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LATTICE APPROACH TO SEMI-LEPTONIC DECAYS OF CHARM MESONS*

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Lattice efforts to compute matrix elements relevant to semi-leptonic form factors are reviewed. The emphasis is on $D \to K^*$ where the two groups seem to find appreciably different results for $A_2(0)/A_1(0)$. Lattice measurments at the end-point region for both the 0^- and the 1^- final states are emphasised. All the lattice results seem to suggest that for D, D_s decays to 1^- final states, the A_1 form factor at the endpoint is always close to unity. We note that the FNAL experiment E691 does not agree with lattice results for A_1 at the endpoint; in addition it tends to disagree with our value for $A_2(0)/A_1(0)$.

1. Introduction

The aim of these efforts is to calculate matrix elements of the form $A \to Be\nu$ where A is a pseudoscalar and B may be a pseudoscalar or a vector. The primary focus in this talk will be on $D \to K^*$ as there are interesting developments on this mode both in the theoretical and in the experimental sector. The basic technique is very similar in all such charge current transitions: $K \to \pi e\nu$, $D \to \pi(\rho)e\nu$ etc. All of these involve matrix elements of the form $\langle A|J|B\rangle$ which are amenable to lattice methods. As is well known, these matrix elements can be used to deduce Cabibbo-Kobayashi-Maskawa (CKM) mixing angles from the experimental data and to test phenomenological models. Since the mixing angles relevant to charm decays, *i.e.*, V_{cs} and V_{cd} are constrained by the Standard Model and three generations [1], lattice calculations at this point do not have an impact on further improvements in the determination of V_{cs} and V_{cd} . Lattice efforts may well, however, lead to an improved understanding of phenomenological models and thereby have an impact on the determination of V_{ub} . Of course, attempts will also be made to study semi-leptonic B decays directly by going to high β with a propagating b quark and/or by the use of static or nonrelativistic heavy quarks. The heavy quark symmetries can help in simplifying such computations.

Lattice studies of semi-leptonic decays have been in progress since 1987. Recent developments by the two groups are documented in [2,3].

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2. The Status at LAT'90

Lubicz *et al.* (ELC) [4,5] reported $A_2(0) = 0.06 \pm 0.40$ in very good agreement with the experimental results of E691 [6] and in sharp contrast with all the continuum models. We [7] reported our preliminary results on A_1 and emphasised the usefulness of examining the ratio A_2/A_1, in view of the poor quality of the signal in A_2.

In the past year both groups have made progress on these issues. Lubicz *et al.* have doubled their statistics from 15 to 30 configurations [3]. Their new value for $A_2(0) = 0.19 \pm 0.21$, is completely consistent with their earlier result. We [2] find $A_2(0)/A_1(0) = 0.70 \pm 0.16^{+0.20}_{-0.15}$, significantly different from zero. This is in conflict with the findings of ELC, but still consistent with the E691 experiment within the large (experimental and lattice) uncertainties. Meantime, the Fermilab experiment E653 [8] has completed its analysis finding $A_2(0)/A_1(0) = 0.82^{+0.22}_{-0.23} \pm 0.11$, not inconsistent with E691 within the large uncertainties, apparently in some disagreement with ELC, and in good agreement with our results.

Although quantitatively the results of Lubicz *et al.* and ours are different by only 1 to 2 σ, qualitatively their implications are significantly different. Lubicz *et al.*'s result implies a serious problem with various continuum models of semi-leptonic form factors since all of them have $A_2(0)/A_1(0)$ of $\mathcal{O}(1)$, whereas our results do not suggest any such serious problem.

3. Experiments and Phenomenology

For $D \to K$, in the helicity representation, there are two form factors, f_+ and f_0 (see [9,10] for definitions and notation). Contribution of f_0 to the rate is proportional to the lepton mass and therefore is very small for $e\nu$ and $\mu\nu$ transitions. Note that the q^2 distribution in the limit of zero lepton mass is given by:

$$\frac{d\Gamma}{dq^2} = G_F^2 \frac{|V|^2 \lambda^{3/2}}{192\pi^3} |f_+(q^2)|^2 \quad , \tag{1}$$

where $\lambda \equiv \lambda(m_A, m_B, q^2) = [m_A^2 - m_B^2 - q^2]^2 - 4m_B^2 q^2$. Thus a precise knowledge of the form factor at a single value of q^2, in conjunction with an experimental measurement of the differential rate at the same value of q^2, can lead to a model independent determination of the relevant mixing angle.

For $D \to K^*$, in the limit of zero lepton mass, there are three contributing form factors. Using pole dominance the rate can be related to the form factors at $q^2 = 0$:

$$\begin{aligned} \Gamma(A \to Bl\nu) = |V_{cs}|^2 [&C_1 {A_1}^2(0) \\ &+ C_2 {A_2}^2(0) - C_{12} A_1(0) A_2(0) \\ &+ C_V V^2(0)] \quad . \end{aligned} \tag{2}$$

Notice that knowledge of just the integrated rate does not give a model independent determination of the mixing angle as three unknown form factors are involved.

There are now four experimental results on this process. E691 and the more recent E653, both extract $A_2(0)/A_1(0)$ and $V(0)/A_1(0)$ using pole dominance and a two parameter maximum likelihood fit (in four phase space variables) to the differential decay rate. E691 then uses the branching ratio for $D \to K^*$ to extract a (pole) model dependent value for $A_1(0)$ through eqn (2). Mark III [11] at SLAC and WA82 at CERN [12] examine the angular distribution to extract Γ_L/Γ_T (which is the ratio for longitudinal to transverse K^* polarization)

The experimental results, along with the theoretical ones are summarized in Table 1.

group	$A_1(0)$	$A_2(0)$	$A_2/A_1(0)$	$V(0)$	$V/A_1(0)$
E691 [6]	0.46 ± 0.05	0.0 ± 0.2	0.0 ± 0.5	0.9 ± 0.3	2.0 ± 0.6
sys. error	± 0.05	± 0.1	± 0.2	± 0.1	± 0.3
E653 [8]			$0.82 {}^{+0.22}_{-0.23}$		$2.0 {}^{+0.34}_{-0.32}$
sys. error			± 0.11		± 0.16
BSW [13]	0.88	1.15		1.27	
KS [14]	0.82	0.82	1.0	0.82	1.0
AW/GS [15]	0.8	0.6		1.5	
BBD [16]	0.50 ± 0.15	0.60 ± 0.15	1.2 ± 0.2	1.10 ± 0.25	2.2 ± 0.2
ELC [3]	0.53 ± 0.03	0.19 ± 0.21		0.86 ± 0.10	
our work [2]	0.83 ± 0.14	0.59 ± 0.14	0.70 ± 0.16	1.43 ± 0.45	1.99 ± 0.22
sys. error	± 0.28	${}^{+0.24}_{-0.23}$	${}^{+0.20}_{-0.15}$	${}^{+0.48}_{-0.49}$	${}^{+0.31}_{-0.35}$

Table 1
The form factors for $D \to K^*$ from various experiments and model calculations

4. Lattice Methodology and Differences in Implementation

In Table 2 we describe the lattice parameters and the key differences. In particular, we note for the discussions to follow that the size of the lattice used by Lubicz *et al.* is substantially smaller than ours. Another important difference is the use of the conserved (non-local) vector current by Lubicz *et al.*, whereas the local vector current is used by our group.

Lubicz *et al.* obtain the form factors as a function of q^2 (in practice for a few values of q^2) for a fixed set of κ's. They then assume pole dominance and obtain the form factors at $q^2 = 0$ for those κ's, extrapolating or interpolating in the κ's to deduce the form factors at $q^2 = 0$ for physical mesons. In addition to the above procedure (which we call method II), we use another method (I): from the lattice we obtain the form factors for a fixed set of injected momenta for several different light κ's. We then extrapolate or interpolate the form factors in the hopping parameters to the set of physical q^2 that corresponds to the set of injected momenta. Finally, pole dominance is used to deduce the form factors at $q^2 = 0$.

For $D \to K^*$ we also extract the form factors from the Green's functions in two different ways (see eqns. (9)-(11) in [2]). The values of the form factors that we quote are an average of these four (2×2) methods and the spread in the methods is included in our estimate of systematic errors.

In our recent work we have also used a new normalization [17] for the fermion field as indicated in Table 2, rather than the conventional normalization used by Lubicz *et al.* With this new normalization, the lattice quark propagator, in free field theory, for both $am \gg 1$ (*i.e.*, $\kappa \to 0$) as well as for $am \ll 1$ ($\kappa \to \kappa_c$) is correctly normalized.

For the 0^- to 0^- case ELC [10] uses a nice trick, namely injecting a minimum unit of (non-zero) momentum to the kaon when they apply the source method to it. This has the advantage that they are able to cover a wide range of q^2. We [9] exploit the symmetry of the situation for the $D \to K$ case and apply the source method 4 times: to K (at time slice ± 11, with the weak operator sitting at $t = 0$ in the middle of the lattice and the kaon always taken to be on the opposite half with respect to the D) and to D (again at $t = \pm 11$). Averaging over the 4 sets of data helps to improve our statistics appreciably.

Quantitiy	Lubicz *et al.*	*Bernard et al.*
Lattice size	$10^3 \times 20$ (gauge)	$24^3 \times 40$ (gauge)
	$10^2 \times 20 \times 40$ (quark)	$24^3 \times 39$ (quark)
spatial vol.	1.9fm^3	13.3fm^3
spat. length	1.0 fm	2.4 fm
κ_{cr}	0.1569(3)	0.157
κ_{charm}	0.135	$0.135, 0.118 \rightarrow 0.128$ for charm
κ_{light}	0.1515, 0.1530, 0.1545	0.152, 0.154, 0.155
a^{-1}	2.25 GeV	2.0 ± 0.4 GeV
$\lvert p \rvert$	$(0, \pm 1, \pm 2)\pi/10a$	$(0, \pm 1, \pm\sqrt{2}, \pm 2)\pi/12a$
systematics	Not Given	Due to finite size, scale
		breaking, extrapolations etc included.
current	non-local vector, local axial	local vector and axial
pert. renorm.	$Z_A^{Loc} \sim 0.87$	$Z_A^{Loc} \sim 0.77, Z_V^{Loc} \sim 0.70$
field renorm.	$\psi^{cont} = \sqrt{2\kappa}\psi^{latt}$	$\psi^{cont} = \sqrt{2\kappa e^{ma}}\psi^{latt}$
		$ma = \ln\left(1 + \frac{1}{2\kappa} - \frac{1}{2\kappa_{cr}}\right)$

Table 2
Lattice parameters and other comparisons

For $D \rightarrow K^*$, ELC apply the source method once to the D, whereas we apply it twice (at $t = \pm 11$) again in an effort to improve statistics.

5. Results

The latest results of the two groups are given in [2,9] and [3]. The ELC result on $D \rightarrow K$ [3,10] is in better agreement with experiment than ours [9] . Our number appears too high but is within (the rather big) 1σ of the systematic error. We also note that although the ELC group does not quote a systematic error it is unlikely to be significantly smaller than ours.

Table 1 shows a summary of the results for $D \rightarrow K^*$. In particular, our result for the ratio $A_2(0)/A_1(0)$ tends to disagree with the experiment E691 and with ELC and is in good agreement with the experiment E653 and also with the continuum models.

In an effort to understand the origin of the different results for A_2 (or A_2/A_1) that the two groups have reported, we show in Figure 1 the ratio A_2/A_1. The direct lattice calculation of this ratio (as we do) has significant advantages over constructing the ratio out of the lattice calculated values of A_1 and A_2: Systematic errors due to scale breaking effects are considerably less on the ratios of form factors than on the form factors themselves. Also, the statistical fluctuations in the ratio tend to cancel leading to an appreciably reduced jacknife error. Furthermore, the ratio is free of the error in the the normalization of the quark field and the uncertainty due to the nonperturbative renormalization of the axial current is very likely reduced. Finally, the ratio has the advantage of being independent of q^2 so long as A_2 and A_1 have the same q^2 dependence. In particular such is of course the case for the pole dominance model. Indeed, the data in Figure 1 do not show any significant q^2 dependence irrespective of the κ used for the different data sets. Note that ELC has not directly calculated the ratio A_2/A_1. For the comparison in Figure 1 we have therefore taken their data for

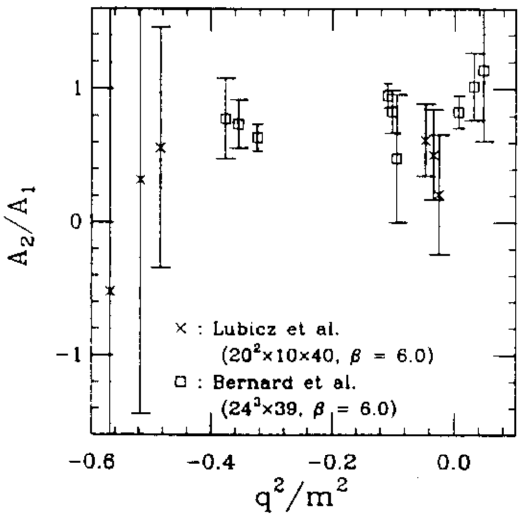

Fig. 1. The ratio A_2/A_1 vs. q^2 in comparison

A_1 and A_2 and constructed the ratio with errors as primarily arising from A_2 alone. Clearly, this tends to overestimate the errors on their results for A_2/A_1.

Note that the data to the left of $q^2 = 0$ is such that the chiral limit is to the left whereas for the data on the right of $q^2 = 0$ the chiral limit is to the right. While, for a heavy "K^*", A_2/A_1 for the two groups tend to agree, there is an apparent disagreement about the value of $A_2(0)/A_1(0)$ in the chiral limit. Note that both sets of ELC data points show that the central value of A_2/A_1 decreases in the chiral limit, although the error bars, especially for the lightest of the three light kappas, are very large. We show three sets of our data points in the Figure. Two of these three sets show a $A_2(0)/A_1(0)$ that increases mildly in the chiral limit; whereas one of our data sets shows a decrease of $A_2(0)/A_1(0)$ in the chiral limit. It would be useful to find out if the difference between the two calculations are physical (*i.e.*, a finite volume effect, especially in view of the significant difference in the spatial volume of the two lattices) . For that purpose it would be better to compare our A_2/A_1 with a direct calculation of that ratio by ELC since we clearly must have overestimated their errors by constructing it from their A_1 and A_2.

6. Remarks on the Pole-Dominance Model and on the End-point Region

Although in lattice calculations we should, in principle, be able to deduce the shape of the form factors by injecting different values of the 3-momenta, in practice numerical limitations have not allowed either groups to do that so far with a great deal of success. Only very few values of momenta can be injected without losing the signal altogether. Thus the quality of the data, from either groups, has not allowed a meaningful test of the pole dominance model . Indeed, the pole dominance model has simply been assumed to extract the form factors at $q^2 = 0$.

For 0^- to 1^- the form factors at $q^2 \approx 0$ can be deduced without extrapolation since in these lattice calculations (see e.g. Figure 1) that value of q^2 is available. However, $q^2 \approx 0$ is not so useful experimentally as lepton detection there can be a problem. The end-point region where $q^2 \approx q^2_{max}$ (with q^2_{max} the maximum lepton momentum) may be more useful experimentally. Clearly, the end-point region is the most suitable for lattice calculations as well since the initial and the final mesons then are both at rest and no momentum is being injected. The differential decay rate for the $0^- \to 1^-$ transition takes on a very simple form for q^2 near q^2_{max}:

$$\frac{d\Gamma}{dq^2} = G_F^2 \frac{|V|^2 \lambda^{1/2}}{64\pi^3} q^2 (m_A + m_B)^2 |A_1(q^2)|^2 \ . \quad (3)$$

A computation of the form factor at q^2_{max} together with an experimental determination of the differential rate near q^2_{max} would thus immediately give a determination of the relevant mixing angle.

Figure 2 shows the form factor $A_1(q^2)$ for several different combination of the relevant κ's at $q^2 = q^2_{max}$ for a subset of our data. From the Figure we see that $A_1(q^2_{max})$ shows little dependence on the κ's and always seems to be close to unity (within about 20%). Indeed, all of our data show this feature; for $D \to K^*$ we find:

$$A_1(q^2_{max}) = 1.26 \pm 0.17 \pm 0.43 \quad . \tag{4}$$

It is interesting that $A_1(q^2_{max})$ is coming out to be close to unity. We note that for a transition from one heavy quark Q to another, say Q', via the charged weak current, the A_1 form factor at the end point should be approximately one [18]. It is rather curious that this aspect of the heavy quark symmetry becomes operational so "precosciously" since the initial and final quark masses involved in these simulations ($\lesssim$1.5 GeV) are not particularly heavy.

In passing we also note that a preliminary examination of the ELC data (by us, from Table 5 in [3]) shows that their value for $A_1(q^2_{max})$ is also rather close to one:

$$A_1(q^2_{max}) = 1.10 \pm 0.15 \quad . \tag{5}$$

In contrast, the result of the experiment E691 for $A_1(0)$ implies using pole dominance [6]:

$$A_1(q^2_{max}) = 0.54 \pm 0.06 \pm 0.06 \quad . \tag{6}$$

At this point there appears to be some disagreement between the experiment E691 and the lattice calculations on this issue. Indeed, this is particularly noteworthy as both lattice groups are in very good agreement on this quantity. Equally noteworthy is the fact that the quoted errors by E691 on A_1 are much smaller than on A_2. We hope that E691, E653 and other experiments will try to directly deduce the form factor $A_1(q^2)$ in the endpoint region from the differential decay rate. This could provide an important check on this unambiguous prediction of the lattice.

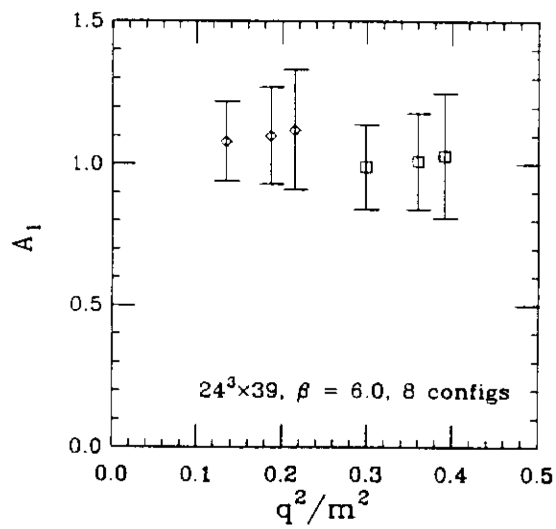

Fig. 2. $A_1(q^2 = q^2_{max})$ is shown. Points on the left are for $\kappa_{charm} = 0.135$, those on the right for $\kappa_{charm} = 0.118$. Data shown is for SU(3) degenerate light quarks. The non-degenerate data show a very similar behavior

7. Conclusions and Summary

The two lattice calculations tend to disagree on $A_2(0)/A_1(0)$. ELC finds $A_2(0)$ vanishingly small in agreement with experiment E691 and in disagreement with experiment E653, with phenomenological models, and with our result for $A_2(0)/A_1(0)$. To make progress on this issue comparison of the directly calculated ratio $A_2(0)/A_1(0)$ should be done.

The two lattice calculations are in very good agreement on the important quantity $A_1(q^2_{max})$, *i.e.*, the only form factor that one needs to know near the end-point for the $D \to K^*$ transition. Experiment E691 seems to disagree with lattice calculations of this crucial form factor.

It is clear that lattice calculations of semi-leptonic form factors are already giving important feedback to experiment and phenomenology. Given the difficulties in the experimental determination of some of these form factors, this is an area in which careful lattice calculations could predict the form factors ahead of experiments, thereby giving us and the non-lattice community additional confidence in lattice results.

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