Spin and spin-flavor oscillations due to neutrino charge radii interaction with an external environment

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Abstract. We derive the effective neutrino evolution Hamiltonian and corresponding expressions for the neutrino flavour and spin-flavour oscillation probabilities accounting for the neutrino interactions with an external electric current though the neutrino charge radius and anapole moment. The obtained results are of interest for neutrino astrophysical applications.

1. Neutrino effective Hamiltonian

This paper is dedicated to the description of the neutrino flavour, spin and spin-flavour oscillations engendered by the neutrino interaction with an external electric current though the diagonal and off-diagonal neutrino charge radii and anapole moments. We consider two flavour neutrinos with two possible helicities $\nu_f = \left(\nu_e^-, \nu_l^-, \nu_e^+, \nu_l^+\right)^T$, where ν_l stands for ν_μ or ν_τ. Also we use the calculations that are analogous to those performed in [1, 2]. In the mass basis the matrix elements of the effective interaction Hamiltonian describing electromagnetic interactions of a neutrino field ν are determined by (see [3] for a detailed derivations)

$$H_J^{(m)ki} = \lim_{q\to 0} \frac{1}{T} \frac{\langle \nu_f(p_k, h_k)| \int d^4x \mathcal{H}_J \, |\nu_i(p_i, h_i)\rangle}{\langle \nu(p,h)|\nu(p,h)\rangle}, \tag{1}$$

where $q = p_i - p_k$ is the difference between initial p_i and final p_k neutrino momenta, T is the normalization time and

$$\langle \nu_k(p_k, h_k)| \, \mathcal{H}_J \, |\nu_i(p_i, h_i)\rangle = \overline{u}_k(p_k, h_k)\Lambda_\mu^{ki}(q)\frac{1}{q^2}u_i(p_i, h_i)J^\mu_{EM}e^{-iqx}. \tag{2}$$

Here $J^\mu_{EM} = e(n_{\mathbb{F}}, n_{\mathbb{F}}\mathbf{v}_{\mathbb{F}})$ is the electric current of fermions $\mathbb{F}$ (protons or electrons) and Λ_μ^{ki} is the neutrino electromagnetic vertex [3] containing charge radii and anapole form factors

$$\Lambda_\mu^{ki}(q) = (q^2\gamma_\mu - q_\mu\gamma_\nu q^\nu)\left[\frac{\langle r^2\rangle^{ki}}{6} + f_A^{ki}\gamma_5\right], \tag{3}$$

where $f_A^{ki}(q^2)$ and $\langle r^2\rangle = 6\left.\dfrac{df_Q(q^2)}{dq^2}\right|_{q^2=0}$ are diagonal and off-diagonal anapole form factors and charge radii is the mass basis correspondingly. This vertex gives the following effective interaction Hamiltonian

$$H_J^{(m)ki} = \frac{1}{\sqrt{E_f E_i}} \overline{u}_f(p_k, h_k)\gamma_\mu \left[\frac{\langle r^2\rangle^{ki}}{6} + f_A^{ki}\gamma_5\right] u_i(p_i, h_i) J^\mu_{EM}(x), \tag{4}$$

where free neutrino mass states spinors are given by

$$u_i(\mathbf{p}, h) = \sqrt{E_i + m_i} \begin{pmatrix} \chi^{(h)} \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m_i}\chi^{(h)} \end{pmatrix}, \tag{5}$$

and $\chi^{(h)}$ defines the neutrino helicity state $\chi^{(+)} = (1 \;\; 0)^T$ and $\chi^{(-)} = (0 \;\; 1)^T$.

Let the neutrino momentum is directed along z-axis. Substituting (5) into equation (4) we get

$$H_J^{(m)ki} = 2\chi^{(h_k)\dagger}\Bigg\{ \boldsymbol{J}_\parallel^{EM}\left(\frac{\langle r^2\rangle^{ki}}{6} + f_A^{ki}\sigma_3\right) + \\ + \boldsymbol{J}_\perp^{EM}\left[\left(\sigma_1\gamma_{ki}^{-1}\cos\alpha + \sigma_2\gamma_{ki}^{-1}\sin\alpha\right) f_A^{ki} + \left(i\sigma_1\tilde{\gamma}_{ki}^{-1}\sin\alpha - i\sigma_2\tilde{\gamma}_{ki}^{-1}\cos\alpha\right)\frac{\langle r^2\rangle^{ki}}{6}\right]\Bigg\}\chi^{(h_i)}, \tag{6}$$

where $\boldsymbol{J}_\parallel^{EM}$ is the longitudinal to the neutrino moving component of the electric current and $\boldsymbol{J}_\perp^{EM}$ is the transversal component and α is the angle between x-axis and $\boldsymbol{J}_\perp^{EM}$. The gamma factors are given by

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}, \quad \gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_\beta^{-1}\right), \quad \tilde{\gamma}_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_\beta^{-1}\right). \tag{7}$$

We will consider contribution of the longitudinal $\boldsymbol{J}_\parallel^{EM}$ and the transversal $\boldsymbol{J}_\perp^{EM}$ components of the electric current separatly.

1.1. $\boldsymbol{J}_\parallel^{EM}$ *contribution*

In the flavour basis $\nu_f = (\nu_e^L, \nu_l^L, \nu_e^R, \nu_l^R)$ the corresponding part of the interaction Hamiltonian

$$H_{J_\parallel}^{(f)} = 2J_\parallel^{EM}\begin{pmatrix} \frac{\langle r^2\rangle^{ee}}{6} - f_A^{ee} & \frac{\langle r^2\rangle^{el}}{6} - f_A^{el} & 0 & 0 \\ \frac{\langle r^2\rangle^{el}}{6} - f_A^{el} & \frac{\langle r^2\rangle^{ll}}{6} - f_A^{ll} & 0 & 0 \\ 0 & 0 & \frac{\langle r^2\rangle^{ee}}{6} + f_A^{ee} & \frac{\langle r^2\rangle^{el}}{6} + f_A^{el} \\ 0 & 0 & \frac{\langle r^2\rangle^{el}}{6} + f_A^{el} & \frac{\langle r^2\rangle^{ll}}{6} + f_A^{ll} \end{pmatrix}, \tag{8}$$

where

$$\langle r^2\rangle^{ee} = \langle r^2\rangle^{11}\cos^2\theta + \langle r^2\rangle^{22}\sin^2\theta + \langle r^2\rangle^{12}\sin 2\theta, \quad \langle r^2\rangle^{ll} = \langle r^2\rangle^{11}\sin^2\theta + \langle r^2\rangle^{22}\cos^2\theta - \langle r^2\rangle^{12}\sin 2\theta, \\ \langle r^2\rangle^{el} = \langle r^2\rangle^{12}\cos 2\theta + \frac{1}{2}\left(\langle r^2\rangle^{22} - \langle r^2\rangle^{11}\right)\sin 2\theta, \tag{9}$$

$$f_A^{ee} = f_A^{11}\cos^2\theta + f_A^{22}\sin^2\theta + f_A^{12}\sin 2\theta, \quad f_A^{ll} = f_A^{11}\sin^2\theta + f_A^{22}\cos^2\theta - f_A^{12}\sin 2\theta, \\ f_A^{el} = f_A^{12}\cos 2\theta + \frac{1}{2}\left(f_A^{22} - f_A^{11}\right)\sin 2\theta. \tag{10}$$

One can see that $\boldsymbol{J}_\parallel^{EM}$ influence on flavour neutrino oscillations.

1.2. $\boldsymbol{J}_\perp^{EM}$ *contribution*

In the flavour basis the part of the Hamiltonian with $\boldsymbol{J}_\perp^{EM}$ is given by

$$H_{J_\perp}^{(f)} = \begin{pmatrix} 0 & \tilde{H}_{J_\perp}^{(f)} \\ \tilde{H}_{J_\perp}^{(f)\dagger} & 0 \end{pmatrix}, \quad \tilde{H}_{J_\perp}^{(f)} = \begin{pmatrix} \left(\frac{f_A}{\gamma}\right)_{ee} e^{i\alpha} & \left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{el}\right] e^{i\alpha} \\ \left[-\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{el}\right] e^{i\alpha} & \left(\frac{f_A}{\gamma}\right)_{ll} e^{i\alpha} \end{pmatrix}, \tag{11}$$

where

$$\left(\frac{f_A}{\gamma}\right)_{ee} = \frac{f_A^{11}}{\gamma_{11}}\cos^2\theta + \frac{f_A^{22}}{\gamma_{22}}\sin^2\theta + \frac{f_A^{12}}{\gamma_{12}}\sin 2\theta, \quad \left(\frac{f_A}{\gamma}\right)_{ll} = \frac{f_A^{11}}{\gamma_{11}}\sin^2\theta + \frac{f_A^{22}}{\gamma_{22}}\cos^2\theta - \frac{f_A^{12}}{\gamma_{12}}\sin 2\theta,$$
$$\left(\frac{f_A}{\gamma}\right)_{el} = \frac{f_A^{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{f_A^{22}}{\gamma_{22}} - \frac{f_A^{11}}{\gamma_{11}}\right)\sin 2\theta. \tag{12}$$

From this expression one can see that the neutrino interactions through charge radius and anapole form factors with transversal electric current can generate neutrino spin and spin-flavour oscillations.

2. Neutrino full interaction Hamiltonian

To consider neutrino evolution in extreme astrophysical environment we have to include neutrino interaction with an external magnetic field and a moving matter. From [2] for the interaction with a moving media

$$H_{mat}^{(f)} = \frac{G_F}{2\sqrt{2}} \begin{pmatrix} 2(2n_e - n_n)\left(1 - v_{||}\right) & 0 & (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{el} \\ 0 & -2n_n\left(1 - v_{||}\right) & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{el} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ll} \\ (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{el} & 0 & 0 \\ (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{el} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ll} & 0 & 0 \end{pmatrix}, \tag{13}$$

where n_n and n_e are the neutron and electron density profiles, $v = v_{||} + v_\perp$ is the matter velocity and

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{ll} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{el} = \frac{\sin 2\theta}{\tilde{\gamma}_{21}}. \tag{14}$$

The Hamiltonian that accounts for the neutrino magnetic moment interaction with parallel and perpendicular components of the magnetic field $B = B_{||} + B_\perp$ has the following form

$$H_B^{(f)} = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & \left(\frac{\mu}{\gamma}\right)_{el} B_{||} & -\mu_{ee}B_\perp e^{i\phi} & -\mu_{el}B_\perp e^{i\phi} \\ \left(\frac{\mu}{\gamma}\right)_{el} B_{||} & \left(\frac{\mu}{\gamma}\right)_{ll} B_{||} & -\mu_{el}B_\perp e^{i\phi} & -\mu_{ll}B_\perp e^{i\phi} \\ -\mu_{ee}B_\perp e^{-i\phi} & -\mu_{el}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{el} B_{||} \\ -\mu_{el}B_\perp e^{-i\phi} & -\mu_{ll}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{el} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{ll} B_{||} \end{pmatrix}, \tag{15}$$

where ϕ is the angle between $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$. Magnetic moments in flavour basis $\mu_{\alpha\beta}$ are expressed through magnetic moments μ_{ij} in mass states

$$\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \quad \mu_{ll} = \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta,$$
$$\mu_{el} = \mu_{12}\cos 2\theta + \frac{1}{2}\left(\mu_{22} - \mu_{11}\right)\sin 2\theta, \tag{16}$$

for the neutrino interaction with $B_\perp$ and

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \quad \left(\frac{\mu}{\gamma}\right)_{ll} = \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta,$$
$$\left(\frac{\mu}{\gamma}\right)_{el} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \tag{17}$$

for the neutrino interaction with $B_{||}$.

To obtain neutrino oscillations probabilities in a real astrophysical environment we should take into account all Hamiltonians and also vacuum part.

3. Neutrino spin oscillations $\nu_e^L \leftrightarrow \nu_e^R$

Consider two states of neutrino (ν_e^L, ν_e^R). The corresponding oscillations are governed by the evolution equation

$$i\frac{d}{dx}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} = (H_J^{(f)} + H_{mat}^{(f)} + H_B^{(f)})\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix}, \tag{18}$$

where $H_J^{(f)}$, $H_{mat}^{(f)}$ and $H_B^{(f)}$ consist only of those matrix elements that correspond to transitions between ν_e^L and ν_e^R states.

For the spin oscillation $\nu_e^L \leftrightarrow \nu_e^R$ probability we get

$$P_{\nu_e^L \to \nu_e^R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2\frac{\pi x}{L_{\text{eff}}}, \quad \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad L_{\text{eff}} = \frac{\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}}, \tag{19}$$

where E_{eff}^2 and Δ_{eff}^2 are expressed in terms of the elements H_{ij} of the Hamiltonian:

$$E_{eff}^2 = 4|H_{12}|^2 = 4\left[\frac{G_F}{2\sqrt{2}}(2n_e - n_n)v_\perp\left(\frac{\eta}{\gamma}\right)_{ee} - \mu_{ee}B_\perp\cos\phi + \right.$$
$$\left. +2J_\perp^{EM}\left(\frac{f_A}{\gamma}\right)_{ee}\cos\alpha\right]^2 + 4\left[\mu_{ee}B_\perp\sin\phi - 2J_\perp^{EM}\left(\frac{f_A}{\gamma}\right)_{ee}\sin\alpha\right]^2, \tag{20}$$

$$\Delta_{eff}^2 = (H_{11} - H_{22})^2 = \left[\frac{G_F}{\sqrt{2}}(2n_e - n_n)\left(1 - v_{||}\right) + 2\left(\frac{\mu}{\gamma}\right)_{ee}B_{||} - 4J_{||}^{EM}f_A^{ee}\right]^2. \tag{21}$$

This formulas can be useful to estimate the impact of neutrino charge radii and anapole form factors to the neutrino oscillations in astrophysical environments.

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