quad \begin{bmatrix} j_1 & j_{23} & j \\ m_1' & m_{23}' & m \end{bmatrix} |j_1 m_1'\rangle |j_2 m_2'\rangle |j_3 m_3'\rangle. \end{aligned} \tag{6}$$

The coupled representations are related by a unitary transformation

$$|j_{12} j_3; jm\rangle = \sum_{j_{23}} U(j_1 j_2 j j_3; j_{12} j_{23}) |j_1 j_{23}; jm\rangle \tag{7}$$

where $U(j_1 j_2 j j_3; j_{12} j_{23})$ is a unitary transformation defined by

$$\begin{aligned} &\sum_{\mu_1 \mu_2} \begin{bmatrix} j_1 & j_2 & j_{12} \\ \mu_1 & \mu_2 & \mu_{12} \end{bmatrix} \begin{bmatrix} j_{12} & j_3 & j \\ \mu_{12} & \mu_3 & m \end{bmatrix} \\ &\begin{bmatrix} j_2 & j_3 & j_{23} \\ \mu_2 & \mu_3 & \mu_{23} \end{bmatrix} \begin{bmatrix} j_1 & j_{23} & j \\ \mu_1 & \mu_{23} & m \end{bmatrix} \end{aligned} \tag{8}$$

which enables one to go from one scheme of coupling to another scheme of coupling.

4. Isospin amplitude decompositions

We express the ket $|I_B + I_H\rangle$ in the uncoupled representation in terms of the coupled representation

$$\begin{aligned} \mathcal{H}_1 |B^+\rangle &= \sqrt{\frac{2}{3}} |3/2, 1/2\rangle - \frac{1}{\sqrt{3}} |1/2, 1/2\rangle, \\ \mathcal{H}_0 |B^+\rangle &= |1/2, 1/2\rangle, \\ \mathcal{H}_1 |B^0\rangle &= \sqrt{\frac{2}{3}} |3/2, -1/2\rangle + \frac{1}{\sqrt{3}} |1/2, -1/2\rangle, \\ \mathcal{H}_0 |B^0\rangle &= |1/2, -1/2\rangle. \end{aligned} \tag{9}$$

The state $|(I_K + I_\pi) + I_\pi\rangle$ is obtained in the $|j_{12} j_3; jm\rangle$ coupled base in terms of the uncoupled base using

Eq.(6). The system of equations obtained is inverted and we get the uncoupled states in terms of the coupled states. Finally, we build the braket $\langle B|\mathcal{H}|K\pi\pi\rangle_{I_{12}}$ and obtained the isospin amplitude decomposition for the charged channels

$$
\begin{aligned}
\mathcal{A}(B^+ \to K^0\pi^0\pi^+) &= -\sqrt{\frac{1}{15}}\bar{A}_{3/2} - \sqrt{\frac{1}{3}}\bar{A}_{1/2}, \\
\mathcal{A}(B^+ \to K^+\pi^0\pi^0) &= \frac{2}{3\sqrt{15}}\bar{A}_{3/2} + \frac{2}{3\sqrt{3}}\bar{A}_{1/2} \\
&+ \frac{1}{3}\sqrt{\frac{2}{3}}A'_{3/2} - \frac{1}{3\sqrt{3}}A'_{1/2} \\
&- \frac{\sqrt{2}}{3}A_{3/2} + \frac{1}{3}A_{1/2}, \\
\mathcal{A}(B^+ \to K^+\pi^+\pi^-) &= \frac{1}{3}\sqrt{\frac{2}{15}}\bar{A}_{3/2} + \frac{1}{3}\sqrt{\frac{2}{3}}\bar{A}_{1/2} \\
&- \frac{2}{3}\sqrt{\frac{1}{3}}A'_{3/2} - \frac{1}{3}\sqrt{\frac{2}{3}}A'_{1/2} \\
&+ \frac{2}{3}A_{3/2} - \frac{\sqrt{2}}{3}A_{1/2},
\end{aligned}
$$

which is expressed in terms of the six amplitude of isospin $\bar{A}_{3/2(1/2)}$, $A'_{3/2(1/2)}$ and $A_{3/2(1/2)}$. The isospin amplitude decomposition for the neutral channels is equivalent.

We realize the same procedure for the ket $|I_K + (I_\pi + I_\pi)\rangle$ and obtain the isospin amplitude decomposition for the amplitudes $\langle B|\mathcal{H}|K\pi\pi\rangle_{I_{23}}$

$$
\begin{aligned}
\mathcal{A}(B^+ \to K^+\pi^+\pi^-) &= \frac{2}{3}\sqrt{\frac{1}{5}}\bar{A}_2 - \frac{\sqrt{2}}{3}A'_0 + \sqrt{\frac{2}{3}}A_0, \\
\mathcal{A}\left(B^+ \to K^+\pi^0\pi^0\right) &= \frac{2}{3}\sqrt{\frac{2}{5}}\bar{A}_2 + \frac{1}{3}A'_0 - \frac{1}{\sqrt{3}}A_0, \\
\mathcal{A}\left(B^+ \to K^0\pi^0\pi^+\right) &= -\frac{1}{\sqrt{5}}\bar{A}_2
\end{aligned}
$$

which is expressed in terms of the three amplitude of isospin $\bar{A}_2$, A'_0 and A_0. We obtain the same amplitudes for the neutral decays.

Since the two isospin amplitude decompositions are equivalent we have the following relations among the isospin amplitudes

$$
\begin{aligned}
A_0 &= \sqrt{\frac{2}{3}}A_{3/2} - \frac{1}{\sqrt{3}}A_{1/2}, \\
A'_0 &= \sqrt{\frac{2}{3}}A'_{3/2} - \frac{1}{\sqrt{3}}A'_{1/2}, \\
\bar{A}_2 &= \frac{1}{\sqrt{6}}\bar{A}_{3/2} + \sqrt{\frac{2}{3}}\bar{A}_{1/2}.
\end{aligned} \tag{10}
$$

5. Triangle relations

The isospin amplitude decompositions for the charged and neutral decays can be cast in the form of a triangle relation between the amplitudes

$$
\frac{1}{\sqrt{2}}\mathcal{A}(B^+ \to K^+\pi^+\pi^-) + \mathcal{A}(B^+ \to K^+\pi^0\pi^0) + \mathcal{A}(B^+ \to K^0\pi^0\pi^+) = 0 \tag{11}
$$

which is depicted in Fig. 1. The same relation is obtained for the neutral channels and its depicted triangle. The two triangles for the B^+ and B^0 decays are identical according to the isospin symmetry.

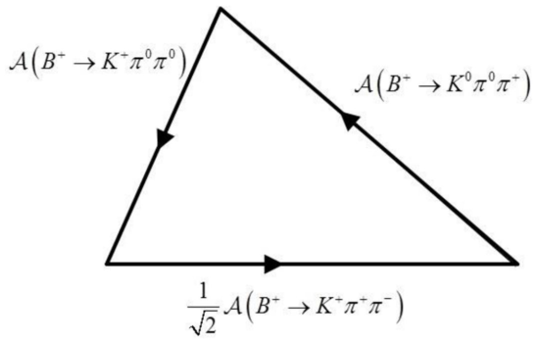

Figure 1: Isospin triangle for $B^+ \to K\pi\pi$ decays.

6. Conclusions

We have obtained the two possible isospin amplitude decompositions of the $B \to K\pi\pi$ decays, using the formalism of quantum mechanical addition of angular momentum. We prove the equivalence of the two isospin amplitude decompositions. A partial result was obtained in Ref. [5]. Additionally, we obtain two triangle relations for the charged and neutral channels, which are equivalent by isospin symmetry.

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